Everything around sl_{2+}^{ϵ} is **DoPeGDO**. So what?

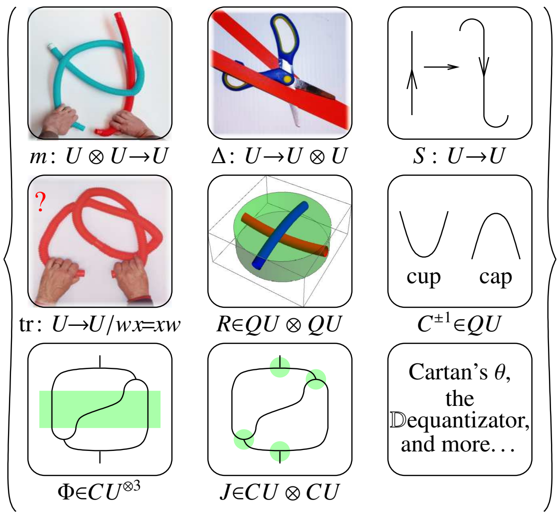
Thanks for inviting me to UCLA!
ωεβ:=http://drorbn.net/la19/
More at ωεβ/talks

Continues Rozansky [Ro1, Ro2, Ro3] and Overbay [Ov], joint with van der Veen [BV].

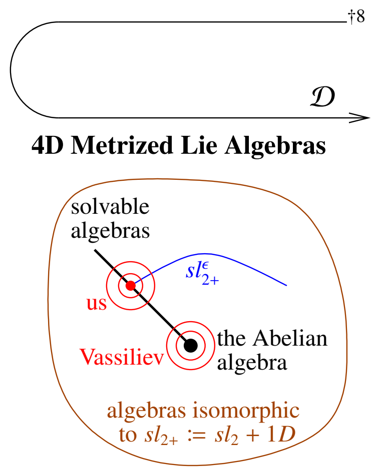
Abstract. I'll explain what "everything around" means: classical and quantum m, Δ, S, tr, R, C, and θ, as well as P, Φ, J, $\mathbb{D}$, and more, and all of their compositions. What **DoPeGDO** means: the category of Docile Perturbed Gaussian Differential Operators. And what sl_{2+}^{ϵ} means: a solvable approximation of the semi-simple Lie algebra sl_2.

Knot theorists should rejoice because all this leads to very powerful and well-behaved poly-time-computable knot invariants. Quantum algebraists should rejoice because it's a realistic playground for testing complicated equations and theories.

Conventions. 1. For a set A, let $z_A := \{z_i\}_{i\in A}$ and let $\zeta_A := \{z_i^* = \zeta_i\}_{i\in A}$.†1 2. Everything converges!



Less Abstract



DoPeGDO := The category with objects finite sets†2 and mor$(A\to B)$:

$$\{\mathcal{F} = \omega\exp(Q+P)\} \subset \mathbb{Q}[\![\zeta_A, z_B, \epsilon]\!]$$

Where: • ω is a scalar.†3 • Q is a "small" ϵ-free quadratic in $\zeta_A\cup z_B$.†4 • P is a "docile perturbation": $P=\sum_{k\geq1}\epsilon^k P^{(k)}$, where $\deg P^{(k)}\leq 2k+2$.†5
• Compositions:†6

$$\mathcal{F}/\!\!/\mathcal{G} = \mathcal{G}\circ\mathcal{F} := \left(\mathcal{G}|_{\zeta_i\to\partial_{z_i}}\mathcal{F}\right)_{z_i=0} = \left(\mathcal{F}|_{z_i\to\partial_{\zeta_i}}\mathcal{G}\right)_{\zeta_i=0}.$$

Cool! $(V^*)^{\otimes\Sigma}\otimes V^{\otimes S}$ explodes; the ranks of quadratics and bounded-degree polynomials grow slowly!†7 Representation theory is over-rated!

Cool! How often do you see a computational toolbox so successful?

Our Algebras. Let $sl_{2+}^{\epsilon} := L\langle y,b,a,x\rangle$ subject to $[a,x]=x$, $[b,y]=-\epsilon y$, $[a,b]=0$, $[a,y]=-y$, $[b,x]=\epsilon x$, and $[x,y]=\epsilon a+b$. So $t:=\epsilon a-b$ is central and if $\exists\epsilon^{-1}$, $sl_{2+}^{\epsilon}/\langle t\rangle\cong sl_2$. ωεβ/oa

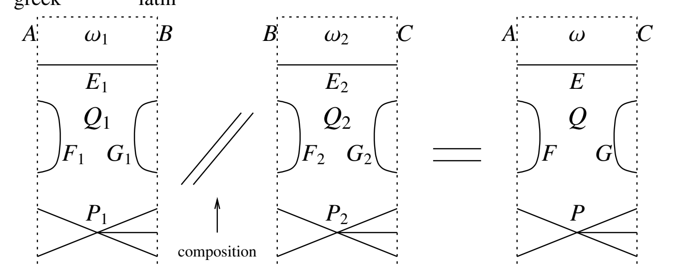
U is either $CU = \mathcal{U}(sl_{2+}^{\epsilon})[\![\hbar]\!]$ or $QU = \mathcal{U}_\hbar(sl_{2+}^{\epsilon}) = A\langle y,b,a,x\rangle[\![\hbar]\!]$ with $[a,x]=x$, $[b,y]=-\epsilon y$, $[a,b]=0$, $[a,y]=-y$, $[b,x]=\epsilon x$, and $xy-qyx=(1-AB)/\hbar$, where $q=\mathbb{e}^{\hbar\epsilon}$, $A=\mathbb{e}^{-\hbar\epsilon a}$, and $B=\mathbb{e}^{-\hbar b}$. Set also $T=A^{-1}B=\mathbb{e}^{\hbar t}$.

The Quantum Leap. Also decree that in QU,

$$\Delta(y,b,a,x) = (y_1+B_1y_2, b_1+b_2, a_1+a_2, x_1+A_1x_2),$$
$$S(y,b,a,x) = (-B^{-1}y, -b, -a, -A^{-1}x),$$

and $R=\sum\hbar^{j+k}y^kb^j\otimes a^jx^k/j![k]_q!$.

