Det and Tr

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3:37 PM

$$\chi_\lambda(A) := \det(\lambda I - A)$$

$$Tr(\lambda I - A)^{-1} = \sum \frac{1}{\lambda - \lambda_i}$$

$$= \sum \frac{d}{d\lambda} \log(\lambda - \lambda_i)$$

$$= \frac{d}{d\lambda} \log \prod (\lambda - \lambda_i)$$

$$= \frac{d}{d\lambda} \log \chi_\lambda(A) = \frac{d}{d\lambda} \log \det(\lambda I - A)$$

Alternatively,

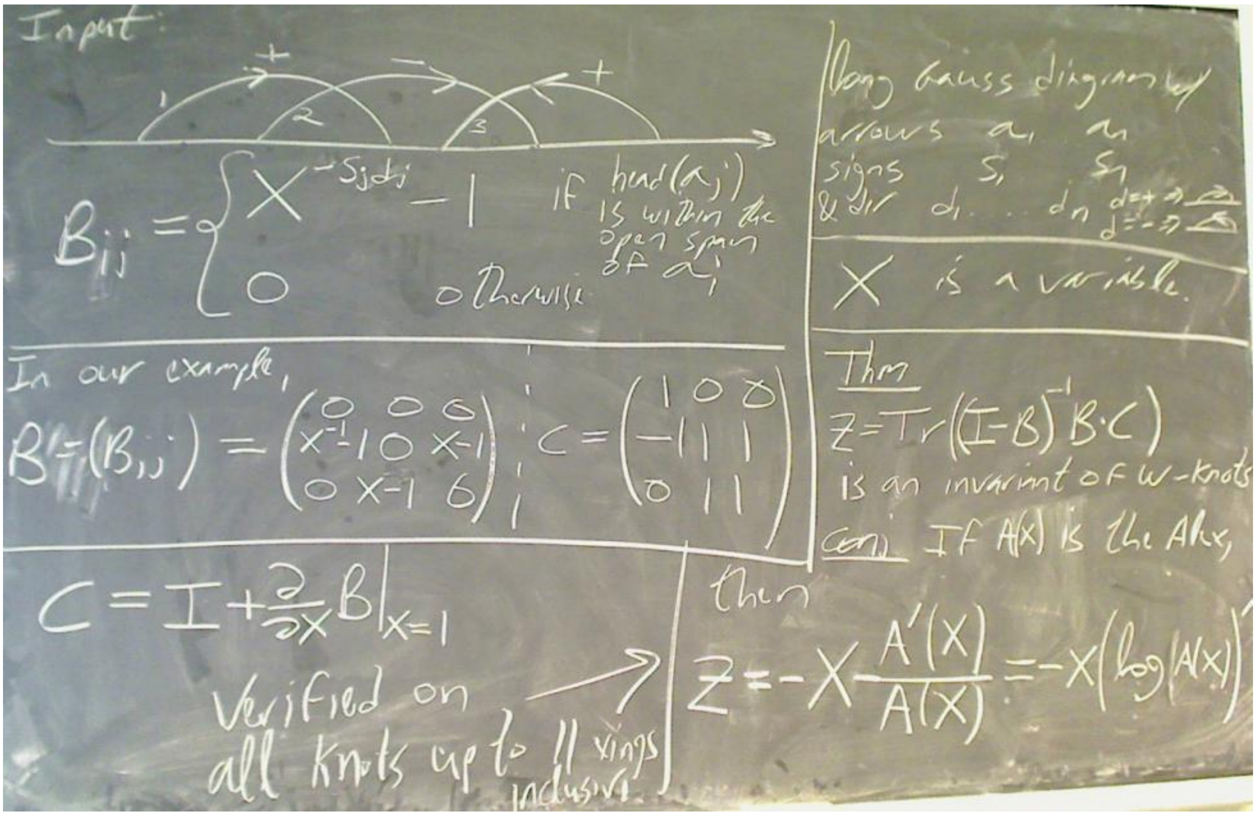
$$\frac{d}{d\lambda} \log(\det(\lambda I - A)) = \frac{\frac{d}{d\lambda} \det(\lambda I - A)}{\det(\lambda I - A)} =$$

$$= \frac{\det(\lambda I - A)\, tr(\lambda I - A)^{-1} I}{\det(\lambda I - A)} = tr(\lambda I - A)^{-1}$$

In general, $\frac{d}{d\epsilon} tr\, A_\epsilon = tr \frac{d}{d\epsilon} A_\epsilon$

$$\frac{d}{d\epsilon} \det A_\epsilon = \frac{d}{d\epsilon} \det A_0 \, \det A_0^{-1} A_\epsilon$$

$$= \det A_0 \, tr\, A_0^{-1} \frac{d}{d\epsilon} A_\epsilon$$



Warning. The first term in the definition of C, written on the blackboard as the identity matrix I, should really be the diagonal matrix whose jj entry is $-s_j d_j$.

Thus the C in the example should be

$$\begin{pmatrix} -1 & 0 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$Z = -X\frac{A'}{A} = -X(\log A)' \iff \int\left(-\frac{Z}{X}\right)dx = \log A$$

$$\iff A = \exp\left(-\int \frac{Z}{X}\,dx\right)$$