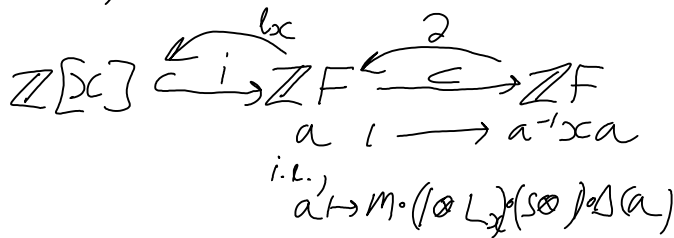


challenge Prove in a categorical way that if $a^{-1}xa = x$, then $a = x^n$.



i is the inclusion c is "conjugation"
 b_x forgets all letters except x .

∂W maps W to the sum of all ways of cutting it at an x and taking the remaining tail. \Rightarrow better take "sum of right cuts + sum of inverses of left cuts"

claim

$$(b_x \circ i) = I$$

$$j \circ b_x \circ \partial \circ c = \text{not working.}$$

Second attempt: Now take $\partial W = \frac{1}{2}(\partial_L W + s(\partial_R W))$
where:

$\partial_L W =$ sum of all ways of cutting W at an x and taking the remaining tail. $(\partial_L x x^{-1} W) = x^{-1} W + (\partial_L x^{-1} W) + \partial_L W$
 $\Rightarrow \partial_L x^{-1} = -x^{-1}$

possibly left or right multiplied by $x^{\pm 1}$

$\partial_R W =$ sum of all ways of cutting W at an x and taking the remaining head.

claim $\partial \circ c + i \circ b_x =$

$$i^{-1} x^{-1} i^{-1} m^{-1} a^{-1} k^{-1} s k m i x^{-1} \mapsto l r + k a m i x^{-1} - x^{-1} i^{-1} n^{-1} \dots$$

$$\begin{array}{cc}
 \swarrow & \searrow \\
 -x^{-1}x^3 & -x^{-1}(x^{-1}x^{-1})^{-1} \\
 -x^2 & -x
 \end{array}
 \qquad
 \begin{array}{cc}
 \swarrow & \searrow \\
 x & (x^{-1}x^{-1})^{-1} \\
 x+x^2 &
 \end{array}$$

$$\partial_L(x^{-1}xx) = -x^{-1}xx + x + 1 = 1$$

$$\partial_L(x) = 1 \qquad \partial_R(x) = 1$$

$$\partial_R(x^{-1}xx) = -x^{-1} + x^{-1} + x^{-1}x = 1$$

$$x^{-1}S(\partial_R(x^{-1}xx)) = -x^{-1}x + x^{-1}x + x^{-1}x^2$$