

# Mathcamp Day 2

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Pensieve Header: Mathcamp Day 2 - Gaussian Elimination.

```
eqns = {x[1]+x[5]-2x[9], x[2]-x[5]+x[9]-x[10], x[3]-x[5]+x[9]-x[10], -x[3]+x[8]+x[11]-x[12], -x[2]+x[8]+x[11]-x[12],
-x[4]-x[8]+2x[12], x[13]+x[17]-2x[21], x[14]-x[17]+x[21]-x[22], x[15]-x[17]+x[21]-x[22], -x[15]+x[20]+x[23]-x[24],
-x[14]+x[20]+x[23]-x[24], -x[16]-x[20]+2x[24], x[25]-x[33]+x[53]-x[57], x[26]-x[34]-x[53]+x[57],
x[27]-x[29]-x[31]+x[33]+x[55]-x[58], -x[27]+x[35]+x[56]-x[60], -x[26]+x[30]+x[32]-x[36]-x[54]+x[59],
-x[28]+x[36]-x[56]+x[60], x[37]-x[45]+x[65]-x[69], x[38]-x[46]-x[65]+x[69], x[39]-x[41]-x[43]+x[45]+x[67]-x[70],
-x[39]+x[47]+x[68]-x[72], -x[38]+x[42]+x[44]-x[48]-x[66]+x[71], -x[40]+x[48]-x[68]+x[72], x[53]-x[57]+x[73]-x[81],
x[54]-x[58]+x[74]-x[77]-x[78]+x[81], -x[53]+x[57]+x[75]-x[82], -x[55]+x[59]-x[75]+x[79]+x[80]-x[84],
x[56]-x[60]-x[74]+x[83], -x[56]+x[60]-x[76]+x[84], x[65]-x[69]+x[85]-x[93], x[66]-x[70]+x[86]-x[89]-x[90]+x[93],
-x[65]+x[69]+x[87]-x[94], -x[67]+x[71]-x[87]+x[91]+x[92]-x[96], x[68]-x[72]-x[86]+x[95], -x[68]+x[72]-x[88]+x[96],
x[97]+x[101]-2x[105], x[98]-x[101]+x[105]-x[106], x[99]-x[101]+x[105]-x[106], -x[99]+x[104]+x[107]-x[108],
-x[98]+x[104]+x[107]-x[108], -x[100]-x[104]+2x[108], x[109]+x[113]-2x[117], x[110]-x[113]+x[117]-x[118],
x[111]-x[113]+x[117]-x[118], -x[111]+x[116]+x[119]-x[120], -x[110]+x[116]+x[119]-x[120], -x[112]-x[116]+2x[120],
x[1]+x[25]-2x[49], x[2]+x[26]-2x[50], x[3]-x[25]+x[49]-x[51], x[4]-x[26]+x[50]-x[52], x[13]-x[25]+x[49]-x[51],
x[14]-x[26]+x[50]-x[52], -x[13]+x[39]+x[61]-x[63], -x[14]+x[40]+x[62]-x[64], -x[3]+x[39]+x[61]-x[63],
-x[4]+x[40]+x[62]-x[64], -x[15]-x[39]+2x[63], -x[16]-x[40]+2x[64], x[5]+x[53]-x[73]-x[77], x[6]+x[54]-x[74]-x[78],
x[7]-x[29]-x[31]+x[55]+x[73]-x[79], x[8]-x[30]-x[32]+x[56]+x[74]-x[80], x[17]-x[53]-x[75]+x[77],
x[18]-x[54]-x[76]+x[78], -x[17]+x[41]+x[43]-x[65]-x[87]+x[89], -x[18]+x[42]+x[44]-x[66]-x[88]+x[90],
-x[7]+x[67]+x[85]-x[91], -x[8]+x[68]+x[86]-x[92], -x[19]-x[67]+x[87]+x[91], -x[20]-x[68]+x[88]+x[92],
x[33]+x[53]-x[77]-x[81], x[35]+x[54]-x[78]-x[83], x[34]-x[53]+x[77]-x[82], x[36]-x[54]+x[78]-x[84],
x[45]-x[57]+x[65]-x[69]-x[79]+x[81], x[47]-x[59]+x[66]-x[71]-x[80]+x[83], -x[45]+x[67]-x[91]+x[93],
-x[47]+x[68]-x[92]+x[95], -x[34]-x[55]+x[58]+x[70]+x[89]-x[94], -x[36]-x[56]+x[60]+x[72]+x[90]-x[96],
-x[46]-x[67]+x[91]+x[94], -x[48]-x[68]+x[92]+x[96], x[9]+x[81]-x[97]-x[105], x[10]+x[82]-x[98]-x[106],
