Dror Bar-Natan: Talks: Fields-0911:

Dror Bar-Natan: Academic Pensieve: 2009-11:

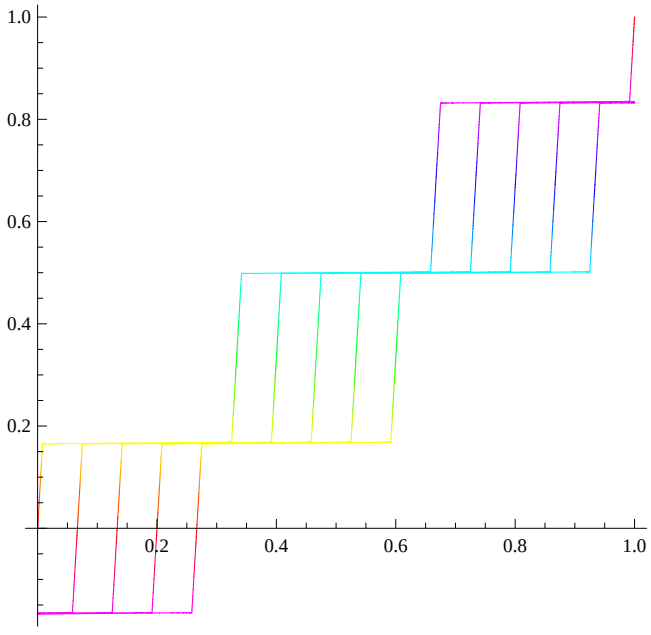
Hilbert's 13th Problem

Pensieve Header: Hilbert 13th, step 3.

```
SetDirectory[
  $MachineName /. {
     "dror-x61" → "C:/drorbn/AcademicPensieve/2009-11"
    } /. $MachineName → "."
 ];
<< Hilbert13th-Program.m
```

```
Do[
  phi2[i] = Phi[Identity, 2, 0.02, 0.95, i / 5];
  g2[i] = G[f, phi2[i]],
  {i, 0, 4}
 ];
Do[
 phi3[i] = Phi[Identity, 3, 0.01, 0.95, i / 5];
 g3[i] = G[f, phi3[i]],
 {i, 0, 4}
]
```

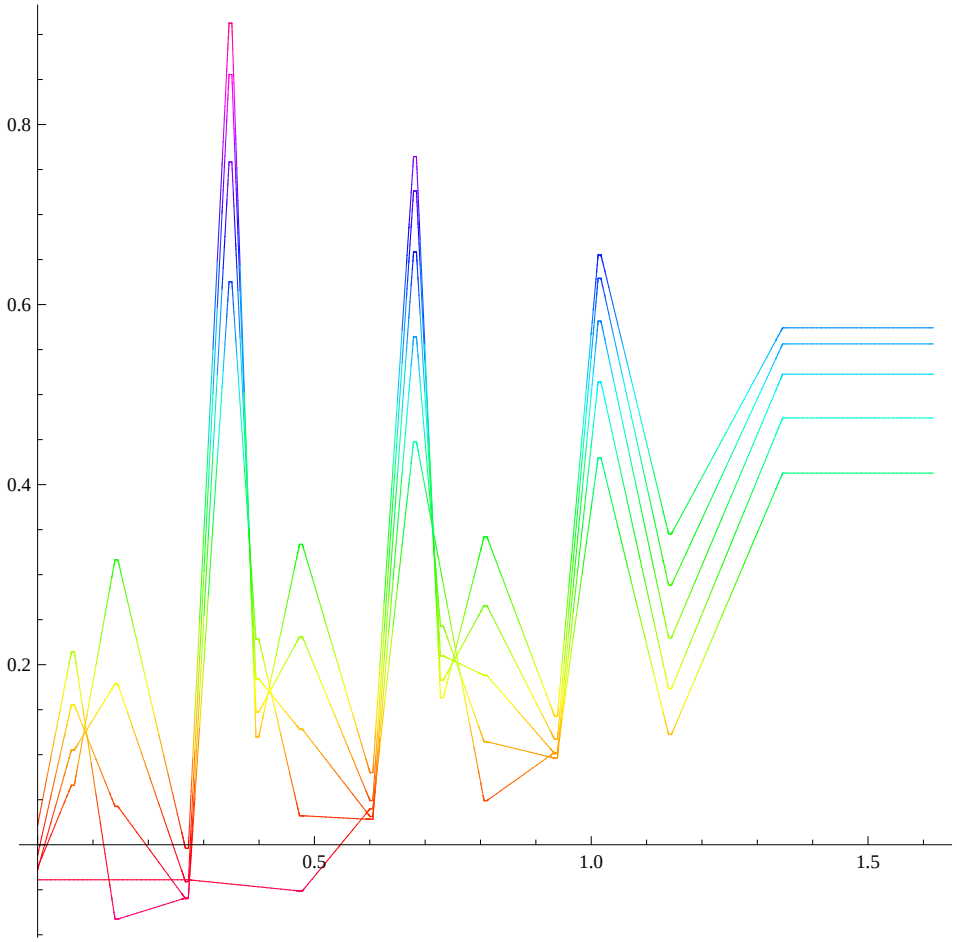
```
Plot[Table[phi3[i][x], {i, 0, 4}] , {x, 0, 1}]
```



```
Plot[
 Table[g3[i][z], {i, 0, 4}], {z, 0, 1 + λ},
 PlotPoints → 200
]
```

