
The Program

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<< KnotTheory`
```

Loading KnotTheory` version of January 18, 2008, 18:17:28.7446.

Read more at <http://katlas.org/wiki/KnotTheory>.

```
wAlex[gc_GC] := {
  MatrixForm[
    AA = DiagonalMatrix[List @@ gc /. Ar[t_, h_, s_] => s]
  ];
  Bij[Ar[ti_, hi_, si_], Ar[tj_, hj_, sj_]] := If[
    ti < hj < hi || hi < hj < ti,
    Sign[hi - ti] * Sign[hj - tj] * (X^(-sj * Sign[hj - tj]) - 1),
    0
  ];
  MatrixForm[BB = Outer[Bij, List @@ gc, List @@ gc]];
  Cij[Ar[ti_, hi_, si_], Ar[tj_, hj_, sj_]] := If[
    ti ≤ hj ≤ hi || hi ≤ hj ≤ ti,
    -Sign[hi - ti] * x,
    0
  ];
  MatrixForm[CC = Outer[Cij, List @@ gc, List @@ gc]];
  MatrixForm[
    DD = AA.Inverse[IdentityMatrix[Length[BB]] - BB].BB.CC
    // Together // ExpandNumerator
  ];
  Tr[DD] // Together // ExpandNumerator // ExpandDenominator
};

wAlex[K_] := Join[{
  pd = PD[K],
  gc = GC @@ pd /. X[i_, j_, k_, l_] => If[PositiveQ[X[i, j, k, l]],
    Ar[l, i, +1], Ar[j, i, -1]
  ]
},
wAlex[gc]
];

wAlexander[K_] := wAlexander[K] = Last[wAlex[K]];

wAlex2[gc_GC] := {
  Bij[Ar[ti_, hi_, si_], Ar[tj_, hj_, sj_]] := If[
    ti < hj < hi || hi < hj < ti,
    X^(-sj * Sign[hj - tj]) - 1,
    0
  ];
  MatrixForm[BB = Outer[Bij, List @@ gc, List @@ gc]],
  Cij[Ar[ti_, hi_, si_], Ar[tj_, hj_, sj_]] := If[
    ti ≤ hj ≤ hi || hi ≤ hj ≤ ti,
    -Sign[hj - tj] * sj * x,
    0
  ];
  MatrixForm[CC = Outer[Cij, List @@ gc, List @@ gc]],
  MatrixForm[
    DD = Inverse[IdentityMatrix[Length[BB]] - BB].BB.CC
  ]
};
```

```

// Together // ExpandNumerator
],
Tr[DD] // Together // ExpandNumerator // ExpandDenominator
];
wAlex2[K_] := Join[{
  pd = PD[K],
  gc = GC @@ pd /. X[i_, j_, k_, l_] => If[PositiveQ[X[i, j, k, l]],
    Ar[1, i, +1], Ar[j, i, -1]
  ]
},
wAlex2[gc]
];
wAlexander2[K_] := Last[wAlex2[K]];

And @@ (
  (wAlexander[#] == wAlexander2[#]) & /@ AllKnots[{3, 9}]
)
True

(# -> wAlex2[#]) & /@ AllKnots[{3, 7}]

{Knot[3, 1] ->
  {PD[X[1, 4, 2, 5], X[3, 6, 4, 1], X[5, 2, 6, 3]], GC[Ar[4, 1, -1], Ar[6, 3, -1], Ar[2, 5, -1]],

$$\left( \begin{array}{ccc} 0 & -1 + \frac{1}{X} & 0 \\ 0 & 0 & -1 + X \\ 0 & -1 + \frac{1}{X} & 0 \end{array} \right), \left( \begin{array}{ccc} -x & -x & 0 \\ 0 & -x & x \\ 0 & -x & x \end{array} \right), \left( \begin{array}{cc} 0 & \frac{-xX+xX^2}{1-X+X^2} \frac{xX-xX^2}{1-X+X^2} \\ 0 & \frac{x-xX}{1-X+X^2} \frac{-x+xX}{1-X+X^2} \\ 0 & \frac{-xX+xX^2}{1-X+X^2} \frac{xX-xX^2}{1-X+X^2} \end{array} \right), \frac{x-xX^2}{1-X+X^2} \},$$

  Knot[4, 1] -> {PD[X[4, 2, 5, 1], X[8, 6, 1, 5], X[6, 3, 7, 4], X[2, 7, 3, 8]],

$$\text{GC}[\text{Ar}[1, 4, 1], \text{Ar}[5, 8, 1], \text{Ar}[3, 6, -1], \text{Ar}[7, 2, -1]], \left( \begin{array}{cccc} 0 & 0 & 0 & -1 + \frac{1}{X} \\ 0 & 0 & -1 + X & 0 \\ -1 + \frac{1}{X} & 0 & 0 & 0 \\ -1 + \frac{1}{X} & 0 & -1 + X & 0 \end{array} \right),$$


$$\left( \begin{array}{ccc} -x & 0 & 0 \\ 0 & -x & x \\ -x & 0 & x \\ -x & 0 & x \end{array} \right), \left( \begin{array}{ccc} \frac{x-xX}{1-3X+X^2} & 0 & \frac{-xX+xX^2}{1-3X+X^2} \frac{2x-3xX+xX^2}{1-3X+X^2} \\ \frac{-x+xX}{1-3X+X^2} & 0 & \frac{xX-xX^2}{1-3X+X^2} \frac{-x+2xX-xX^2}{X(1-3X+X^2)} \\ \frac{2x-3xX+xX^2}{1-3X+X^2} & 0 & \frac{-x+2xX-xX^2}{1-3X+X^2} \frac{x-xX}{X(1-3X+X^2)} \\ \frac{x-2xX+xX^2}{1-3X+X^2} & 0 & \frac{-x+3xX-2xX^2}{1-3X+X^2} \frac{x-xX}{1-3X+X^2} \end{array} \right), \frac{x-xX^2}{1-3X+X^2} \},$$

  Knot[5, 1] -> {PD[X[1, 6, 2, 7], X[3, 8, 4, 9], X[5, 10, 6, 1], X[7, 2, 8, 3], X[9, 4, 10, 5]],
  GC[Ar[6, 1, -1], Ar[8, 3, -1], Ar[10, 5, -1], Ar[2, 7, -1], Ar[4, 9, -1]],

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$$\begin{pmatrix} 0 & -1 + \frac{1}{X} & -1 + \frac{1}{X} & 0 & 0 \\ 0 & 0 & -1 + \frac{1}{X} & -1 + X & 0 \\ 0 & 0 & 0 & -1 + X & -1 + X \\ 0 & -1 + \frac{1}{X} & -1 + \frac{1}{X} & 0 & 0 \\ 0 & 0 & -1 + \frac{1}{X} & -1 + X & 0 \end{pmatrix}, \begin{pmatrix} -x & -x & -x & 0 & 0 \\ 0 & -x & -x & x & 0 \\ 0 & 0 & -x & x & x \\ 0 & -x & -x & x & 0 \\ 0 & 0 & -x & x & x \end{pmatrix},$$

$$\begin{pmatrix} 0 & \frac{-xX^3 + xX^4}{1 - X + X^2 - X^3 + X^4} & \frac{-xX + xX^2 - xX^3 + xX^4}{1 - X + X^2 - X^3 + X^4} & \frac{xX - xX^2 + xX^3 - xX^4}{1 - X + X^2 - X^3 + X^4} & \frac{xX - xX^2}{1 - X + X^2 - X^3 + X^4} \\ 0 & \frac{-xX^3 + xX^4}{1 - X + X^2 - X^3 + X^4} & \frac{-xX + xX^2 - xX^3 + xX^4}{1 - X + X^2 - X^3 + X^4} & \frac{-x + xX + xX^3 - xX^4}{1 - X + X^2 - X^3 + X^4} & \frac{xX^3 - xX^4}{1 - X + X^2 - X^3 + X^4} \\ 0 & \frac{xX^2 - xX^3}{1 - X + X^2 - X^3 + X^4} & \frac{-xX + xX^2 - xX^3}{1 - X + X^2 - X^3 + X^4} & \frac{-x + xX - xX^2 + xX^3}{1 - X + X^2 - X^3 + X^4} & \frac{-x + xX}{1 - X + X^2 - X^3 + X^4} \\ 0 & \frac{-xX^3 + xX^4}{1 - X + X^2 - X^3 + X^4} & \frac{-xX + xX^2 - xX^3 + xX^4}{1 - X + X^2 - X^3 + X^4} & \frac{xX - xX^2 + xX^3 - xX^4}{1 - X + X^2 - X^3 + X^4} & \frac{xX - xX^2}{1 - X + X^2 - X^3 + X^4} \\ 0 & \frac{-xX^3 + xX^4}{1 - X + X^2 - X^3 + X^4} & \frac{-xX + xX^2 - xX^3 + xX^4}{1 - X + X^2 - X^3 + X^4} & \frac{-x + xX + xX^3 - xX^4}{1 - X + X^2 - X^3 + X^4} & \frac{xX^3 - xX^4}{1 - X + X^2 - X^3 + X^4} \end{pmatrix}, \left. \frac{2x - xX + xX^3 - 2xX^4}{1 - X + X^2 - X^3 + X^4} \right\},$$

