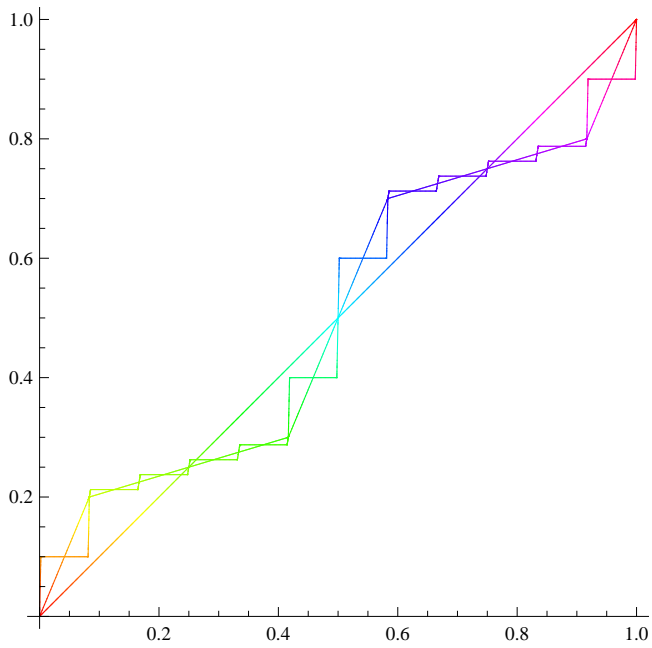# Hilbert's 13th Problem

Pensieve Header: Some pictures for the Kolmogorov-Arnol'd solution of Hilbert's 13th problem.

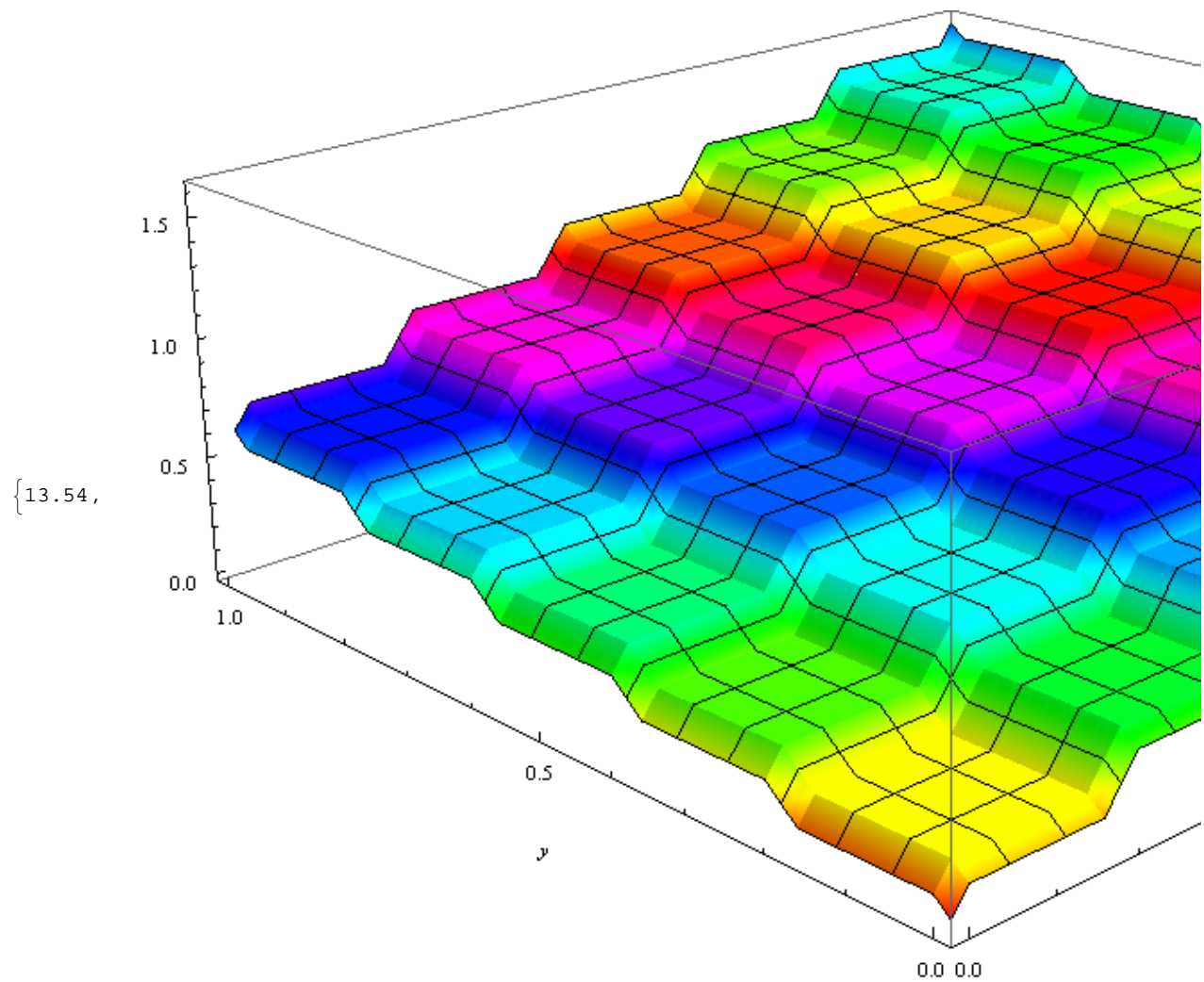```
<< C:\drorbn\AcademicPensieve\2009-11\Hilbert13th-Program.m

phi2 := Phi[phi0, 2, 0.3, 2 / 3];
phi26 := Phi[phi2, 12, 0, 0.95];
Plot[{ phi0[x], phi2[x], phi26[x]}, {x, 0, 1},
 PlotPoints → 479
]
```

```
phi5 := Phi[ϕ];
Timing[
 Cascade5 = Rasterize[
   Plot3D[phi5[x] + λ * phi5[y], {x, 0, 1}, {y, 0, 1},
    PlotPoints → 51, Mesh → 14, Exclusions → None, ViewPoint → {-2, -2, 1},
