The Unitary Alternative

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5:54 AM

$$M = VmV^{-1}$$

Suppose $V^* = V^{-1}$ and $V^{-1}MV = m$. Then

$$\int mf = \langle 1, mf \rangle = \langle V1, Vmf \rangle$$
$$= \langle V1, MVf \rangle =$$
$$= \langle 1, Mf \rangle = \int Mf = \int (V1) M(Vf) = \int Mf\,(V1)^2$$

Warmup

Assuming $V1 = 1$, $Vf = f$

What if $V^* \hat{W}(x+y) V = \hat{W}(x)\hat{W}(y)$? Then

$$\int m W(x) W(y) f = \langle 1, m W(x) W(y) f \rangle$$
$$= \langle V1, \hat{W}(x+y) V m f \rangle$$
$$= \langle 1, \hat{W}(x+y) M V f \rangle$$
$$= \langle 1, \hat{W}(x+y) M f \rangle$$
$$= \int W(x+y) M f$$

Equiv. to $V^*V = W^{-1}(\log e^x e^y) \cdot W(x)W(y)$

(but here we can't expect $V1 = 1$)

In general, suppose $V^* \alpha V = \beta$ and $\beta V = V\beta$. Set $U = \sqrt{\frac{\alpha}{\beta}}\, V$ and get

$$U^*U = V^* \sqrt{\frac{\alpha}{\beta}} \sqrt{\frac{\alpha}{\beta}}\, V = V^* \frac{\alpha}{\beta} V = \frac{1}{\beta} V^* \alpha V = I$$

and also

$$U1 = \sqrt{\frac{\alpha}{\beta}} \quad (\text{assuming } V1 = 1)$$

The structural property of A^ω-operators: If f is invariant, then

$$Vf = (V1)f$$