Hilbert's 13th revisited

November-01-09
9:27 AM

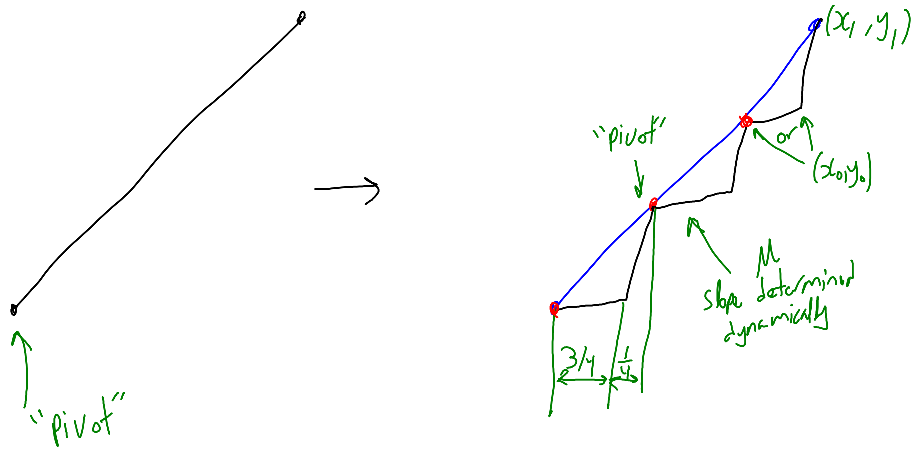
Proposed Title. "Dessert: Hilbert's 13th Problem, in Full Colour."

Abstract. To end a week of deep thinking with a nice colourful light dessert, we will present the Kolmogorov-Arnol'd solution of Hilbert's 13th problem with lots of computer-generated rainbow-painted 3D pictures.

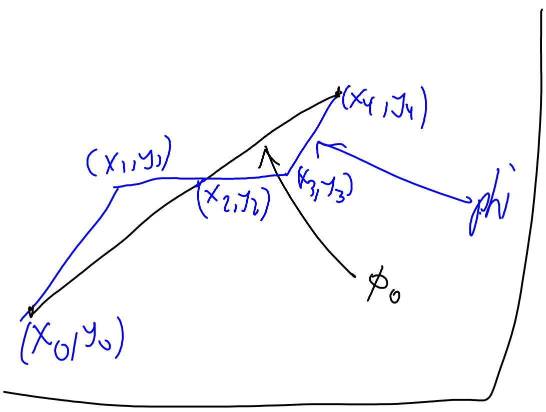
In short, Hilbert asked if a certain specific function of three variables can be written as a multiple (yet finite) composition of continuous functions of just two variables. Kolmogorov and Arnol'd showed him silly (ok, it took 60-70 years, so it is a bit tricky) by showing that *any* continuous function f of any finite number of variables is such a finite composition of continuous functions of one variable and several instances of the binary function "+" (addition). For f(x,y)=xy, this may be xy=exp(log x + log y). For f(x,y,z)=x^y/z, this may be exp(exp(log y + log log x) + (-log z)). What might it be for the real part (say) of the Riemann zeta function?

The only original material in this talk will be the pictures. The math was known in the 60s. It was the first seminar lecture I ever gave, back as an undergraduate in Tel Aviv in 1983.

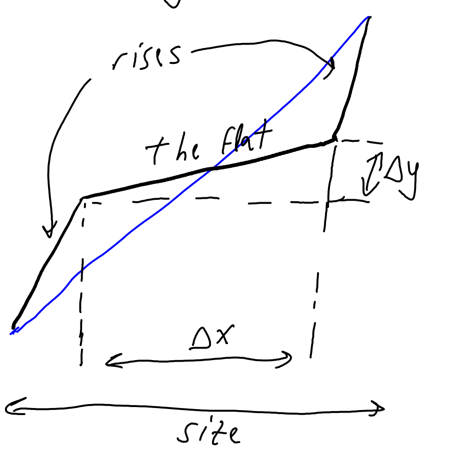
Scheme: $\phi_{\ldots,3,4}$ comes from $\phi_{\ldots,3}$ via



Failed; see Mathematica / Hilbert13th - Failure1.nb



Drawing for handout:



$\text{slope} = \frac{\Delta y}{\Delta x}$

$\text{Fill Factor} = \frac{\Delta x}{\text{size}}$

A Political statement.

Bad mouthing: I hate power point!

* No good feelings towards Impress, beamer, etc.
* Can't sync with speaker, can't look back at key points, don't know what to look forward too. ["striptease"'s even worse].
* No upper bound to useless content.
* Nothing to take home.

Propaganda: I love handouts!

* I have nothing to hide and you can take what you want, forward, backwards, here and at home.
* What doesn't fit on one sheet can't be done in one hour.
* It costs hours and pennies. The audience's worth it.
* Can put hyperbolic geometry in every talk!

The error:

$\frac{2}{5}+\left(1-\frac{3}{5}\right)=\frac{4}{5}$	or	$\frac{1}{5}+\left(1-\frac{4}{5}\right)=\frac{2}{5}$	or	0	The $1/5$ option
$\frac{2}{3}$	or	$\frac{1}{3}+$ solid $1/3$	or	solid $2/3$!	The $1/3$ option
2 bad		1 bad		all good	

Set $Tg := \sum_{i=1}^{5} g(\phi_i(x)+\lambda\phi_i(y))$, $f_1 := f$, $M := \|f\|$, and iterate "shooting and adjusting". Find g_1 with $\|g_1\| \leq M$ and $\|f_2 := f_1 - Tg_1\| \leq \frac{3}{4}M$. Find g_2 with $\|g_2\| \leq \frac{3}{4}M$ and $\|f_3 := f_2 - Tg_2\| \leq (\frac{3}{4})^2 M$. Find g_3 with $\|g_3\| \leq (\frac{3}{4})^2 M$ and $\|f_4 := f_3 - Tg_3\| \leq (\frac{3}{4})^3 M$. Continue to eternity. When done, set $g = \sum g_k$ and note that $f = Tg$ as required.

$Tg_k = f_k - f_{k+1}$ so

$\|T(\sum_1^n g_k) - f\| = \|f_1 - f_{n+1} - f\| = \|f_{n+1}\| \leq (\frac{3}{4})^n M \to 0$

Talk Plan: 1. Mention video.

2. Mention "giving myself a candy".

3. Go through handout.

4. And then propaganda.

Nice ref (from Mike Shub): A.G. Vitushkin, "On Representations of Functions by Means of Superpositions and Related Topics".