    ImageSize → 800 , AxesLabel → Automatic, NormalsFunction → None
   ]
  ]
]
```
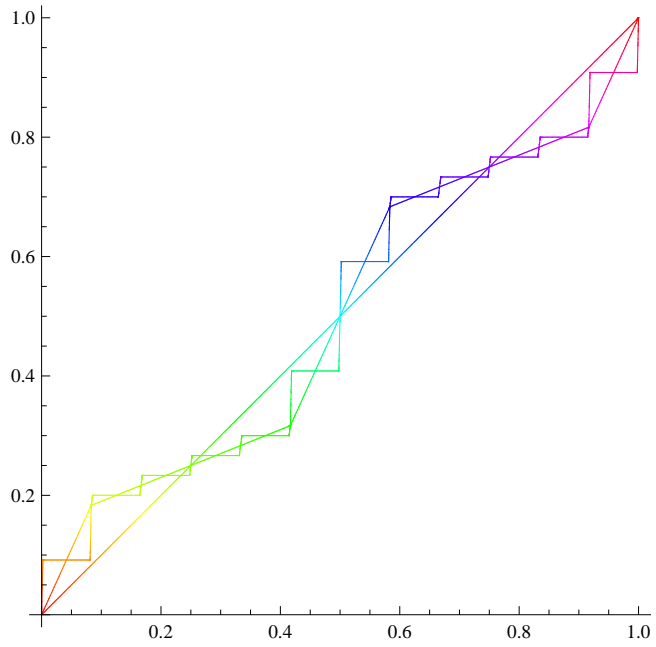
```
phi2 := Phi[ϕ, Subdivisions → 2, Slope → 0.4, FlatFraction → 2 / 3];
phi26 := Phi[ϕ, ϕ0 → phi2, Subdivisions → 12, FlatFraction → 0.95];
Plot[{ x, phi2[x], phi26[x]}, {x, 0, 1},
 PlotPoints → 479
]
```

```
Xs = Union @@ Phi[{X0, X1, X3}, ϕ0 → phi2, Subdivisions → 12, FlatFraction → 0.95];
Ys = Union @@ Phi[{Y0, Y1, Y3}, ϕ0 → phi2, Subdivisions → 12, FlatFraction → 0.95];
l = Length[Ys];
Cascade26 = Rasterize[
  ListPlot3D[
    Join @@ Table[{Xs[[i]], Xs[[j]], Ys[[i]] + λ * Ys[[j]]}, {i, l}, {j, l}],