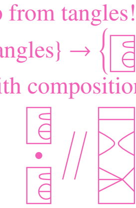
Compositions (1). In mor$(A\to B)$, $Q=\sum_{i\in A,j\in B}E_{ij}\zeta_iz_j+\frac12\sum_{i,j\in A}F_{ij}\zeta_i\zeta_j+\frac12\sum_{i,j\in B}G_{ij}z_iz_j$



Where • $E=E_1(I-F_2G_1)^{-1}E_2$.
• $F=F_1+E_1F_2(I-G_1F_2)^{-1}E_1^T$.
• $G=G_2+E_2^TG_1(I-F_2G_1)^{-1}E_2$.
• $\omega=\omega_1\omega_2\det(I-F_2G_1)^{-1}$.
• P is computed using "connected Feynman diagrams" or as the solution of a messy PDE (yet we're still in algebra!).

Alexander lives here

One abstraction level up from tangles!
{tangles} → {…}
with compositions:

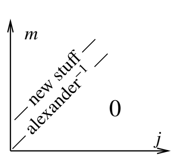


Mid-Talk Debts. • What is this good for in quantum algebra?
• In knot theory?
• How does the "inclusion" $\mathcal{D}$: Hom$(U^{\otimes\Sigma}\to U^{\otimes S})\rightsquigarrow$ **DoPeGDO** work?
• Proofs that everything around sl_{2+}^{ϵ} really is **DoPeGDO**.
• Relations with prior art.
• The rest of the "compositions" story.

Theorem ([BG], conjectured [MM], elucidated [Ro1]). Let $J_d(K)$ be the coloured Jones polynomial of K, in the d-dimensional representation of sl_2. Writing

$$\left.\frac{(q^{1/2}-q^{-1/2})J_d(K)}{q^{d/2}-q^{-d/2}}\right|_{q=e^\hbar} = \sum_{j,m\geq0}a_{jm}(K)d^j\hbar^m,$$

"below diagonal" coefficients vanish, $a_{jm}(K)=0$ if $j>m$, and "on diagonal" coefficients give the inverse of the Alexander polynomial: $\left(\sum_{m=0}^\infty a_{mm}(K)\hbar^m\right)\cdot\omega(K)(e^\hbar)=1$.



"Above diagonal" we have Rozansky's Theorem [Ro3, (1.2)]:

$$J_d(K)(q) = \frac{q^d-q^{-d}}{(q-q^{-1})\omega(K)(q^d)}\left(1+\sum_{k=1}^\infty\frac{(q-1)^k\rho_k(K)(q^d)}{\omega^{2k}(K)(q^d)}\right).$$

DoPeGDO Footnotes. †1. Each variable has a "weight"$\in\{0,1,2\}$, and always wt z_i + wt $\zeta_i=2$.
†2. Really, "weight-graded finite sets" $A=A_0\sqcup A_1\sqcup A_2$.
†3. Really, a power series in the weight-0 variables†9.
†4. The weight of Q must be 2, so it decomposes as $Q=Q_{20}+Q_{11}$. The coefficients of Q_{20} are rational numbers while the coefficients of Q_{11} may be weight-0 power series†9.
†5. Setting wt $\epsilon=-2$, the weight of P is ≤ 2 (so the powers of the weight-0 variables are not constrained†9).
†6. There's also an obvious product

$$\mathrm{mor}(A_1\to B_1)\times\mathrm{mor}(A_2\to B_2)\to\mathrm{mor}(A_1\sqcup A_2\to B_1\sqcup B_2).$$

†7. That is, if the weight-0 variables are ignored. Otherwise more care is needed yet the conclusion remains.
†8. Hom$(U^{\otimes\Sigma}\to U^{\otimes S})\rightsquigarrow$ mor$(\{\eta_i,\beta_i,\tau_i,\alpha_i,\xi_i\}_{i\in\Sigma}\to\{y_i,b_i,t_i,a_i,x_i\}_{i\in S})$, where wt$(\eta_i,\xi_i,y_i,x_i)=1$ and wt$(\beta_i,\tau_i,\alpha_i;b_i,t_i,a_i)=(2,2,0;0,0,2)$.
†9. For tangle invariants the wt-0 power series are always rational functions in the exponentials of the wt-0 variables (for knots: just one variable), with degrees bounded linearly by the crossing number.

$\mathcal{D}\colon \operatorname{Hom}(U^{\otimes\Sigma}\to U^{\otimes S})\to\mathbb{Q}[\![\eta_\Sigma,\beta_\Sigma,\alpha_\Sigma,\xi_\Sigma,y_S,b_S,a_S,x_S]\!]$. The PBW theorem for CU (always in the *ybax* order), or its quantum analog for QU, say that if $U=CU$ or QU then $U^{\otimes S}$ is isomorphic as a vector space to $\mathbb{Q}[y_i,b_i,a_i,x_i]_{i\in S}[\![\hbar]\!]$; so it is enough to understand $\operatorname{Hom}(\mathbb{Q}[z_A]\to\mathbb{Q}[z_B])$ for finite sets A and B.

Claim. $F\in\operatorname{Hom}(\mathbb{Q}[z_A]\to\mathbb{Q}[z_B]) \xrightarrow[\mathcal{D}]{\sim} \mathbb{Q}[z_B][\![\zeta_A]\!]\ni\mathcal{F}$ via

$$\mathcal{D}(F):=\sum_{n\in\mathbb{N}^A}\frac{\zeta_A^n}{n!}F(z_A^n)=F\left(\mathbb{e}^{\sum_{a\in A}\zeta_a z_a}\right)=\mathcal{F},$$

$$\mathcal{D}^{-1}(\mathcal{F})(p)=\left(p|_{z_a\to\partial_{\zeta_a}}\mathcal{F}\right)_{\zeta_a=0}\quad\text{for } p\in\mathbb{Q}[z_A].$$

Claim. Assuming convergence, if $F\in\operatorname{Hom}(\mathbb{Q}[z_A]\to\mathbb{Q}[z_B])$, $G\in\operatorname{Hom}(\mathbb{Q}[z_B]\to\mathbb{Q}[z_C])$, $\mathcal{F}=\mathcal{D}(F)$, and $\mathcal{G}=\mathcal{D}(G)$, then

$$\mathcal{D}(F/\!\!/G)=\left(\mathcal{F}|_{z_i\to\partial_{\zeta_i}}\mathcal{G}\right)_{\zeta_i=0}.$$

And so the title of the talk finally makes sense!

