Dror Bar-Natan: Talks: KSU-090407:

Homomorphic Expansions

The general setup:

$$\mathcal{K} \supset \mathcal{K}_0 \supset \mathcal{K}_1 \supset \mathcal{K}_2 \supset \mathcal{K}_3 \supset \dots$$

$$\Downarrow \qquad \downarrow Z$$

$$\operatorname{gr}\mathcal{K} := \mathcal{K}/\mathcal{K}_0 \oplus \mathcal{K}_0/\mathcal{K}_1 \oplus \mathcal{K}_1/\mathcal{K}_2 \oplus \mathcal{K}_2/\mathcal{K}_3 \oplus \mathcal{K}_3/\mathcal{K}_4 \oplus \dots$$

An expansion is a filtration-respecting map $\mathcal{K} \to \operatorname{gr}\mathcal{K}$ that "covers" the identity map $\operatorname{gr}\mathcal{K} \to \operatorname{gr}\mathcal{K}$.

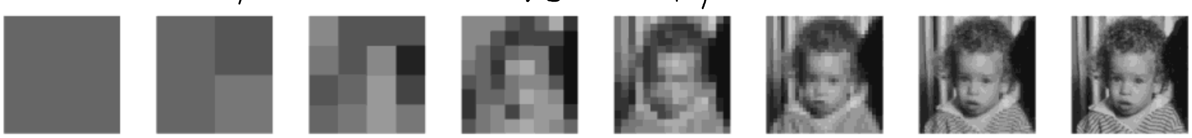
A homomorphic expansion is an expansion that respects all relevant "extra" operations.

A concrete example:

$\mathcal{K} = \{$  $\}$ = The set of all b/w 2D projections of reality

$\mathcal{K}$ ... $\mathcal{K}/\mathcal{K}_0$ $\mathcal{K}/\mathcal{K}_1$ $\mathcal{K}/\mathcal{K}_2$ $\mathcal{K}/\mathcal{K}_3$ $\mathcal{K}/\mathcal{K}_4$ . . .

Crop$_0$ Adjoin$_0$



$\Downarrow$ an "expansion" Z is a choice of a "progressive scan" algorithm.

$$\mathcal{K}/\mathcal{K}_0 \oplus \frac{\mathcal{K}/\mathcal{K}_1}{\mathcal{K}/\mathcal{K}_0} \oplus \frac{\mathcal{K}/\mathcal{K}_2}{\mathcal{K}/\mathcal{K}_1} \oplus \mathcal{K}_2/\mathcal{K}_3 \oplus \mathcal{K}_3/\mathcal{K}_4 \oplus \dots$$

$$\frac{\mathcal{K}/\mathcal{K}_1}{\mathcal{K}/\mathcal{K}_0} = \mathcal{K}_0/\mathcal{K}_1, \qquad \frac{\mathcal{K}/\mathcal{K}_2}{\mathcal{K}/\mathcal{K}_1} = \mathcal{K}_1/\mathcal{K}_2$$

added May 5, 2009: I was a bit confused.... See Talks/Trieste.

Our Case:

$$\mathcal{K} \xrightarrow{Z:\ \text{High Algebra}} \operatorname{gr}\mathcal{K} \xrightarrow[\text{algebra } \mathfrak{g}]{\text{given a "Lie"}} \text{"}\mathcal{U}(\mathfrak{g})\text{"}$$

$\mathcal{K}$ = {Knot Theory, Topology}

Finding Z is solving finitely many eq'ns in finitely many unknowns

$\operatorname{gr}\mathcal{K}$ = {Finite combinatorics: bounded-complexity diagrams mod simple relations}

Low Algebra: Pictures represent formulas