InterpolatingFunction::dmval :

Input value $\{8.13895\times10^{-6}\}$ lies outside the range of data in the interpolating function. Extrapolation will be used. ≫




```
Rasterize[
 Plot3D[phi3[0][x] + λ * phi3[0][y], {x, 0, 1}, {y, 0, 1},
  Mesh → 11, ViewPoint → {-2, -2, 1}, AxesLabel → Automatic,
  NormalsFunction → None, ColorFunction → (Hue[g3[0][#3]] &)
 ]
]
```

InterpolatingFunction::dmval :

Input value $\{2.15123\times10^{-7}\}$ lies outside the range of data in the interpolating function. Extrapolation will be used. ≫

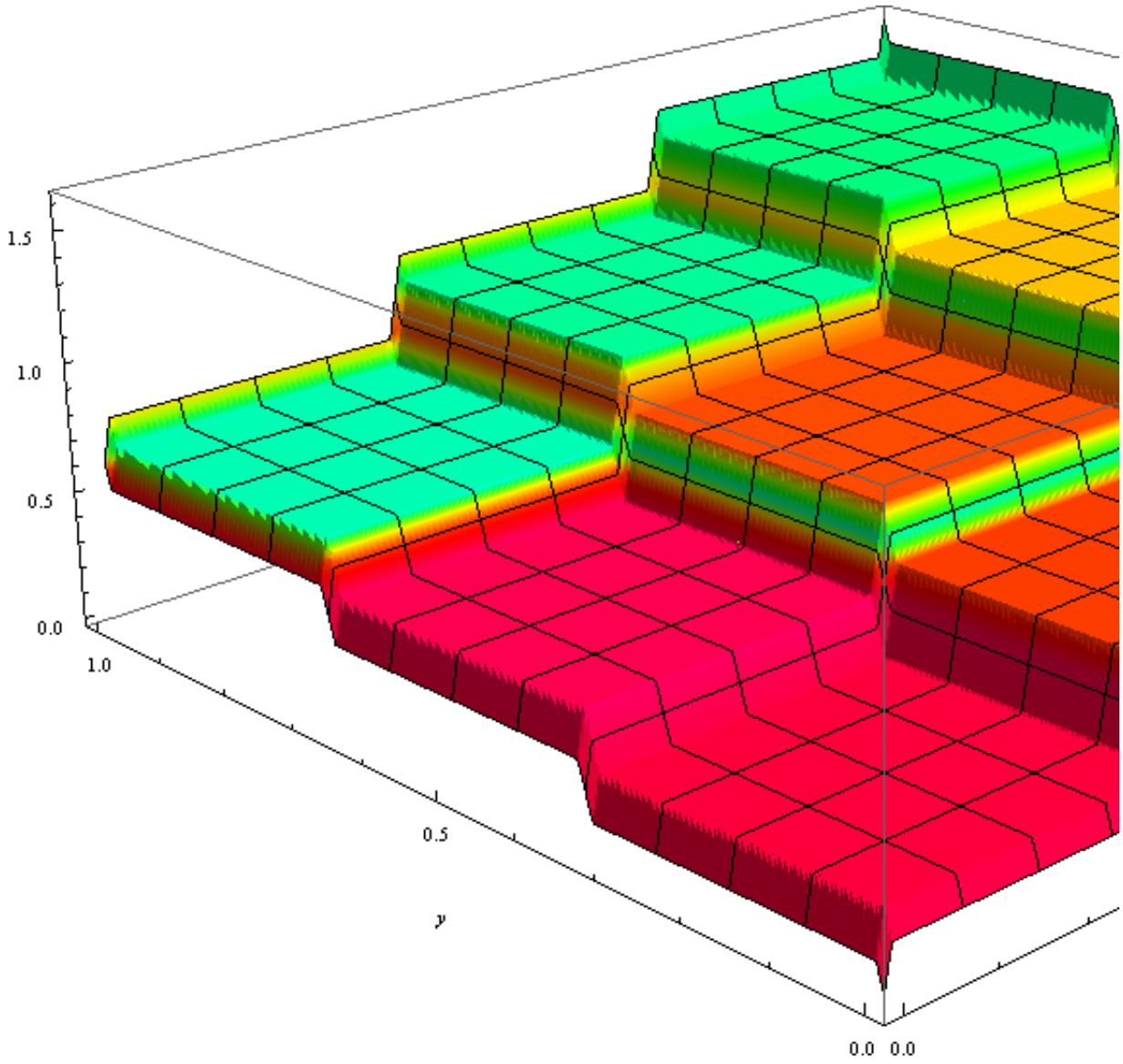
InterpolatingFunction::dmval :