Example. $\mathcal{D}(id\colon U\to U)=\mathbb{e}^{\eta y+\beta b+\alpha a+\xi x}$.

Example. Let $c\Delta^i_{jk}\colon CU^{\otimes\{i\}}\to CU^{\otimes\{j,k\}}$ be the standard co-product, given by $c\Delta^i_{jk}(y_i,b_i,a_i,x_i)=(y_j+y_k,b_j+b_k,a_j+a_k,x_j+x_k)$. Then

$$\mathcal{D}(c\Delta^i_{jk})=c\Delta^i_{jk}(\mathbb{e}^{\eta_i y_i+\beta_i b_i+\alpha_i a_i+\xi_i x_i})$$
$$=\mathbb{e}^{\eta_i(y_j+y_k)+\beta_i(b_j+b_k)+\alpha_i(a_j+a_k)+\xi_i(x_j+x_k)}.$$

Example. The standard commutative product m^{ij}_k of polynomials is given by $z_i,z_j\to z_k$. Hence $\mathcal{D}(m^{ij}_k)=m^{ij}_k(\mathbb{e}^{\zeta_i z_i+\zeta_j z_j})=\mathbb{e}^{(\zeta_i+\zeta_j)z_k}$.

$$\begin{array}{ccc}\mathbb{Q}[z]_i\otimes\mathbb{Q}[z]_j & \xrightarrow{m^{ij}_k} & \mathbb{Q}[z]_k\\ \| & & \|\\ \mathbb{Q}[z_i,z_j] & \xrightarrow{m^{ij}_k} & \mathbb{Q}[z_k]\end{array}$$

A real DoPeGDO Example. Let $cm^{ij}_k\colon CU_i\otimes CU_j\to CU_k$ be "classical multiplication" for sl^ϵ_{2+}, and let $\mathbb{O}_i\colon\mathbb{Q}[y_i,b_i,a_i,x_i]\to CU_i$ be the PBW ordering map.

$$\begin{array}{ccc}CU_i\otimes CU_j & \xrightarrow{cm^{ij}_k} & CU_k\\ \uparrow\mathbb{O}_{i,j} & & \uparrow\mathbb{O}_k\\ \mathbb{Q}[y_i,b_i,a_i,x_i,y_j,b_j,a_j,x_j] & & \mathbb{Q}[y_k,b_k,a_k,x_k]\end{array}$$

Claim. Let (all brawn and no brains)

$$\Lambda=\left(\eta_i+\frac{e^{-\alpha_i-\epsilon\beta_i}\eta_j}{1+\epsilon\eta_j\xi_i}\right)y_k+\left(\beta_i+\beta_j+\frac{\log\left(1+\epsilon\eta_j\xi_i\right)}{\epsilon}\right)b_k+$$
$$\left(\alpha_i+\alpha_j+\log\left(1+\epsilon\eta_j\xi_i\right)\right)a_k+\left(\frac{e^{-\alpha_j-\epsilon\beta_j}\xi_i}{1+\epsilon\eta_j\xi_i}+\xi_j\right)x_k$$

Then $\mathbb{e}^{\eta_i y_i+\beta_i b_i+\alpha_i a_i+\xi_i x_i+\eta_j y_j+\beta_j b_j+\alpha_j a_j+\xi_j x_j}/\!\!/\mathbb{O}_{i,j}/\!\!/cm^{ij}_k=\mathbb{e}^\Lambda/\!\!/\mathbb{O}_k$, and hence $\mathcal{D}(cm^{ij}_k)=\mathbb{e}^\Lambda$ and cm^{ij}_k is DoPeGDO.

Proof. We compute in a faithful 2D representation $z\mapsto\hat{z}$ of CU: (ωεβ/cm)

```
HL[ℰ_] := Style[ℰ, Background → If[TrueQ@ℰ, ■, ■]];
{ŷ = (0 0; ϵ 0), b̂ = (0 0; 0 -ϵ), â = (1 0; 0 0), x̂ = (0 1; 0 0)};
HL /@ {â.x̂ - x̂.â == x̂, â.ŷ - ŷ.â == -ŷ, b̂.ŷ - ŷ.b̂ == -ϵ ŷ,
   b̂.x̂ - x̂.b̂ == ϵ x̂, x̂.ŷ - ŷ.x̂ == b̂ + ϵ â}

{True, True, True, True, True}
HL@Simplify@With[{𝔼 = MatrixExp},
    𝔼[ηi ŷ].𝔼[βi b̂].𝔼[αi â].𝔼[ξi x̂].𝔼[ηj ŷ].𝔼[βj b̂].
      𝔼[αj â].𝔼[ξj x̂] == 𝔼[ŷ ∂yk Λ].𝔼[b̂ ∂bk Λ].𝔼[â ∂ak Λ].
      𝔼[x̂ ∂xk Λ]]

True
Series[Λ, {ϵ, 0, 1}]
(ak (αi + αj) + yk (ηi + e^-αi ηj) +
    bk (βi + βj + ηj ξi) + xk (e^-αj ξi + ξj)) +
  (ak ηj ξi - 1/2 bk ηj^2 ξi^2 - e^-αi yk ηj (βi + ηj ξi) -
    e^-αj xk ξi (βj + ηj ξi)) ϵ + O[ϵ]^2
```

(Shame, but this technique fails for QU).

Claim. In QU, R is DoPeGDO.

Proof. Recall that with $q=\mathbb{e}^{\hbar\epsilon}$,

$$R=\sum\hbar^{j+k}y^kb^j\otimes a^jx^k/j![k]_q!=\mathbb{O}\left(\mathbb{e}^{\hbar b_1a_2}\mathbb{e}_q^{\hbar y_1x_2}\right).$$

Now expand $\mathbb{e}_q^{\hbar y_1x_2}$ in powers of ϵ using:

Faddeev's Formula (In as much as we can tell, first appeared without proof in Faddeev [Fa], rediscovered and proven in Quesne [Qu], and again with easier proof, in Zagier [Za]). With $[n]_q:=\frac{q^n-1}{q-1}$, with $[n]_q!:=[1]_q[2]_q\cdots[n]_q$ and with $\mathbb{e}_q^x:=\sum_{n\geq0}\frac{x^n}{[n]_q!}$, we have

$$\log\mathbb{e}_q^x=\sum_{k\geq1}\frac{(1-q)^kx^k}{k(1-q^k)}=x+\frac{(1-q)^2x^2}{2(1-q^2)}+\dots.$$