Knot[5, 2] \rightarrow {PD[X[1, 4, 2, 5], X[3, 8, 4, 9], X[5, 10, 6, 1], X[9, 6, 10, 7], X[7, 2, 8, 3]],
GC[Ar[4, 1, -1], Ar[8, 3, -1], Ar[10, 5, -1], Ar[6, 9, -1], Ar[2, 7, -1]],

$$\begin{pmatrix} 0 & -1 + \frac{1}{X} & 0 & 0 & 0 \\ 0 & 0 & -1 + \frac{1}{X} & 0 & -1 + X \\ 0 & 0 & 0 & -1 + X & -1 + X \\ 0 & 0 & 0 & 0 & -1 + X \\ 0 & -1 + \frac{1}{X} & -1 + \frac{1}{X} & 0 & 0 \end{pmatrix}, \begin{pmatrix} -x & -x & 0 & 0 & 0 \\ 0 & -x & -x & 0 & x \\ 0 & 0 & -x & x & x \\ 0 & 0 & 0 & x & x \\ 0 & -x & -x & 0 & x \end{pmatrix},$$

$$\begin{pmatrix} 0 & \frac{-xX + xX^2}{2 - 3X + 2X^2} & \frac{-x + xX}{2 - 3X + 2X^2} & \frac{xX - 2xX^2 + xX^3}{2 - 3X + 2X^2} & \frac{2xX - 3xX^2 + xX^3}{2 - 3X + 2X^2} \\ 0 & \frac{2x - 3xX + xX^2}{2 - 3X + 2X^2} & \frac{2x - 4xX + 2xX^2}{2 - 3X + 2X^2} & \frac{xX^2 - xX^3}{2 - 3X + 2X^2} & \frac{-2x + 3xX - xX^3}{2 - 3X + 2X^2} \\ 0 & \frac{xX - xX^2}{2 - 3X + 2X^2} & \frac{x - xX}{2 - 3X + 2X^2} & \frac{-2x + 4xX - 3xX^2 + xX^3}{2 - 3X + 2X^2} & \frac{-2x + 3xX - 2xX^2 + xX^3}{2 - 3X + 2X^2} \\ 0 & \frac{x - xX}{2 - 3X + 2X^2} & \frac{x - xX}{x(2 - 3X + 2X^2)} & \frac{-x + 2xX - xX^2}{2 - 3X + 2X^2} & \frac{-2x + 3xX - xX^2}{2 - 3X + 2X^2} \\ 0 & \frac{x - 3xX + 2xX^2}{2 - 3X + 2X^2} & \frac{-x + 2xX - 3xX^2 + 2xX^3}{x(2 - 3X + 2X^2)} & \frac{x - xX}{2 - 3X + 2X^2} & \frac{2xX - 2xX^2}{2 - 3X + 2X^2} \end{pmatrix}, \left. \frac{2x - 2xX^2}{2 - 3X + 2X^2} \right\},$$

Knot[6, 1] \rightarrow {PD[X[1, 4, 2, 5], X[7, 10, 8, 11], X[3, 9, 4, 8], X[9, 3, 10, 2], X[5, 12, 6, 1],
X[11, 6, 12, 7]], GC[Ar[4, 1, -1], Ar[10, 7, -1], Ar[8, 3, 1], Ar[2, 9, 1], Ar[12, 5, -1]],

$$\text{Ar}[6, 11, -1], \begin{pmatrix} 0 & 0 & -1 + X & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 + \frac{1}{X} & 0 & 0 \\ 0 & -1 + \frac{1}{X} & 0 & 0 & -1 + \frac{1}{X} & 0 \\ 0 & -1 + \frac{1}{X} & -1 + X & 0 & -1 + \frac{1}{X} & 0 \\ 0 & -1 + \frac{1}{X} & 0 & -1 + \frac{1}{X} & 0 & -1 + X \\ 0 & -1 + \frac{1}{X} & 0 & -1 + \frac{1}{X} & 0 & 0 \end{pmatrix}, \begin{pmatrix} -x & 0 & x & 0 & 0 & 0 \\ 0 & -x & 0 & -x & 0 & 0 \\ 0 & -x & x & 0 & -x & 0 \\ 0 & -x & x & -x & -x & 0 \\ 0 & -x & 0 & -x & -x & x \\ 0 & -x & 0 & -x & 0 & x \end{pmatrix},$$

$$\left(\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \begin{array}{l} \frac{x-xX}{-1+2X} \\ \frac{-x+xX}{-1+2X} \\ \frac{-2x+2xX}{-1+2X} \\ \frac{-x+xX}{-1+2X} \\ \frac{-x+xX}{-1+2X} \\ \frac{-2x+2xX}{-1+2X} \\ \frac{-2x+2xX}{-1+2X} \end{array} \begin{array}{l} \frac{xX-xX^2}{2-5X+2X^2} \\ \frac{-xX+xX^2}{2-5X+2X^2} \\ \frac{-2x+4xX-2xX^2}{2-5X+2X^2} \\ \frac{-2x+5xX-3xX^2}{2-5X+2X^2} \\ \frac{-xX+xX^2}{2-5X+2X^2} \\ \frac{-xX+xX^2}{2-5X+2X^2} \\ \frac{-xX+xX^2}{2-5X+2X^2} \end{array} \begin{array}{l} \frac{-2x+4xX-2xX^2}{(-2+X)X(-1+2X)} \\ \frac{3x-5xX+2xX^2}{(-2+X)(-1+2X)} \\ \frac{2x-2xX}{(-2+X)X(-1+2X)} \\ \frac{2x-2xX}{(-2+X)(-1+2X)} \\ \frac{3x-5xX+2xX^2}{(-2+X)(-1+2X)} \\ \frac{x+2xX-5xX^2+2xX^3}{(-2+X)X(-1+2X)} \\ \frac{x+2xX-5xX^2+2xX^3}{(-2+X)X(-1+2X)} \end{array} \begin{array}{l} \frac{-x+xX}{2-5X+2X^2} \\ \frac{x-xX}{2-5X+2X^2} \\ \frac{3x-5xX+2xX^2}{2-5X+2X^2} \\ \frac{x-xX}{2-5X+2X^2} \\ \frac{x-xX}{2-5X+2X^2} \\ \frac{x-xX}{X(2-5X+2X^2)} \\ \frac{x-xX}{X(2-5X+2X^2)} \end{array} \begin{array}{l} \frac{xX-2xX^2+xX^3}{2-5X+2X^2} \\ \frac{-xX+2xX^2-xX^3}{2-5X+2X^2} \\ \frac{-xX+xX^2}{2-5X+2X^2} \\ \frac{-xX^2+xX^3}{2-5X+2X^2} \\ \frac{-2x+6xX-5xX^2+xX^3}{2-5X+2X^2} \\ \frac{-x+2xX-xX^2}{2-5X+2X^2} \\ \frac{-x+2xX-xX^2}{2-5X+2X^2} \end{array} \right) \cdot \left. \begin{array}{l} \\ \\ \\ \\ \\ \\ \\ \end{array} \right\} \frac{2x-2xX^2}{2-5X+2X^2},$$

Knot[6, 2] → {PD[X[1, 4, 2, 5], X[5, 10, 6, 11], X[3, 9, 4, 8], X[9, 3, 10, 2],
X[7, 12, 8, 1], X[11, 6, 12, 7]], GC[Ar[4, 1, -1], Ar[10, 5, -1], Ar[8, 3, 1],

Ar[2, 9, 1], Ar[12, 7, -1], Ar[6, 11, -1]],

$$\left(\begin{array}{cccccc} 0 & 0 & -1+X & 0 & 0 & 0 \\ 0 & 0 & 0 & -1+\frac{1}{X} & -1+\frac{1}{X} & 0 \\ 0 & -1+\frac{1}{X} & 0 & 0 & -1+\frac{1}{X} & 0 \\ 0 & -1+\frac{1}{X} & -1+X & 0 & -1+\frac{1}{X} & 0 \\ 0 & 0 & 0 & -1+\frac{1}{X} & 0 & -1+X \\ 0 & 0 & 0 & -1+\frac{1}{X} & -1+\frac{1}{X} & 0 \end{array} \right),$$

$$\left(\begin{array}{ccccc} -x & 0 & x & 0 & 0 & 0 \\ 0 & -x & 0 & -x & -x & 0 \\ 0 & -x & x & 0 & -x & 0 \\ 0 & -x & x & -x & -x & 0 \\ 0 & 0 & 0 & -x & -x & x \\ 0 & 0 & 0 & -x & -x & x \end{array} \right) \cdot \left(\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \begin{array}{l} \frac{-x+2xX-2xX^2+xX^3}{1-3X+3X^2-3X^3+X^4} \\ \frac{x-xX}{1-3X+3X^2-3X^3+X^4} \\ \frac{2x-4xX+4xX^2-3xX^3+xX^4}{1-3X+3X^2-3X^3+X^4} \\ \frac{x-2xX+2xX^2-2xX^3+xX^4}{1-3X+3X^2-3X^3+X^4} \\ \frac{xX^2-xX^3}{1-3X+3X^2-3X^3+X^4} \\ \frac{x-xX}{1-3X+3X^2-3X^3+X^4} \end{array} \begin{array}{l} \frac{xX-2xX^2+2xX^3-xX^4}{1-3X+3X^2-3X^3+X^4} \\ \frac{-xX+xX^2}{1-3X+3X^2-3X^3+X^4} \\ \frac{-x+2xX-2xX^2+2xX^3-xX^4}{1-3X+3X^2-3X^3+X^4} \\ \frac{-x+3xX-4xX^2+4xX^3-2xX^4}{1-3X+3X^2-3X^3+X^4} \\ \frac{-xX^3+xX^4}{1-3X+3X^2-3X^3+X^4} \\ \frac{-xX+xX^2}{1-3X+3X^2-3X^3+X^4} \end{array} \begin{array}{l} \frac{-x+2xX-2xX^2+2xX^3-xX^4}{X(1-3X+3X^2-3X^3+X^4)} \\ \frac{2x-3xX+3xX^2-3xX^3+xX^4}{1-3X+3X^2-3X^3+X^4} \\ \frac{x-xX+xX^2-xX^3}{X(1-3X+3X^2-3X^3+X^4)} \\ \frac{x-xX+xX^2-xX^3}{1-3X+3X^2-3X^3+X^4} \\ \frac{x-3xX+4xX^2-3xX^3+xX^4}{1-3X+3X^2-3X^3+X^4} \\ \frac{2x-3xX+3xX^2-3xX^3+xX^4}{1-3X+3X^2-3X^3+X^4} \end{array} \begin{array}{l} \frac{-x+xX}{1-3X+3X^2-3X^3+X^4} \\ \frac{x-xX+2xX^2-2xX^3}{1-3X+3X^2-3X^3+X^4} \\ \frac{2x-3xX+3xX^2-3xX^3}{1-3X+3X^2-3X^3+X^4} \\ \frac{x-2xX+3xX^2-2xX^3}{1-3X+3X^2-3X^3+X^4} \\ \frac{x-3xX+3xX^2-2xX^3}{1-3X+3X^2-3X^3+X^4} \\ \frac{x-xX+2xX^2-2xX^3}{1-3X+3X^2-3X^3+X^4} \end{array} \right),$$