x[11]-x[33]-x[35]+x[83]+x[97]-x[107], x[12]-x[34]-x[36]+x[84]+x[98]-x[108], x[21]-x[81]-x[99]+x[105],
x[22]-x[82]-x[100]+x[106], -x[21]+x[45]+x[47]-x[93]-x[111]+x[117], -x[22]+x[46]+x[48]-x[94]-x[112]+x[118],
-x[11]+x[95]+x[109]-x[119], -x[12]+x[96]+x[110]-x[120], -x[23]-x[95]+x[111]+x[119], -x[24]-x[96]+x[112]+x[120],
x[29]-x[53]+x[57]+x[77]-2x[101], x[31]-x[55]+x[58]+x[79]-x[102]-x[103], x[30]-x[54]-x[57]+x[78]+x[101]-x[102],
x[32]-x[56]-x[58]+x[80]+x[102]-x[104], x[41]-x[65]+x[69]-x[77]+x[101]-x[103],
x[43]-x[67]+x[70]-x[79]+x[103]-x[104], -x[41]+x[65]+x[71]-x[89]+x[113]-x[115],
-x[43]+x[67]+x[72]-x[91]+x[115]-x[116], -x[30]+x[54]-x[59]+x[90]+x[113]-x[114],
-x[32]+x[56]-x[60]+x[92]+x[114]-x[116], -x[42]+x[66]-x[71]-x[90]+x[114]+x[115], -x[44]+x[68]-x[72]-x[92]+2x[116],
x[49]+x[81]-x[97]-x[105], x[51]+x[82]-x[99]-x[106], x[50]-x[81]-x[98]+x[105], x[52]-x[82]-x[100]+x[106],
x[61]-x[73]-x[85]+x[93]+x[97]-x[107], x[63]-x[75]-x[87]+x[94]+x[99]-x[108], -x[61]+x[95]+x[109]-x[119],
-x[63]+x[96]+x[111]-x[120], -x[50]+x[74]-x[83]+x[86]-x[110]+x[117], -x[52]+x[76]-x[84]+x[88]-x[112]+x[118],
-x[62]-x[95]+x[110]+x[119], -x[64]-x[96]+x[112]+x[120], x[5]-x[9], -x[3]+x[7], x[8]-x[12], x[17]-x[21],
-x[15]+x[19], x[20]-x[24], x[53]-x[57], -x[27]+x[31], x[32]-x[36], x[65]-x[69], -x[39]+x[43], x[44]-x[48],
x[77]-x[81], -x[75]+x[79], x[56]-x[60], x[89]-x[93], -x[87]+x[91], x[68]-x[72], x[101]-x[105], -x[99]+x[103],
x[104]-x[108], x[113]-x[117], -x[111]+x[115], x[116]-x[120], x[25]-x[49], x[26]-x[50], -x[13]+x[37], -x[14]+x[38],
x[39]-x[63], x[40]-x[64], x[29]-x[73], x[30]-x[74], -x[17]+x[41], -x[18]+x[42], x[67]-x[91], x[68]-x[92],
x[53]-x[77], x[54]-x[78], -x[45]+x[69], -x[47]+x[71], x[70]-x[94], x[72]-x[96], x[33]-x[97], x[34]-x[98],
-x[21]+x[45], -x[22]+x[46], x[95]-x[119], x[96]-x[120], x[57]-x[101], x[58]-x[102], -x[41]+x[65], -x[43]+x[67],
x[90]-x[114], x[92]-x[116], x[81]-x[105], x[82]-x[106], -x[61]+x[85], -x[63]+x[87], x[86]-x[110], x[88]-x[112];
```

```
{m = Length[eqns], n = Max[Cases[eqns, x[k_] => k, Infinity]]}
```

```
{180, 120}
```

```

Clear[t];
Feed[0] = Null;
Feed[eqn_] := Module[{i, eqn1},
  i = Min[Cases[eqn, x[k_] => k, Infinity]];
  eqn1 = Expand[eqn / Coefficient[eqn, x[i]]];
  If[Head[t[i]] != t,
    (*then*) Feed[t[i] - eqn1],
    (*else*) t[i] = eqn1
  ];
];

```

```
Feed /@ eqns;
```

```
Table[t[i], {i, n}] // ColumnForm
```