Proof. We have that $\mathbb{e}_q^x=\frac{\mathbb{e}_q^{qx}-\mathbb{e}_q^x}{qx-x}$ ("the q-derivative of $\mathbb{e}_q^x$ is itself"), and hence $\mathbb{e}_q^{qx}=(1+(1-q)x)\mathbb{e}_q^x$, and

$$\log\mathbb{e}_q^{qx}=\log(1+(1-q)x)+\log\mathbb{e}_q^x.$$

Writing $\log\mathbb{e}_q^x=\sum_{k\geq1}a_kx^k$ and comparing powers of x, we get $q^ka_k=-(1-q)^k/k+a_k$, or $a_k=\frac{(1-q)^k}{k(1-q^k)}$. □

Compositions (2). Recall that with all indices i running in some set B,

$$\mathcal{F}/\!\!/\mathcal{G}=\left(\mathcal{F}|_{z_i\to\partial_{\zeta_i}}\mathcal{G}\right)_{\zeta_i=0}\overset{(1)}{=}\mathbb{e}^{\sum\partial_{z_i}\partial_{\zeta_i}}(\mathcal{F}\mathcal{G})\Big|_{z_i=\zeta_i=0},$$

(1) Strictly speaking, true only when $B\cap(A\cup C)=\emptyset$.

so in general we wish to understand

$$[F:\mathcal{E}]_B:=\mathbb{e}^{\frac12\sum_{i,j\in B}F_{ij}\partial_{z_i}\partial_{z_j}}\mathcal{E}\quad\text{and}\quad\langle F:\mathcal{E}\rangle_B:=[F:\mathcal{E}]_B|_{z_B\to0},$$

where $\mathcal{E}$ is a docile perturbed Gaussian. The following lemma allows us to restrict to the case where $\mathcal{E}$ has no B-B quadratic part:

Lemma 1. With convergences left to the reader,

$$\left\langle F:\mathcal{E}\mathbb{e}^{\frac12\sum_{i,j\in B}G_{ij}z_iz_j}\right\rangle_B=\det(1-GF)^{-1/2}\left\langle F(1-GF)^{-1}:\mathcal{E}\right\rangle_B.$$

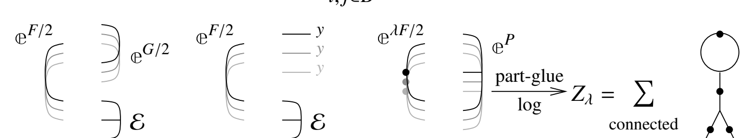
The next lemma dispatches the case where $\mathcal{E}$ has a B-linear part:

Lemma 2. $\left\langle F:\mathcal{E}\mathbb{e}^{\sum_{i\in B}y_iz_i}\right\rangle_B=\mathbb{e}^{\frac12\sum_{i,j\in B}F_{ij}y_iy_j}\left\langle F:\ \mathcal{E}|_{z_B\to z_B+Fy_B}\right\rangle_B$.

Finally, we deal with the docile perturbation case:

Lemma 3. With an extra variable λ, $Z_\lambda:=\log[\lambda F:\mathbb{e}^P]_B$ satisfies and is determined by the following PDE / IVP:

$$Z_0=P\quad\text{and}\quad\partial_\lambda Z_\lambda=\frac12\sum_{i,j\in B}F_{ij}\left(\partial_{z_i}\partial_{z_j}Z_\lambda+(\partial_{z_i}Z_\lambda)(\partial_{z_j}Z_\lambda)\right).$$



Lemma 1 Lemma 2 Lemma 3

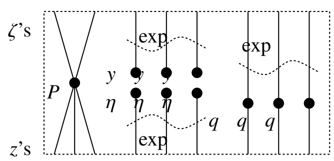
Complexity to ϵ^k, for an n-xing width w knot (by [LT], $w\in O(\sqrt{n})$), is $O(n^2w^{2k+2}\log n)=O(n^{k+3}\log n)$ integer operations.

A Partial To Do List.

- Understand tr and links.
- Implement Φ, J. Determine the appropriate wt-0 ground ring.
- Implement the “dequantizators”.
- Understand denominators and get rid of them.
- Implement zipping at the log-level.
- Clean the program and make it efficient.
- Run it for all small knots and links, at $k = 3, 4$.
- Understand the centre and figure out how to read the output.
- Is the “+” really necessary in sl^{ϵ}_{2+}? Why?
- Extend to sl_3 and beyond.
- Describe a genus bound and a Seifert formula.
- Obtain “Gauss-Gassner formulas” (ωεβ/NCSU).
- Relate with the representation theory dogma, with Melvin-Morton-Rozansky and with Rozansky-Overbay.
- Understand the braid group representations that arise.
- Relate with finite-type (Vassiliev) invariants.
- Find a topological interpretation/foundation. The Garoufalidis - Rozansky “loop expansion” [GR]?
- Figure out the action of the Cartan automorphism.
- Understand “the subspace of classical knots / tangles”.
- Disprove the ribbon-slice conjecture!
- Figure out the action of the Weyl group.
- Use to study “Ševera quantization”.
- Do everything at the “arrow diagram” level of finite-type invariants of (rotational) virtual tangles.
- Find “internal” proofs of consistency.
- What else can you do with the “solvable approximations”?
- And with the “Gaussian compositions” technology?

Warning. Some implementation details match earlier versions of the theory.