Knot[6, 3] → {PD[X[4, 2, 5, 1], X[8, 4, 9, 3], X[12, 9, 1, 10], X[10, 5, 11, 6],
X[6, 11, 7, 12], X[2, 8, 3, 7]], GC[Ar[1, 4, 1], Ar[3, 8, 1], Ar[9, 12, -1],

Ar[5, 10, -1], Ar[11, 6, -1], Ar[7, 2, 1]],

$$\left(\begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & -1+X \\ -1+\frac{1}{X} & 0 & 0 & 0 & -1+\frac{1}{X} & 0 \\ 0 & 0 & 0 & -1+X & 0 & 0 \\ 0 & -1+\frac{1}{X} & 0 & 0 & -1+\frac{1}{X} & 0 \\ 0 & -1+\frac{1}{X} & 0 & -1+X & 0 & 0 \\ -1+\frac{1}{X} & 0 & 0 & 0 & -1+\frac{1}{X} & 0 \end{array} \right),$$

$$\begin{pmatrix} -x & 0 & 0 & 0 & 0 & x \\ -x & -x & 0 & 0 & -x & 0 \\ 0 & 0 & x & x & 0 & 0 \\ 0 & -x & 0 & x & -x & 0 \\ 0 & -x & 0 & x & -x & 0 \\ -x & 0 & 0 & 0 & -x & x \end{pmatrix}, \begin{pmatrix} \frac{x-2xX+2xX^2-xX^3}{1-3X+5X^2-3X^3+X^4} & \frac{xX-2xX^2+xX^3}{1-3X+5X^2-3X^3+X^4} & 0 & \frac{-xX^2+2xX^3-xX^4}{1-3X+5X^2-3X^3+X^4} & \frac{xX-xX^2}{1-3X+5X^2-3X^3+X^4} \\ \frac{-2xX+4xX^2-3xX^3+xX^4}{1-3X+5X^2-3X^3+X^4} & \frac{-xX+xX^2}{1-3X+5X^2-3X^3+X^4} & 0 & \frac{xX^2-xX^3}{1-3X+5X^2-3X^3+X^4} & \frac{x-4xX+5xX^2-3xX^3}{1-3X+5X^2-3X^3+X^4} \\ \frac{x-2xX+xX^2}{X(1-3X+5X^2-3X^3+X^4)} & \frac{x-2xX+2xX^2-xX^3}{X(1-3X+5X^2-3X^3+X^4)} & 0 & \frac{-x+2xX-2xX^2+xX^3}{1-3X+5X^2-3X^3+X^4} & \frac{2x-4xX+3xX^2-2xX^3}{1-3X+5X^2-3X^3+X^4} \\ \frac{-x+xX}{X(1-3X+5X^2-3X^3+X^4)} & \frac{-x+2xX-4xX^2+5xX^3-3xX^4+xX^5}{X(1-3X+5X^2-3X^3+X^4)} & 0 & \frac{2xX-4xX^2+3xX^3-xX^4}{1-3X+5X^2-3X^3+X^4} & \frac{-x-xX+4xX^2-3xX^3}{1-3X+5X^2-3X^3+X^4} \\ \frac{-xX+xX^2}{1-3X+5X^2-3X^3+X^4} & \frac{x-4xX+6xX^2-4xX^3+xX^4}{1-3X+5X^2-3X^3+X^4} & 0 & \frac{-x+3xX-4xX^2+2xX^3}{1-3X+5X^2-3X^3+X^4} & \frac{x-3xX+3xX^2-xX^3}{1-3X+5X^2-3X^3+X^4} \\ \frac{-2xX+4xX^2-3xX^3+xX^4}{1-3X+5X^2-3X^3+X^4} & \frac{-xX+xX^2}{1-3X+5X^2-3X^3+X^4} & 0 & \frac{xX^2-xX^3}{1-3X+5X^2-3X^3+X^4} & \frac{x-4xX+5xX^2-3xX^3}{1-3X+5X^2-3X^3+X^4} \end{pmatrix}$$

Knot[7, 1] → {PD[X[1, 8, 2, 9], X[3, 10, 4, 11], X[5, 12, 6, 13],
 X[7, 14, 8, 1], X[9, 2, 10, 3], X[11, 4, 12, 5], X[13, 6, 14, 7]],
 GC[Ar[8, 1, -1], Ar[10, 3, -1], Ar[12, 5, -1], Ar[14, 7, -1], Ar[2, 9, -1],

$$\text{Ar}[4, 11, -1], \text{Ar}[6, 13, -1]], \begin{pmatrix} 0 & -1 + \frac{1}{X} & -1 + \frac{1}{X} & -1 + \frac{1}{X} & 0 & 0 & 0 \\ 0 & 0 & -1 + \frac{1}{X} & -1 + \frac{1}{X} & -1 + X & 0 & 0 \\ 0 & 0 & 0 & -1 + \frac{1}{X} & -1 + X & -1 + X & 0 \\ 0 & 0 & 0 & 0 & -1 + X & -1 + X & -1 + X \\ 0 & -1 + \frac{1}{X} & -1 + \frac{1}{X} & -1 + \frac{1}{X} & 0 & 0 & 0 \\ 0 & 0 & -1 + \frac{1}{X} & -1 + \frac{1}{X} & -1 + X & 0 & 0 \\ 0 & 0 & 0 & -1 + \frac{1}{X} & -1 + X & -1 + X & 0 \end{pmatrix},$$

$$\begin{pmatrix} -x & -x & -x & -x & 0 & 0 & 0 \\ 0 & -x & -x & -x & x & 0 & 0 \\ 0 & 0 & -x & -x & x & x & 0 \\ 0 & 0 & 0 & -x & x & x & x \\ 0 & -x & -x & -x & x & 0 & 0 \\ 0 & 0 & -x & -x & x & x & 0 \\ 0 & 0 & 0 & -x & x & x & x \end{pmatrix}, \begin{pmatrix} 0 & \frac{-xX^5+xX^6}{1-X+X^2-X^3+X^4-X^5+X^6} & \frac{-xX^3+xX^4-xX^5+xX^6}{1-X+X^2-X^3+X^4-X^5+X^6} & \frac{-xX+xX^2-xX^3+xX^4-xX^5+xX^6}{1-X+X^2-X^3+X^4-X^5+X^6} & \frac{xX-xX^2+xX^3-xX^4+xX^5}{1-X+X^2-X^3+X^4-X^5+X^6} \\ 0 & \frac{x-xX}{1-X+X^2-X^3+X^4-X^5+X^6} & \frac{x-xX-xX^5+xX^6}{1-X+X^2-X^3+X^4-X^5+X^6} & \frac{x-xX-xX^3+xX^4-xX^5+xX^6}{1-X+X^2-X^3+X^4-X^5+X^6} & \frac{-x+xX+xX^3-xX^4+xX^5}{1-X+X^2-X^3+X^4-X^5+X^6} \\ 0 & \frac{xX^2-xX^3}{1-X+X^2-X^3+X^4-X^5+X^6} & \frac{x-xX+xX^2-xX^3}{1-X+X^2-X^3+X^4-X^5+X^6} & \frac{x-xX+xX^2-xX^3-xX^5+xX^6}{1-X+X^2-X^3+X^4-X^5+X^6} & \frac{-x+xX-xX^2+xX^3+xX^4}{1-X+X^2-X^3+X^4-X^5+X^6} \\ 0 & \frac{xX^4-xX^5}{1-X+X^2-X^3+X^4-X^5+X^6} & \frac{xX^2-xX^3+xX^4-xX^5}{1-X+X^2-X^3+X^4-X^5+X^6} & \frac{x-xX+xX^2-xX^3+xX^4-xX^5}{1-X+X^2-X^3+X^4-X^5+X^6} & \frac{-x+xX-xX^2+xX^3-xX^4}{1-X+X^2-X^3+X^4-X^5+X^6} \\ 0 & \frac{-xX^5+xX^6}{1-X+X^2-X^3+X^4-X^5+X^6} & \frac{-xX^3+xX^4-xX^5+xX^6}{1-X+X^2-X^3+X^4-X^5+X^6} & \frac{-xX+xX^2-xX^3+xX^4-xX^5+xX^6}{1-X+X^2-X^3+X^4-X^5+X^6} & \frac{xX-xX^2+xX^3-xX^4+xX^5}{1-X+X^2-X^3+X^4-X^5+X^6} \\ 0 & \frac{x-xX}{1-X+X^2-X^3+X^4-X^5+X^6} & \frac{x-xX-xX^5+xX^6}{1-X+X^2-X^3+X^4-X^5+X^6} & \frac{x-xX-xX^3+xX^4-xX^5+xX^6}{1-X+X^2-X^3+X^4-X^5+X^6} & \frac{-x+xX+xX^3-xX^4+xX^5}{1-X+X^2-X^3+X^4-X^5+X^6} \\ 0 & \frac{xX^2-xX^3}{1-X+X^2-X^3+X^4-X^5+X^6} & \frac{x-xX+xX^2-xX^3}{1-X+X^2-X^3+X^4-X^5+X^6} & \frac{x-xX+xX^2-xX^3-xX^5+xX^6}{1-X+X^2-X^3+X^4-X^5+X^6} & \frac{-x+xX-xX^2+xX^3+xX^4}{1-X+X^2-X^3+X^4-X^5+X^6} \end{pmatrix}$$

