

A Determinant for W_{alex} ?

By Dror Bar-Natan; with some code borrowed from a joint project with Louis Leung, Arrow_Diagrams_and_gl (N).

Pensieve Header: Testing if the determinant formula for the w-Alexander weight system satisfies TC, 6T and Slide.

Program

Diagrams

```

SetAttributes[Diag, Orderless];
Place[{ar}, {i_, j_}] := {Diag[ar[i, j]], Diag[ar[j, i]]};
Place[{ar, objs__}, {i_, rest__}] := Flatten[Table[
  Outer[Join,
    Place[{ar}, {i, {rest}[[k]]}],
    Place[{objs}, Delete[{rest}, k]]
  ],
  {k, Length[{rest]}]
]];
Diagrams[k_. * ar] := Place[Table[ar, {k}], Range[2 k]];

```

Relators

```

Place[{r: (TC | R6T | Slide), objs__}, {i_, rest__}] := Flatten[Table[
  Outer[Join,
    Place[{r}, {i, {rest}[[j]], {rest}[[k]]}],
    Place[{objs}, Delete[{rest}, {j}, {k}]]
  ],
  {k, 2, Length[{rest]}], {j, 1, k - 1}
]];

Place[{R6T}, {i_, j_, k_}] :=
  Permutations[{i, j, k}] /. {i1_, j1_, k1_} => Diag[R6T[i1, j1, k1]];
Diagrams[R6T] := Place[{R6T}, {1, 2, 3}];
Diagrams[R6T + k_. * ar] /; k > 0 := Flatten[
  Place[#, Range[2 k + 3]] & /@ Permutations[Table[ar, {k}] ~Append~ R6T]
];
Diagrams[R6T + k_. * ar] /; k < 0 := {};

Place[{TC}, {i_, j_, k_}] := Diag /@ {TC[i, j, k], TC[j, k, i], TC[k, i, j]};
Diagrams[TC] := Place[{TC}, {1, 2, 3}];
Diagrams[TC + k_. * ar] /; k > 0 := Flatten[
  Place[#, Range[2 k + 3]] & /@ Permutations[Table[ar, {k}] ~Append~ TC]
];
Diagrams[TC + k_. * ar] /; k < 0 := {};

```

```

Place[{Slide}, {i_, j_, k_}] := Diag /@ {Slide[i, j, k], Slide[j, k, i], Slide[k, i, j]};
Diagrams[Slide] := Place[{Slide}, {1, 2, 3}];
Diagrams[Slide + k_. * ar] /; k > 0 := Flatten[
  Place[#, Range[2 k + 3]] & /@ Permutations[Table[ar, {k}] ~Append~ Slide]
];
Diagrams[Slide + k_. * ar] /; k < 0 := {};

```

Relations

```

NormalizeDiag[diag_Diag] := Module[
  {indices = Union@@ (List @@ diag /. ar → List)},
  diag /. Thread[indices → Range[Length[indices]]]
];
R[Diag[lft___, R6T[i_, j_, k_], rgt___]] := (
  +NormalizeDiag[Diag[lft, ar[i, j], ar[i + 0.5, k], rgt]]
  + NormalizeDiag[Diag[lft, ar[i, j], ar[j + 0.5, k], rgt]]
  + NormalizeDiag[Diag[lft, ar[i, k], ar[j, k + 0.5], rgt]]
  - NormalizeDiag[Diag[lft, ar[i, k], ar[i + 0.5, j], rgt]]
  - NormalizeDiag[Diag[lft, ar[i, j + 0.5], ar[j, k], rgt]]
  - NormalizeDiag[Diag[lft, ar[i, k + 0.5], ar[j, k], rgt]]
);
R[Diag[lft___, TC[i_, j_, k_], rgt___]] := (
  +NormalizeDiag[Diag[lft, ar[i, j], ar[i + 0.5, k], rgt]]
  - NormalizeDiag[Diag[lft, ar[i + 0.5, j], ar[i, k], rgt]]
);
R[Diag[lft___, Slide[i_, j_, k_], rgt___]] := (
  +NormalizeDiag[Diag[lft, ar[i, j - 0.5], ar[j, k], rgt]]
  - NormalizeDiag[Diag[lft, ar[i, k + 0.5], ar[j, k], rgt]]
);

```

Dimensions

```

DimAwLong[m_] /; m < 2 := Length[Diagrams[m ar]];
DimAwLong[m_] /; m ≥ 2 := Module[
  {diags, rels, mat, rel, i},
  diags = Diagrams[m ar];
  rels = R /@ Join[Diagrams[R6T + (m - 2) ar], Diagrams[TC + (m - 2) ar]];
  mat = SparseArray[
    Join @@ Table[
      rel = rels[[i]];
      {i, Position[diags, #][[1, 1]]} → Coefficient[rel, #] & /@
      Cases[{rel}, diag_Diag, Infinity],
      {i, Length[rels]}
    ],
    {Length[rels], Length[diags]}
  ];
  Length[diags] - MatrixRank[mat]
];

