

Mathcamp Day 2

By Dror Bar-Natan; with some code borrowed from a joint project with Louis Leung, Arrow_Diagrams_and_gl (N).

Pensieve Header: Generating the equations for Mathcamp Day 2.

Program

Diagrams

```

SetAttributes[Diag, Orderless];
Place[{ar}, {i_, j_}] := {Diag[ar[i, j]], Diag[ar[j, i]]};
Place[{ar, objs__}, {i_, rest__}] := Flatten[Table[
  Outer[Join,
    Place[{ar}, {i, {rest}[[k]]}],
    Place[{objs}, Delete[{rest}, k]]
  ],
  {k, Length[{rest]}]
]];
Diagrams[k_.*ar] := Place[Table[ar, {k}], Range[2k]];

```

Relators

```

Place[{r: (TC | R6T | Slide), objs__}, {i_, rest__}] := Flatten[Table[
  Outer[Join,
    Place[{r}, {i, {rest}[[j]], {rest}[[k]]}],
    Place[{objs}, Delete[{rest}, {j}, {k}]]
  ],
  {k, 2, Length[{rest]}}, {j, 1, k - 1}
]];

Place[{R6T}, {i_, j_, k_}] :=
  Permutations[{i, j, k}] /. {i1_, j1_, k1_} => Diag[R6T[i1, j1, k1]];
Diagrams[R6T] := Place[{R6T}, {1, 2, 3}];
Diagrams[R6T + k_.*ar] /; k > 0 := Flatten[
  Place[#, Range[2k + 3]] & /@ Permutations[Table[ar, {k}] ~Append~ R6T]
];
Diagrams[R6T + k_.*ar] /; k < 0 := {};

Place[{TC}, {i_, j_, k_}] := Diag /@ {TC[i, j, k], TC[j, k, i], TC[k, i, j]};
Diagrams[TC] := Place[{TC}, {1, 2, 3}];
Diagrams[TC + k_.*ar] /; k > 0 := Flatten[
  Place[#, Range[2k + 3]] & /@ Permutations[Table[ar, {k}] ~Append~ TC]
];
Diagrams[TC + k_.*ar] /; k < 0 := {};

```

```

Place[{Slide}, {i_, j_, k_}] := Diag /@ {Slide[i, j, k], Slide[j, k, i], Slide[k, i, j]};
Diagrams[Slide] := Place[{Slide}, {1, 2, 3}];
Diagrams[Slide + k_. * ar] /; k > 0 := Flatten[
  Place[#, Range[2 k + 3]] & /@ Permutations[Table[ar, {k}] ~Append~ Slide]
];
Diagrams[Slide + k_. * ar] /; k < 0 := {};

```

Relations

```

NormalizeDiag[diag_Diag] := Module[
  {indices = Union@@ (List @@ diag /. ar → List)},
  diag /. Thread[indices → Range[Length[indices]]]
];
R[Diag[lft___, R6T[i_, j_, k_], rgt___]] := (
  +NormalizeDiag[Diag[lft, ar[i, j], ar[i + 0.5, k], rgt]]
  + NormalizeDiag[Diag[lft, ar[i, j], ar[j + 0.5, k], rgt]]
  + NormalizeDiag[Diag[lft, ar[i, k], ar[j, k + 0.5], rgt]]
  - NormalizeDiag[Diag[lft, ar[i, k], ar[i + 0.5, j], rgt]]
  - NormalizeDiag[Diag[lft, ar[i, j + 0.5], ar[j, k], rgt]]
  - NormalizeDiag[Diag[lft, ar[i, k + 0.5], ar[j, k], rgt]]
);
R[Diag[lft___, TC[i_, j_, k_], rgt___]] := (
  +NormalizeDiag[Diag[lft, ar[i, j], ar[i + 0.5, k], rgt]]
  - NormalizeDiag[Diag[lft, ar[i + 0.5, j], ar[i, k], rgt]]
);