Knot[7, 2] → {PD[X[1, 4, 2, 5], X[3, 10, 4, 11], X[5, 14, 6, 1],
 X[7, 12, 8, 13], X[11, 8, 12, 9], X[13, 6, 14, 7], X[9, 2, 10, 3]],
 GC[Ar[4, 1, -1], Ar[10, 3, -1], Ar[14, 5, -1], Ar[12, 7, -1], Ar[8, 11, -1],

$$\text{Ar}[6, 13, -1], \text{Ar}[2, 9, -1]], \begin{pmatrix} 0 & -1 + \frac{1}{X} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 + \frac{1}{X} & -1 + \frac{1}{X} & 0 & 0 & -1 + X \\ 0 & 0 & 0 & -1 + \frac{1}{X} & -1 + X & -1 + X & -1 + X \\ 0 & 0 & 0 & 0 & -1 + X & 0 & -1 + X \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 + X \\ 0 & 0 & 0 & -1 + \frac{1}{X} & -1 + X & 0 & -1 + X \\ 0 & -1 + \frac{1}{X} & -1 + \frac{1}{X} & -1 + \frac{1}{X} & 0 & 0 & 0 \end{pmatrix},$$

$$\begin{pmatrix} -x & -x & 0 & 0 & 0 & 0 & 0 \\ 0 & -x & -x & -x & 0 & 0 & x \\ 0 & 0 & -x & -x & x & x & x \\ 0 & 0 & 0 & -x & x & 0 & x \\ 0 & 0 & 0 & 0 & x & 0 & x \\ 0 & 0 & 0 & -x & x & x & x \\ 0 & -x & -x & -x & 0 & 0 & x \end{pmatrix}, \begin{pmatrix} 0 & \frac{-xX+xX^2}{3-5X+3X^2} & \frac{-x+xX}{3-5X+3X^2} & \frac{-2x+3xX-xX^2}{3-5X+3X^2} & \frac{2xX-4xX^2+2xX^3}{3-5X+3X^2} & \frac{xX-2x}{3-5X+3X^2} \\ 0 & \frac{3x-5xX+2xX^2}{3-5X+3X^2} & \frac{3x-6xX+3xX^2}{3-5X+3X^2} & \frac{3x-7xX+4xX^2}{3-5X+3X^2} & \frac{2xX^2-2xX^3}{3-5X+3X^2} & \frac{xX^2-x}{3-5X+3X^2} \\ 0 & \frac{xX-xX^2}{3-5X+3X^2} & \frac{x-xX}{3-5X+3X^2} & \frac{2x-3xX+xX^2}{3-5X+3X^2} & \frac{-3x+6xX-4xX^2+xX^3}{3-5X+3X^2} & \frac{-3x+x}{3-5X+3X^2} \\ 0 & \frac{xX-xX^2}{3-5X+3X^2} & \frac{x-xX}{3-5X+3X^2} & \frac{2x-3xX+xX^2}{3-5X+3X^2} & \frac{-3x+6xX-4xX^2+xX^3}{3-5X+3X^2} & \frac{-xX+x}{3-5X+3X^2} \\ 0 & \frac{x-xX}{3-5X+3X^2} & \frac{x-xX}{X(3-5X+3X^2)} & \frac{2x-3xX+xX^2}{X(3-5X+3X^2)} & \frac{-2x+4xX-2xX^2}{3-5X+3X^2} & \frac{-x+2x}{3-5X+3X^2} \\ 0 & \frac{x-xX}{3-5X+3X^2} & \frac{x-xX}{X(3-5X+3X^2)} & \frac{-x+5xX-7xX^2+3xX^3}{X(3-5X+3X^2)} & \frac{-2x+4xX-2xX^2}{3-5X+3X^2} & \frac{-x+2x}{3-5X+3X^2} \\ 0 & \frac{2x-5xX+3xX^2}{3-5X+3X^2} & \frac{-x+3xX-5xX^2+3xX^3}{X(3-5X+3X^2)} & \frac{-2x+4xX-5xX^2+3xX^3}{X(3-5X+3X^2)} & \frac{2x-2xX}{3-5X+3X^2} & \frac{x-x}{3-5X+3X^2} \end{pmatrix},$$

Knot[7, 3] → {PD[X[6, 2, 7, 1], X[10, 4, 11, 3], X[14, 8, 1, 7], X[8, 14, 9, 13], X[12, 6, 13, 5], X[2, 10, 3, 9], X[4, 12, 5, 11]], GC[Ar[1, 6, 1], Ar[3, 10, 1], Ar[7, 14, 1], Ar[13, 8, 1],

$$\text{Ar}[5, 12, 1], \text{Ar}[9, 2, 1], \text{Ar}[11, 4, 1]], \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -1+X & -1+X \\ -1+\frac{1}{X} & 0 & 0 & -1+X & 0 & 0 & -1+X \\ 0 & -1+\frac{1}{X} & 0 & -1+X & -1+\frac{1}{X} & 0 & 0 \\ 0 & -1+\frac{1}{X} & 0 & 0 & -1+\frac{1}{X} & 0 & 0 \\ -1+\frac{1}{X} & -1+\frac{1}{X} & 0 & -1+X & 0 & 0 & 0 \\ -1+\frac{1}{X} & 0 & 0 & -1+X & 0 & 0 & -1+X \\ -1+\frac{1}{X} & -1+\frac{1}{X} & 0 & -1+X & 0 & 0 & 0 \end{pmatrix},$$

$$\begin{pmatrix} -x & 0 & 0 & 0 & 0 & x & x \\ -x & -x & 0 & x & 0 & 0 & x \\ 0 & -x & -x & x & -x & 0 & 0 \\ 0 & -x & 0 & x & -x & 0 & 0 \\ -x & -x & 0 & x & -x & 0 & 0 \\ -x & 0 & 0 & x & 0 & x & x \\ -x & -x & 0 & x & 0 & 0 & x \end{pmatrix}, \begin{pmatrix} \frac{x-xX+xX^2-xX^3}{2-3X+3X^2-3X^3+2X^4} & \frac{-x+2xX-xX^2+xX^3-xX^4}{2-3X+3X^2-3X^3+2X^4} & 0 & \frac{-xX^2+xX^3-xX^4+xX^5}{2-3X+3X^2-3X^3+2X^4} \\ \frac{2x-3xX+2xX^2-3xX^3+2xX^4}{2-3X+3X^2-3X^3+2X^4} & \frac{2x-3xX+2xX^2-xX^3}{2-3X+3X^2-3X^3+2X^4} & 0 & \frac{-2x+3xX-3xX^2+3xX^3-xX^4}{2-3X+3X^2-3X^3+2X^4} \\ \frac{-x+xX-xX^2+xX^3}{2-3X+3X^2-3X^3+2X^4} & \frac{x-2xX+xX^2-xX^3+xX^4}{2-3X+3X^2-3X^3+2X^4} & 0 & \frac{-2x+5xX-5xX^2+5xX^3-4xX^4}{2-3X+3X^2-3X^3+2X^4} \\ \frac{-x+xX-xX^2+xX^3}{X(2-3X+3X^2-3X^3+2X^4)} & \frac{-x+3xX-5xX^2+5xX^3-4xX^4+2xX^5}{X(2-3X+3X^2-3X^3+2X^4)} & 0 & \frac{xX-xX^2+xX^3-xX^4}{2-3X+3X^2-3X^3+2X^4} \\ \frac{x-3xX+3xX^2-3xX^3+2xX^4}{2-3X+3X^2-3X^3+2X^4} & \frac{x-xX+xX^2-3xX^3+2xX^4}{2-3X+3X^2-3X^3+2X^4} & 0 & \frac{-2x+3xX-2xX^2+3xX^3-2xX^4}{2-3X+3X^2-3X^3+2X^4} \\ \frac{2x-3xX+2xX^2-3xX^3+2xX^4}{2-3X+3X^2-3X^3+2X^4} & \frac{2x-3xX+2xX^2-xX^3}{2-3X+3X^2-3X^3+2X^4} & 0 & \frac{-2x+3xX-3xX^2+3xX^3-xX^4}{2-3X+3X^2-3X^3+2X^4} \\ \frac{x-3xX+3xX^2-3xX^3+2xX^4}{2-3X+3X^2-3X^3+2X^4} & \frac{x-xX+xX^2-3xX^3+2xX^4}{2-3X+3X^2-3X^3+2X^4} & 0 & \frac{-2x+3xX-2xX^2+3xX^3-2xX^4}{2-3X+3X^2-3X^3+2X^4} \end{pmatrix},$$

