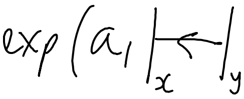
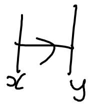
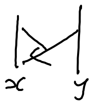
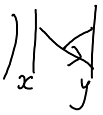
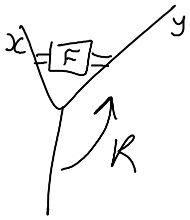
Mod CC Cyclic Symmetry of the Vertex, II

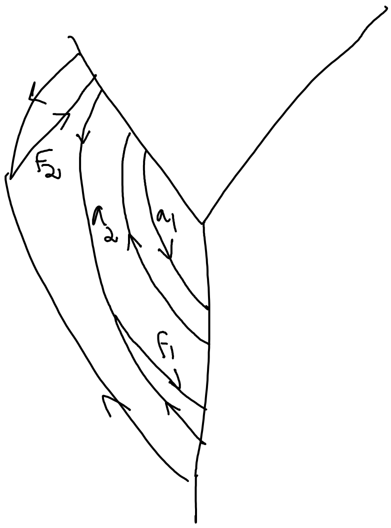
February-08-09
5:26 PM

The vertex is

$F = \exp\Big(a_1$  $+ a_2$  $+ H(f_1)$  $+ H(f_2)$  $\Big)$



Q: How does R, the $120°$ counterclockwise rotation, acts on F?



After linearization, ignoring wheels:

$$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \longmapsto \begin{pmatrix} -f_1(-x-y,x)+f_2(-x-y,x) \\ -f_1(-x-y,x) \end{pmatrix}$$

needs re-check, I may have confused hair w/ antenas.

TestingLinearisedR.nb:

```
In[1]:= R[{f1_, f2_}] := {-f1 + f2, -f1} /. {x → -x - y, y → x}

In[2]:= R@{f1[x, y], f2[x, y]}
Out[2]= {-f1[-x - y, x] + f2[-x - y, x], -f1[-x - y, x]}

In[3]:= R@R@{f1[x, y], f2[x, y]}
Out[3]= {-f2[y, -x - y], f1[y, -x - y] - f2[y, -x - y]}

In[4]:= R@R@R@{f1[x, y], f2[x, y]}
Out[4]= {f1[x, y], f2[x, y]}
```

Uniquness of F: From 2009-01/The Two F Eqns:

$f_1 = x_2 g$ & $f_2 = -x_1 g$

$x_1 = x$
$x_2 = y$

where g might be any function of $x_1 + x_2$ & $x_1 \cdot x_2$.

Now add to this $f_2(x,y) = -f_1(-x-y,x)$

$$\Rightarrow -xg(x,y) = -x\, g(-x-y,x)$$

$$\Rightarrow g(x,y) = g(-x-y,x)$$

$$\underset{1,2}{\begin{pmatrix} x \\ y \end{pmatrix}} \sim \underset{1\,2}{\begin{pmatrix} y \\ x \end{pmatrix}} \sim \underset{1\,2}{\begin{pmatrix} -x-y \\ x \end{pmatrix}} \sim \underset{1\,2}{\begin{pmatrix} -x-y \\ y \end{pmatrix}} \sim \underset{1\,2}{\begin{pmatrix} y \\ -x-y \end{pmatrix}} \sim \underset{1\,2}{\begin{pmatrix} x \\ -x-y \end{pmatrix}}$$

$$x^2+y^2+2(x+y)^2+x^2+y^2 \rightsquigarrow x^2+y^2+(x+y)^2$$

$$\rightsquigarrow x^2+y^2+xy$$

So even with cyclic symmetry imposed [ignoring

So even with cyclic symmetry imposed [ignoring wheels], F is not unique.

What with wheels? The first two F equations are wheel-oblivious. [Though $w(x,y)=w(y,x)$ is forced by the Θ-equation]. On linearized level, we have:

$$R: \begin{pmatrix} f_1 \\ f_2 \\ w \end{pmatrix} \longmapsto \begin{pmatrix} -f_1(-x-y,x)+f_2(-x-y,x) \\ -f_1(-x-y,x) \\ w(-x-y,x) + \dots \end{pmatrix}$$