```

Equations

```

m = 3;
diags = Diagrams[m ar];
rels = R /@ Join[Diagrams[R6T + (m - 2) ar], Diagrams[TC + (m - 2) ar]];
eqns = rels /. Thread[diags → Table[x[i], {i, Length[diags]}]]
{x[1] + x[5] - 2 x[9], x[2] - x[5] + x[9] - x[10], x[3] - x[5] + x[9] - x[10],
 -x[3] + x[8] + x[11] - x[12], -x[2] + x[8] + x[11] - x[12], -x[4] - x[8] + 2 x[12],
 x[13] + x[17] - 2 x[21], x[14] - x[17] + x[21] - x[22], x[15] - x[17] + x[21] - x[22],
 -x[15] + x[20] + x[23] - x[24], -x[14] + x[20] + x[23] - x[24],
 -x[16] - x[20] + 2 x[24], x[25] - x[33] + x[53] - x[57], x[26] - x[34] - x[53] + x[57],
 x[27] - x[29] - x[31] + x[33] + x[55] - x[58], -x[27] + x[35] + x[56] - x[60],
 -x[26] + x[30] + x[32] - x[36] - x[54] + x[59], -x[28] + x[36] - x[56] + x[60],
 x[37] - x[45] + x[65] - x[69], x[38] - x[46] - x[65] + x[69],
 x[39] - x[41] - x[43] + x[45] + x[67] - x[70], -x[39] + x[47] + x[68] - x[72],
 -x[38] + x[42] + x[44] - x[48] - x[66] + x[71], -x[40] + x[48] - x[68] + x[72],
 x[53] - x[57] + x[73] - x[81], x[54] - x[58] + x[74] - x[77] - x[78] + x[81],
 -x[53] + x[57] + x[75] - x[82], -x[55] + x[59] - x[75] + x[79] + x[80] - x[84],
 x[56] - x[60] - x[74] + x[83], -x[56] + x[60] - x[76] + x[84], x[65] - x[69] + x[85] - x[93],
 x[66] - x[70] + x[86] - x[89] - x[90] + x[93], -x[65] + x[69] + x[87] - x[94],
 -x[67] + x[71] - x[87] + x[91] + x[92] - x[96], x[68] - x[72] - x[86] + x[95],
 -x[68] + x[72] - x[88] + x[96], x[97] + x[101] - 2 x[105], x[98] - x[101] + x[105] - x[106],
 x[99] - x[101] + x[105] - x[106], -x[99] + x[104] + x[107] - x[108],
 -x[98] + x[104] + x[107] - x[108], -x[100] - x[104] + 2 x[108], x[109] + x[113] - 2 x[117],
 x[110] - x[113] + x[117] - x[118], x[111] - x[113] + x[117] - x[118],