The Zipping Theorem. If P has a finite ζ-degree and $\tilde{q}$ is the inverse matrix of $1-q$: $(\delta^i_j - q^i_j)\tilde{q}^j_k = \delta^i_k$, then



$$\left\langle P(z_i,\zeta^j)e^{c+\eta^i z_i + y_j\zeta^j + q^i_j z_i \zeta^j}\right\rangle = |\tilde{q}| e^{c+\eta^i \tilde{q}^k_i y_k} \left\langle P\left(\tilde{q}^k_i(z_k+y_k), \zeta^j + \eta^i \tilde{q}^j_i\right)\right\rangle.$$

The “Speedy” Engine

ωεβ/engine

Internal Utilities

Canonical Form:

```
CCF[ℰ_] :=
  PP_CCF@ExpandDenominator@
   ExpandNumerator@PP_Together@Together[PP_Exp[
      Expand[ℰ] //. 𝕖^x_ 𝕖^y_ :→ 𝕖^(x+y) /. 𝕖^x_ :→ 𝕖^CCF[x]]];
CF[ℰ_List] := CF /@ ℰ;
CF[sd_SeriesData] := MapAt[CF, sd, 3];
CF[ℰ_] := PP_CF@Module[
    {vs = Cases[ℰ, (y | b | t | a | x | η | β | τ | α | ξ)_, ∞] ⋃
       {y, b, t, a, x, η, β, τ, α, ξ}},
    Total[CoefficientRules[Expand[ℰ], vs] /.
      (ps_ → c_) :→ CCF[c] (Times @@ vs^ps)]
   ];
CF[ℰ_𝔼] := CF /@ ℰ;
CF[𝔼_sp___[ℰs___]] := CF /@ 𝔼_sp[ℰs];
```

The Kronecker δ:

```
Kδ /: Kδ_{i_,j_} := If[i === j, 1, 0];
```

Equality, multiplication, and degree-adjustment of perturbed Gaussians; $\mathbb{E}[L, Q, P]$ stands for $e^{L+Q}P$:

```
𝔼 /: 𝔼[L1_, Q1_, P1_] ≡ 𝔼[L2_, Q2_, P2_] :=
  CF[L1 == L2] ∧ CF[Q1 == Q2] ∧ CF[Normal[P1 - P2] == 0];
𝔼 /: 𝔼[L1_, Q1_, P1_] × 𝔼[L2_, Q2_, P2_] :=
  𝔼[L1 + L2, Q1 + Q2, P1 * P2];
𝔼[L_, Q_, P_]_{$k_} := 𝔼[L, Q, Series[Normal@P, {ϵ, 0, $k}]];
```

Zip and Bind

Variables and their duals:

```
{t*, b*, y*, a*, x*, z*} = {τ, β, η, α, ξ, 𝔷};
{τ*, β*, η*, α*, ξ*, 𝔷*} = {t, b, y, a, x, z};
(u_i_)* := (u*)_i;
```

Upper to lower and lower to Upper:

```
U2l = {B_{i_}^{p_.} :→ 𝕖^(-p ℏ γ b_i), B^{p_.} :→ 𝕖^(-p ℏ γ b), T_{i_}^{p_.} :→ 𝕖^(p ℏ t_i),
   T^{p_.} :→ 𝕖^(p ℏ t), 𝒜_{i_}^{p_.} :→ 𝕖^(p γ α_i), 𝒜^{p_.} :→ 𝕖^(p γ α)};
l2U = {𝕖^(c_. b_{i_} + d_.) :→ B_i^(-c/(ℏ γ)) 𝕖^d, 𝕖^(c_. b + d_.) :→ B^(-c/(ℏ γ)) 𝕖^d,
   𝕖^(c_. t_{i_} + d_.) :→ T_i^(c/ℏ) 𝕖^d, 𝕖^(c_. t + d_.) :→ T^(c/ℏ) 𝕖^d,
   𝕖^(c_. α_{i_} + d_.) :→ 𝒜_i^(c/γ) 𝕖^d, 𝕖^(c_. α + d_.) :→ 𝒜^(c/γ) 𝕖^d,
   𝕖^ℰ_ :→ 𝕖^Expand@ℰ};
```

Derivatives in the presence of exponentiated variables:

```
D_b[f_] := ∂_b f - ℏ γ B ∂_B f; D_{b_i_}[f_] := ∂_{b_i} f - ℏ γ B_i ∂_{B_i} f;
D_t[f_] := ∂_t f + ℏ T ∂_T f; D_{t_i_}[f_] := ∂_{t_i} f + ℏ T_i ∂_{T_i} f;
D_α[f_] := ∂_α f + γ 𝒜 ∂_𝒜 f; D_{α_i_}[f_] := ∂_{α_i} f + γ 𝒜_i ∂_{𝒜_i} f;
D_v_[f_] := ∂_v f; D_{v_,0}[f_] := f; D_{}[f_] := f;
D_{v_,n_Integer}[f_] := D_v[D_{v,n-1}[f]];
D_{l_List,ls___}[f_] := D_{ls}[D_l[f]];
```

Finite Zips:

```
collect[sd_SeriesData, ζ_] :=
  MapAt[collect[#, ζ] &, sd, 3];
collect[ℰ_, ζ_] := PP_Collect@Collect[ℰ, ζ];
Zip_{}[P_] := P;
Zip_ζs_[Ps_List] := Zip_ζs /@ Ps;
Zip_{ζ_,ζs___}[P_] := PP_Zip[
  (collect[P // Zip_{ζs}, ζ] /. f_. ζ^d_. :→ (D_{ζ*,d}[f])) /.
   ζ* → 0 /. ((ζ* /. {b → B, t → T, α → 𝒜}) → 1)]
```

QZip implements the “Q-level zips” on $\mathbb{E}(L, Q, P) = e^{L+Q}P(\epsilon)$. Such zips regard the L variables as scalars.

```
QZip𝜁s_List@𝔼[L_, Q_, P_] :=
  PPQZip@Module[{ζ, z, zs, c, ys, ηs, qt, zrule, ζrule, out},
    zs = Table[ζ*, {ζ, 𝜁s}];
    c = CF[Q /. Alternatives @@ (𝜁s ⋃ zs) → 0];
    ys = CF@Table[∂ζ (Q /. Alternatives @@ zs → 0),
      {ζ, 𝜁s}];
    ηs = CF@Table[∂z (Q /. Alternatives @@ 𝜁s → 0), {z, zs}];
    qt = CF@Inverse@Table[Kδz,ζ* - ∂z,ζ Q, {ζ, 𝜁s}, {z, zs}];
    zrule = Thread[zs → CF[qt.(zs + ys)]];
    ζrule = Thread[𝜁s → 𝜁s + ηs.qt];
    CF /@ 𝔼[L, c + ηs.qt.ys,
      Det[qt] Zip𝜁s[P /. (zrule ⋃ ζrule)]]];
```