```


Wimm[(R /@ Diagrams[R6T + 1 ar]) /. Plus → plus]

```
{plus[0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0],
plus[0, 0, 0], plus[0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0],
plus[0, 0, 0, 0], plus[0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0, 0, 0],
plus[0, 0, YT[3], -YT[3]], plus[0, 0, 0, 0, YT[3], -YT[3]], plus[0, 0, -YT[3], YT[3]],
plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0, 0, 0], plus[0, 0, 0, 0],
plus[0, 0, 0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[-YT[3], 0, 0, 0, YT[3], 0],
plus[0, 0, 0, 0], plus[-YT[3], 0, 0, 0, YT[3], 0], plus[YT[3], -YT[3], 0, 0],
plus[-YT[3], YT[3], 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0, 0, 0], plus[0, 0, 0, 0],
plus[0, 0, 0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0], plus[0, 0, 0, 0],
plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0], plus[0, 0, 0],
plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0],
plus[0, 0, 0], plus[0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0],
plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0],
plus[0, 0, 0], plus[0, 0, 0], plus[0, 0, 0, 0], plus[0, -YT[3], 0, YT[3]],
plus[0, 0, 0, 0, 0, 0], plus[0, 0, 0, YT[3], 0, -YT[3]], plus[0, 0, 0, 0],
plus[YT[3], -YT[3], 0, 0], plus[0, 0, 0, 0, 0, 0], plus[0, 0, 0, 0, 0, 0],
plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0],
plus[0, -YT[3], YT[3], 0], plus[0, 0, 0, 0], plus[0, YT[3], -YT[3], 0],
plus[0, 0, 0, 0, 0, 0], plus[YT[3], 0, -YT[3], 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0],
plus[0, 0, 0, 0, 0, 0], plus[0, -YT[3], YT[3], 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0],
plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0, 0, 0], plus[0, 0, 0, 0, 0, 0],
plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0, 0],
plus[0, 0, 0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0],
plus[0, 0, 0, 0, 0, 0], plus[0, 0, 0, 0, 0, 0], plus[0, 0, 0, 0, 0, 0], plus[0, 0, 0, 0, 0, 0],
plus[0, -YT[3], YT[3], 0, 0, 0], plus[0, YT[3], -YT[3], 0, 0, 0], plus[0, 0, 0, 0, 0, 0],
plus[0, 0, 0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0],
plus[0, 0, 0, 0, 0, 0], plus[0, 0, 0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0],
plus[0, 0, 0, 0, 0, 0], plus[0, 0, 0, 0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, 0, 0]}
```

