Moskovich on Words

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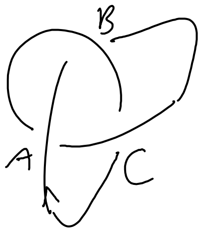
10:05 AM



$\to ABCABC$

some 2 xing virt. knot. $\to ABAB$

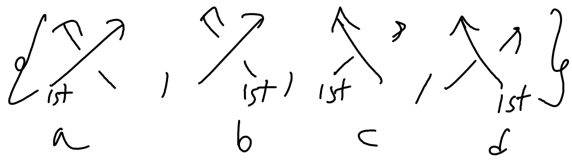
Gausswords: every letter appears twice.

Turaev's words: Words in which each letter appears <u>at least</u> twice.

Gauss word: Each letter has multiplicity exactly 2.

Étale word: Gauss word over $\mathcal{A}$, with a "labeling" map $\underbrace{\mathcal{A} \to \alpha}_{\text{such a map is an "}\alpha\text{-alphabet"}}$, where α is some fixed alphabet.

<u>Example</u> Knots are Étale words over the alphabet

$\{$  $\}$

The trefoil: $\begin{matrix} A & B & C & A & B & C \\ \downarrow & \downarrow & \downarrow & & & \\ a & b & a & & & \end{matrix}$

α-Morphism of α-alphabets:

$$\begin{tikzcd} A_1 \arrow[rr, "f"] \arrow[dr] & & A_2 \arrow[dl] \\ & \alpha & \end{tikzcd}$$

Two α-words are "isomorphic" if $\exists\, \alpha$-isomo. of their alphabets which carries one to the other.

There is a multiplication operation of α-words.

<u>Desingularization:</u> (map into Gauss words)

$ABABA \longrightarrow A_{12}A_{13}BA_{23}A_{12}BA_{13}A_{23}$

Reidemeister moves: (for knots)

R1: $xAAy \sim xy$

R2 $xAByABz \sim xyz$

$xAByBAz \sim xyz$

R3 $xAByACzBCt \sim xBAyCAzCBt$

Homotopy of nano-words. [nano-word := α-Gauss word]

Homotopy data: $\tau: \alpha \to \alpha$ an involution [when is an R2 allowed]

$S \subset \alpha \times \alpha \times \alpha$ of "allowed R3's".

Group theoretic invariant:

$$\pi = \langle (z_a)_{a \in \alpha} : z_\alpha z_{\tau(\alpha)} = 1 \rangle$$