Knot[7, 4] → {PD[X[6, 2, 7, 1], X[12, 6, 13, 5], X[14, 8, 1, 7], X[8, 14, 9, 13], X[2, 12, 3, 11], X[10, 4, 11, 3], X[4, 10, 5, 9]], GC[Ar[1, 6, 1], Ar[5, 12, 1], Ar[7, 14, 1], Ar[13, 8, 1],

$$\text{Ar}[11, 2, 1], \text{Ar}[3, 10, 1], \text{Ar}[9, 4, 1]], \begin{pmatrix} 0 & 0 & 0 & 0 & -1+X & 0 & -1+X \\ -1+\frac{1}{X} & 0 & 0 & -1+X & 0 & -1+\frac{1}{X} & 0 \\ 0 & -1+\frac{1}{X} & 0 & -1+X & 0 & -1+\frac{1}{X} & 0 \\ 0 & -1+\frac{1}{X} & 0 & 0 & 0 & -1+\frac{1}{X} & 0 \\ -1+\frac{1}{X} & 0 & 0 & -1+X & 0 & -1+\frac{1}{X} & -1+X \\ -1+\frac{1}{X} & 0 & 0 & -1+X & 0 & 0 & -1+X \\ -1+\frac{1}{X} & 0 & 0 & -1+X & 0 & 0 & 0 \end{pmatrix},$$

$$\begin{pmatrix} -x & 0 & 0 & 0 & x & 0 & x \\ -x & -x & 0 & x & 0 & -x & 0 \\ 0 & -x & -x & x & 0 & -x & 0 \\ 0 & -x & 0 & x & 0 & -x & 0 \\ -x & 0 & 0 & x & x & -x & x \\ -x & 0 & 0 & x & 0 & -x & x \\ -x & 0 & 0 & x & 0 & 0 & x \end{pmatrix}, \begin{pmatrix} \frac{2x-2xX}{4-7X+4X^2} & \frac{-2x+4xX-2xX^2}{4-7X+4X^2} & 0 & \frac{-2xX^2+2xX^3}{4-7X+4X^2} & \frac{-4x+9xX-7xX^2+2xX^3}{4-7X+4X^2} & - \\ \frac{2x-6xX+4xX^2}{4-7X+4X^2} & \frac{2x-3xX+xX^2}{4-7X+4X^2} & 0 & \frac{-4x+7xX-2xX^2-xX^3}{4-7X+4X^2} & \frac{2xX-3xX^2+xX^3}{4-7X+4X^2} & \frac{3}{4-7X+4X^2} \\ \frac{-2x+2xX}{4-7X+4X^2} & \frac{2x-4xX+2xX^2}{4-7X+4X^2} & 0 & \frac{-4x+11xX-9xX^2+2xX^3}{4-7X+4X^2} & \frac{2xX-4xX^2+2xX^3}{4-7X+4X^2} & \frac{x}{4-7X+4X^2} \\ \frac{-2x+2xX}{x(4-7X+4X^2)} & \frac{-2x+7xX-9xX^2+4xX^3}{x(4-7X+4X^2)} & 0 & \frac{2xX-2xX^2}{4-7X+4X^2} & \frac{2x-4xX+2xX^2}{4-7X+4X^2} & - \\ \frac{3x-7xX+4xX^2}{4-7X+4X^2} & \frac{x-xX}{4-7X+4X^2} & 0 & \frac{-4x+7xX-3xX^2}{4-7X+4X^2} & \frac{xX-xX^2}{4-7X+4X^2} & - \\ \frac{4x-8xX+4xX^2}{4-7X+4X^2} & \frac{xX-xX^2}{4-7X+4X^2} & 0 & \frac{-4x+7xX-4xX^2+xX^3}{4-7X+4X^2} & \frac{xX^2-xX^3}{4-7X+4X^2} & \frac{2}{4-7X+4X^2} \\ \frac{3x-7xX+4xX^2}{4-7X+4X^2} & \frac{x-xX}{4-7X+4X^2} & 0 & \frac{-4x+7xX-3xX^2}{4-7X+4X^2} & \frac{xX-xX^2}{4-7X+4X^2} & \frac{2}{4-7X+4X^2} \end{pmatrix}$$

Knot [7, 5] → {PD[X[1, 4, 2, 5], X[3, 10, 4, 11], X[5, 12, 6, 13],
 X[7, 14, 8, 1], X[13, 6, 14, 7], X[11, 8, 12, 9], X[9, 2, 10, 3]],
 GC[Ar[4, 1, -1], Ar[10, 3, -1], Ar[12, 5, -1], Ar[14, 7, -1], Ar[6, 13, -1],

$$\text{Ar}[8, 11, -1], \text{Ar}[2, 9, -1]], \begin{pmatrix} 0 & -1 + \frac{1}{X} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 + \frac{1}{X} & -1 + \frac{1}{X} & 0 & 0 & -1 + X \\ 0 & 0 & 0 & -1 + \frac{1}{X} & 0 & -1 + X & -1 + X \\ 0 & 0 & 0 & 0 & -1 + X & -1 + X & -1 + X \\ 0 & 0 & 0 & -1 + \frac{1}{X} & 0 & -1 + X & -1 + X \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 + X \\ 0 & -1 + \frac{1}{X} & -1 + \frac{1}{X} & -1 + \frac{1}{X} & 0 & 0 & 0 \end{pmatrix},$$

$$\begin{pmatrix} -x & -x & 0 & 0 & 0 & 0 & 0 \\ 0 & -x & -x & -x & 0 & 0 & x \\ 0 & 0 & -x & -x & 0 & x & x \\ 0 & 0 & 0 & -x & x & x & x \\ 0 & 0 & 0 & -x & x & x & x \\ 0 & 0 & 0 & 0 & 0 & x & x \\ 0 & -x & -x & -x & 0 & 0 & x \end{pmatrix}, \begin{pmatrix} 0 & \frac{-xX+2xX^2-2xX^3+xX^4}{2-4X+5X^2-4X^3+2X^4} & \frac{-x+2xX-2xX^2+xX^3}{2-4X+5X^2-4X^3+2X^4} & \frac{-x+xX}{2-4X+5X^2-4X^3+2X^4} \\ 0 & \frac{2x-4xX+4xX^2-3xX^3+xX^4}{2-4X+5X^2-4X^3+2X^4} & \frac{2x-5xX+6xX^2-5xX^3+2xX^4}{2-4X+5X^2-4X^3+2X^4} & \frac{2x-5xX+5xX^2-4xX^3+2xX^4}{2-4X+5X^2-4X^3+2X^4} \\ 0 & \frac{xX-xX^2}{2-4X+5X^2-4X^3+2X^4} & \frac{x-xX}{2-4X+5X^2-4X^3+2X^4} & \frac{x-2xX+3xX^2-4xX^3+2xX^4}{2-4X+5X^2-4X^3+2X^4} \\ 0 & \frac{xX^3-xX^4}{2-4X+5X^2-4X^3+2X^4} & \frac{xX^2-xX^3}{2-4X+5X^2-4X^3+2X^4} & \frac{2x-4xX+4xX^2-2xX^3}{2-4X+5X^2-4X^3+2X^4} \\ 0 & \frac{xX-xX^2}{2-4X+5X^2-4X^3+2X^4} & \frac{x-xX}{2-4X+5X^2-4X^3+2X^4} & \frac{x-2xX+3xX^2-4xX^3+2xX^4}{2-4X+5X^2-4X^3+2X^4} \\ 0 & \frac{x-2xX+2xX^2-xX^3}{2-4X+5X^2-4X^3+2X^4} & \frac{x-2xX+2xX^2-xX^3}{x(2-4X+5X^2-4X^3+2X^4)} & \frac{x-xX}{x(2-4X+5X^2-4X^3+2X^4)} \\ 0 & \frac{x-3xX+4xX^2-4xX^3+2xX^4}{2-4X+5X^2-4X^3+2X^4} & \frac{-x+3xX-5xX^2+5xX^3-4xX^4+2xX^5}{x(2-4X+5X^2-4X^3+2X^4)} & \frac{-x+2xX-4xX^2+5xX^3-4xX^4+2xX^5}{x(2-4X+5X^2-4X^3+2X^4)} \end{pmatrix}$$