```

x[1] + x[5] - 2 x[9]
x[2] - x[5] + x[9] - x[10]
x[3] - x[5] + x[9] - x[10]
x[4] + x[8] - 2 x[12]
x[5] - x[8] - x[9] + x[10] - x[11] + x[12]
x[6] + x[54] - x[74] - x[78]
x[7] - x[29] - x[31] + x[55] + x[73] - x[79]
x[8] - x[9] - x[10] + x[11] - x[12] - x[25] + 2 x[49]
x[9] + x[10] + x[25] + x[26] - 2 x[49] - 2 x[50]
x[10] +  $\frac{x[25]}{2}$  + x[26] - x[49] - 2 x[50] -  $\frac{x[53]}{2}$  +  $\frac{x[73]}{2}$  +  $\frac{x[77]}{2}$ 
x[11] + x[12] - x[50] - x[52]
x[12] -  $\frac{x[26]}{2}$  -  $\frac{x[30]}{2}$  -  $\frac{x[32]}{2}$  +  $\frac{x[50]}{2}$  -  $\frac{x[52]}{2}$  +  $\frac{x[56]}{2}$  +  $\frac{x[74]}{2}$  -  $\frac{x[80]}{2}$ 
x[13] + x[17] - 2 x[21]
x[14] - x[17] + x[21] - x[22]
x[15] - x[17] + x[21] - x[22]
x[16] + x[20] - 2 x[24]
x[17] - x[20] - x[21] + x[22] - x[23] + x[24]
x[18] - x[54] - x[76] + x[78]
x[19] + x[67] - x[87] - x[91]
x[20] - x[21] - x[22] + x[23] - x[24] + x[25] - x[49] + x[51]
x[21] + x[22] - x[25] - x[26] + x[49] + x[50] - x[51] - x[52]
x[22] -  $\frac{x[25]}{2}$  - x[26] +  $\frac{x[49]}{2}$  + x[50] -  $\frac{x[51]}{2}$  - x[52] +  $\frac{x[53]}{2}$  +  $\frac{x[75]}{2}$  -  $\frac{x[77]}{2}$ 
x[23] + x[24] - x[26] + x[40] + x[50] - x[52] - 2 x[64]
x[24] +  $\frac{x[40]}{2}$  - x[64] +  $\frac{x[68]}{2}$  -  $\frac{x[88]}{2}$  -  $\frac{x[92]}{2}$ 
x[25] - x[33] + x[53] - x[57]
x[26] - x[34] - x[53] + x[57]
x[27] - x[29] - x[31] + x[33] + x[55] - x[58]
x[28] - x[36] + x[56] - x[60]
x[29] + x[31] - x[33] - x[35] - x[55] - x[56] + x[58] + x[60]
x[30] + x[32] - x[34] - x[36] - x[53] - x[54] + x[57] + x[59]
x[31] - x[33] - x[35] + x[53] - x[55] - x[56] - x[57] + x[58] + x[60] - x[77] + 2 x[101]
x[32] - x[34] - x[36] - x[53] + 2 x[57] + x[59] - x[78] - x[101] + x[102]
x[33] + x[34] - x[49] - 2 x[50] + x[51]
x[34] + x[39] - 2 x[50] + x[53] - x[57] + x[61] - x[63]
x[35] + x[39] + x[49] - x[51] + x[53] + x[56] - x[57] - x[58] - x[60] + x[61] - x[63] - x[67] - x[73] + x[79] - x[85] + x[91]
x[36] - x[39] + 2 x[50] + x[54] - x[56] - x[59] - x[61] + x[63] - x[68] - x[74] + x[80] - x[86] + x[92]
x[37] - x[45] + x[65] - x[69]
x[38] - x[46] - x[65] + x[69]
x[39] - x[41] - x[43] + x[45] + x[67] - x[70]
x[40] - x[48] + x[68] - x[72]
x[41] + x[43] - x[45] - x[47] - x[67] - x[68] + x[70] + x[72]
x[42] + x[44] - x[46] - x[48] - x[65] - x[66] + x[69] + x[71]
x[43] - x[45] - x[47] + x[65] - x[67] - x[68] - x[69] + x[70] + x[72] + x[77] - x[101] + x[103]
x[44] - x[46] - x[48] - x[65] + x[69] - x[90] + x[114] + x[115]
x[45] + x[47] - x[53] - x[65] + x[67] + x[68] - x[70] - x[72] - x[75] + x[77] - x[87] + x[89]
x[46] + x[48] - x[54] + x[65] - x[69] - x[71] - x[76] + x[78] - x[88] + x[90]
x[47] + x[48] - x[50] - x[52] + x[61] + x[62] - x[63] - x[64]
x[48] - x[49] - x[50] + x[51] - x[52] - 2 x[53] + x[57] + x[62] - x[64] - x[68] + x[72] + x[77] + x[81]
x[49] + 2 x[50] - x[51] - x[81] - x[82]
x[50] + x[52] - x[61] - x[63]
x[51] + 2 x[52] +  $\frac{x[53]}{2}$  -  $\frac{x[57]}{2}$  - 2 x[61] - 2 x[63] +  $\frac{x[73]}{2}$  +  $\frac{3x[81]}{2}$  + x[82] - x[97] - x[105]
x[52] +  $\frac{3x[53]}{2}$  - x[57] -  $\frac{x[61]}{2}$  -  $\frac{3x[63]}{2}$  + x[65] -  $\frac{x[67]}{2}$  -  $\frac{x[69]}{2}$  +  $\frac{x[70]}{2}$  +  $\frac{x[75]}{2}$  - x[77] -  $\frac{x[79]}{2}$  +  $\frac{x[81]}{2}$  +  $\frac{x[82]}{2}$  +  $\frac{x[87]}{2}$  -  $\frac{x[89]}{2}$ 
x[53] - x[57] + x[73] - x[81]
x[54] - x[58] + x[74] - x[77] - x[78] + x[81]
x[55] - x[59] + x[75] - x[79] - x[80] + x[84]
x[56] - x[60] - x[74] + x[83]
x[57] + 2 x[58] + x[67] - 2 x[74] - x[79] - x[81] + x[85] - x[91]
x[58] +  $\frac{x[65]}{2}$  + x[67] -  $\frac{x[69]}{2}$  - x[74] - x[79] +  $\frac{x[85]}{2}$  - x[91] +  $\frac{x[93]}{2}$ 
x[59] + x[60] + x[67] + x[68] + 2 x[73] + 2 x[74] - x[77] - x[78] - x[79] - x[80] - x[81] - x[82] - x[83] - x[84] + x[85] + x[86] - x[91] - x[92]

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$$\begin{aligned}
& x[60] + 2x[65] + x[66] - x[69] + x[70] - x[71] + x[72] + x[73] + 2x[74] + x[75] - 2x[77] - x[78] - 2x[79] - 2x[80] + x[81] - x[82] - x[84] + x[85] + x \\
& x[61] - x[63] + 3x[65] - 2x[69] + x[70] - 3x[73] + x[75] - 2x[77] - 2x[79] + 3x[81] + x[82] + x[87] - x[89] - x[91] + x[93] + x[97] - x[98] + x[105] \\
& x[62] - 2x[63] - x[64] + \frac{x[65]}{2} - x[67] - x[68] + \frac{x[69]}{2} + x[70] + x[71] + x[72] - x[73] + x[75] + x[76] + x[81] - \frac{x[83]}{2} + x[87] + x[88] - x[89] - x[90] - \frac{x[9]}{2} \\
& x[63] - \frac{3x[65]}{2} - \frac{x[67]}{2} - \frac{x[68]}{2} + x[69] - \frac{x[70]}{2} + \frac{3x[73]}{2} - \frac{x[75]}{2} + \frac{3x[77]}{2} + \frac{3x[79]}{2} - 2x[81] - x[82] - \frac{x[85]}{2} - \frac{x[86]}{2} - \frac{x[87]}{2} + \frac{x[89]}{2} + x[91] + \frac{x[92]}{2} - \frac{x[93]}{2} + x[98] - x \\
& x[64] + \frac{7x[65]}{4} + 2x[67] + 2x[68] - \frac{7x[69]}{4} - \frac{x[71]}{2} - \frac{x[72]}{2} - 2x[73] - \frac{x[76]}{2} - \frac{5x[77]}{2} - \frac{5x[79]}{2} + \frac{5x[81]}{2} + 2x[82] + \frac{5x[85]}{4} + x[86] - x[88] + \frac{x[90]}{2} - 2x[91] - \frac{3x[9]}{2} \\