Union[Wimm[(R /@ Diagrams[R6T + 2 ar]) /. Plus → plus]]

```
{plus[0, 0, 0], plus[0, 0, 0, 0], plus[0, 0, -YT[4], YT[4]],
plus[0, 0, YT[4], -YT[4]], plus[0, 0, -3YT[4] - 2YT[2, 2], 3YT[4] + 2YT[2, 2]],
plus[0, 0, 3YT[4] + 2YT[2, 2], -3YT[4] - 2YT[2, 2]], plus[0, -YT[4], 0, YT[4]],
plus[0, -YT[4], YT[4], 0], plus[0, YT[4], 0, -YT[4]],
plus[0, YT[4], -YT[4], 0], plus[0, -3YT[4] - 2YT[2, 2], 3YT[4] + 2YT[2, 2], 0],
plus[0, 3YT[4] + 2YT[2, 2], 0, -3YT[4] - 2YT[2, 2]],
plus[0, 3YT[4] + 2YT[2, 2], -3YT[4] - 2YT[2, 2], 0], plus[-YT[4], YT[4], 0, 0],
plus[YT[4], -YT[4], 0, 0], plus[-3YT[4] - 2YT[2, 2], 3YT[4] + 2YT[2, 2], 0, 0],
plus[3YT[4] + 2YT[2, 2], -3YT[4] - 2YT[2, 2], 0, 0], plus[0, 0, 0, 0, 0],
plus[0, 0, 0, 0, 0, 0], plus[0, 0, 0, 0, -YT[4], YT[4]], plus[0, 0, 0, 0, YT[4], -YT[4]],
plus[0, 0, 0, 0, -YT[4] - YT[2, 2], YT[4] + YT[2, 2]], plus[0, 0, 0, -YT[4], 0, YT[4]],
plus[0, 0, 0, YT[4], 0, -YT[4]], plus[0, 0, 0, YT[4] + YT[2, 2], 0, -YT[4] - YT[2, 2]],
plus[0, 0, YT[4], -YT[4], 0, 0], plus[0, 0, -YT[4] - YT[2, 2], 2YT[4] + YT[2, 2], 0, -YT[4]],
plus[0, 0, -YT[2, 2], YT[2, 2], 0, 0], plus[0, 0, -YT[2, 2], YT[4] + YT[2, 2], 0, -YT[4]],
plus[0, 0, YT[2, 2], -YT[2, 2], YT[4], -YT[4]],
plus[0, 0, YT[2, 2], -YT[2, 2], -YT[2, 2], YT[2, 2]],
plus[0, 0, YT[4] + YT[2, 2], -YT[4] - YT[2, 2], YT[4], -YT[4]],
plus[0, -YT[4], YT[4], 0, 0, 0]}
```

```

plus[0, -YT[4], YT[4], -YT[4], YT[4], 0], plus[0, YT[4], 0, 0, -YT[4], 0],
plus[0, YT[4], 0, -YT[4], 0, 0], plus[0, YT[4], -YT[4], 0, 0, 0],
plus[0, -2 YT[4] - YT[2, 2], YT[4], -YT[4], 2 YT[4] + YT[2, 2], 0],
plus[0, -2 YT[4] - YT[2, 2], YT[4] + YT[2, 2], YT[4], 0, 0],
plus[0, -YT[4] - YT[2, 2], 0, -YT[4], YT[4] + YT[2, 2], YT[4]],
plus[0, -YT[4] - YT[2, 2], 0, YT[4] + YT[2, 2], 0, 0],
plus[0, -YT[4] - YT[2, 2], 0, 3 YT[4] + 2 YT[2, 2], 0, -2 YT[4] - YT[2, 2]],
plus[0, -YT[4] - YT[2, 2], YT[4], -YT[4], YT[4] + YT[2, 2], 0],
plus[0, -YT[4] - YT[2, 2], YT[4] + YT[2, 2], 0, 0, 0],
plus[0, -YT[2, 2], 0, -YT[4], YT[2, 2], YT[4]],
plus[0, YT[2, 2], 0, 0, -YT[4] - YT[2, 2], YT[4]], plus[0, YT[2, 2], -YT[2, 2], 0, 0, 0],
plus[0, YT[4] + YT[2, 2], 0, 0, -3 YT[4] - 2 YT[2, 2], 2 YT[4] + YT[2, 2]],
plus[0, YT[4] + YT[2, 2], 0, 0, -2 YT[4] - YT[2, 2], YT[4]],
plus[0, YT[4] + YT[2, 2], 0, 0, -YT[4] - YT[2, 2], 0],
plus[0, YT[4] + YT[2, 2], -YT[4] - YT[2, 2], 0, 0, 0],
plus[0, 2 YT[4] + YT[2, 2], -YT[4], YT[4], -2 YT[4] - YT[2, 2], 0],
plus[-YT[4], 0, 0, 0, YT[4], 0], plus[-YT[4], 0, 0, 0, 2 YT[4] + YT[2, 2], -YT[4] - YT[2, 2]],
plus[-YT[4], 0, 0, YT[4], 0, 0], plus[-YT[4], 0, YT[4], 0, 0, 0],
plus[-YT[4], -YT[4], YT[4], 0, YT[4], 0], plus[-YT[4], YT[4], 0, 0, 0, 0],
plus[-YT[4], YT[4], -YT[4], -YT[4], YT[4], YT[4]], plus[-YT[4], YT[4],
  -YT[4] - YT[2, 2], -3 YT[4] - 2 YT[2, 2], 3 YT[4] + 2 YT[2, 2], YT[4] + YT[2, 2]],