Knot [7, 6] → {PD[X[1, 4, 2, 5], X[3, 8, 4, 9], X[5, 12, 6, 13], X[9, 1, 10, 14], X[13, 11, 14, 10],
 X[11, 6, 12, 7], X[7, 2, 8, 3]], GC[Ar[4, 1, -1], Ar[8, 3, -1], Ar[12, 5, -1], Ar[14, 9, 1],

$$\text{Ar}[10, 13, 1], \text{Ar}[6, 11, -1], \text{Ar}[2, 7, -1]], \begin{pmatrix} 0 & -1 + \frac{1}{X} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 + \frac{1}{X} & 0 & 0 & 0 & -1 + X \\ 0 & 0 & 0 & -1 + X & 0 & -1 + X & -1 + X \\ 0 & 0 & 0 & 0 & -1 + \frac{1}{X} & -1 + X & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 + X & 0 \\ 0 & 0 & 0 & -1 + X & 0 & 0 & -1 + X \\ 0 & -1 + \frac{1}{X} & -1 + \frac{1}{X} & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\begin{pmatrix} -x & -x & 0 & 0 & 0 & 0 & 0 \\ 0 & -x & -x & 0 & 0 & 0 & x \\ 0 & 0 & -x & x & 0 & x & x \\ 0 & 0 & 0 & x & -x & x & 0 \\ 0 & 0 & 0 & 0 & -x & x & 0 \\ 0 & 0 & 0 & x & 0 & x & x \\ 0 & -x & -x & 0 & 0 & 0 & x \end{pmatrix}, \begin{pmatrix} 0 & \frac{-x X+4 x X^2-4 x X^3+x X^4}{1-5 X+7 X^2-5 X^3+X^4} & \frac{-x+4 x X-4 x X^2+x X^3}{1-5 X+7 X^2-5 X^3+X^4} & \frac{-x X^3+2 x X^4-x X^5}{1-5 X+7 X^2-5 X^3+X^4} & \frac{-x X+}{1-5 X} \\ 0 & \frac{x-5 x X+6 x X^2-2 x X^3}{1-5 X+7 X^2-5 X^3+X^4} & \frac{x-6 x X+10 x X^2-6 x X^3+x X^4}{1-5 X+7 X^2-5 X^3+X^4} & \frac{-x X^4+x X^5}{1-5 X+7 X^2-5 X^3+X^4} & \frac{-x X}{1-5 X} \\ 0 & \frac{-x X^2+x X^3}{1-5 X+7 X^2-5 X^3+X^4} & \frac{-x X+x X^2}{1-5 X+7 X^2-5 X^3+X^4} & \frac{-x+5 x X-7 x X^2+4 x X^3-x X^5}{1-5 X+7 X^2-5 X^3+X^4} & \frac{-x X+}{1-5 X} \\ 0 & \frac{x-2 x X+x X^2}{1-5 X+7 X^2-5 X^3+X^4} & \frac{x-2 x X+x X^2}{X(1-5 X+7 X^2-5 X^3+X^4)} & \frac{x X-2 x X^2+2 x X^3-x X^4}{1-5 X+7 X^2-5 X^3+X^4} & \frac{3 x-8}{1} \\ 0 & \frac{x X-2 x X^2+x X^3}{1-5 X+7 X^2-5 X^3+X^4} & \frac{x-2 x X+x X^2}{1-5 X+7 X^2-5 X^3+X^4} & \frac{x X^2-2 x X^3+2 x X^4-x X^5}{1-5 X+7 X^2-5 X^3+X^4} & \frac{x-3 x}{1} \\ 0 & \frac{-x X+x X^2}{1-5 X+7 X^2-5 X^3+X^4} & \frac{-x+x X}{1-5 X+7 X^2-5 X^3+X^4} & \frac{-x+5 x X-8 x X^2+6 x X^3-2 x X^4}{1-5 X+7 X^2-5 X^3+X^4} & \frac{-x+2}{1-5 X} \\ 0 & \frac{-2 x X+6 x X^2-5 x X^3+x X^4}{1-5 X+7 X^2-5 X^3+X^4} & \frac{-x+4 x X-6 x X^2+7 x X^3-5 x X^4+x X^5}{X(1-5 X+7 X^2-5 X^3+X^4)} & \frac{-x X^2+x X^3}{1-5 X+7 X^2-5 X^3+X^4} & \frac{-x}{1-5 X} \end{pmatrix}$$

Knot[7, 7] → {PD[X[1, 4, 2, 5], X[5, 10, 6, 11], X[3, 9, 4, 8], X[9, 3, 10, 2], X[11, 14, 12, 1], X[7, 13, 8, 12], X[13, 7, 14, 6]], GC[Ar[4, 1, -1], Ar[10, 5, -1], Ar[8, 3, 1], Ar[2, 9, 1],

$$\text{Ar}[14, 11, -1], \text{Ar}[12, 7, 1], \text{Ar}[6, 13, 1]], \begin{pmatrix} 0 & 0 & -1+X & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1+\frac{1}{X} & 0 & -1+X & 0 \\ 0 & -1+\frac{1}{X} & 0 & 0 & 0 & -1+X & 0 \\ 0 & -1+\frac{1}{X} & -1+X & 0 & 0 & -1+X & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1+\frac{1}{X} \\ 0 & 0 & 0 & -1+\frac{1}{X} & -1+\frac{1}{X} & 0 & 0 \\ 0 & 0 & 0 & -1+\frac{1}{X} & -1+\frac{1}{X} & -1+X & 0 \end{pmatrix},$$

$$\begin{pmatrix} -x & 0 & x & 0 & 0 & 0 & 0 \\ 0 & -x & 0 & -x & 0 & x & 0 \\ 0 & -x & x & 0 & 0 & x & 0 \\ 0 & -x & x & -x & 0 & x & 0 \\ 0 & 0 & 0 & 0 & -x & 0 & -x \\ 0 & 0 & 0 & -x & -x & x & 0 \\ 0 & 0 & 0 & -x & -x & x & -x \end{pmatrix}, \begin{pmatrix} 0 & \frac{-x+4 x X-4 x X^2+x X^3}{1-5 X+9 X^2-5 X^3+X^4} & \frac{x X-4 x X^2+4 x X^3-x X^4}{1-5 X+9 X^2-5 X^3+X^4} & \frac{-x+4 x X-6 x X^2+4 x X^3-x X^4}{X(1-5 X+9 X^2-5 X^3+X^4)} & \frac{-x}{1-5 X} \\ 0 & \frac{x-3 x X+2 x X^2}{1-5 X+9 X^2-5 X^3+X^4} & \frac{-x X+3 x X^2-2 x X^3}{1-5 X+9 X^2-5 X^3+X^4} & \frac{2 x-7 x X+9 x X^2-5 x X^3+x X^4}{1-5 X+9 X^2-5 X^3+X^4} & \frac{2 x}{1-5 X} \\ 0 & \frac{2 x-8 x X+10 x X^2-5 x X^3+x X^4}{1-5 X+9 X^2-5 X^3+X^4} & \frac{-x+4 x X-6 x X^2+4 x X^3-x X^4}{1-5 X+9 X^2-5 X^3+X^4} & \frac{x-3 x X+3 x X^2-x X^3}{X(1-5 X+9 X^2-5 X^3+X^4)} & \frac{-x}{1-5 X} \\ 0 & \frac{x-4 x X+6 x X^2-4 x X^3+x X^4}{1-5 X+9 X^2-5 X^3+X^4} & \frac{-x+5 x X-10 x X^2+8 x X^3-2 x X^4}{1-5 X+9 X^2-5 X^3+X^4} & \frac{x-3 x X+3 x X^2-x X^3}{1-5 X+9 X^2-5 X^3+X^4} & \frac{-x}{1-5 X} \\ 0 & \frac{-x X+2 x X^2-x X^3}{1-5 X+9 X^2-5 X^3+X^4} & \frac{x X^2-2 x X^3+x X^4}{1-5 X+9 X^2-5 X^3+X^4} & \frac{-x X+x X^2}{1-5 X+9 X^2-5 X^3+X^4} & \frac{x-4}{1-5 X} \\ 0 & \frac{-x X+x X^2}{1-5 X+9 X^2-5 X^3+X^4} & \frac{x X^2-x X^3}{1-5 X+9 X^2-5 X^3+X^4} & \frac{x-6 x X+9 x X^2-5 x X^3+x X^4}{1-5 X+9 X^2-5 X^3+X^4} & \frac{2 x-}{1-5 X} \\ 0 & \frac{-x X^2+x X^3}{1-5 X+9 X^2-5 X^3+X^4} & \frac{x X^3-x X^4}{1-5 X+9 X^2-5 X^3+X^4} & \frac{x-5 x X+8 x X^2-5 x X^3+x X^4}{1-5 X+9 X^2-5 X^3+X^4} & \frac{x-4}{1-5 X} \end{pmatrix}$$

```
{
Length[Ks = AllKnots[{3, 12}]],
Length[Union[Alexander[#][t] & /@ Ks]],
Length[Union[wAlexander[#] & /@ Ks]],
Length[Union[Pair[Alexander[#][t], wAlexander[#]] & /@ Ks]]
}
```

KnotTheory::loading: Loading precomputed data in KnotTheory/12A.dts.

KnotTheory::loading: Loading precomputed data in KnotTheory/12N.dts.