Input value {0.132953} lies outside the range of data in the interpolating function. Extrapolation will be used. ≫

InterpolatingFunction::dmval :

Input value {0.165134} lies outside the range of data in the interpolating function. Extrapolation will be used. ≫

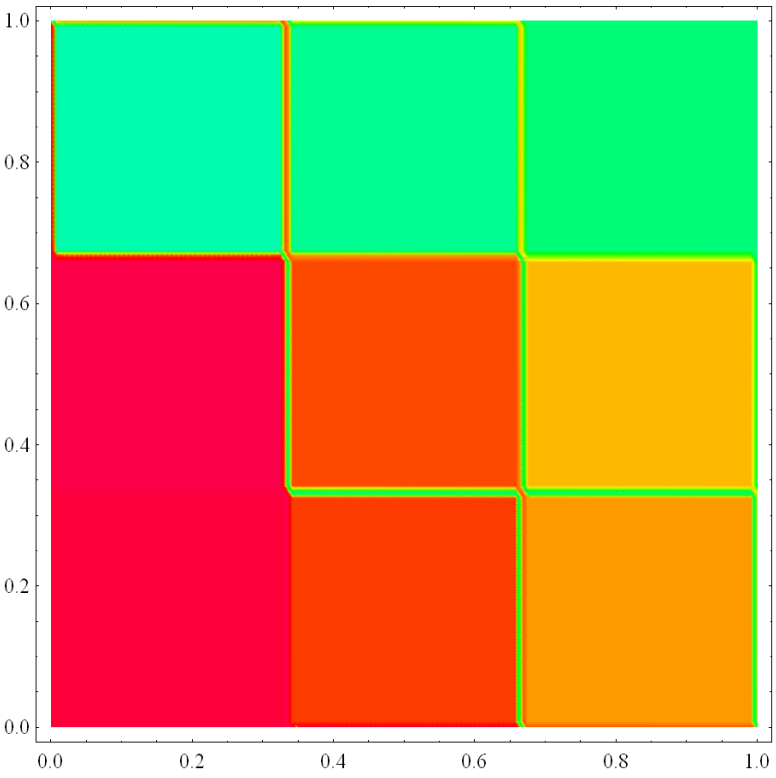
General::stop : Further output of InterpolatingFunction::dmval will be suppressed during this calculation. ≫



```
Rasterize[
 DensityPlot[
  g3[0][phi3[0][x] + λ * phi3[0][y]],
  {x, 0, 1}, {y, 0, 1}
 ]
]
```

InterpolatingFunction::dmval :

Input value {0.000215123} lies outside the range of data in the interpolating function. Extrapolation will be used. ≫



```
Step3FiveCascades = Rasterize[
  GraphicsRow[
   Table[
    Plot3D[phi3[i][x] + λ * phi3[i][y], {x, 0, 1}, {y, 0, 1},
     Mesh → None, Boxed → False, ViewPoint → {-2, -2, 1}, Axes → None, AxesLabel → None,
     Ticks → None, NormalsFunction → None, ColorFunction → (Hue[g3[i][#3]] &)
    ], {i, 0, 4}
   ], Spacings → 0
  ], ImageSize → 900
 ]
```

