Ozsvath in Trieste

May-18-09
9:12 AM

Day 1

Knot Floer Homology (Descendant of Gauge theory)

Physics → PDE's → ASD-YME → moduli spaces $\mathcal{M}_{ASDYME}(X,g)$

→ Donaldson theory of (X^4, g)

→ integer or $\mathbb{Z}/2$-valued invariants of smooth 4-manifolds

→ Some homeomorphic 4-manifolds are not diffeomorphic (~ 1981)

94': Seiberg & Witten → SW^4_X

2000': Ozs - Szabó: Heegaard Floer Homology

$Y^3 \longrightarrow HF(Y)$ a Graded Abelian group (really, the homology of a chain complex over $\mathbb{Z}[U]$)

$Y_1 \; W^4 \; Y_2 \longrightarrow F: HF(Y_1) \to HF(Y_2)$

$HF(Y)$ is $H_*(CF(Y))$ where in (CF, d), CF is combinatorial but d is not.

2002': Oz-Sz, Rasmussen: an invariant of knots inside Y^3 $K \hookrightarrow Y^3$:

$HFK(K)$

Our topic is mostly HFK for knots in S^3.

Formal properties of HFK:

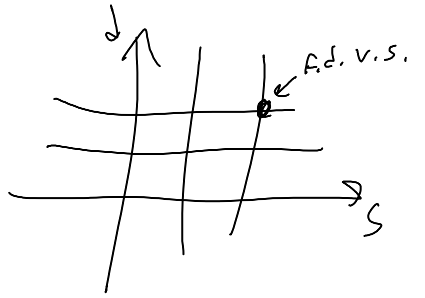
HFK is a bi-graded Abelian group

(for simplicity, $\mathbb{Z}/2$-vector space)

$\widehat{HFK}(K)$: bi-graded f.d. $\mathbb{Z}/2$ v.s.:

$$\widehat{HFK}(K) = \bigoplus_{d,s\in\mathbb{Z}} \widehat{HFK}_d(K,s)$$

Maslov grading   Alexander grading.



Relation to Alexander:

$$\sum (-1)^d \operatorname{rk} \widehat{HFK}_d(K,s)\, T^s \in \mathbb{Z}/2[T,T^{-1}]$$

is $\Delta_K(T)$, the Alexander polynomial.

(There is a filtered version and a version over $\mathbb{Z}/2[U]$)

The "skein relation" for HFK: K_+: , K_-: , K_0:

$$\begin{tikzcd} \widehat{HFK}(K_\pm) \arrow[rr] & & \widehat{HFK}(K_\mp) \arrow[dl] \\ & \widehat{HFK}(K_0) \arrow[ul] & \end{tikzcd}$$

if both strands belong to same component.

speaker forgot signs.

and also:

$$\begin{tikzcd} \widehat{HFK}(K_\pm) \arrow[rr] & & \widehat{HFK}(K_\mp) \arrow[dl] \\ & \widehat{HFK}(K_0)\otimes H_*(T^2) \arrow[ul] & \end{tikzcd}$$

if the strands belong to different components.

$$\widehat{HFK}(K_1 \# K_2) = \widehat{HFK}(K_1)\otimes \widehat{HFK}(K_2)$$

What's HFK good for?

3 notions of complexity of knots:

1. unknotting number $u(K)$.

Q is $u(K_1 \# K_2) = u(K_1) + u(K_2)$

($\leq$ is easy)

2. The Seifert genus $g(K)$

* additive under connect sum.
* $g(K)=0 \iff K=U$.

3. The slice genus $g_4(K)$

$g_4 \leq g$ trivially.

$g_4(K \# \overline{K}) = 0$

$g_4(K) \leq u(K)$

HFK has something to say about all 3!

Thm (2003, Oz-Sz)

$g(K) = \max\{s \mid \widehat{HFK}_*(K,s) \neq 0\}$

$\Rightarrow \operatorname{rk} \widehat{HFK} = 1$ iff K is U.

Thm K is fibered iff at highest s, $\operatorname{rk} \widehat{HFK} = 1$

(Ghiggini $g=1$, Y. Ni $g>1$ ~2006 later also Juhász)