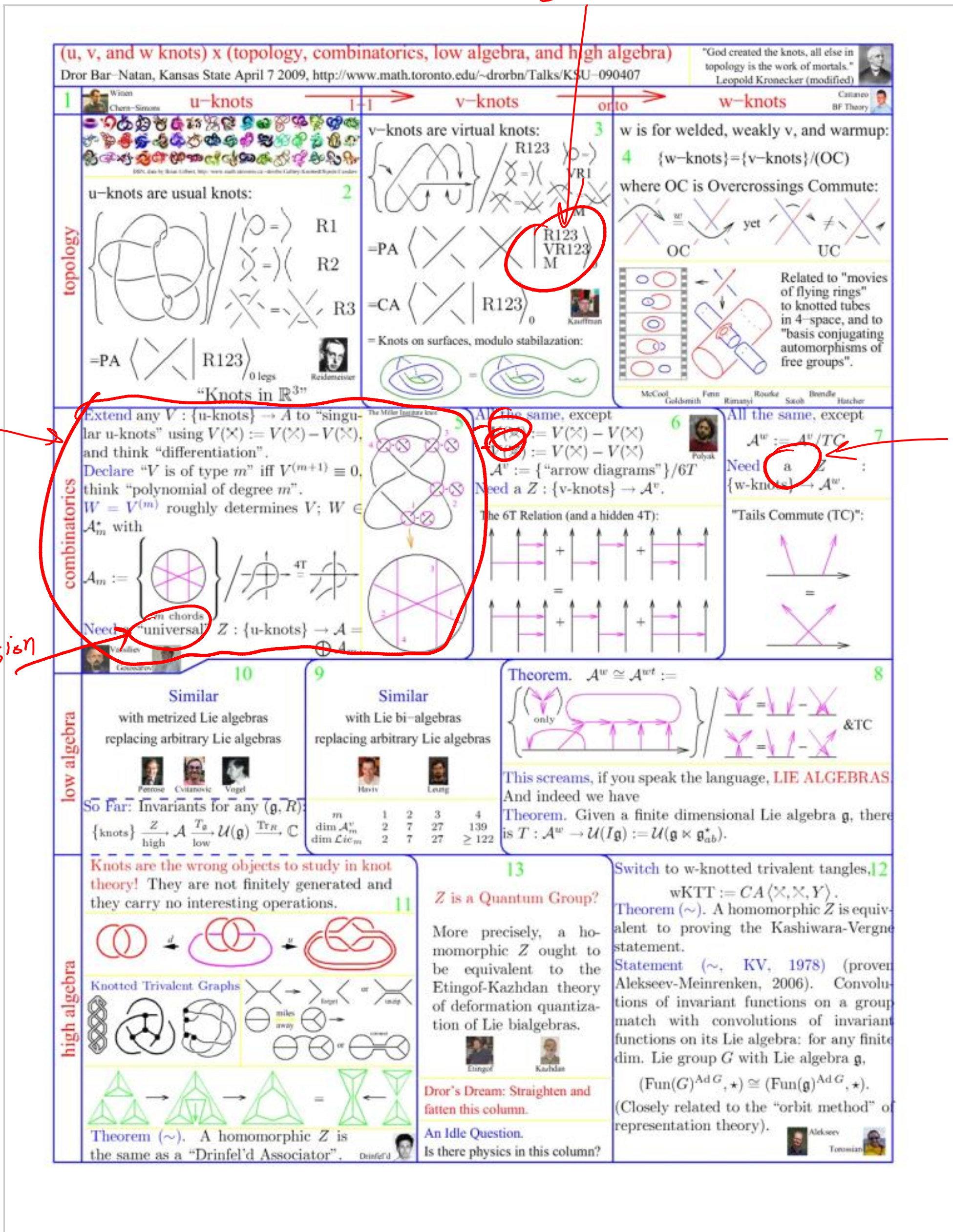
Main Handout

April-03-09
4:49 PM

center



(u, v, and w knots) x (topology, combinatorics, low algebra, and high algebra)

Dror Bar-Natan, Kansas State April 7 2009, http://www.math.toronto.edu/~drorbn/Talks/KSU-090407

"God created the knots, all else in topology is the work of mortals." Leopold Kronecker (modified)