& x[65] - x[69] + x[85] - x[93] \\
& x[66] - x[70] + x[86] - x[89] - x[90] + x[93] \\
& x[67] - x[71] + x[87] - x[91] - x[92] + x[96] \\
& x[68] - x[72] - x[86] + x[95] \\
& x[69] + x[70] - x[71] - x[72] - x[73] + x[75] - x[77] - x[79] + 2x[81] - 2x[85] - 2x[86] + 2x[87] - x[89] - x[91] + 2x[93] + x[95] + x[96] \\
& x[70] - 2x[71] - x[72] - x[73] + x[75] - x[79] + 2x[81] - 2x[85] - 2x[86] + 2x[87] - x[91] + 2x[93] + x[95] + x[96] - x[101] + x[103] - x[113] + x[1] \\
& x[71] + x[72] - x[73] - x[74] - x[75] - x[77] - x[79] + 2x[81] + x[82] + x[83] - x[84] - x[85] - 2x[87] + x[90] + x[92] + 2x[93] + x[94] - 2x[96] \\
& x[72] + x[73] - x[74] - x[75] + x[81] + x[82] + x[83] - x[84] - x[87] + x[90] + x[94] - x[96] - x[97] - x[105] \\
& x[73] + x[75] - x[81] - x[82] \\
& x[74] + x[76] - x[83] - x[84] \\
& x[75] - x[81] - x[82] - x[85] + x[93] + x[95] + x[97] - x[107] + x[109] - x[119] \\
& x[76] + x[81] - 2x[84] - 2x[85] - 2x[86] - x[87] + x[90] + x[92] + 2x[93] + x[94] + x[95] - x[96] - x[105] + x[106] - x[107] + x[108] \\
& x[77] + x[79] - x[81] + x[84] + x[85] + \frac{x[86]}{2} + x[87] + \frac{x[88]}{2} - x[90] - x[92] - 2x[93] - x[94] - \frac{x[95]}{2} + \frac{x[96]}{2} \\
& x[78] + x[80] + x[82] - 2x[84] + 5x[85] + 3x[86] + 3x[87] + x[88] - x[89] - 2x[90] - x[91] - 2x[92] - 5x[93] - 2x[94] - 4x[95] + x[96] - 2 \\
& x[79] + x[84] + x[85] + \frac{x[86]}{2} + x[87] + \frac{x[88]}{2} - x[90] - x[92] - 2x[93] - x[94] - \frac{x[95]}{2} + \frac{x[96]}{2} \\
& t[80] \\
& x[81] + x[82] - x[84] - 2x[85] - \frac{3x[86]}{2} - x[87] - \frac{x[88]}{2} + x[90] + x[92] + 2x[93] + x[94] + \frac{x[95]}{2} - \frac{3x[96]}{2} - \frac{x[97]}{2} - \frac{3x[98]}{2} - \frac{x[105]}{2} + \frac{x[106]}{2} + 2x[108] + x[112] \\
& x[82] - x[83] + x[84] + 2x[85] + x[86] + x[87] - x[90] - x[92] - 3x[93] - x[94] - 3x[95] - \frac{5x[97]}{2} - \frac{3x[98]}{2} + 2x[101] + x[102] + x[103] + \frac{x[105]}{2} - \frac{x[106]}{2} + \\
& x[83] - x[84] - 3x[85] - 2x[86] - x[87] + 2x[90] + 2x[92] + 4x[93] + 2x[94] + 4x[95] + 2x[97] - 2x[101] - x[102] - x[103] - 2x[107] + 2x[109] - x \\
& x[84] + 2x[85] + \frac{3x[86]}{2} + x[87] + \frac{x[88]}{2} - x[90] - x[92] - 2x[93] - x[94] - \frac{3x[95]}{2} + \frac{x[96]}{2} - \frac{x[97]}{2} + \frac{x[98]}{2} + \frac{x[105]}{2} - \frac{x[106]}{2} + x[107] - x[108] + x[111] - x[117] \\
