Schleimer: Introduction to three-manifolds, triangulations, and normal surfaces

November-06-09
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<u>Plan:</u> Topology, Geometry, Normal Surfaces, S^3 recognition, lies in NP.

References: Gordon: "3 dim'l top up to 1960"
Scott: "The geometries of 3-mnflds".
Gordon: "Notes on normal surfaces"
Schleimer: "S^3 recognition lies in NP"

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3 mflds: Every pt. has a neighborhood that looks like $\mathbb{R}^3$ or $\mathbb{R}^3_+$.

<u>Examples:</u> $\mathbb{R}^3$, B^3, S^3, $T^3 = \mathbb{R}^3/\mathbb{Z}^3$

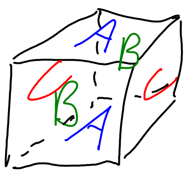
<u>A goal of 3D topology</u> classify 3 mflds up to homeomorphisms.

<u>Homeomorphism problem:</u> Decide if $M^3 \cong N^3$.

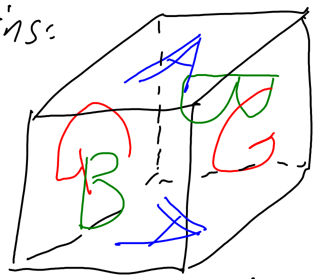
<u>Model theorem</u> In dim 2 $F^2 \cong G^2$ (connected, oriented, compact) iff they have same genus & number of bndry components.

<u>Constructing 3-mflds:</u>

<u>Gluing</u> fundamental domains:


$= T^3$

dihedral angles $= 90°$


$=: Q$

$= ?$

dihedral angles $= 120°$

<u>Cutting</u> If $F \subset M$, M_F = "M cut along F" = $M \setminus \binom{\text{open}}{\text{nbd of } F}$

$S^3_{S^2}$ = a pair of 3-balls ("Schönflies thm")
(if the embedding is "nice")

[illegible]

<u>The Alexander Trick</u>: The gluing of two B^3's along their bndries is always S^3.

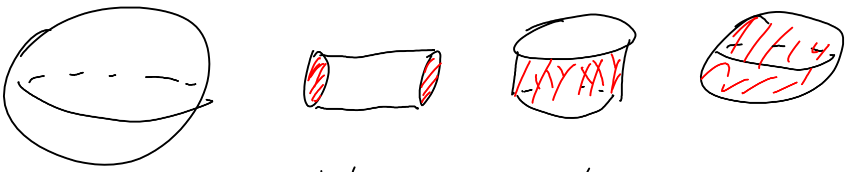
... Surgery along a knot/link. ...

Gordon Luecke: If $S^3 \setminus K_1 \cong S^3 \setminus K_2$ then $K_1 = K_2$ for knots K_1, K_2.

<u>Alexander's Theorem</u> Every T^2 in S^3 bounds a solid torus.

<u>Handles:</u>

glue area in red



0-handle B_0 | 1-handle B_1 | 2-handle B_2 | 3-handle B_3

S_g in S^3 is "standard" if $S^3 \setminus S_g$ is a pair of handlbodies.

genus g surface

<u>Exercise</u> Find S_2 in S^3 so that neither component of $S^3 \setminus S_2$ is a handlebody.

$$V_g := B_0 \cup \bigcup_g B_1$$

<u>Thm</u> (Waldhausen) Any two standard embeddings of V_g in S^3 are ambient isotopic.

JFF, Example: The Alexander Horned Sphere.

<u>Thm</u> Every 3-mnfld has a Heegaard decomposition

The fundamental group & some basic facts.

<u>claim</u> $\pi_1(Q)$ shows that Q ain't S^3 or T^3.

twisted cube above

$\pi_1(V \cup_S W)$ has a presentation with one generator
a Heegaard splitting for each one handle and one relator
for each 2-handle.

The poincaré homology
sphere: