Vassiliev Invariants to Degree 6

Pensieve Header: Vassiliev invariants to degree 6, using data files, interface programs, and examples by Petr Dunin-Barkowski of barkovs@itep.ru, Andrey Smirnov of asmirnov@itep.ru, and Alexei Sleptsov of sleptsov@itep.ru.

Reference manual

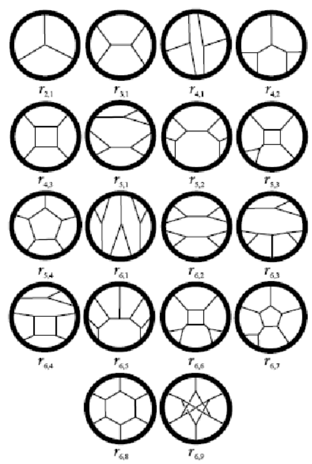
The Vassiliev invariants can be defined as the coefficients of the universal Kontsevich integral. The Kontsevich integral is defined for oriented links (and knots in particular) and take values in the space of Chinese characters:

$$I(K)=\sum_{i=0}^{\infty}\sum_{j=0}^{d_i}\alpha_{i,j}\,r_{i,j}$$

In this expression $\alpha_{i,j}$ and $r_{i,j}$ are the independent Vassiliev invariants and Chinese characters of degree i, respectively and d_i are the numbers of independent Chinese characters of degree i. Up to degree 6 we have

i	1	2	3	4	5	6
d_i	1	1	1	3	4	9

Up to degree 6 we use the basis in the space of Chinese characters suggested in [1]:



In this basis the Vassiliev invariants $\alpha_{2,1}$, $\alpha_{3,1}$, $\alpha_{4,1}$, $\alpha_{4,2}$, $\alpha_{5,2}$, $\alpha_{5,3}$, $\alpha_{5,4}$, $\alpha_{6,5}$, $\alpha_{6,6}$, $\alpha_{6,7}$, $\alpha_{6,8}$, $\alpha_{6,9}$ are primitive, and the rest ones can be expressed through them:

$$\alpha_{4,1}=\frac{1}{2}\alpha_{2,1}^2,\quad \alpha_{5,1}=\alpha_{2,1}\alpha_{3,1}\quad \alpha_{6,1}=\frac{1}{6}\alpha_{2,1}^3$$

$$\alpha_{6,2}=\frac{1}{2}\alpha_{3,1}^2,\quad \alpha_{6,3}=\alpha_{2,1}\alpha_{4,2},\quad \alpha_{6,4}=\alpha_{2,1}\alpha_{4,3}.$$

Example: For trefoil knot 3_1 we have the following table:

$\alpha_{2,1}$	$\alpha_{3,1}$	$\alpha_{4,1}$	$\alpha_{4,2}$	$\alpha_{4,3}$	$\alpha_{5,1}$	$\alpha_{5,2}$	$\alpha_{5,3}$	$\alpha_{5,4}$	$\alpha_{6,1}$	$\alpha_{6,2}$	$\alpha_{6,3}$	$\alpha_{6,4}$	$\alpha_{6,5}$	$\alpha_{6,6}$	$\alpha_{6,7}$	$\alpha_{6,8}$	$\alpha_{6,9}$
4	-8	8	$\frac{62}{3}$	$\frac{10}{3}$	-32	$-\frac{176}{3}$	$-\frac{32}{3}$	-8	$\frac{32}{3}$	32	$\frac{248}{3}$	$\frac{40}{3}$	$\frac{5071}{30}$	$\frac{58}{15}$	$\frac{3062}{45}$	$\frac{17}{18}$	$\frac{271}{30}$

The first two Vassiliev invariants from "Katlas" are $V_1=\alpha_{21}/4$ and $V_2=\alpha_{31}/8$.

References

[1] M.Alvarez, and J.M.F.Labastida, Numerical knot invariants of finite type from Chern-Simons perturbation theory, Nucl.Phys. B433:555-596, 1995, Erratum-ibid. B441:403-404,1995, e-Print: hep-th/9407076

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In this expression $\alpha_{i,j}$ and $r_{i,j}$ are the independent Vassiliev invariants and Chinese characters of degree i, respectively and d_i are the numbers of independent Chinese characters of degree i. Up to degree 6 we have

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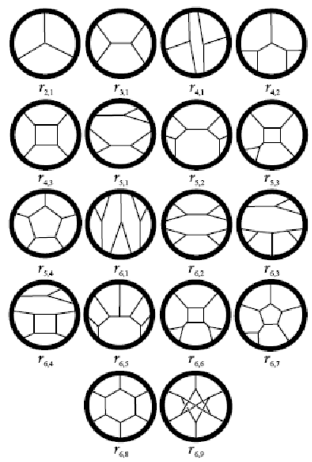