InterpolatingFunction::dmval :

Input value $\{2.15123\times10^{-7}\}$ lies outside the range of data in the interpolating function. Extrapolation will be used. ≫

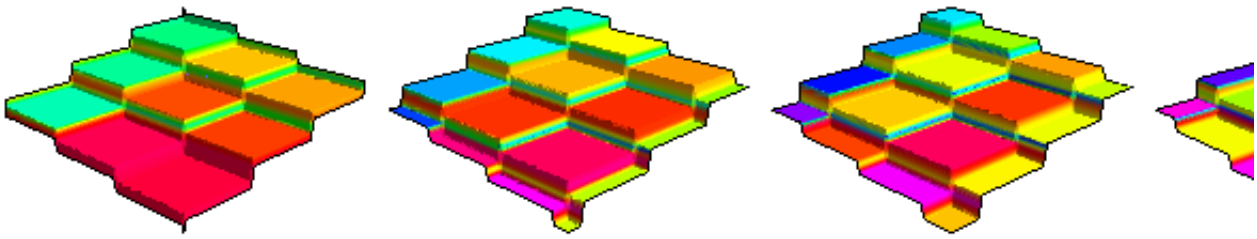
InterpolatingFunction::dmval :

Input value {0.132953} lies outside the range of data in the interpolating function. Extrapolation will be used. ≫

InterpolatingFunction::dmval :

Input value {0.165134} lies outside the range of data in the interpolating function. Extrapolation will be used. ≫

General::stop : Further output of InterpolatingFunction::dmval will be suppressed during this calculation. ≫



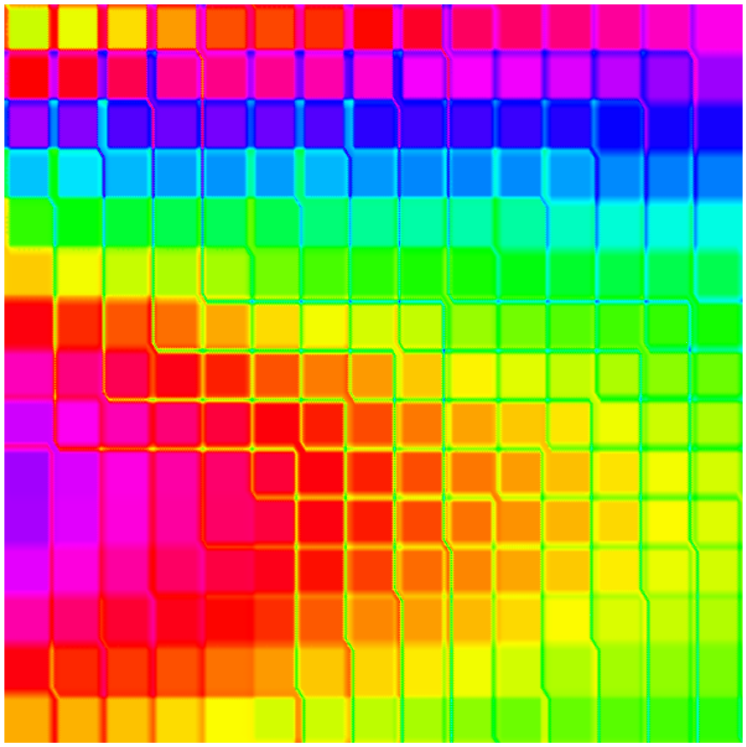
```
Export[
 "Step3FiveCascades.png",
 ImageCrop[Step3FiveCascades]
]
```