\$Aborted


```

Ks = AllKnots[10];
As = Alexander[#][t] & /@ Ks;
Ps = Position[As, #][[1, 1]] & /@ As;
Position[MapThread[Equal, {Ps, Range[Length[Ps]]}], False]

{{24}, {37}, {52}, {54}, {56}, {68}, {74}, {75}, {77}, {98}, {103}, {135}, {150}, {162}, {164}}

Ks = AllKnots[10];
As = wAlexander[#] & /@ Ks;
Ps = Position[As, #][[1, 1]] & /@ As;
Position[MapThread[Equal, {Ps, Range[Length[Ps]]}], False]

{{24}, {37}, {52}, {54}, {56}, {68}, {74}, {75}, {77}, {98}, {103}, {135}, {150}, {162}, {164}}

Alexander[#][t] & /@ {Knot[10, 42], Knot[10, 75]}

{27 - 1/t^3 + 7/t^2 - 19/t - 19t + 7t^2 - t^3, 27 - 1/t^3 + 7/t^2 - 19/t - 19t + 7t^2 - t^3}

wAlexander /@ {Knot[10, 42], Knot[10, 75]}

KnotTheory::loading: Loading precomputed data in PD4Knots`.

{3x - 14xX + 19xX^2 - 19xX^4 + 14xX^5 - 3xX^6, 3x - 14xX + 19xX^2 - 19xX^4 + 14xX^5 - 3xX^6}
{1 - 7X + 19X^2 - 27X^3 + 19X^4 - 7X^5 + X^6, 1 - 7X + 19X^2 - 27X^3 + 19X^4 - 7X^5 + X^6}

test[K_] := (
  wAlexander[K] == ExpandDenominator[ExpandNumerator[Factor[
    -x * X * D[Alexander[K][X], X] / Alexander[K][X]
  ]]]
)

test[Knot[10, 75]]

True

And @@ (test /@ AllKnots[{3, 11}])

True

And @@ (test /@ AllKnots[12, Alternating])

$Aborted

And @@ (test /@ AllKnots[12, NonAlternating])

```

The smallest former breakdown :

```

wAlex[GC[Ar[5, 3, -1], Ar[6, 2, +1]]]

{0}

wAlex[GC[Ar[1, 4, +1], Ar[5, 3, -1], Ar[6, 2, +1]]]

{0}

```

A long 4 crossing positive knot is a long trefoil

```

Compare[common_GC, extras_List] := Thread[
  wAlex[common~Join~#] & /@ extras
];

Compare[GC[Ar[1, 8, +1]],
{
  GC[Ar[7, 2, +1], Ar[12, 3, +1], Ar[6, 11, +1]],
  GC[Ar[6, 3, +1], Ar[11, 2, +1], Ar[7, 12, +1]]
}
]

{ {  $\frac{x - x X^2}{1 - X + X^2}$ ,  $\frac{x - x X^2}{1 - X + X^2}$  } }

wAlex[GC[Ar[1, 6, +1], Ar[5, 2, +1], Ar[8, 3, +1], Ar[4, 7, +1]]]

{  $\frac{x - x X^2}{1 - X + X^2}$  }

wAlex[GC[Ar[1, 8, +1], Ar[7, 2, +1],
  Ar[12, 3, +1], Ar[4, 9, +1], Ar[5, 10, -1], Ar[6, 11, +1]]]

{  $\frac{x - x X^2}{1 - X + X^2}$  }

wAlex[GC[Ar[1, 8, +1], Ar[11, 2, +1],
  Ar[6, 3, +1], Ar[4, 9, +1], Ar[5, 10, -1], Ar[7, 12, +1]]]

{  $\frac{x - x X^2}{1 - X + X^2}$  }

```

Other checks :

```

wAlex[GC[Ar[1, 3, +1], Ar[4, 2, +1]]]

{  $\frac{x - x X^2}{1 - X + X^2}$  }

wAlex[GC[Ar[1, 5, +1], Ar[6, 4, -1], Ar[7, 3, +1], Ar[8, 2, +1]]]

{  $\frac{x - x X^2}{1 - X + X^2}$  }

wAlex[BR[2, {1}]]

{PD[X[1, 1, 2, 2]], GC[Ar[1, 1, -1]], 0}

wAlex[BR[2, {1, 1, -1}]]

{PD[X[3, 1, 4, 6], X[1, 5, 2, 4], X[2, 5, 3, 6]], GC[Ar[6, 3, 1], Ar[4, 1, 1], Ar[5, 2, -1]], 0}

```

wAlex[BR[2, {1, 1, 1}]]

$$\left\{ \text{PD}[X[3, 1, 4, 6], X[1, 5, 2, 4], X[5, 3, 6, 2]], \right. \\ \left. \text{GC}[\text{Ar}[6, 3, 1], \text{Ar}[4, 1, 1], \text{Ar}[2, 5, 1]], \frac{x - x X^2}{1 - X + X^2} \right\}$$

wAlex[BR[3, {1, 2, 1, 2}]]

$$\left\{ \text{PD}[X[8, 6, 1, 5], X[3, 7, 4, 6], X[4, 2, 5, 1], X[7, 3, 8, 2]], \right. \\ \left. \text{GC}[\text{Ar}[5, 8, 1], \text{Ar}[6, 3, 1], \text{Ar}[1, 4, 1], \text{Ar}[2, 7, 1]], \frac{x - x X^2}{1 - X + X^2} \right\}$$

wAlex[BR[3, {-1, 2, -1, 2}]]

$$\left\{ \text{PD}[X[5, 8, 6, 1], X[3, 7, 4, 6], X[1, 4, 2, 5], X[7, 3, 8, 2]], \right. \\ \left. \text{GC}[\text{Ar}[8, 5, -1], \text{Ar}[6, 3, 1], \text{Ar}[4, 1, -1], \text{Ar}[2, 7, 1]], \frac{x - x X^2}{1 - 3 X + X^2} \right\}$$

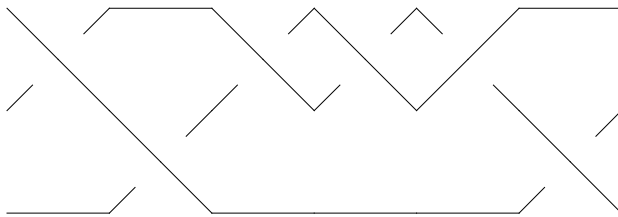
wAlex[BR[3, {1, -2, 1, -2}]]

$$\left\{ \text{PD}[X[8, 6, 1, 5], X[6, 3, 7, 4], X[4, 2, 5, 1], X[2, 7, 3, 8]], \right. \\ \left. \text{GC}[\text{Ar}[5, 8, 1], \text{Ar}[3, 6, -1], \text{Ar}[1, 4, 1], \text{Ar}[7, 2, -1]], \frac{x - x X^2}{1 - 3 X + X^2} \right\}$$

wAlex[BR[3, {-2, 1, -2, 1}]]

$$\left\{ \text{PD}[X[6, 3, 7, 4], X[4, 2, 5, 1], X[2, 7, 3, 8], X[8, 6, 1, 5]], \right. \\ \left. \text{GC}[\text{Ar}[3, 6, -1], \text{Ar}[1, 4, 1], \text{Ar}[7, 2, -1], \text{Ar}[5, 8, 1]], \frac{x - x X^2}{1 - 3 X + X^2} \right\}$$

BraidPlot[BR[3, {1, 2, 1, 1, -1, 2}]]



wAlex[BR[3, {-1, 2, 1, 2, 1, 1}]]

$$\left\{ \text{PD}[X[7, 12, 8, 1], X[3, 9, 4, 8], X[4, 2, 5, 1], X[9, 3, 10, 2], X[10, 6, 11, 5], X[6, 12, 7, 11]], \right. \\ \left. \text{GC}[\text{Ar}[12, 7, -1], \text{Ar}[8, 3, 1], \text{Ar}[1, 4, 1], \text{Ar}[2, 9, 1], \text{Ar}[5, 10, 1], \text{Ar}[11, 6, 1]], \frac{x - x X^2}{1 - X + X^2} \right\}$$

wAlex[BR[3, {1, 2, 1, 1, -1, 2}]]

$$\left\{ \text{PD}[X[5, 3, 6, 2], X[10, 4, 11, 3], X[11, 7, 12, 6], X[7, 1, 8, 12], X[8, 1, 9, 2], X[4, 10, 5, 9]], \right. \\ \left. \text{GC}[\text{Ar}[2, 5, 1], \text{Ar}[3, 10, 1], \text{Ar}[6, 11, 1], \text{Ar}[12, 7, 1], \text{Ar}[1, 8, -1], \text{Ar}[9, 4, 1]], \frac{x - x X^2}{1 - X + X^2} \right\}$$

```

Table[
  {k, wAlex[
    GC[Ar[2, 5, 1], Ar[3, 10, 1], Ar[6, 11, 1], Ar[12, 7, 1], Ar[1, 8, -1], Ar[9, 4, 1]] /.
    Ar[t_, h_, s_] => Ar[(k+t)~Mod~12, (k+h)~Mod~12, s]
  ]},
  {k, 5, 6}
]

{{5, { $\frac{x - x X^2}{1 - X + X^2}$ }}, {6, { $\frac{x - x X^2}{1 - X + X^2}$ }}}

wAlex[BR[3, {1, 2, 1, 2, -2, 2}]]

{PD[X[10, 6, 11, 5], X[3, 7, 4, 6], X[4, 12, 5, 11], X[7, 1, 8, 12], X[8, 1, 9, 2], X[9, 3, 10, 2]],
 GC[Ar[5, 10, 1], Ar[6, 3, 1], Ar[11, 4, 1], Ar[12, 7, 1], Ar[1, 8, -1], Ar[2, 9, 1]],  $\frac{x - x X^2}{1 - X + X^2}$ }

wAlex[Knot[4, 1]]

{PD[X[4, 2, 5, 1], X[8, 6, 1, 5], X[6, 3, 7, 4], X[2, 7, 3, 8]],
 GC[Ar[1, 4, 1], Ar[5, 8, 1], Ar[3, 6, -1], Ar[7, 2, -1]],  $\frac{x - x X^2}{1 - 3 X + X^2}$ }

wAlex[Mirror[Knot[4, 1]]]

{PD[X[1, 4, 2, 5], X[5, 8, 6, 1], X[3, 7, 4, 6], X[7, 3, 8, 2]],
 GC[Ar[4, 1, -1], Ar[8, 5, -1], Ar[6, 3, 1], Ar[2, 7, 1]],  $\frac{x - x X^2}{1 - 3 X + X^2}$ }

wAlex[Knot[5, 1]]

{PD[X[1, 6, 2, 7], X[3, 8, 4, 9], X[5, 10, 6, 1], X[7, 2, 8, 3], X[9, 4, 10, 5]],
 GC[Ar[6, 1, -1], Ar[8, 3, -1], Ar[10, 5, -1], Ar[2, 7, -1], Ar[4, 9, -1]],  $\frac{2 x - x X + x X^3 - 2 x X^4}{1 - X + X^2 - X^3 + X^4}$ }

wAlex[Knot[10, 132]]

{PD[X[4, 2, 5, 1], X[8, 4, 9, 3], X[5, 12, 6, 13], X[15, 18, 16, 19], X[9, 16, 10, 17],
 X[17, 10, 18, 11], X[13, 20, 14, 1], X[19, 14, 20, 15], X[11, 6, 12, 7], X[2, 8, 3, 7]],
 GC[Ar[1, 4, 1], Ar[3, 8, 1], Ar[12, 5, -1], Ar[18, 15, -1], Ar[16, 9, -1], Ar[10, 17, -1],
 Ar[20, 13, -1], Ar[14, 19, -1], Ar[6, 11, -1], Ar[7, 2, 1]],  $\frac{2 x - x X + x X^3 - 2 x X^4}{1 - X + X^2 - X^3 + X^4}$ }

wAlex[BR[2, {-1, -1, -1}]]

{PD[X[6, 3, 1, 4], X[4, 1, 5, 2], X[2, 5, 3, 6]],
 GC[Ar[3, 6, -1], Ar[1, 4, -1], Ar[5, 2, -1]],  $\frac{x - x X^2}{1 - X + X^2}$ }

