

Post Paris notes on KV handout

June-18-09  
8:10 AM

sh.. CA's are better

S1  
S2  
S3  
S4  
S5  
S6

The handout contains the following sections and content:

- Convolutions statement:** Convolutions of invariant functions on a Lie group agree with convolutions of invariant functions on its Lie algebra.
- Group-Algebra statement:** There exists  $\omega \in \text{Fun}(g)^{\otimes 2}$  so that for every  $\phi, \psi \in \text{Fun}(g)^{\otimes 2}$  (with small support), the following holds in  $\mathcal{U}(g)$ :
- Unitary statement:** There exists  $\omega \in \text{Fun}(g)$  and an (infinite order) tangential differential operator  $V$  defined on  $\text{Fun}(g, \mathfrak{g})$  so that:
- Algebraic statement:** With  $\mathfrak{g} = \mathfrak{g}^* \oplus \mathfrak{g}$ , with  $\circ: \mathcal{U}(\mathfrak{g}) \rightarrow \mathcal{U}(\mathfrak{g})/\mathcal{U}(g) = \text{Sig}^*$  the obvious projection, with  $S$  the antipode of  $\mathcal{U}(g)$ , with  $W$  the automorphism of  $\mathcal{U}(g)$  induced by flipping the sign of  $\mathfrak{g}^*$ , with  $\tau \in \mathfrak{g}^* \otimes \mathfrak{g}$  the identity element and with  $R = e^{\tau} \in \mathcal{U}(g) \otimes \mathcal{U}(g)$  there exist  $\omega \in \mathcal{S}(g^*)$  and  $V \in \mathcal{U}(g)^{\otimes 2}$  so that:
- Diagrammatic statement:** Let  $R = \exp(\tau) \in \mathcal{A}^*(\mathbb{1})$ . There exist  $\omega \in \mathcal{A}^*(\mathbb{1})$  and  $V \in \mathcal{A}^*(\mathbb{1})$  so that:
- Unitary  $\Leftrightarrow$  Group-Algebra:**  $\int \omega(x_1) \omega(x_2) e^{x_1 x_2} = \int \omega(x) \omega(y) e^{xy}$
- Algebraic  $\Leftrightarrow$  Unitary:** The key is to interpret  $\mathcal{U}(g)$  as tangential differential operators on  $\text{Fun}(g)$ .
- Diagrammatic to Algebraic:** With  $\{x_i\}$  and  $\{y_i\}$  dual bases of  $\mathfrak{g}$  and  $\mathfrak{g}^*$  and with  $\{z_i, x_j\} = \sum b_{ij}^k z_k$ , we have  $\mathcal{A}^* \rightarrow \mathcal{U}$  via

Possible alternative order:

0. Do the two maps.
1. state S1 "side 1"
2. Go through "homomorphic expansions" to "top, comb, high, low" "side 2"
3. S1, S2, S3 in detail.
4. S4, S5, S6 briefly
5. The full definition of WTT in detail.
6. Back to S5, S4, S3 in detail.

"The relationship with the work of Alek-Mein & The work of bricker"