

Diff-Op to Group-Ring

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9:40 PM

Present (unhappy) state:

$$\begin{aligned} \int \omega(x+y) e^{x+y} \phi(x) \psi(y) &= \langle \omega(x+y), \widehat{e^{x+y}} \phi(x) \psi(y) \rangle \\ &= \langle V \omega(x+y), V \widehat{e^{x+y}} \phi(x) \psi(y) \rangle = \langle \omega(x) \omega(y), \widehat{e^x e^y} V \phi(x) \psi(y) \rangle \\ &= \langle \omega(x) \omega(y), \widehat{e^x e^y} \phi(x) \psi(y) \rangle = \int \omega(x) \omega(y) e^x e^y \phi(x) \psi(y) \end{aligned}$$

Relies on: $V \omega(x+y) = \omega(x) \omega(y)$ \leftarrow not happy
 $V 1 = 1$ \leftarrow together

Now assume only $V \omega^{1/2}(x+y) = \omega^{1/2}(x) \omega^{1/2}(y)$

$$\begin{aligned} \int \omega(x+y) e^{x+y} \phi(x) \psi(y) &= \langle \omega^{1/2}(x+y), \widehat{e^{x+y}} \omega^{1/2}(x+y) \phi(x) \psi(y) \rangle \\ &= \langle V \omega^{1/2}(x+y), V \widehat{e^{x+y}} \omega^{1/2}(x+y) \phi(x) \psi(y) \rangle \\ &= \langle \omega^{1/2}(x) \omega^{1/2}(y), \widehat{e^x e^y} V \omega^{1/2}(x+y) \phi(x) \psi(y) \rangle \\ &= \langle \omega^{1/2}(x) \omega^{1/2}(y), \widehat{e^x e^y} \omega^{1/2}(x) \omega^{1/2}(y) \phi(x) \psi(y) \rangle \\ &= \int \omega(x) \omega(y) e^x e^y \phi(x) \psi(y) \quad ! \end{aligned}$$