LZip implements the “L-level zips” on $\mathbb{E}(L, Q, P) = Pe^{L+Q}$. Such zips regard all of Pe^{Q} as a single”P”. Here the z’s are b and α and the ζ’s are β and a.

```
LZip𝜁s_List@𝔼[L_, Q_, P_] :=
  PPLZip@Module[{ζ, z, zs, Zs, c, ys, ηs, lt, zrule,
     Zrule, ζrule, Q1, EEQ, EQ},
    zs = Table[ζ*, {ζ, 𝜁s}];
    Zs = zs /. {b → B, t → T, α → 𝒜};
    c = L /. Alternatives @@ (𝜁s ⋃ zs) → 0 /.
      Alternatives @@ Zs → 1;
    ys = Table[∂ζ (L /. Alternatives @@ zs → 0), {ζ, 𝜁s}];
    ηs = Table[∂z (L /. Alternatives @@ 𝜁s → 0), {z, zs}];
    lt = Inverse@Table[Kδz,ζ* - ∂z,ζ L, {ζ, 𝜁s}, {z, zs}];
    zrule = Thread[zs → lt.(zs + ys)];
    Zrule = Join[zrule,
      zrule /.
       r_Rule ⧴ ((U = r〚1〛 /. {b → B, t → T, α → 𝒜}) →
         (U /. U2l /. r //. l2U))];
    ζrule = Thread[𝜁s → 𝜁s + ηs.lt];
    Q1 = Q /. (Zrule ⋃ ζrule);
    EEQ[ps___] :=
     EEQ[ps] =
      PP"EEQ"@(CF[e^-Q1 DThread[{zs,{ps}}][e^Q1]] /.
         {Alternatives @@ zs → 0, Alternatives @@ Zs → 1});
    CF@𝔼[c + ηs.lt.ys,
      Q1 /. {Alternatives @@ zs → 0, Alternatives @@ Zs → 1},
      Det[lt]
       (Zip𝜁s[(EQ @@ zs) (P /. (Zrule ⋃ ζrule))] /.
         Derivative[ps___][EQ][___] ⧴ EEQ[ps] /.
        _EQ → 1) ]];
```

```
B{}[L_, R_] := L R;
B{is__}[L_𝔼, R_𝔼] := PPB@Module[{n},
   Times[
     L /. Table[(v : b | B | t | T | a | x | y)i → vn@i,
       {i, {is}}],
     R /. Table[(v : β | τ | α | 𝒜 | ξ | η)i → vn@i, {i, {is}}]
    ] // LZipJoin@@Table[{βn@i,τn@i,an@i},{i,{is}}] //
   QZipJoin@@Table[{ξn@i,yn@i},{i,{is}}]];
Bis___[L_, R_] := B{is}[L, R];
```

𝔼 morphisms with domain and range.

```
Bis_List[𝔼d1_→r1_[L1_, Q1_, P1_], 𝔼d2_→r2_[L2_, Q2_, P2_]] :=
  𝔼(d1⋃Complement[d2,is])→(r2⋃Complement[r1,is]) @@
   Bis[𝔼[L1, Q1, P1], 𝔼[L2, Q2, P2]];
𝔼d1_→r1_[L1_, Q1_, P1_] // 𝔼d2_→r2_[L2_, Q2_, P2_] :=
  Br1⋂d2[𝔼d1→r1[L1, Q1, P1], 𝔼d2→r2[L2, Q2, P2]];
𝔼d1_→r1_[L1_, Q1_, P1_] ≡ 𝔼d2_→r2_[L2_, Q2_, P2_] ^:=
  (d1 == d2) ∧ (r1 == r2) ∧ (𝔼[L1, Q1, P1] ≡ 𝔼[L2, Q2, P2]);
𝔼d1_→r1_[L1_, Q1_, P1_] 𝔼d2_→r2_[L2_, Q2_, P2_] ^:=
  𝔼(d1⋃d2)→(r1⋃r2) @@ (𝔼[L1, Q1, P1] × 𝔼[L2, Q2, P2]);
𝔼dr_[L_, Q_, P_]$k_ := 𝔼dr @@ 𝔼[L, Q, P]$k;
𝔼_[ℰ___][i_] := {ℰ}〚i〛;
```

𝔼[Λ]

```
𝔼dr_[Λ_] :=
  CF@Module[{L, Λ0 = Limit[Λ, ϵ → 0]},
    𝔼dr[L = Λ0 /. (η | y | ξ | x)_ → 0, Λ0 - L, e^(Λ-Λ0)]$k /. l2U]
```

Exponentials as needed.

Task. Define $\mathrm{Exp}_{m,i,k}[P]$ to compute $e^{\mathbb{O}(P)}$ to ϵ^k in the using the $m_{i,j\to i}$ multiplication, where P is an ϵ-dependent near-docile element, giving the answer in $\mathbb{E}$-form.

Methodology. If $P_0 := P_{\epsilon=0}$ and $e^{\lambda\,\mathbb{O}(P)} = \mathbb{O}\big(e^{\lambda P_0} F(\lambda)\big)$, then $F(\lambda=0)=1$ and we have:

$$\mathbb{O}\big(e^{\lambda P_0}(P_0 F(\lambda) + \partial_\lambda F)\big) = \mathbb{O}\big(\partial_\lambda e^{\lambda P_0} F(\lambda)\big) =$$

$$\partial_\lambda \mathbb{O}\big(e^{\lambda P_0} F(\lambda)\big) = \partial_\lambda e^{\lambda\,\mathbb{O}(P)} = e^{\lambda\,\mathbb{O}(P)}\,\mathbb{O}(P) = \mathbb{O}\big(e^{\lambda P_0} F(\lambda)\big)\,\mathbb{O}(P)$$