In this basis the Vassiliev invariants $\alpha_{2,1}$, $\alpha_{3,1}$, $\alpha_{4,1}$, $\alpha_{4,2}$, $\alpha_{5,2}$, $\alpha_{5,3}$, $\alpha_{5,4}$, $\alpha_{6,5}$, $\alpha_{6,6}$, $\alpha_{6,7}$, $\alpha_{6,8}$, $\alpha_{6,9}$ are primitive, and the rest ones can be expressed through them:

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Example: For trefoil knot $\mathbf{3_1}$ we have the following table:

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4	-8	8	$\frac{62}{3}$	$\frac{10}{3}$	-32	$-\frac{176}{3}$	$-\frac{32}{3}$	-8	$\frac{32}{3}$	32	$\frac{248}{3}$	$\frac{40}{3}$	$\frac{5071}{30}$	$\frac{58}{15}$	$\frac{3062}{45}$	$\frac{17}{18}$	$\frac{271}{30}$

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References

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Loading ...

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2010-10"];
<< KnotTheory`
```

```
Loading KnotTheory` version of August 22, 2010, 13:36:57.55.
Read more at http://katlas.org/wiki/KnotTheory.
```

```
BeginPackage["KnotTheory`Vass4Knots`", {"KnotTheory`"}];
VI;
Begin["`Private`"];
Vs = Import["VassilievTo6Data.m"];
nar = {0, 1, 1, 3, 4, 9};
num[n_, i_] := Module[{j}, Return[Sum[nar[[j]], {j, 1, n - 1}] + i];];
numkn[n_, i_] := Module[{j}, Return[Sum[NumberOfKnots[j], {j, 3, n - 1}] + i];];
For[n = 2, n ≤ 6, n++, For[i = 1, i ≤ nar[[n]], i++,
   For[k1 = 3, k1 ≤ 14, k1++, For[k2 = 1, k2 ≤ NumberOfKnots[k1], k2++,
     VI[n, i, Knot[k1, k2]] = Vs[[numkn[k1, k2], num[n, i]]];]]]];
Clear[Ks, Vs];
End[]; EndPackage[]
```

```
Vassiliev invariants data to degree 6 by Petr Dunin-Barkowski <barkovs@itep.ru>,
  Andrey Smirnov <asmirnov@itep.ru>, and Alexei Sleptsov <sleptsov@itep.ru>.
```

```
VI[_, _, Knot[0, 1]] = "?";
VI[m_, j_, Knot[n_, Alternating, k_]] := VI[m, j, Knot[n, k]];
VI[m_, j_, Knot[n_, NonAlternating, k_]] :=
  VI[m, j, Knot[n, NumberOfKnots[n, Alternating] + k]];
```

```
Total[(4 Vassiliev[2][#] == VI[2, 1, #]) & /@ AllKnots[{3, 11}]]
```

```
801 True
```

```
Total[(8 Vassiliev[3][#] == VI[3, 1, #]) & /@ AllKnots[{3, 11}]]
```

KnotTheory::loading : Loading precomputed data in Jones4Knots11`.

