

Question Is there a basis conjugating automorphism F_2 of $F(a,b)$ s.t. $F_2(a^2b^2) = (ab)^2$?

And in general, F_n s.t.

$$F_n(a^n b^n) = (ab)^n \quad ?$$

The interest comes from Trotter's formula,

$$e^{x+y} = \lim_{n \rightarrow \infty} (e^{x/n} e^{y/n})^n$$

By BCH, $\exists F$ s.t. $F(e^x e^y) = e^{x+y} \underset{\text{large } n}{\sim} (e^{x/n} e^{y/n})^n$

taking $a = e^{x/n}, b = e^{y/n}$ this is

$$= F_n^{-1}(e^x e^y)$$

So it look like $F = \lim_{n \rightarrow \infty} F_n^{-1}$

Thus knowing the F_n 's will give us F .

Are $abab$ and $aabb$ conjugates? No, they are different cyclic words.