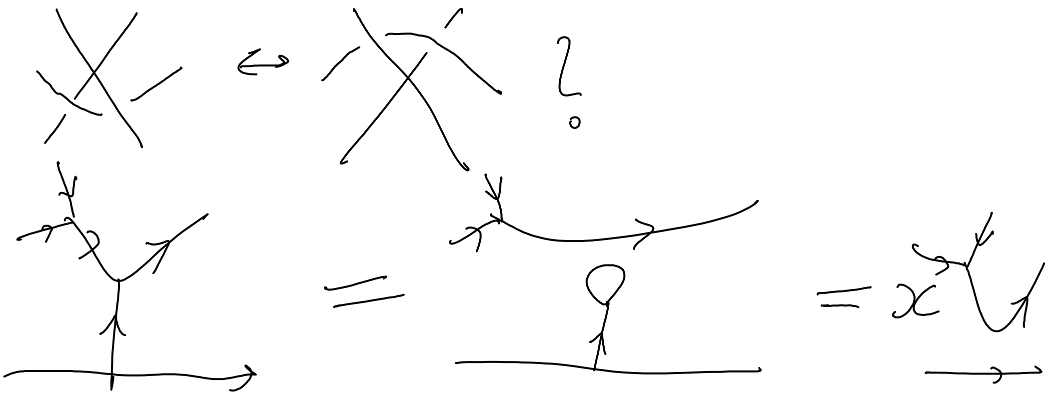
Alexander Notes

June-09-09
8:55 AM



From the MMR paper:

this matrix satisfies $a_{ij}=0 \Leftrightarrow a_{ji}=0$, which is not the case for us.

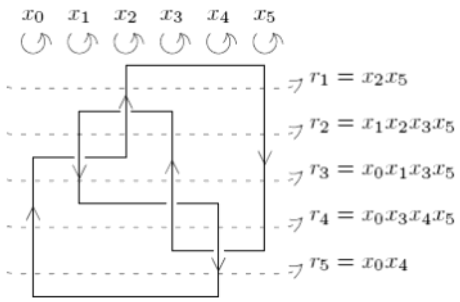
(hopefully this got broken by the removal of the wun rows/cols)

Exercise 3.10. Deduce theorem 3 from the fact (see e.g. [Kau, chapter 7]) that the Alexander polynomial of a knot K is given by $\det(z^{-1}\theta - z\theta^T)$, where θ is Seifert pairing matrix for some Seifert surface for K, and θ^T is its transpose.

Exercise 3.9. (Melvin) Let F be the surface obtained by thickening a chord diagram D (that is, thicken all chords *and* the base line), and let ∂F be its boundary. $W_C(D) = 1$ if $H_0(\partial F) = \mathbf{Z}$, and otherwise, $W_C(D) = 0$. Now consider the following long exact sequence:

$$\begin{array}{ccccccccc} H_1(F) & \xrightarrow{p_\star} & H_1(F, \partial F) & \xrightarrow{\delta} & H_0(\partial F) & \xrightarrow{i_\star} & H_0(F) = \mathbf{Z} & \longrightarrow & 0 \\ & & \big\downarrow \gamma \ \binom{\text{Poincaré}}{\text{duality}} & & & & & & \\ & & H^1(F) & & & & & & \end{array}$$

We are interested in knowing when $H_0(\partial F) = \mathbf{Z}$, which is when $p_\star$ is an epimorphism, which is when $\gamma \circ p_\star$ is an epimorphism. Show that in the basis suggested by the chords of D, $\gamma \circ p_\star$ is given by the matrix $\mathrm{IM}(D)$, and use this to deduce theorem 3. (We wish to thank C. Kassel for reminding us that the determinant of an anti-symmetric matrix is always non-negative).



Likely irrelevant.

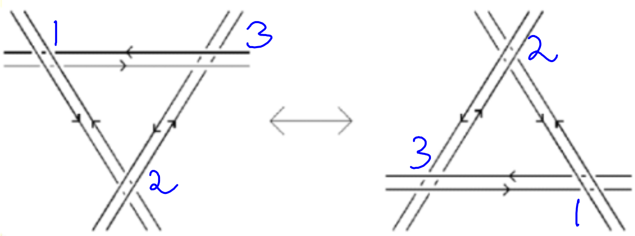
[7] L. P. Neuwirth, $*$ *projections of knots*, in *Algebraic and Differential Topology—Global Differential Geometry*, Teubner, Leipzig, volume 70 of *Teubner-Texte Math.*, pp. 198–205, 1984.

Screen clippings taken: 11/06/2009, 3:10 PM from http://front.math.ucdavis.edu/0610.5559 by Ciprian Manolescu, Peter Ozsvath, Zoltan Szabo, Dylan Thurston

Perhaps the trick will be to derive some presentation from the GPV maximal tree? (Likely not)

Image from http://www.math.toronto.edu/~drorbn/Talks/Sandbjerg-0810/pAHandout.html

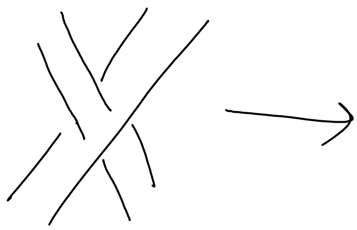
The Naik–Stanford Double Delta Relation

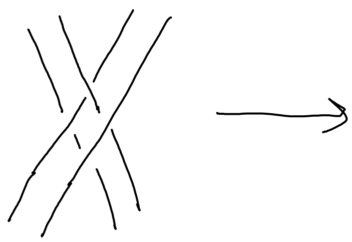


Note – This relation follows trivially from the Seifert formula for Alexander if a Seifert surface "along the ribbons" can be found, for then the Seifert pairing matrix is unchanged by the move.

Another idea: Let K be a knot diagram and K' be its descending version. Construct a spanning disk D for $K \# K'$ by drawing K & K' on two parallel planes and tracing an annulus of connections between them (with some exception near the connect-sum point). De-singularize D to get a Seifert surface, and use the Seifert formula.

Question Can this process be "factored" at D?

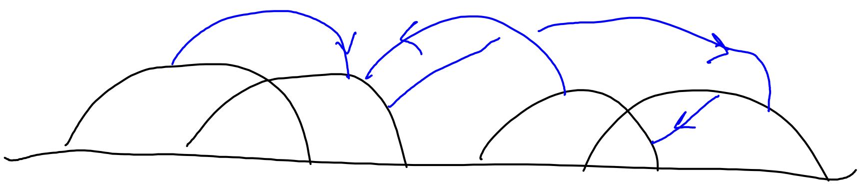


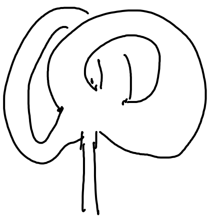


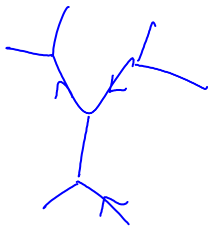
Perhaps the way to go is to get a w-formula for any knot presented as the boundary of a skeletal Seifert formula?

Even better would be to find a direct relationship with the knot group. Will surgery be relevant?

<u>Assertion</u> After ∂_A, heads commute and tails commute with heads, if they are on the same hump:





 $=0$