plus[-YT[4], YT[4], -YT[4] - YT[2, 2], 2 YT[4] + YT[2, 2], -YT[4] - YT[2, 2], YT[2, 2]],
plus[-YT[4], YT[4], -YT[4] - YT[2, 2], 3 YT[4] + 2 YT[2, 2],
  -3 YT[4] - 2 YT[2, 2], YT[4] + YT[2, 2]], plus[YT[4], 0, 0, 0, -YT[4], 0],
plus[YT[4], 0, 0, 0, -2 YT[4] - YT[2, 2], YT[4] + YT[2, 2]], plus[YT[4], 0, -YT[4], 0, 0, 0],
plus[YT[4], 0, YT[4] + YT[2, 2], 0, -2 YT[4] - YT[2, 2], 0],
plus[YT[4], -YT[4], 0, 0, 0, 0], plus[YT[4], -YT[4], 0, -YT[4], YT[4], 0],
plus[YT[4], -YT[4], -YT[4], YT[4], -YT[4], YT[4]],
plus[YT[4], -YT[4], -YT[4], YT[4], YT[4], -YT[4]],
plus[YT[4], -YT[4], YT[4], -YT[4], 0, 0],
plus[YT[4], -YT[4], YT[4], -YT[4], YT[4], -YT[4]],
plus[YT[4], -YT[4], YT[4], -YT[4], -YT[4]],
plus[YT[4], -YT[4], YT[4], -YT[4], YT[4], -YT[4]],
plus[-2 YT[4] - YT[2, 2], YT[4] + YT[2, 2], YT[4], 0, 0, 0],
plus[-2 YT[4] - YT[2, 2], 2 YT[4] + YT[2, 2], 0, YT[4], -YT[4], 0],
plus[-YT[4] - YT[2, 2], YT[4] + YT[2, 2], 0, 0, 0, 0],
plus[-YT[4] - YT[2, 2], YT[4] + YT[2, 2], -YT[2, 2], YT[4] + YT[2, 2], -YT[4], 0],
plus[-YT[4] - YT[2, 2], YT[4] + YT[2, 2], -YT[2, 2], YT[4] + YT[2, 2], -2 YT[4] - YT[2, 2],
  YT[4] + YT[2, 2]], plus[-YT[4] - YT[2, 2], 2 YT[4] + YT[2, 2], -YT[4], 0, 0, 0],
plus[-YT[4] - YT[2, 2], 3 YT[4] + 2 YT[2, 2], -2 YT[4] - YT[2, 2], 0, 0, 0],
plus[-YT[2, 2], YT[2, 2], -YT[2, 2], YT[2, 2], 0, 0],
plus[-YT[2, 2], YT[2, 2], YT[2, 2], -YT[2, 2], 0, 0],
plus[YT[2, 2], YT[2, 2], -YT[2, 2], 0, -YT[2, 2], 0],
plus[YT[2, 2], YT[4] + YT[2, 2], -YT[4] - YT[2, 2], YT[4], -YT[4] - YT[2, 2], 0],
plus[YT[2, 2], YT[4] + YT[2, 2], -YT[4] - YT[2, 2], 2 YT[4] + YT[2, 2], -YT[4] - YT[2, 2],
  -YT[4] - YT[2, 2]], plus[YT[4] + YT[2, 2], 0, 0, 0, -YT[4] - YT[2, 2], 0],
plus[YT[4] + YT[2, 2], 0, -2 YT[4] - YT[2, 2], YT[4], 0, 0],
plus[YT[4] + YT[2, 2], 0, -YT[4] - YT[2, 2], 0, 0, 0],
plus[YT[4] + YT[2, 2], YT[4], -2 YT[4] - YT[2, 2], YT[4] + YT[2, 2], -YT[4], -YT[2, 2]],
plus[YT[4] + YT[2, 2], -3 YT[4] - 2 YT[2, 2], 0, 2 YT[4] + YT[2, 2], 0, 0],
plus[YT[4] + YT[2, 2], -3 YT[4] - 2 YT[2, 2], YT[4], 3 YT[4] + 2 YT[2, 2], -YT[4],
  -YT[4] - YT[2, 2]], plus[YT[4] + YT[2, 2], -2 YT[4] - YT[2, 2], 0, YT[4], 0, 0],

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plus [YT [4] + YT [2, 2], -YT [4] - YT [2, 2], 0, 0, 0, 0],
plus [YT [4] + YT [2, 2], -YT [4] - YT [2, 2], 0, 0, YT [4], -YT [4]],
plus [YT [4] + YT [2, 2], -YT [4] - YT [2, 2], 0, -YT [4], YT [4], 0],
plus [2 YT [4] + YT [2, 2], 0, YT [4] + YT [2, 2], 0, -3 YT [4] - 2 YT [2, 2], 0],
plus [2 YT [4] + YT [2, 2], -YT [4], 0, 0, -YT [4] - YT [2, 2], 0],
plus [2 YT [4] + YT [2, 2], -2 YT [4] - YT [2, 2], 0, -YT [4], YT [4], 0],
plus [2 YT [4] + YT [2, 2], -YT [4] - YT [2, 2], 0, 0, -YT [4], 0],
plus [3 YT [4] + 2 YT [2, 2], -2 YT [4] - YT [2, 2], 0, 0, -YT [4] - YT [2, 2], 0],
plus [3 YT [4] + 2 YT [2, 2], -YT [4] - YT [2, 2],
  YT [4] + YT [2, 2], YT [4], -3 YT [4] - 2 YT [2, 2], -YT [4]] }

```