

consider expanding

<p>Convolutions on Lie Groups and Lie Algebras and Ribbon 2-Knots</p> <p>Deor Bar-Natan, Paris June 2009, http://www.math.utoronto.edu/~deorbar/Talks/Paris-0906</p> <p>Combinatorial statement (Kashiwara-Vergne). Convolutions of invariant functions on a Lie group agree with convolutions of invariant functions on its Lie algebra. More accurately, let G be a finite dimensional Lie group and let \mathfrak{g} be its Lie algebra, let $j, \gamma : \mathfrak{g} \rightarrow \mathbb{R}$ be the Jacobian of the exponential map $\exp : \mathfrak{g} \rightarrow G$, and let $\Phi : \text{Fun}(G) \rightarrow \text{Fun}(\mathfrak{g})$ be given by $\Phi(f)(x) := \int_{\mathbb{R}} f(\exp(x)) \gamma(x) dx$. Then if $f, g \in \text{Fun}(G)$ are \mathbb{R}-invariant and supported near the identity, then $\Phi(f) * \Phi(g) = \Phi(f * g)$.</p> <p>Group-Algebra statement. There exists $\omega \in \text{Fun}(\mathfrak{g})^{\otimes 2}$ so that for every $\phi, \psi \in \text{Fun}(\mathfrak{g})^{\otimes 2}$ (with small support), the following holds in $\mathcal{U}(\mathfrak{g})$:</p> $\int \int \phi(x)\psi(y) \omega(x, y) e^{ix} e^{iy} = \int \int \phi(x)\psi(y) \omega(x, y) e^{ix} e^{iy}$ <p>(ohh, this is subtle)</p> <p>Unitary statement. There exists $\omega \in \text{Fun}(\mathfrak{g})^{\otimes 2}$ and an (infinite order) tangential differential operator V defined on $\text{Fun}(\mathfrak{g}, x, y)$ so that</p> <ol style="list-style-type: none"> $V e^{ix} e^{iy} = e^{ix} e^{iy} V$ (allowing $\mathcal{U}(\mathfrak{g})$-valued functions) $V^2 = 1$ $\omega(x, y) = \omega(y, x)$ <p>Algebraic statement. With $\mathfrak{g} := \mathfrak{g}^* * \mathfrak{g}$, with $\omega \in \mathcal{U}(\mathfrak{g}) \rightarrow \mathcal{U}(\mathfrak{g})/\mathcal{U}(\mathfrak{g}) = \mathcal{S}(\mathfrak{g}^*)$ the obvious projection, with S the antipode of $\mathcal{U}(\mathfrak{g})$, with Π the automorphism of $\mathcal{U}(\mathfrak{g})$ induced by flipping the sign of \mathfrak{g}^*, with $r \in \mathfrak{g}^* \otimes \mathfrak{g}$ the identity element and with $R = e^r \in \mathcal{U}(\mathfrak{g}) \otimes \mathcal{U}(\mathfrak{g})$ there exist $\omega \in \mathcal{S}(\mathfrak{g}^*)$ and $V \in \mathcal{U}(\mathfrak{g})^{\otimes 2}$ so that</p> <ol style="list-style-type: none"> $V(\Delta \otimes 1)(R) = R^{\otimes 2} R^{\otimes 2} V$ in $\mathcal{U}(\mathfrak{g})^{\otimes 2} \otimes \mathcal{U}(\mathfrak{g})$ $V^2 = S^{\otimes 2} V = 1$ $\omega(x, y) = \omega(\Pi(x), \Pi(y)) = \omega(y, x)$ <p>Diagrammatic statement. Let $R = \exp(r) \in \mathcal{A}^{\otimes 2}(\mathbb{1})$. There exist $\omega \in \mathcal{A}^{\otimes 2}(\mathbb{1})$ and $V \in \mathcal{A}^{\otimes 2}(\mathbb{1})$ so that</p> <ol style="list-style-type: none"> <p>Knot-Theoretic statement. There exists a homomorphic expansion Z for trivalent w-tangles. In particular, Z should respect R and intertwine annulus and disk maps:</p> <ol style="list-style-type: none"> 	<p>Free Lie statement (Kashiwara-Vergne). There exist convergent Lie series F and G so that with $z = \log e^x e^y$</p> $z + y - \log e^x e^y = (1 - e^{-ad_x})F + (e^{ad_y} - 1)G$ $\text{tr}(ad_x)^2 F + \text{tr}(ad_y)^2 G = \frac{1}{2} \left(\frac{ad_x^2 + ad_y^2}{ad_x^2 + ad_y^2 + [ad_x, ad_y]} - 1 \right)$ <p>Akshay-Josephson statement. There is an element $F \in \text{TAut}_2$ with $F(x + y) = \log e^x e^y$</p> <p>and $j(F) \in \text{Im} \mathfrak{d} \subset \mathfrak{t}_2$, where for $u \in \mathfrak{t}_2$:</p> $\frac{1}{2} \left(\frac{ad_x^2 + ad_y^2}{ad_x^2 + ad_y^2 + [ad_x, ad_y]} - 1 \right)$ <p>Convolutions and Group Algebras (ignoring all Jacobians). If G is finite, A is an algebra, $\tau : G \rightarrow A$ is multiplicative then $\text{Fun}(G, *) \cong (A, *)$ via $L : f \mapsto \sum f(a)\tau(a)$. For Lie (G, \mathfrak{g}),</p> $(\mathfrak{g}, +) \otimes \mathfrak{g} \xrightarrow{\text{conv}} \mathfrak{g}^* \in \mathcal{S}(\mathfrak{g}) \quad \text{so} \quad \text{Fun}(G) \xrightarrow{\text{conv}} \mathcal{S}(\mathfrak{g})$ <p>$(G, *) \otimes e^r \xrightarrow{\text{conv}} e^r \in \mathcal{U}(\mathfrak{g}) \quad \text{so} \quad \text{Fun}(G) \xrightarrow{\text{conv}} \mathcal{U}(\mathfrak{g})$</p> <p>with $L\psi = \int \psi(x)e^x dx \in \mathcal{S}(\mathfrak{g})$ and $L\phi = \int \phi(x)e^x dx \in \mathcal{U}(\mathfrak{g})$. Given $\psi \in \text{Fun}(G)$ compare $\Phi^{-1}(\psi) = \Phi^{-1}(\psi)$ and $\Phi^{-1}(\psi) = \psi$ in $\mathcal{U}(\mathfrak{g})$.</p> <p>Unitary \Rightarrow Group-Algebra. $\int \int \psi_1(x)\psi_2(y) e^{ix} e^{iy} = \int \int \psi_1(x)\psi_2(y) e^{ix} e^{iy}$</p> <p>$= \int \int \psi_1(x)\psi_2(y) e^{ix} e^{iy} - \int \int \psi_1(x)\psi_2(y) e^{ix} e^{iy} = \int \int \psi_1(x)\psi_2(y) e^{ix} e^{iy}$</p>	<p>Convolutions on Lie Groups and Lie Algebras and Ribbon 2-Knots, Page 2</p> <p>June 7, 2009 (DRAFT)</p> <p>Jacobi diagrams and $\mathcal{A}^{\otimes 2}(\mathbb{1}) \cong \mathcal{A}^{\otimes 2}(\mathbb{1})$ is $\mathcal{A}^{\otimes 2}(\mathbb{1})$ and with $\{x, x\} = \sum \partial_i^2 x_i$, we have $\mathcal{A}^{\otimes 2} \rightarrow \mathcal{U}$ via \mathfrak{g} and \mathfrak{g}^*</p> <p>Diagrammatic to Algebraic. With $\{x, x\}$ and $\{y, y\}$ dual bases of \mathfrak{g} and \mathfrak{g}^* and with $\{x, x\} = \sum \partial_i^2 x_i$, we have $\mathcal{A}^{\otimes 2} \rightarrow \mathcal{U}$ via \mathfrak{g} and \mathfrak{g}^*</p> <p>What are w-trivalent tangles?</p> <p>$\{ \text{knots} \} = \text{PA} \langle \text{R123}, \text{R123}, \text{M}, \text{OC} \rangle$</p> <p>$\{ \text{tangles} \} = \text{PA} \langle \text{R123}, \text{R14}, \text{M}, \text{OC} \rangle$</p> <p>$\{ \text{trivalent w-tangles} \} = \text{PA} \langle \text{R123}, \text{R14}, \text{M}, \text{OC} \rangle$</p> <p>From WT to $\mathcal{A}^{\otimes 2}$. $\text{gr}_w \text{WT} := \{m\text{-cubes}\} / \{(m+1)\text{-cubes}\}$</p> <p>Forget topology</p> <p>Homomorphic expansion for a filtered algebraic structure K.</p> <p>$\text{open } K := K_0 \supset K_1 \supset K_2 \supset K_3 \supset \dots$</p> <p>$\text{gr } K := K_0/K_1 \oplus K_1/K_2 \oplus K_2/K_3 \oplus \dots$</p> <p>An expansion is a filtration respecting $Z : K \rightarrow \text{gr } K$ that "covers" the identity on $\text{gr } K$. A homomorphic expansion is an expansion that respects all relevant "extra" operations.</p> <p>Filtered algebraic structures are clump and plant. In any K, allow formal linear combinations, let K_1 be the ideal generated by differences (the "augmentation ideal"), and let $K_0 := (K_1)^{\otimes 0}$ (using all available "products").