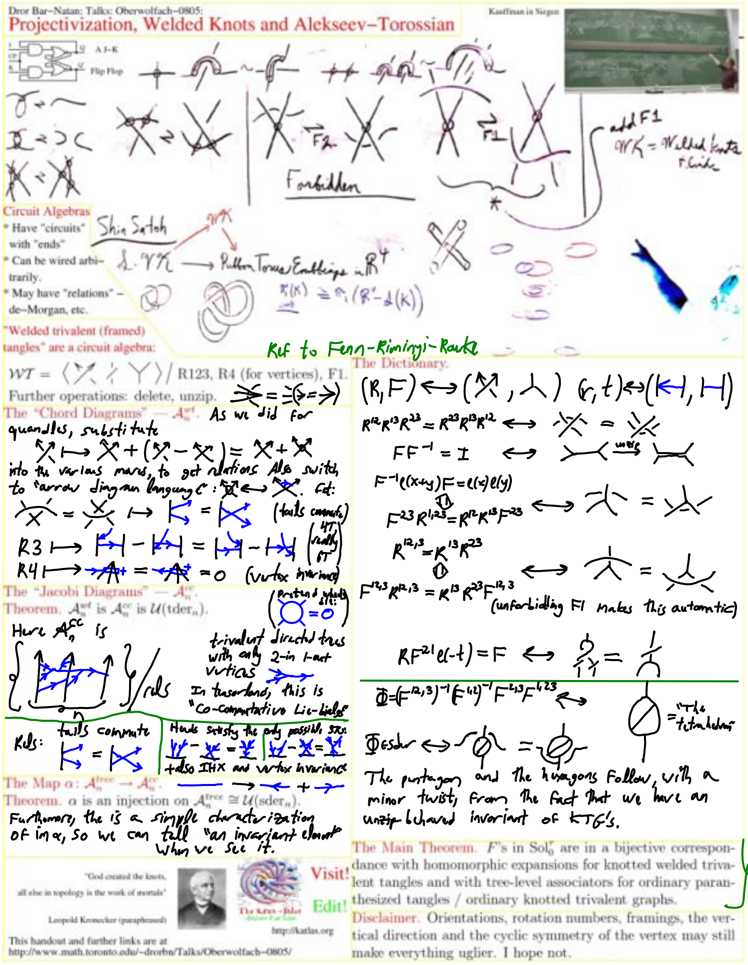
Ober handout

May-05-08
2:29 AM



Dror Bar-Natan: Talks: Oberwolfach-0805:

Projectivization, Welded Knots and Alekseev–Torossian

Kauffman in Siegen

A J-K Flip Flop

Forbidden

add F1

Circuit Algebras
* Have "circuits" with "ends"
* Can be wired arbitrarily.
* May have "relations" – de-Morgan, etc.

Shin Satoh

Ribbon Torus Embeddings in $\mathbb{R}^4$

"Welded trivalent (framed) tangles" are a circuit algebra:

Ref to Fenn-Rimanyi-Rourke

$\mathcal{WT} = \langle \ldots \rangle /$ R123, R4 (for vertices), F1.

Further operations: delete, unzip.

The "Chord Diagrams" — $\mathcal{A}_n^{wt}$. As we did for quandles, substitute ... into the various moves, to get relations. Also switch to "arrow diagram language". Get: (tails commute), R3 ↦ ... (4T, really 6T), R4 ↦ ... = 0 (vertex invariance)

The "Jacobi Diagrams" — $\mathcal{A}_n^{cc}$.

Theorem. $\mathcal{A}_n^{wt}$ is $\mathcal{A}_n^{cc}$ is $\mathcal{U}(\mathrm{tder}_n)$.

(Pretend wheels die: ... = 0)

Here $\mathcal{A}_n^{cc}$ is { ... } / rels — trivalent directed trees with only 2-in 1-out vertices. In tensorland, this is "co-commutative Lie-bialg".

Rels: tails commute | Heads satisfy the only possible STU: ... + also IHX and vertex invariance

The Map $\alpha: \mathcal{A}_n^{tree} \to \mathcal{A}_n^{cc}$.

Theorem. α is an injection on $\mathcal{A}_n^{tree} \cong \mathcal{U}(\mathrm{sder}_n)$.

Furthermore, the is a simple characterization of im α, so we can tell "an invariant element" when we see it.

The Dictionary.

$(R, F) \leftrightarrow (\ldots)$ $(r, t) \leftrightarrow (\ldots)$

$R^{12}R^{13}R^{23} = R^{23}R^{13}R^{12} \leftrightarrow \ldots$

$FF^{-1} = I \leftrightarrow \ldots$

$F^{-1}e(x+y)F = e(x)e(y)$

$F^{23}R^{1,23} = R^{12}R^{13}F^{23} \leftrightarrow \ldots$

$R^{12,3} = R^{13}R^{23}$

$F^{12,3}R^{12,3} = R^{13}R^{23}F^{12,3}$ (unforbidding F1 makes this automatic)

$RF^{21}e(-t) = F \leftrightarrow \ldots$

$\Phi = (F^{12,3})^{-1}(F^{12})^{-1}F^{1,23}F^{23} \leftrightarrow$ "the tetrahedron"

$\Phi \in \mathrm{sder} \leftrightarrow \ldots$

The pentagon and the hexagons follow, with a minor twist, from the fact that we have an unzip-behaved invariant of KTG's.

"God created the knots, all else in topology is the work of mortals" Leopold Kronecker (paraphrased)

Visit! Edit! The Knot Atlas http://katlas.org

This handout and further links are at http://www.math.toronto.edu/~drorbn/Talks/Oberwolfach-0805/

The Main Theorem. F's in Sol_0^F are in a bijective correspondance with homomorphic expansions for knotted welded trivalent tangles and with tree-level associators for ordinary parenthesized tangles / ordinary knotted trivalent graphs.

Rephrase

Disclaimer. Orientations, rotation numbers, framings, the vertical direction and the cyclic symmetry of the vertex may still make everything uglier. I hope not.

To do.

√1. Add $t \leftrightarrow$ ⊢⊣
$r \leftrightarrow$ ⊢→⊣

2. Rearrange views

√3. Add the vertex invariance relations.

√4. Draw a picture of R4.

√5. Re-order the formulas in "The Dictionary".

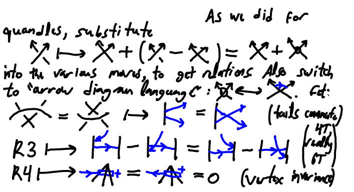
~~6. Add "This is a standard 3-part math talk, with a handout".~~

√7. Fix the wheels issue.

~~8. Change "Theorems" to "Dreams".~~

√9. Better study unzip.

10. Start with the disclaimer.



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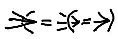

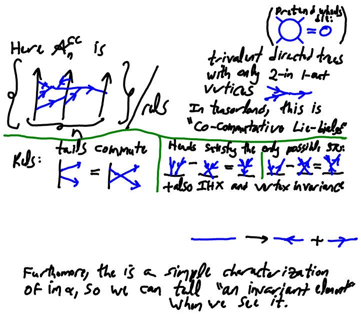

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