

A parity  $p$  on a "knot theory" (a theory of words in the sense of Turner) is an assignment of a parity in  $\mathbb{Z}_2$  for every knot diagram, so that

1.  $p$  is invariant relative to the "identity of xing" partial connection [so it is local and has a  $\begin{matrix} 3 \\ \diagdown \\ \diagup \\ 1 \end{matrix} \leftrightarrow \begin{matrix} 2 \\ \diagdown \\ \diagup \\ 3 \end{matrix}$  property w.r.  $R_3$ ].
2. The sum of the parities of the xings involved in an  $R_1, R_2$  or  $R_3$  move is even.

Examples 1. The number of chords (mod 2) intersecting a given chord, for  $v$ -knots.

2. For links, the parity of the number of components involved in a given xing.

Question Is there a non-trivial parity for honest 1-component  $u$ -knots?

Thm "odd & irreducible" is minimal in a strong sense; follows from  

$$\text{prop } \sum_{\text{mod } 2} (\text{1-component Kauffman smoothings of even xing}) \Big/ \text{R}_2 \text{ moves}$$

is invariant also under  $R_3$ .

Another corollary Non-trivial free knots exist

$$\text{Free knots} = \text{CA} \langle \text{X} \rangle / R_1 R_2 R_3 \ \& \ \text{X}$$

Question classify free knots.