Step3FiveCascades.png

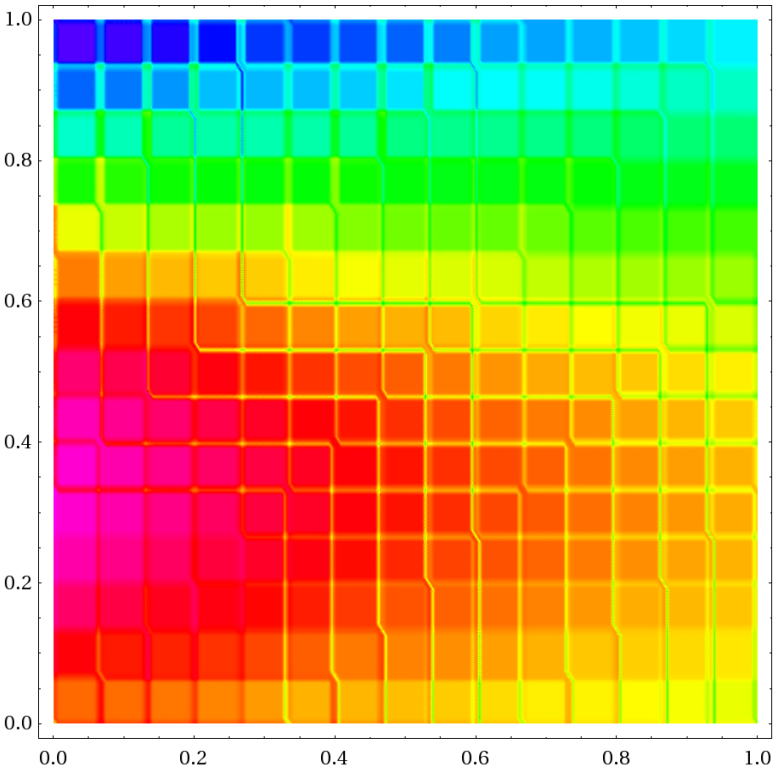
```
Rasterize[
 DensityPlot[
  (1 / 3) * Sum[g3[k][phi3[k][x] + λ * phi3[k][y]], {k , 0, 4}],
  {x, 0, 1}, {y, 0, 1}, Frame → False
 ]
]
```

InterpolatingFunction::dmval :

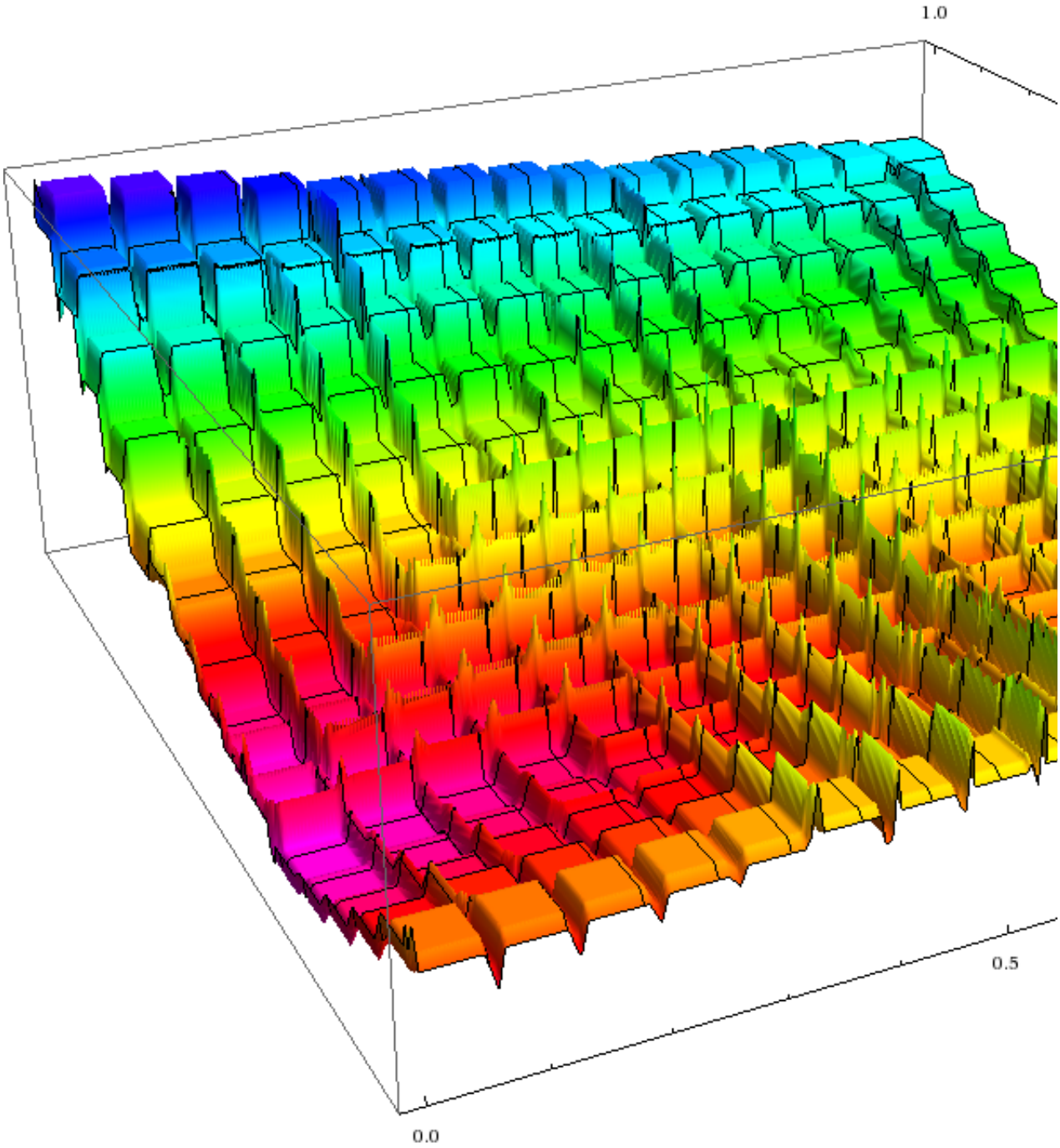
Input value {0.000215123} lies outside the range of data in the interpolating function. Extrapolation will be used. ≫



```
Rasterize[
 DensityPlot[
  (1 / 5) * Sum[g3[k][phi3[k][x] + λ * phi3[k][y]], {k , 0, 4}],
  {x, 0, 1}, {y, 0, 1}
 ]
]
```