& x[85] + x[87] - x[93] - x[94] \\
& x[86] + x[88] - x[95] - x[96] \\
& x[87] + x[88] + \frac{x[90]}{2} + \frac{x[92]}{2} - \frac{x[93]}{2} - \frac{x[94]}{2} - x[95] - x[96] - \frac{x[97]}{2} - \frac{x[98]}{2} + \frac{x[105]}{2} + \frac{x[106]}{2} + x[107] + x[108] - x[109] - \frac{x[111]}{2} + \frac{x[112]}{2} - \frac{x[114]}{2} - \frac{x[115]}{2} - x[116] \\
& x[88] + x[90] + x[92] + x[94] - \frac{x[97]}{2} - \frac{x[98]}{2} + \frac{x[105]}{2} - \frac{x[106]}{2} + x[107] - x[109] - x[111] - x[114] - x[115] - 2x[116] + x[117] + x[119] \\
& x[89] + \frac{3x[90]}{2} + x[91] + \frac{x[92]}{2} - \frac{x[93]}{2} + \frac{x[94]}{2} - 3x[95] - x[96] - 2x[97] - 2x[98] - x[101] - x[103] + 3x[105] + x[106] + 5x[107] + 3x[108] - x[109] + 2x \\
& x[90] + x[92] + x[93] + x[94] + x[111] + x[112] - x[114] - x[115] - 2x[116] - x[117] - x[118] \\
& x[91] - x[92] - x[93] - x[94] - 3x[95] - x[96] - 2x[97] - 2x[98] - x[101] - x[103] + 3x[105] + x[106] + 5x[107] + 3x[108] - x[109] + 2x[111] + 2x[1] \\
& x[92] + x[93] + x[94] + 3x[95] + 2x[96] + x[97] + x[98] - x[105] - x[106] - 2x[107] - 2x[108] + x[109] - x[111] - x[112] - x[113] - x[115] - 2x[116] \\
& x[93] + x[94] - x[95] - x[96] + x[111] + x[112] - x[117] - x[118] \\
& x[94] + 2x[95] + 3x[96] - x[106] - x[108] - x[111] - x[112] - 2x[115] - 2x[116] + x[117] + x[118] \\
& x[95] + x[96] + \frac{x[97]}{2} + \frac{x[98]}{2} - \frac{x[105]}{2} - \frac{x[106]}{2} - x[107] - x[108] - x[111] - x[112] + x[117] + x[118] \\
& x[96] + x[98] + x[101] - x[105] - x[107] - x[108] - x[109] - x[111] - x[112] + x[117] + x[118] + x[119] \\
& x[97] + x[101] - 2x[105] \\
& x[98] - x[101] + x[105] - x[106] \\
& x[99] - x[101] + x[105] - x[106] \\
& x[100] + x[104] - 2x[108] \\
& x[101] - x[104] - x[105] + x[106] - x[107] + x[108] \\
& t[102] \\
& x[103] + x[104] - x[107] - x[108] \\
& x[104] - x[106] + x[107] - x[108] \\
& t[105] \\
& x[106] - x[107] \\
& x[107] + x[108] - x[109] - x[111] \\
& x[108] + x[109] + \frac{3x[111]}{2} + x[112] - x[115] - x[116] - x[117] - x[118] - \frac{x[119]}{2} \\
& x[109] + x[113] - 2x[117] \\
& x[110] - x[113] + x[117] - x[118] \\
& x[111] - x[113] + x[117] - x[118] \\
& x[112] + x[116] - 2x[120] \\
& x[113] - x[116] - x[117] + x[118] - x[119] + x[120] \\
& t[114] \\
& x[115] + x[116] - 2x[118] + x[119] - x[120] \\
& x[116] - x[118] + x[119] - x[120] \\
& t[117] \\
& x[118] - x[119] \\
& t[119] \\
& t[120]
\end{aligned}$$