The algebra of tagged Groups

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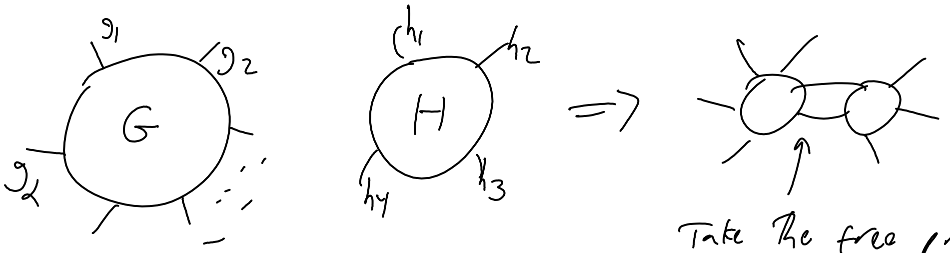
12:06 PM

A tagged group $G=(G, g_1 \dots g_\alpha)$ is a group with a choice of α elements in it.

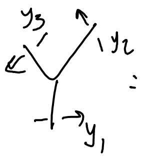
Question what operations exist on tagged groups?

Tagged groups form a circuit algebra:

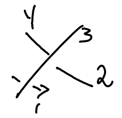


Take the free product of G & H and identify some of the tags.

Examples:

$Y :=$ 

$:= (\langle y_1, y_2, y_3 : y_1 y_2 y_3 = 1 \rangle, y_1, y_2, y_3)$

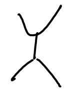
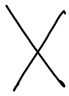
$X :=$ 

$:= (\langle x_1 .. x_4 : x_1 = x_3^{-1}, x_1 x_2 x_1^{-1} x_4 = 1 \rangle, x_i)$

Question what is the circuit algebra generated by X? By X and Y?

How close is it to wTTG? ← "w-Tangled Trivalent Graphs"

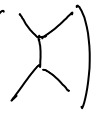
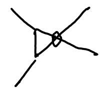
Question In the theory of finite type invariants of graphs, does it make sense to consider differences of objects with different skeleta?

I.e.,  $-$ , or even  $-$)(?

Answer(?) This is probably too restrictive — in Lie-language, it corresponds to forcing all strands to carry the same representation.

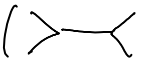
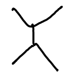
Question Is there a way to insert the notion of "skeleton" to tagged groups?

Question

$$\text{Is}\quad Ab\Big($$  $$\Big) \cong Ab\Big($$  $$\Big) ?$$

(Yes; both are $\langle y_1, y_2, y_3, y_4 : \sum y_i = 0 \rangle$, tagged $y_1 \dots y_4$)

Problem It is not true that "the fundamental group of the complement" detects skeleta - indeed,

$$\pi_1\Big($$  $$\Big) \cong \pi_1\Big($$  $$\Big)$$

$\longrightarrow$ So even the theory of wTTG's does not have a simple-minded tagged-group description.