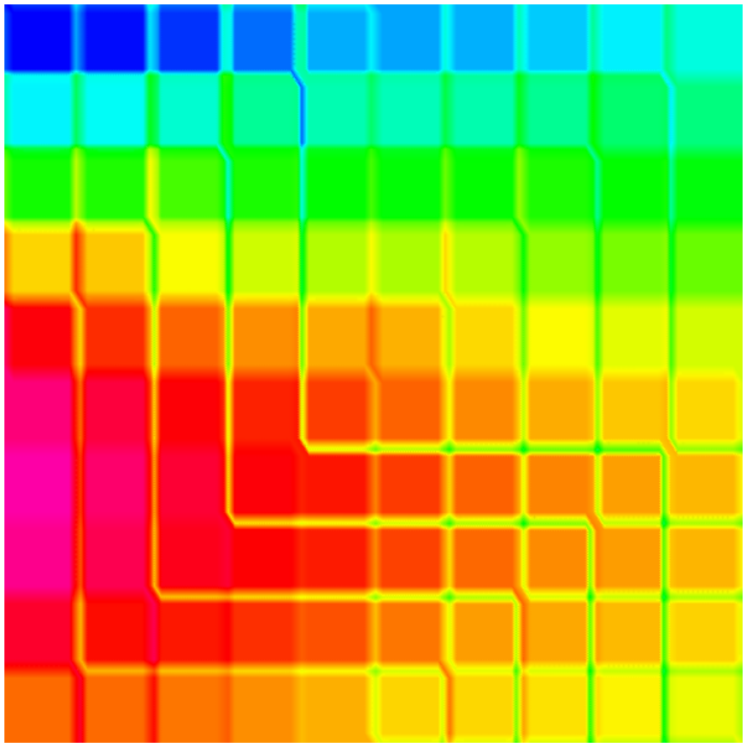
```
Rasterize[
 Plot3D[
  (1 / 5) * Sum[g3[k][phi3[k][x] + λ * phi3[k][y]], {k , 0, 4}],
  {x, 0, 1}, {y, 0, 1}
 ]
]
```



```
Step3Density = Rasterize[
  DensityPlot[
   (1 / 5) * Sum[g2[k][phi2[k][x] + λ * phi2[k][y]], {k , 0, 4}],
   {x, 0, 1}, {y, 0, 1}, PlotPoints → 150, Frame → False
  ]
 ]
```

InterpolatingFunction::dmval :