</p> <p>Our covers:</p> <ul style="list-style-type: none"> $K = \mathcal{A}^{\otimes 2}$ high algebra $\rightarrow \mathcal{U}(\mathfrak{g})$ gives a "low" algebra \mathfrak{g} adding finitely many \mathfrak{g} \rightarrow low algebra \mathfrak{g} (two \mathfrak{g}'s represent $\mathfrak{g}^* \otimes \mathfrak{g}$) <p>$K$ is knot theory or topology; $\text{gr } K$ is finite combinatorics: bounded-complexity diagrams modulo simple relations.</p> <p>But we have (at least) three knot theories, $\mathfrak{u} \rightarrow \mathfrak{u} \rightarrow \mathfrak{u}$, and thus their "high algebras" are related!</p> <p>We skipped:</p> <ul style="list-style-type: none"> The Alexander \bullet-Knots, quantum groups and polynomial and Milnor numbers, Group-Knots. \bullet-Knots and Drinfeld assoc \bullet-BF theory and the successful \bullet-topology of path integrals. 	<p>Diagrammatic to Algebraic. With $\{x, x\}$ and $\{y, y\}$ dual bases of \mathfrak{g} and \mathfrak{g}^* and with $\{x, x\} = \sum \partial_i^2 x_i$, we have $\mathcal{A}^{\otimes 2} \rightarrow \mathcal{U}$ via \mathfrak{g} and \mathfrak{g}^*</p> <p>What are w-trivalent tangles?</p> <p>$\{ \text{knots} \} = \text{PA} \langle \text{R123}, \text{R123}, \text{M}, \text{OC} \rangle$</p> <p>$\{ \text{tangles} \} = \text{PA} \langle \text{R123}, \text{R14}, \text{M}, \text{OC} \rangle$</p> <p>$\{ \text{trivalent w-tangles} \} = \text{PA} \langle \text{R123}, \text{R14}, \text{M}, \text{OC} \rangle$</p> <p>From WT to $\mathcal{A}^{\otimes 2}$. $\text{gr}_w \text{WT} := \{m\text{-cubes}\} / \{(m+1)\text{-cubes}\}$</p> <p>Forget topology</p> <p>Homomorphic expansion for a filtered algebraic structure K.</p> <p>$\text{open } K := K_0 \supset K_1 \supset K_2 \supset K_3 \supset \dots$</p> <p>$\text{gr } K := K_0/K_1 \oplus K_1/K_2 \oplus K_2/K_3 \oplus \dots$</p> <p>An expansion is a filtration respecting $Z : K \rightarrow \text{gr } K$ that "covers" the identity on $\text{gr } K$. A homomorphic expansion is an expansion that respects all relevant "extra" operations.</p> <p>Filtered algebraic structures are clump and plant. In any K, allow formal linear combinations, let K_1 be the ideal generated by differences (the "augmentation ideal"), and let $K_0 := (K_1)^{\otimes 0}$ (using all available "products").</p> <p>Our covers:</p> <ul style="list-style-type: none"> $K = \mathcal{A}^{\otimes 2}$ high algebra $\rightarrow \mathcal{U}(\mathfrak{g})$ gives a "low" algebra \mathfrak{g} adding finitely many \mathfrak{g} \rightarrow low algebra \mathfrak{g} (two \mathfrak{g}'s represent $\mathfrak{g}^* \otimes \mathfrak{g}$) <p>$K$ is knot theory or topology; $\text{gr } K$ is finite combinatorics: bounded-complexity diagrams modulo simple relations.</p> <p>But we have (at least) three knot theories, $\mathfrak{u} \rightarrow \mathfrak{u} \rightarrow \mathfrak{u}$, and thus their "high algebras" are related!</p> <p>We skipped:</p> <ul style="list-style-type: none"> The Alexander \bullet-Knots, quantum groups and polynomial and Milnor numbers, Group-Knots. \bullet-Knots and Drinfeld assoc \bullet-BF theory and the successful \bullet-topology of path integrals.