```
801 True
```

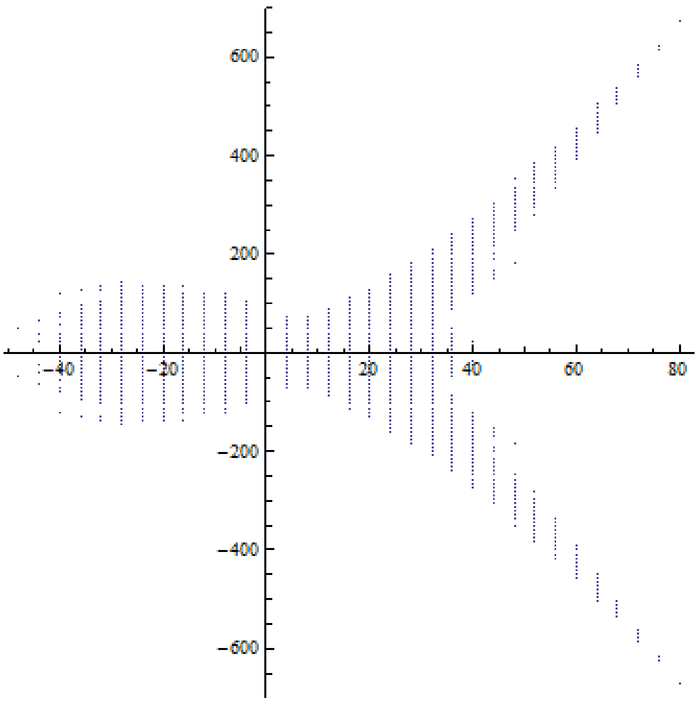
```
{VI[5, 2, Knot[14, 1]], VI[3, 1, Knot[3, 1]],
 Vassiliev[3][Knot[3, 1]], VI[3, 1, Knot[8, 1]], Vassiliev[3][Knot[8, 1]]}
```

KnotTheory::loading : Loading precomputed data in Jones4Knots`.

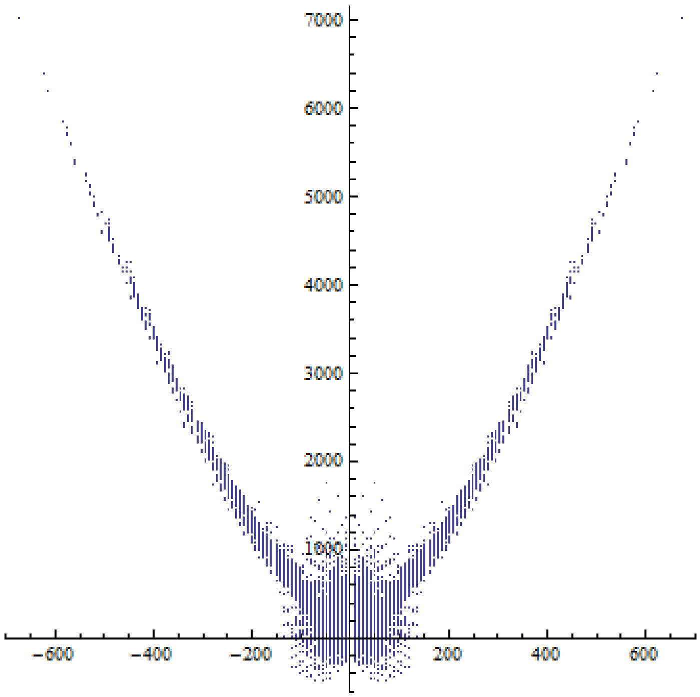
$$\left\{\frac{160}{3}, -8, -1, 24, 3\right\}$$

Drawing ...

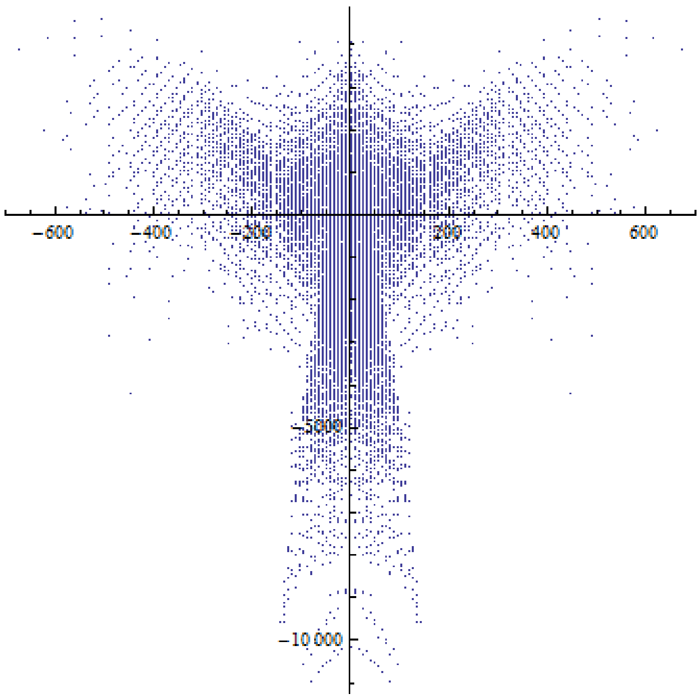
```
(*"The fish" (i.e. Vassiliev invarian of
   degree 3 plotted against Vassiliev invariant of degree 2),
plotted for all knots up to 14 crossings*)
For[n = 3, n ≤ 14, n++,
  fish = Join @@ Table[
      {{VI[2, 1, Knot[n, i]], VI[3, 1, Knot[n, i]]},
       {VI[2, 1, Knot[n, i]], -VI[3, 1, Knot[n, i]]}}, {i, NumberOfKnots[n]}];
 ];
ListPlot[fish, PlotStyle → PointSize[0.004],
  PlotRange → All, AspectRatio → 1] // Rasterize
```



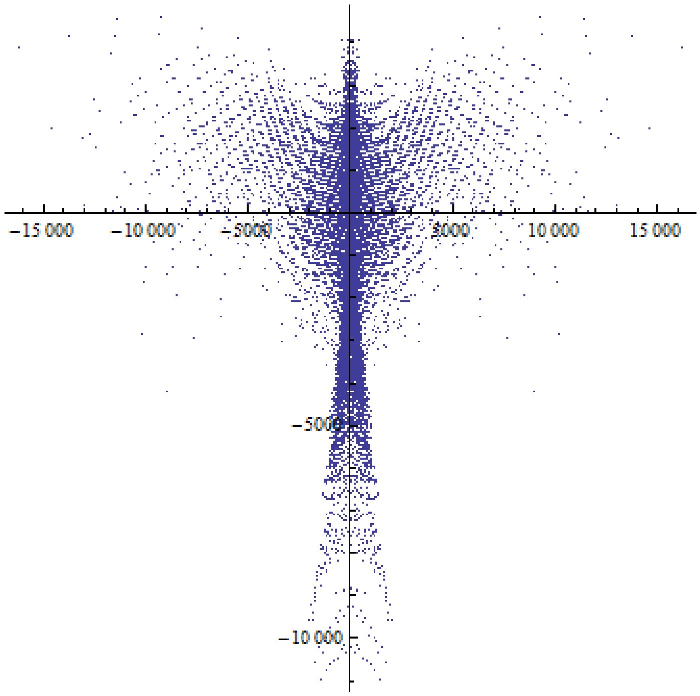
```
(*"The horns" (i.e. VI4,2 plotted against VI2,1),
plotted for all knots up to 14 crossings*)
For[n = 3, n ≤ 14, n++,
  horns = Join @@ Table[
      {{VI[3, 1, Knot[n, i]], VI[4, 2, Knot[n, i]] - 31 / 6 * VI[2, 1, Knot[n, i]]},
       {-VI[3, 1, Knot[n, i]], VI[4, 2, Knot[n, i]] - 31 / 6 * VI[2, 1, Knot[n, i]]}}, {i,
       NumberOfKnots[n]}];
 ];
ListPlot[horns, PlotStyle → PointSize[0.004],
  PlotRange → All, AspectRatio → 1] // Rasterize
```



