KAL of August 6 Summary and expansion

August-06-08
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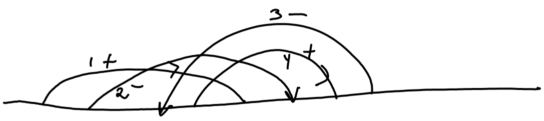

$a_i :=$ arrow $\# i$

$d_i :=$ direction of a_i here: $(++-+)$

$s_i :=$ sign of a_i here: $(+--+)$

T - the trapping matrix:

$$T_{ij} = \begin{cases} 1 & a_j \text{ ends within the open span of } a_i \\ 0 & \text{otherwise} \end{cases}$$

$$S = \mathrm{diag}(s_i d_i)$$

$$\boxed{\frac{d}{dt}\log\det(M) = \\ = tr\left(M^{-1}\frac{d}{dt}M\right)}$$

Let $C = (I+T)S$

what a horrible idea!

$$B = \frac{1}{2}\left((X-1)T(I+S) + (X^{-1}-1)T(I-S)\right)$$

$$Z = Tr(I-B)^{-1}BC = Tr\left[(I-B)^{-1}C - C\right]$$

$$\frac{d}{dt}\log\det M\left(-\frac{t^2}{2}\right) = -t\,tr\left(M^{-1}\dot{M}\right)\left(-\frac{t^2}{2}\right)$$

Conjecture $Z(X) = -X\dfrac{A'(X)}{A(X)}$, with $A(X)$ being the Alexander polynomial.

WAIT!

$$B = \frac{1}{2}T\left((X-1)(I+S) + (X^{-1}-1)(I-S)\right) =$$

$$= \frac{T}{2}\left[(X+X^{-1}-2)I + (X-X^{-1})S\right]$$

$$= \frac{T}{2}\left((X^{\frac12}-X^{-\frac12})^2 I + (X-X^{-1})S\right) =$$

$$= (X^{\frac12}-X^{-\frac12})\frac{T}{2}\left((X^{\frac12}-X^{-\frac12})I + (X^{\frac12}+X^{-\frac12})S\right)$$

$$= (t-t^{-1})\frac{T}{2}\left[(t-t^{-1})I + (t+t^{-1})S\right]$$

Guess The M that works is $I-B$ or a close relative.

Set $X^{\frac12} = t$

Alternatively,

$$C = (I+T)S, \quad B = T\left(e^{-xS}-1\right)$$

$$\frac{\partial}{\partial x}(I-B) = Te^{-xS}S = BS + TS = BS + C - S = C - (I-B)S$$

$$\frac{d}{dx}\log(\det(I-B)) = tr\left((I-B)^{-1}\frac{d}{dx}(I-B)\right)$$

$$= tr\left((I-B)^{-1}(C-(I-B)S)\right)$$

$$= tr\left((I-B)^{-1}C - S\right)$$

$$= tr\left((I-B)^{-1}C - C\right) \quad \text{as } tr\,C = tr\,S$$

$$= tr\left((I-B)^{-1}C - C\right) \qquad \text{as } tr\, S = tr\, C$$

$$= tr\left((I-B)^{-1}\left(C-(I-B)C\right)\right)$$

$$= tr\left((I-B)^{-1}BC\right)$$

QED (part I)

It remains to explain why $\det(I-B)$ is the Alexander polynomial!