```

Morals

wAlex2[Knot[10, 132]]

{PD[X[4, 2, 5, 1], X[8, 4, 9, 3], X[5, 12, 6, 13], X[15, 18, 16, 19], X[9, 16, 10, 17],
 X[17, 10, 18, 11], X[13, 20, 14, 1], X[19, 14, 20, 15], X[11, 6, 12, 7], X[2, 8, 3, 7]],
 GC[Ar[1, 4, 1], Ar[3, 8, 1], Ar[12, 5, -1], Ar[18, 15, -1], Ar[16, 9, -1],
 Ar[10, 17, -1], Ar[20, 13, -1], Ar[14, 19, -1], Ar[6, 11, -1], Ar[7, 2, 1]],

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 + X \\ -1 + \frac{1}{X} & 0 & -1 + \frac{1}{X} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 + \frac{1}{X} & 0 & 0 & -1 + \frac{1}{X} & 0 & 0 & 0 & -1 + X & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 + X & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 + \frac{1}{X} & 0 & 0 & -1 + \frac{1}{X} & 0 & -1 + X & 0 & 0 \\ 0 & 0 & 0 & -1 + \frac{1}{X} & 0 & 0 & -1 + \frac{1}{X} & 0 & -1 + X & 0 & 0 \\ 0 & 0 & 0 & -1 + \frac{1}{X} & 0 & -1 + X & 0 & -1 + X & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 + \frac{1}{X} & 0 & -1 + X & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 + \frac{1}{X} & 0 & 0 & -1 + \frac{1}{X} & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 + \frac{1}{X} & 0 & -1 + \frac{1}{X} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\begin{pmatrix} -x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x \\ -x & -x & -x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -x & -x & 0 & -x & 0 & 0 & 0 & x & 0 & 0 \\ 0 & 0 & 0 & -x & 0 & x & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -x & -x & 0 & -x & 0 & x & 0 & 0 \\ 0 & 0 & 0 & -x & 0 & x & -x & 0 & x & 0 & 0 \\ 0 & 0 & 0 & -x & 0 & x & -x & x & 0 & 0 & 0 \\ 0 & 0 & 0 & -x & 0 & x & 0 & x & 0 & 0 & 0 \\ 0 & -x & 0 & 0 & -x & 0 & 0 & 0 & x & 0 & 0 \\ -x & 0 & -x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x \end{pmatrix},$$

$$\begin{pmatrix} \frac{3x-8xX+8xX^2-3xX^3}{1-X+X^2-X^3+X^4} & \frac{2x-7xX+10xX^2-7xX^3+2xX^4}{1-X+X^2-X^3+X^4} & \frac{3x-8xX+8xX^2-3xX^3}{1-X+X^2-X^3+X^4} \\ \frac{-2x+4xX-2xX^2-xX^3+xX^4}{1-X+X^2-X^3+X^4} & \frac{-2x+5xX-5xX^2+2xX^3}{1-X+X^2-X^3+X^4} & \frac{-2x+4xX-2xX^2-xX^3+x}{1-X+X^2-X^3+X^4} \\ \frac{-2x+5xX-5xX^2+2xX^3}{1-X+X^2-X^3+X^4} & \frac{-x+4xX-6xX^2+4xX^3-xX^4}{1-X+X^2-X^3+X^4} & \frac{-2x+5xX-5xX^2+2xX^3}{1-X+X^2-X^3+X^4} \\ \frac{-x+3xX-3xX^2+xX^3}{1-X+X^2-X^3+X^4} & \frac{-x+3xX-4xX^2+3xX^3-xX^4}{1-X+X^2-X^3+X^4} & \frac{-x+3xX-3xX^2+xX^3}{1-X+X^2-X^3+X^4} \\ \frac{x-2xX+xX^2}{1-X+X^2-X^3+X^4} & \frac{x-2xX+2xX^2-xX^3}{1-X+X^2-X^3+X^4} & \frac{x-2xX+xX^2}{1-X+X^2-X^3+X^4} \\ \frac{x-2xX+xX^2}{1-X+X^2-X^3+X^4} & \frac{x-2xX+2xX^2-xX^3}{1-X+X^2-X^3+X^4} & \frac{x-2xX+xX^2}{1-X+X^2-X^3+X^4} \\ \frac{-x+3xX-3xX^2+xX^3}{1-X+X^2-X^3+X^4} & \frac{-x+3xX-4xX^2+3xX^3-xX^4}{1-X+X^2-X^3+X^4} & \frac{-x+3xX-3xX^2+xX^3}{1-X+X^2-X^3+X^4} \\ \frac{-x+3xX-3xX^2+xX^3}{X(1-X+X^2-X^3+X^4)} & \frac{-x+3xX-4xX^2+3xX^3-xX^4}{X(1-X+X^2-X^3+X^4)} & \frac{-x+3xX-3xX^2+xX^3}{X(1-X+X^2-X^3+X^4)} \\ \frac{-2x+5xX-5xX^2+2xX^3}{X(1-X+X^2-X^3+X^4)} & \frac{-2x+6xX-8xX^2+6xX^3-3xX^4+xX^5}{X(1-X+X^2-X^3+X^4)} & \frac{-2x+5xX-5xX^2+2xX^3}{X(1-X+X^2-X^3+X^4)} \\ \frac{-2x+4xX-2xX^2-xX^3+xX^4}{1-X+X^2-X^3+X^4} & \frac{-2x+5xX-5xX^2+2xX^3}{1-X+X^2-X^3+X^4} & \frac{-2x+4xX-2xX^2-xX^3+x}{1-X+X^2-X^3+X^4} \end{pmatrix}$$


```

{CC/x, -IdentityMatrix[Length[BB]] + (D[BB, X] /. X -> 1)}
{{{ -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}, {-1, -1, -1, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, -1, -1, 0, -1, 0, 0, 0, 1, 0}, {0, 0, 0, -1, 0, 1, 0, 0, 0, 0, 0},
 {0, 0, 0, -1, -1, 0, -1, 0, 1, 0}, {0, 0, 0, -1, 0, 1, -1, 0, 1, 0},
 {0, 0, 0, -1, 0, 1, -1, 1, 0, 0}, {0, 0, 0, -1, 0, 1, 0, 1, 0, 0},
 {0, -1, 0, 0, -1, 0, 0, 0, 1, 0}, {-1, 0, -1, 0, 0, 0, 0, 0, 0, 1}},
{{ -1, 0, 0, 0, 0, 0, 0, 0, 0, 1}, {-1, -1, -1, 0, 0, 0, 0, 0, 0, 0},
 {0, -1, -1, 0, -1, 0, 0, 0, 1, 0}, {0, 0, 0, -1, 0, 1, 0, 0, 0, 0},
 {0, 0, 0, -1, -1, 0, -1, 0, 1, 0}, {0, 0, 0, -1, 0, -1, -1, 0, 1, 0},
 {0, 0, 0, -1, 0, 1, -1, 1, 0, 0}, {0, 0, 0, -1, 0, 1, 0, -1, 0, 0},
 {0, -1, 0, 0, -1, 0, 0, -1, 0}, {-1, 0, -1, 0, 0, 0, 0, 0, 0, -1}}}

{1, -1}.%

{{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2}}

% // MatrixForm


$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$


```

- The example on <http://katlas.math.toronto.edu/drorbn/bbs/show?shot=Archibald-080709-110259.jpg> is :

```
wAlex2[GC[Ar[1, 3, +1], Ar[2, 5, -1], Ar[6, 4, +1]]]
```

$$\left\{ \begin{pmatrix} 0 & 0 & 0 \\ -1 + \frac{1}{x} & 0 & -1 + X \\ 0 & -1 + X & 0 \end{pmatrix}, \begin{pmatrix} -x & 0 & 0 \\ -x & x & x \\ 0 & x & x \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ \frac{x-2xX^2+xX^3}{(-2+x)X^2} & \frac{x-xX}{-2+x} & \frac{x-xX}{-2+x} \\ \frac{-x+xX}{(-2+x)X^2} & \frac{x-xX}{-2+x} & \frac{x-xX}{-2+x} \end{pmatrix}, \frac{2x-2xX}{-2+X} \right\}$$