```

$-x[111] + x[116] + x[119] - x[120], -x[110] + x[116] + x[119] - x[120],$
 $-x[112] - x[116] + 2x[120], x[1] + x[25] - 2x[49], x[2] + x[26] - 2x[50],$
 $x[3] - x[25] + x[49] - x[51], x[4] - x[26] + x[50] - x[52], x[13] - x[25] + x[49] - x[51],$
 $x[14] - x[26] + x[50] - x[52], -x[13] + x[39] + x[61] - x[63], -x[14] + x[40] + x[62] - x[64],$
 $-x[3] + x[39] + x[61] - x[63], -x[4] + x[40] + x[62] - x[64], -x[15] - x[39] + 2x[63],$
 $-x[16] - x[40] + 2x[64], x[5] + x[53] - x[73] - x[77], x[6] + x[54] - x[74] - x[78],$
 $x[7] - x[29] - x[31] + x[55] + x[73] - x[79], x[8] - x[30] - x[32] + x[56] + x[74] - x[80],$
 $x[17] - x[53] - x[75] + x[77], x[18] - x[54] - x[76] + x[78],$
 $-x[17] + x[41] + x[43] - x[65] - x[87] + x[89], -x[18] + x[42] + x[44] - x[66] - x[88] + x[90],$
 $-x[7] + x[67] + x[85] - x[91], -x[8] + x[68] + x[86] - x[92], -x[19] - x[67] + x[87] + x[91],$
 $-x[20] - x[68] + x[88] + x[92], x[33] + x[53] - x[77] - x[81],$
 $x[35] + x[54] - x[78] - x[83], x[34] - x[53] + x[77] - x[82], x[36] - x[54] + x[78] - x[84],$
 $x[45] - x[57] + x[65] - x[69] - x[79] + x[81], x[47] - x[59] + x[66] - x[71] - x[80] + x[83],$
 $-x[45] + x[67] - x[91] + x[93], -x[47] + x[68] - x[92] + x[95],$
 $-x[34] - x[55] + x[58] + x[70] + x[89] - x[94], -x[36] - x[56] + x[60] + x[72] + x[90] - x[96],$
 $-x[46] - x[67] + x[91] + x[94], -x[48] - x[68] + x[92] + x[96], x[9] + x[81] - x[97] - x[105],$
 $x[10] + x[82] - x[98] - x[106], x[11] - x[33] - x[35] + x[83] + x[97] - x[107],$
 $x[12] - x[34] - x[36] + x[84] + x[98] - x[108], x[21] - x[81] - x[99] + x[105],$
 $x[22] - x[82] - x[100] + x[106], -x[21] + x[45] + x[47] - x[93] - x[111] + x[117],$
 $-x[22] + x[46] + x[48] - x[94] - x[112] + x[118], -x[11] + x[95] + x[109] - x[119],$
 $-x[12] + x[96] + x[110] - x[120], -x[23] - x[95] + x[111] + x[119],$
 $-x[24] - x[96] + x[112] + x[120], x[29] - x[53] + x[57] + x[77] - 2x[101],$
 $x[31] - x[55] + x[58] + x[79] - x[102] - x[103], x[30] - x[54] - x[57] + x[78] + x[101] - x[102],$
 $x[32] - x[56] - x[58] + x[80] + x[102] - x[104], x[41] - x[65] + x[69] - x[77] + x[101] - x[103],$
 $x[43] - x[67] + x[70] - x[79] + x[103] - x[104], -x[41] + x[65] + x[71] - x[89] + x[113] - x[115],$
 $-x[43] + x[67] + x[72] - x[91] + x[115] - x[116], -x[30] + x[54] - x[59] +$
 $x[90] + x[113] - x[114], -x[32] + x[56] - x[60] + x[92] + x[114] - x[116],$
 $-x[42] + x[66] - x[71] - x[90] + x[114] + x[115], -x[44] + x[68] - x[72] - x[92] + 2x[116],$
 $x[49] + x[81] - x[97] - x[105], x[51] + x[82] - x[99] - x[106], x[50] - x[81] - x[98] + x[105],$
 $x[52] - x[82] - x[100] + x[106], x[61] - x[73] - x[85] + x[93] + x[97] - x[107],$
 $x[63] - x[75] - x[87] + x[94] + x[99] - x[108], -x[61] + x[95] + x[109] - x[119],$
 $-x[63] + x[96] + x[111] - x[120], -x[50] + x[74] - x[83] + x[86] - x[110] + x[117],$
 $-x[52] + x[76] - x[84] + x[88] - x[112] + x[118], -x[62] - x[95] + x[110] + x[119],$
 $-x[64] - x[96] + x[112] + x[120], x[5] - x[9], -x[3] + x[7], x[8] - x[12], x[17] - x[21],$
 $-x[15] + x[19], x[20] - x[24], x[53] - x[57], -x[27] + x[31], x[32] - x[36], x[65] - x[69],$
 $-x[39] + x[43], x[44] - x[48], x[77] - x[81], -x[75] + x[79], x[56] - x[60],$
 $x[89] - x[93], -x[87] + x[91], x[68] - x[72], x[101] - x[105], -x[99] + x[103],$
 $x[104] - x[108], x[113] - x[117], -x[111] + x[115], x[116] - x[120], x[25] - x[49],$
 $x[26] - x[50], -x[13] + x[37], -x[14] + x[38], x[39] - x[63], x[40] - x[64],$
 $x[29] - x[73], x[30] - x[74], -x[17] + x[41], -x[18] + x[42], x[67] - x[91],$
 $x[68] - x[92], x[53] - x[77], x[54] - x[78], -x[45] + x[69], -x[47] + x[71],$
 $x[70] - x[94], x[72] - x[96], x[33] - x[97], x[34] - x[98], -x[21] + x[45],$
 $-x[22] + x[46], x[95] - x[119], x[96] - x[120], x[57] - x[101], x[58] - x[102],$
 $-x[41] + x[65], -x[43] + x[67], x[90] - x[114], x[92] - x[116], x[81] - x[105],$
 $x[82] - x[106], -x[61] + x[85], -x[63] + x[87], x[86] - x[110], x[88] - x[112]$

Length [eqns]

180