    InterpolationOrder → 1, Mesh → 14,
    ViewPoint → {-2, -2, 1}, ImageSize → 800 , AxesLabel → Automatic,
    NormalsFunction → None, ColorFunction → Hue, ColorFunctionScaling → False
  ]
 ]
```
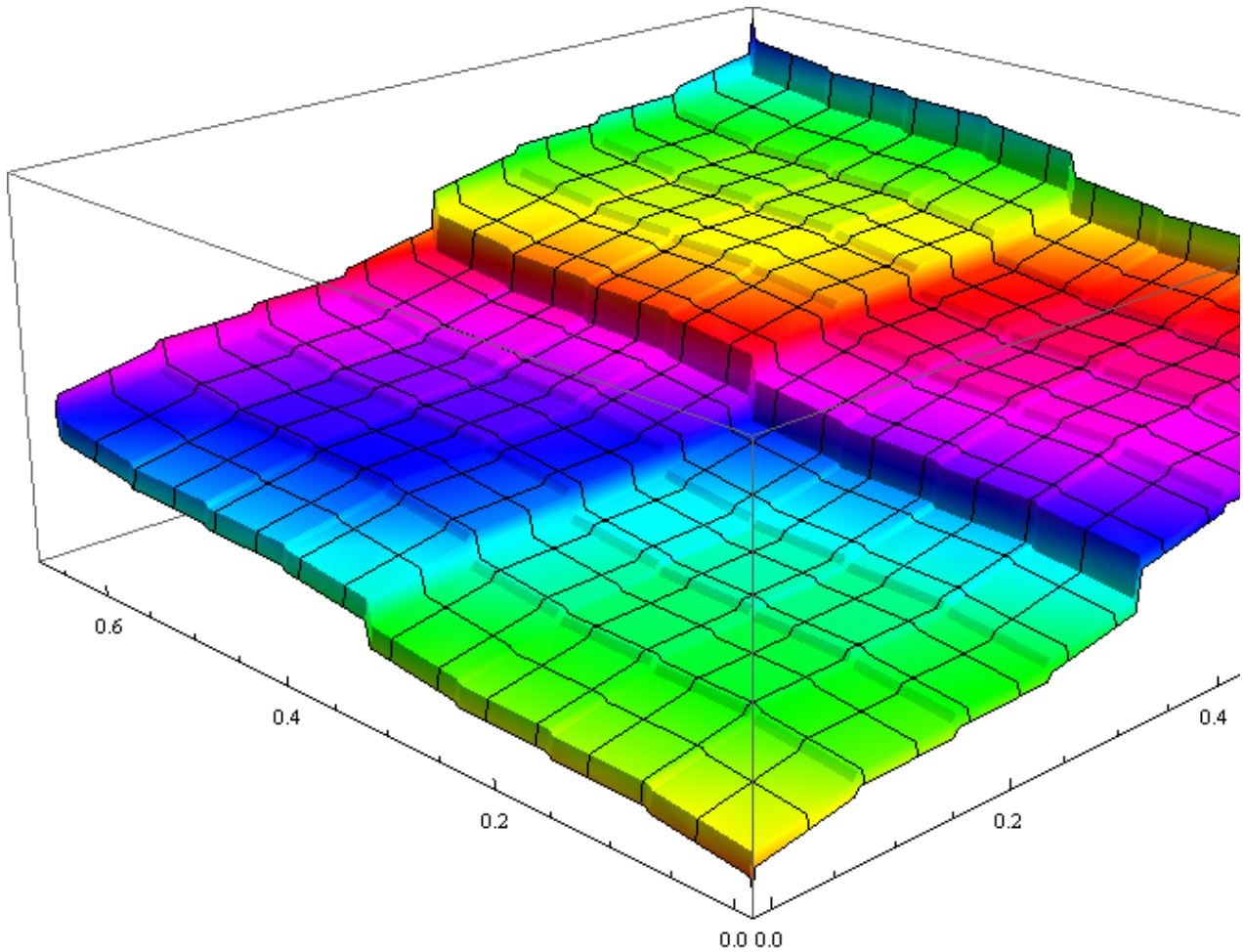
Xs

$\Big\{0, 0.00208333, 0.08125, \dfrac{1}{12}, 0.0854167, 0.164583, \dfrac{1}{6}, 0.16875, 0.247917,$

$\dfrac{1}{4}, 0.252083, 0.33125, \dfrac{1}{3}, 0.335417, 0.414583, \dfrac{5}{12}, 0.41875, 0.497917, \dfrac{1}{2},$

$0.502083, 0.58125, \dfrac{7}{12}, 0.585417, 0.664583, \dfrac{2}{3}, 0.66875, 0.747917, \dfrac{3}{4}, 0.752083,$