This is a linear ODE for F. Setting inductively $F_k = F_{k-1} + \epsilon^k \varphi$ we find that $F_0 = 1$ and solve for φ.

```
(* Bug: The first line is valid only if 𝕆(e^P0)==e^𝕆(P0). *)
Expm_,i_,0[P_] := Module[{LQ = Normal@P /. ϵ → 0},
   𝔼[LQ /. (x | y)i → 0, LQ /. (b | a | t)i → 0, 1]];
```

```
Expm_,i_,k_[P_] := Block[{$k = k},
  Module[{P0, λ, φ, φs, F, j, rhs, eqn, pows, at0, atλ},
   P0 = Normal@P /. ϵ → 0;
   F = Normal@Last@Expm,i,k-1[λ P];
   While[
    rhs =
     mi,j→i[
       𝔼{}→{i}[λ P0 /. (x | y)i → 0, λ P0 /. (b | a | t)i → 0,
         F]k sσi→j@𝔼{}→{i}[0, 0, P]k] // Last // Normal;
    eqn = CF[(∂λ F) + P0 F - rhs];
    eqn =!= 0, (*do*)
    pows = First /@ CoefficientRules[eqn, {yi, bi, ai, xi}];
    F += Sum[ϵ^k φjs[λ] Times @@ {yi, bi, ai, xi}^js,
      {js, pows}];
    rhs =
     mi,j→i[
       𝔼{}→{i}[λ P0 /. (x | y)i → 0, λ P0 /. (b | a | t)i → 0,
         F]k sσi→j@𝔼{}→{i}[0, 0, P]k] // Last // Normal;
    eqn = CF[(∂λ F) + P0 F - rhs];
    φs = Table[φjs[λ], {js, pows}];
    at0 = Table[φjs[0] == 0, {js, pows}];
    atλ = (# == 0) & /@
      (pows /. CoefficientRules[eqn, {yi, bi, ai, xi}]);
    F = F /. DSolve[And @@ (at0 ⋃ atλ), φs, λ]〚1〛
   ];
   𝔼{}→{i}[P0 /. (x | y)i → 0, P0 /. (b | a | t)i → 0,
    F + O[ϵ]^(k+1) /. λ → 1]]]
```

“Define” Code

Define[lhs = rhs, ...] defines the lhs to be rhs, except that rhs is computed only once for each value of $k. Fancy Mathematica not for the faint of heart. Most readers should ignore.

```
SetAttributes[Define, HoldAll];
Define[def_, defs__] := (Define[def]; Define[defs];);
Define[op_is__ = ℰ_] :=
 Module[{SD, ii, jj, kk, isp, nis, nisp, sis},
  Block[{i, j, k},
   ReleaseHold[Hold[
      SD[op_{nisp,$k_Integer}, PP_Boot@Block[{i, j, k}, op_{isp,$k} = ℰ;
        op_{nis,$k}]];
      SD[op_isp, op_{{is},$k}]; SD[op_sis__, op_{sis}];
     ] /. {SD → SetDelayed,
      isp → {is} /. {i → i_, j → j_, k → k_},
      nis → {is} /. {i → ii, j → jj, k → kk},
      nisp → {is} /. {i → ii_, j → jj_, k → kk_}
     }] ]]
```

The Objects

ωεβ/objects

Symmetric Algebra Objects

```
sm_{i_,j_→k_} :=
  𝔼_{{i,j}→{k}}[b_k (β_i + β_j) + t_k (τ_i + τ_j) + a_k (α_i + α_j) +
    y_k (η_i + η_j) + x_k (ξ_i + ξ_j)];
sΔ_{i_→j_,k_} :=
  𝔼_{{i}→{j,k}}[β_i (b_j + b_k) + τ_i (t_j + t_k) + α_i (a_j + a_k) +
    η_i (y_j + y_k) + ξ_i (x_j + x_k)];
sS_{i_} := 𝔼_{{i}→{i}}[-β_i b_i - τ_i t_i - α_i a_i - η_i y_i - ξ_i x_i];
sε_{i_} := 𝔼_{{}→{i}}[0];
sη_{i_} := 𝔼_{{i}→{}}[0];
sσ_{i_→j_} := 𝔼_{{i}→{j}}[β_i b_j + τ_i t_j + α_i a_j + η_i y_j + ξ_i x_j];
sΥ_{i_→j_,k_,l_,m_} := 𝔼_{{i}→{j,k,l,m}}[β_i b_k + τ_i t_k + α_i a_l + η_i y_j + ξ_i x_m];
```

The CU Definitions

```
cΛ = (η_i + e^(-γ α_i - ϵ β_i) η_j / (1 + γ ϵ η_j ξ_i)) y_k + (β_i + β_j + Log[1 + γ ϵ η_j ξ_i] / ϵ) b_k +
   (α_i + α_j + Log[1 + γ ϵ η_j ξ_i] / γ) a_k + (e^(-γ α_j - ϵ β_j) ξ_i / (1 + γ ϵ η_j ξ_i) + ξ_j) x_k;
Define[cm_{i,j→k} = 𝔼_{{i,j}→{k}}[cΛ]]
Define[cσ_{i→j} = sσ_{i,j} /. τ_i → 0, cε_i = sε_i, cη_i = sη_i,
  cΔ_{i→j,k} = sΔ_{i→j,k},
  cS_i = sS_i // sΥ_{i→1,2,3,4} // cm_{4,3→i} // cm_{i,2→i} // cm_{i,1→i}];
```

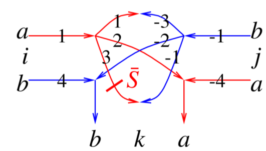
Booting Up QU

```
Define[aσ_{i→j} = 𝔼_{{i}→{j}}[a_j α_i + x_j ξ_i],
  bσ_{i→j} = 𝔼_{{i}→{j}}[b_j β_i + y_j η_i]]
Define[am_{i,j→k} = 𝔼_{{i,j}→{k}}[(α_i + α_j) a_k + (𝒜_j^{-1} ξ_i + ξ_j) x_k],
  bm_{i,j→k} = 𝔼_{{i,j}→{k}}[(β_i + β_j) b_k + (η_i + e^(-ϵ β_i) η_j) y_k]]
Define[R_{i,j} = 𝔼_{{}→{i,j}}[ℏ a_j b_i + Sum_{k=1}^{$k+1} (1 - e^(γ ϵ ℏ))^k (ℏ y_i x_j)^k / (k (1 - e^(k γ ϵ ℏ)))],
  R̄_{i,j} = CF@𝔼_{{}→{i,j}}[-ℏ a_j b_i, -ℏ x_j y_i / B_i,
    1 + If[$k == 0, 0, (R̄_{{i,j},$k-1})_{$k}[3] -
       (((R̄_{{i,j},0})_{$k} R_{1,2} (R̄_{{3,4},$k-1})_{$k}) // (bm_{i,1→i} am_{j,2→j}) //
          (bm_{i,3→i} am_{j,4→j}))[3]]],
  P_{i,j} = 𝔼_{{i,j}→{}}[β_i α_j / ℏ, η_i ξ_j / ℏ,
   1 + If[$k == 0, 0, (P_{{i,j},$k-1})_{$k}[3] -
     (R_{1,2} // ((P_{{1,j},0})_{$k} (P_{{i,2},$k-1})_{$k}))[3]]]]
```

```
Define[aS_i = (aσ_{i→2} R̄_{1,i}) // P_{1,2},
  aS̄_i = 𝔼_{{i}→{i}}[-a_i α_i, -x_i 𝒜_i ξ_i,
    1 + If[$k == 0, 0, (aS̄_{{i},$k-1})_{$k}[3] -
       ((aS̄_{{i},0})_{$k} // aS_i // (aS̄_{{i},$k-1})_{$k})[3]]]]
```

```
Define[bS_i = bσ_{i→1} R_{i,2} // aS_2 // P_{1,2},
  bS̄_i = bσ_{i→1} R_{i,2} // aS̄_2 // P_{1,2},
  aΔ_{i→j,k} = (R_{1,j} R_{2,k}) // bm_{1,2→3} // P_{3,i},
  bΔ_{i→j,k} = (R_{j,1} R_{k,2}) // am_{1,2→3} // P_{i,3}]
```