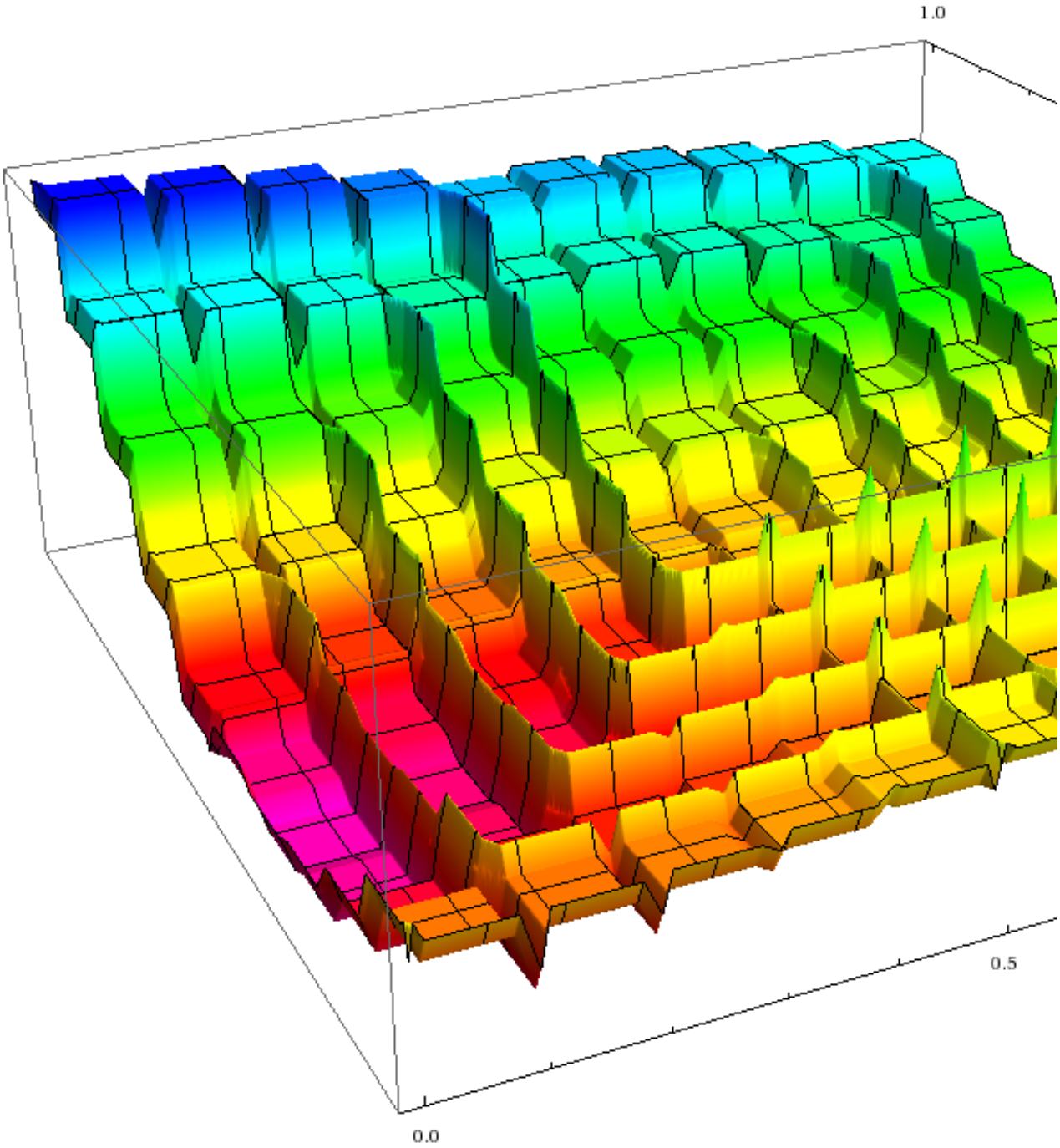
Input value {0.000213059} lies outside the range of data in the interpolating function. Extrapolation will be used. ≫



```
Export[
 "Step3Density.png",
 Step3Density
]
```

Step3Density.png

```
Step3Trenches = Rasterize[
  Plot3D[
   (1 / 5) * Sum[g2[k][phi2[k][x] + λ * phi2[k][y]],  {k , 0,  4}],
   {x, 0, 1}, {y, 0, 1}, PlotPoints → 50, Axes → None, Boxed → False
  ]
 ]
```



```
Export[
 "Step3Trenches.png",
 Step3Trenches
]
```

```
Step3Trenches.png
```