$0.83125, \dfrac{5}{6}, 0.835417, 0.914583, \dfrac{11}{12}, 0.91875, 0.997917, 1, 1.00208, 1.08125\Big\}$

Ys

$\Big\{1.01779 \times 10^{-17}, 0.0916667, 0.183333, 0.2, 0.216667, 0.233333, 0.25, 0.266667,$
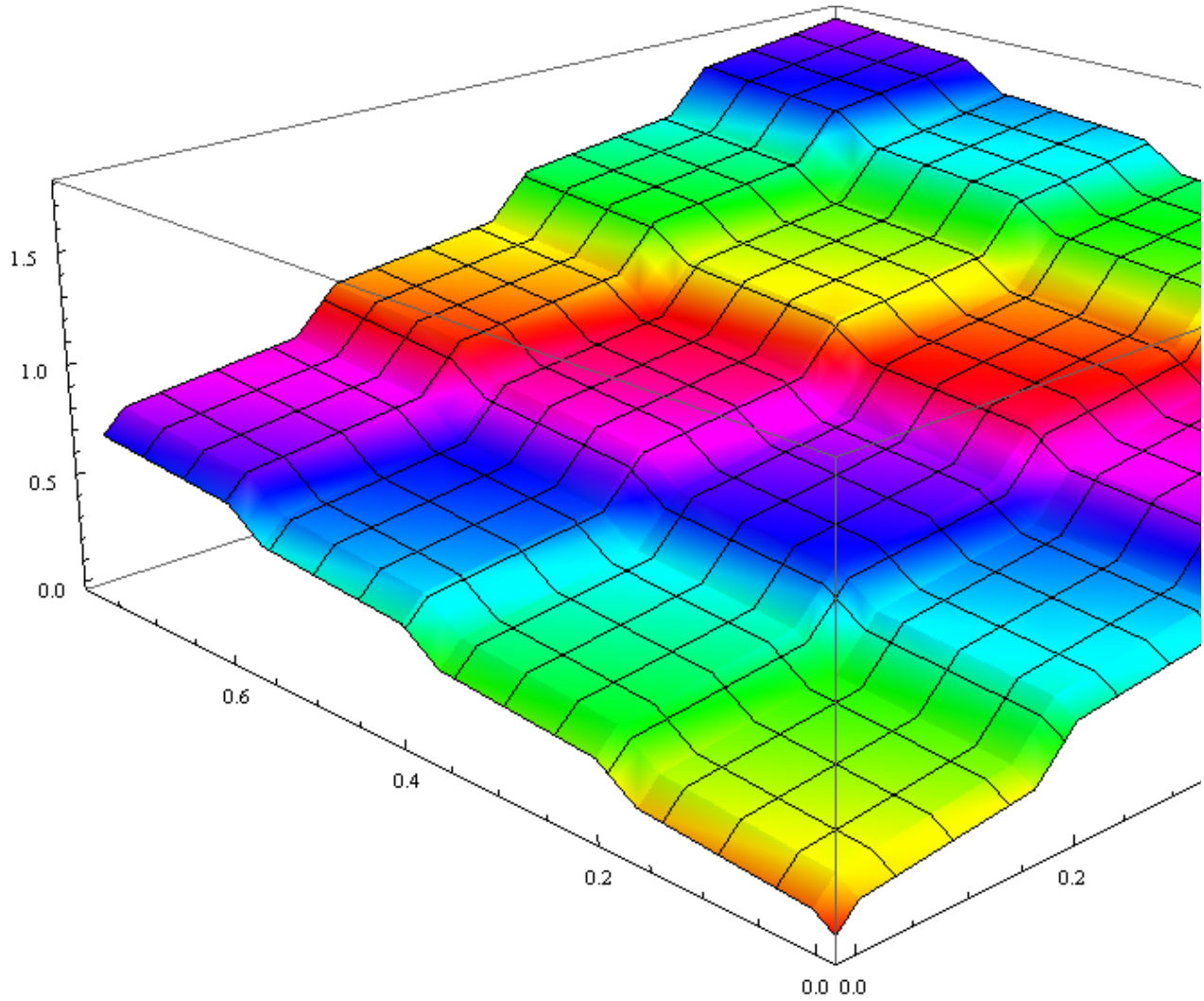
$0.283333, 0.3, 0.316667, 0.408333, 0.5, 0.591667, 0.683333, 0.7, 0.716667,$

$0.733333, 0.75, 0.766667, 0.783333, 0.8, 0.816667, 0.908333, 1., 1.09167\Big\}$

```
Xs = Union @@ Phi[{X0, X1, X3}];
Ys = Union @@ Phi[{Y0, Y1, Y3}];
l = Length[Ys];
Cascade = Rasterize[
  ListPlot3D[
    Join @@ Table[{Xs[[i]], Xs[[j]], Ys[[i]] + λ * Ys[[j]]}, {i, l}, {j, l}],
    InterpolationOrder → 1, Mesh → 14, ViewPoint → {-2, -2, 1}, ImageSize → 800,
    NormalsFunction → None, ColorFunction → Hue, ColorFunctionScaling → False
  ]
]
```


Xs

$$\left\{0,\, 0.02,\, 0.18,\, \frac{1}{5},\, 0.22,\, 0.38,\, \frac{2}{5},\, 0.42,\, 0.58,\, \frac{3}{5},\, 0.62,\, 0.78,\, \frac{4}{5},\, 0.82,\, 0.98,\, 1,\, 1.02,\, 1.18\right\}$$

Ys

$$\left\{0,\, \frac{1}{10},\, \frac{1}{5},\, \frac{3}{10},\, \frac{2}{5},\, \frac{1}{2},\, \frac{3}{5},\, \frac{7}{10},\, \frac{4}{5},\, \frac{9}{10},\, 1,\, \frac{11}{10}\right\}$$

phi26

InterpolatingFunction [{{0., 1.08125}}, <>]