BEER-5 by Peter Lee, November 9

November-09-09
9:37 AM

Recall: P_n is the convex hull of all partitions. The faces of P_n correspond to ordered partitions of $[n] := \{1, \dots, n\}$

Also,

$$QC_n = \frac{P_n \times S_r}{\sim}$$

$(S,\tau) \sim (S,\sigma)$ means $\tau(a) < \tau(b) \Leftrightarrow \sigma(a) < \sigma(b)$ $\forall a,b \in [r]$, where S is a partition of $[n]$

$$C_n = P_n / \{S_r\}$$

Homology Consider the chain complex spanned by all faces of P_n, with

$$\partial(S_1 \sqcup \dots \sqcup S_r) = \sum_{i=1}^{r} S_1 \sqcup \dots \partial S_i \dots \sqcup S_r$$

$$\partial S = \sum_{S_1 \sqcup S_2 = S} \pm S_1 \sqcup S_2$$

Prop The differential descends to C_n as 0; The homology of C_n is

$$\dim H_r(C_n) = \left\{ {n \atop n-r} \right\} = \left| \begin{matrix} \text{unordered partitions of} \\ [n] \text{ into } (n-r) \text{ unordered parts} \end{matrix} \right|$$

$$\dim H_r(QC_n) = L(n, n-r) = \left| \begin{matrix} \text{unordered partitions of } [n] \\ \text{into } (n-r) \text{ ordered subsets} \end{matrix} \right|$$