1 u–knots 1-1 v–knots onto w–knots

topology

u–knots are usual knots: 2

R1 R2 R3

=PA ⟨ R123 ⟩ 0 legs

"Knots in $\mathbb{R}^3$"

v–knots are virtual knots: 3

R123 VR1 VR123 M

=PA ⟨ R123 VR123 M ⟩

=CA ⟨ R123 ⟩ 0

= Knots on surfaces, modulo stabilazation:

w is for welded, weakly v, and warmup:

4 {w–knots}={v–knots}/(OC)

where OC is Overcrossings Commute:

OC yet UC

Related to "movies of flying rings" to knotted tubes in 4–space, and to "basis conjugating automorphisms of free groups".

combinatorics

Extend any V : {u-knots} $\to A$ to "singular u-knots" using $V(\times) := V(\times) - V(\times)$, and think "differentiation".

Declare "V is of type m" iff $V^{(m+1)} \equiv 0$, think "polynomial of degree m".

$W = V^{(m)}$ roughly determines V; $W \in \mathcal{A}_m^\star$ with

$\mathcal{A}_m :=$ { m chords } / 4T

Need a "universal" Z : {u-knots} $\to \mathcal{A} = \bigoplus \mathcal{A}_m$.

All the same, except 6

$:= V(\times) - V(\times)$

$:= V(\times) - V(\times)$

$\mathcal{A}^v :=$ {"arrow diagrams"}/6T

Need a Z : {v-knots} $\to \mathcal{A}^v$.

The 6T Relation (and a hidden 4T):

All the same, except 7

$\mathcal{A}^w := \mathcal{A}^v/TC$

Need a Z : {w-knots} $\to \mathcal{A}^w$.

"Tails Commute (TC)":

low algebra

10 Similar with metrized Lie algebras replacing arbitrary Lie algebras

So Far: Invariants for any $(\mathfrak{g}, R)$:

{knots} $\xrightarrow[\text{high}]{Z} \mathcal{A} \xrightarrow[\text{low}]{T_\mathfrak{g}} \mathcal{U}(\mathfrak{g}) \xrightarrow{\mathrm{Tr}_R} \mathbb{C}$

9 Similar with Lie bi–algebras replacing arbitrary Lie algebras

m	1	2	3	4
$\dim \mathcal{A}_m^v$	2	7	27	139
$\dim \mathcal{L}ie_m$	2	7	27	≥ 122

Theorem. $\mathcal{A}^w \cong \mathcal{A}^{wt} :=$ 8

only &TC

This screams, if you speak the language, LIE ALGEBRAS. And indeed we have

Theorem. Given a finite dimensional Lie algebra $\mathfrak{g}$, there is $T : \mathcal{A}^w \to \mathcal{U}(I\mathfrak{g}) := \mathcal{U}(\mathfrak{g} \ltimes \mathfrak{g}^\star_{ab})$.

high algebra

Knots are the wrong objects to study in knot theory! They are not finitely generated and they carry no interesting operations. 11

Knotted Trivalent Graphs

forget, unzip, miles away, connect

Theorem (∼). A homomorphic Z is the same as a "Drinfel'd Associator".

13 Z is a Quantum Group?

More precisely, a homomorphic Z ought to be equivalent to the Etingof-Kazhdan theory of deformation quantization of Lie bialgebras.

Dror's Dream: Straighten and fatten this column.

An Idle Question. Is there physics in this column?

Switch to w-knotted trivalent tangles, 12

wKTT $:= CA\langle \times, \times, Y \rangle$.

Theorem (∼). A homomorphic Z is equivalent to proving the Kashiwara-Vergne statement.

Statement (∼, KV, 1978) (proven Alekseev-Meinrenken, 2006). Convolutions of invariant functions on a group match with convolutions of invariant functions on its Lie algebra: for any finite dim. Lie group G with Lie algebra $\mathfrak{g}$,

$$(\mathrm{Fun}(G)^{\mathrm{Ad}\, G}, \star) \cong (\mathrm{Fun}(\mathfrak{g})^{\mathrm{Ad}\, G}, \star).$$

(Closely related to the "orbit method" of representation theory).

rewrite without duality

expansion

an expansion