Kamnitzer: Categorical sl(2) actions and equivalences of categories

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4:12 PM

(Joint w/ Sabin Cautis & Anthony Licata)

X – smooth variety / $\mathbb{C}$

$D(X)$ = bounded derived category of coherent sheaves on X

objects: complexes of coherent sheaves (vector bundles)

<u>Questions:</u> 1. When is $D(X) \cong D(Y)$

2. What is $\mathrm{Aut}(D(X))$

<u>Goal</u> Find interesting autoequivalences of $D(X)$ (for certain X)

$$K_0(D(X))_{\mathbb{C}} = \left\{ \begin{array}{c} \mathbb{C}\text{-vector space} \\ \text{with basis } [A] \\ A \in Coh(X) \end{array} \right\} \Big/ \begin{array}{l} [A] = [B] + [C] \\ \text{whenever} \\ \exists\ 0 \to B \to A \to C \to 0 \end{array}$$

(sometimes $K_0 \cong H_0$)

<u>Motivations</u> * Purely alg. geometry

* homological mirror symmetry $D(X) \rightleftarrows Fuk(M)$
* Can make braid group actions on $D(X)$
  $\rightsquigarrow$ get homological knot invariants
* Can lift automorphisms of $H(X)$ to autoequivs of $D(X)$

<u>spherical twist</u> (Seidel & Thomas)

X, Y smooth varieties

A functor $F : D(X) \to D(Y)$ is "spherical"

if (i) $F_R = F_L[2]$

(ii) $F_R \cdot F \cong I \oplus I[2]$

In this case $T_F(A) = \mathrm{Cone}(FF_R(A) \to A)$

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$A \in D(Y)$

<u>Theorem</u> (Seidel–Thomas, ...) T_F is an equivalence of categories.

<u>Example</u> If $X = \mathrm{pt}$ $D(X) = D(\mathrm{Vect})$, F is determined by $\mathcal{E} \in D(Y)$, $F(V) = V \otimes \mathcal{E}$

$F_R(A) = \mathrm{Ext}(\mathcal{E}, A)$

$T_F(A) = \mathrm{Cone}(\mathcal{E} \otimes \mathrm{Ext}(\mathcal{E}, A) \to A)$

on $K_0(D(Y))$, this is

$$[A] \longrightarrow \dim \mathrm{Ext}(\mathcal{E}, A) \cdot [\mathcal{E}]$$

$$= [A] - \langle \mathcal{E}, A \rangle [\mathcal{E}]$$ reflection on $\mathcal{E}$

Last at $T_0 + 28$ mins.