```
(*"The Reaper" (i.e. "pure" Vassilev invariant of degree 4, 5*VI4,
  2-31*VI4,3, plotted against Vassiliev invariant of degree 3)*)
For[n = 3, n ≤ 14, n++,
  reaper = Join @@ Table[
      {{VI[3, 1, Knot[n, i]], 5 * VI[4, 2, Knot[n, i]] - 31 * VI[4, 3, Knot[n, i]]},
       {-VI[3, 1, Knot[n, i]], 5 * VI[4, 2, Knot[n, i]] - 31 * VI[4, 3, Knot[n, i]]}},
      {i, NumberOfKnots[n]}];
 ];
ListPlot[reaper, PlotStyle → PointSize[0.004],
  PlotRange → All, AspectRatio → 1] // Rasterize
```



```
For[n = 3, n ≤ 14, n++,
  pterfiddlestick = Join @@ Table[
      {{VI[5, 3, Knot[n, i]], 5 * VI[4, 2, Knot[n, i]] - 31 * VI[4, 3, Knot[n, i]]},
       {-VI[5, 3, Knot[n, i]], 5 * VI[4, 2, Knot[n, i]] - 31 * VI[4, 3, Knot[n, i]]}},
      {i, NumberOfKnots[n]}];
 ];
ListPlot[pterfiddlestick, PlotStyle → PointSize[0.004],
  PlotRange → All, AspectRatio → 1] // Rasterize
```



Uploading to the Knot Atlas ...

```
WikiUser = "DrorsRobot";
WikiPassword = InputString["Enter wiki password for " <> WikiUser];
CreateWikiConnection[
  "http://katlas.math.toronto.edu/w/index.php",
  WikiUser,
  WikiPassword
];
WikiUserName[]
```

```
DrorsRobot
```

```
UploadVI[K_Knot, n_, k_] := WikiSetPageText[
  "Data:" <> NameString[K] <> "/V_" <> ToString[n] <> "," <> ToString[k],
  "<math>" <> ToString[VI[n, k, K], TeXForm] <> "</math>"
 ]
```

```
VI[6, 9, Knot[3, 1]]
```

$$\frac{271}{30}$$

```
UploadVI[Knot[3, 1], 6, 9]
```

True

```
UploadVIs[K_Knot] := (
  doing = K;
  And @@ Flatten[
    Table[
     UploadVI[K, n, k],
     {n, 2, 6}, {k, 1, {Null, 1, 1, 3, 4, 9}[[n]]}
    ]
   ]
 )
```

```
Dynamic[doing]
```

doing

```
UploadVIs[Knot[3, 1]]
```

True

```
UploadVIs /@ AllKnots[{0, 10}]
```

```
UploadVIs /@ AllKnots[11]
```

```
{True, True, True, True, True, True, True, True, True, True, True, True, True, True,
 True, True, True, True, True, True, True, True, True, True, True, True, True, True,
 True, True, True, True, True, True, True, True, True, True, True, True, True, True,
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```