Schleimer: Introduction to three-manifolds, triangulations, and normal surfaces, meeting 4

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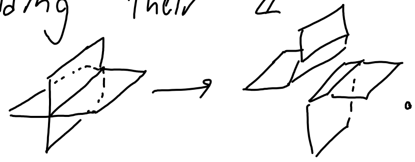
Ben Burton has a program "Regina" to do normal surfaces in 3-manifolds, including S^3 recognition.

http://regina.sourceforge.net/

A normal surface in a triangulation T is specified by a polarization in T and by a vector in $\mathbb{Z}_{\geq 0}^{5|T|}$ satisfying some linear equation.

The choice of polarization gives the exponential trouble; beyond this we minimize Euler char. using linear programming.

"Haken Sum": If F & G are normal surfaces which are compatible (have same polarization) their Haken Sum is obtained by adding their $\mathbb{Z}^{5|T|}$ vector. Geometrically this is  .

Def A normal surface is "fundamental" if it does not decompose as a Haken Sum. It is a vertex if $[S]$ is a vertex of $P(T)$.

Meta Thm A topologically interesting property of surfaces in M will have a vertex / fundamental representative.

Example:

Thm (Haken) If (M, T) has ∂M compressible, Thm (Haken) If (M, T) has ∂M compressible, then there is a normal compressing disk which is fundamental.

- - - - This solves the "unknot recognition problem".

Hass et al: Unknotting $\in NP$

Lemma If K is the unknot there is a compressing disk among the vertices of $P(T)$.

So the "unknotting certificate" will be a compressing disk plus a sufficient quantity of supporting hyperplanes.