<p>Unitary \Rightarrow Group-Algebra. $\int \int \psi_1(x)\psi_2(y) e^{ix} e^{iy} = \int \int \psi_1(x)\psi_2(y) e^{ix} e^{iy}$</p> <p>$= \int \int \psi_1(x)\psi_2(y) e^{ix} e^{iy} - \int \int \psi_1(x)\psi_2(y) e^{ix} e^{iy} = \int \int \psi_1(x)\psi_2(y) e^{ix} e^{iy}$</p> <p>Knot-Theoretic statement. There exists a homomorphic expansion Z for trivalent w-tangles. In particular, Z should respect R and intertwine annulus and disk maps:</p> <ol style="list-style-type: none"> 	<p>Unitary \Rightarrow Group-Algebra. $\int \int \psi_1(x)\psi_2(y) e^{ix} e^{iy} = \int \int \psi_1(x)\psi_2(y) e^{ix} e^{iy}$</p> <p>$= \int \int \psi_1(x)\psi_2(y) e^{ix} e^{iy} - \int \int \psi_1(x)\psi_2(y) e^{ix} e^{iy} = \int \int \psi_1(x)\psi_2(y) e^{ix} e^{iy}$</p> <p>Knot-Theoretic statement. There exists a homomorphic expansion Z for trivalent w-tangles. In particular, Z should respect R and intertwine annulus and disk maps:</p> <ol style="list-style-type: none"> 	<p>Unitary \Rightarrow Group-Algebra. $\int \int \psi_1(x)\psi_2(y) e^{ix} e^{iy} = \int \int \psi_1(x)\psi_2(y) e^{ix} e^{iy}$</p> <p>$= \int \int \psi_1(x)\psi_2(y) e^{ix} e^{iy} - \int \int \psi_1(x)\psi_2(y) e^{ix} e^{iy} = \int \int \psi_1(x)\psi_2(y) e^{ix} e^{iy}$</p> <p>Knot-Theoretic statement. There exists a homomorphic expansion Z for trivalent w-tangles. In particular, Z should respect R and intertwine annulus and disk maps:</p> <ol style="list-style-type: none"> 	<p>Unitary \Rightarrow Group-Algebra. $\int \int \psi_1(x)\psi_2(y) e^{ix} e^{iy} = \int \int \psi_1(x)\psi_2(y) e^{ix} e^{iy}$</p> <p>$= \int \int \psi_1(x)\psi_2(y) e^{ix} e^{iy} - \int \int \psi_1(x)\psi_2(y) e^{ix} e^{iy} = \int \int \psi_1(x)\psi_2(y) e^{ix} e^{iy}$</p> <p>Knot-Theoretic statement. There exists a homomorphic expansion Z for trivalent w-tangles. In particular, Z should respect R and intertwine annulus and disk maps:</p> <ol style="list-style-type: none">

Perhaps insert a quick PBN example.

Consider adding the 3xy philosophy at end or into "map of the field".

insert into vllws