The Drinfel'd double:



```
Define[
  dm_{i,j→k} =
   ((sΥ_{i→4,4,1,1} // aΔ_{1→1,2} // aΔ_{2→2,3} // aS̄_3)
      (sΥ_{j→-1,-1,-4,-4} // bΔ_{-1→-1,-2} // bΔ_{-2→-2,-3})) //
    (P_{-1,3} P_{-3,1} am_{2,-4→k} bm_{4,-2→k}) ]
```

```
Define[dσ_{i→j} = aσ_{i→j} bσ_{i→j},
  dε_i = sε_i, dη_i = sη_i,
  dS_i = sΥ_{i→1,1,2,2} // (bS̄_1 aS_2) // dm_{2,1→i},
  dS̄_i = sΥ_{i→1,1,2,2} // (bS_1 aS̄_2) // dm_{2,1→i},
  dΔ_{i→j,k} = (bΔ_{i→3,1} aΔ_{i→2,4}) // (dm_{3,4→k} dm_{1,2→j}) ]
```

```
Define[C_i = 𝔼_{{}→{i}}[0, 0, B_i^{1/2} e^(-ℏ ϵ a_i/2)]_{$k},
  C̄_i = 𝔼_{{}→{i}}[0, 0, B_i^{-1/2} e^(ℏ ϵ a_i/2)]_{$k},
  Kink_i = (R_{1,3} C̄_2) // dm_{1,2→1} // dm_{1,3→i},
  K̄ink_i = (R̄_{1,3} C_2) // dm_{1,2→1} // dm_{1,3→i}]
```

Note. $t == \epsilon a - \gamma b$ and $b == -t/\gamma + \epsilon a/\gamma$.

```
Define[b2t_i = 𝔼_{{i}→{i}}[α_i a_i + β_i (ϵ a_i - t_i) / γ + ξ_i x_i + η_i y_i],
  t2b_i = 𝔼_{{i}→{i}}[α_i a_i + τ_i (ϵ a_i - γ b_i) + ξ_i x_i + η_i y_i]]
```

The Knot Tensors

```
Define[kR_{i,j} = R_{i,j} // (b2t_i b2t_j) /. t_{i|j} → t,
  kR̄_{i,j} = R̄_{i,j} // (b2t_i b2t_j) /. {t_{i|j} → t, T_{i|j} → T},
  km_{i,j→k} = (t2b_i t2b_j) // dm_{i,j→k} //
    b2t_k /. {t_k → t, T_k → T, τ_{i|j} → 0},
  kC_i = C_i // b2t_i /. T_i → T,
  kC̄_i = C̄_i // b2t_i /. T_i → T,
  kKink_i = Kink_i // b2t_i /. {t_i → t, T_i → T},
  kK̄ink_i = K̄ink_i // b2t_i /. {t_i → t, T_i → T}]
```

Some of the Atoms.

ωεβ/atoms

With $\mathcal{A}_i := e^{\alpha_i}$ and $B_i = e^{-b_i}$,

```
PP_ := Identity; $k = 1; ℏ = γ = 1;
Column[
 (# → (ℰ = ToExpression[#];
      Normal@Simplify[ℰ[[1]] + ℰ[[2]] + Log@ℰ[[3]]])) & /@
  {"dm_{i,j→k}", "dΔ_{i→j,k}", "dS_i", "R_{i,j}", "P_{i,j}"}]
```

$$\mathrm{dm}_{i,j\to k}\to a_k\,(\alpha_i+\alpha_j)+b_k\,(\beta_i+\beta_j)+y_k\,\eta_i+\frac{y_k\,\eta_j}{\mathcal{A}_i}+\frac{x_k\,\xi_i}{\mathcal{A}_j}+\eta_j\,\xi_i-B_k\,\eta_j\,\xi_i+\frac{1}{4\,\mathcal{A}_i\,\mathcal{A}_j}\epsilon\,\Big(2\,y_k\,\eta_j\,\big(2\,x_k\,\xi_i+\mathcal{A}_j\,(-2\,\beta_i+(1-3\,B_k)\,\eta_j\,\xi_i)\big)+\mathcal{A}_i\,\xi_i\,\Big(x_k\,(-4\,\beta_j+2\,(1-3\,B_k)\,\eta_j\,\xi_i)+\mathcal{A}_j\,\eta_j\,\big(4\,a_k\,B_k+(1-4\,B_k+3\,B_k^2)\,\eta_j\,\xi_i\big)\Big)\Big)+x_k\,\xi_j$$

$$\mathrm{d}\Delta_{i\to j,k}\to a_j\,\alpha_i+a_k\,\alpha_i+b_j\,\beta_i+b_k\,\beta_i+y_j\,\eta_i+B_j\,y_k\,\eta_i+x_j\,\xi_i+x_k\,\xi_i+\frac{1}{2}\,\epsilon\,\big(B_j\,y_j\,y_k\,\eta_i^2+x_k\,\xi_i\,(-2\,a_j+x_j\,\xi_i)\big)$$

$$\mathrm{dS}_i\to -a_i\,\alpha_i-b_i\,\beta_i-\frac{\mathcal{A}_i\,(y_i\,\eta_i+(-\eta_i+B_i\,(x_i+\eta_i))\,\xi_i)}{B_i}-\frac{1}{4\,B_i^2}\,\epsilon\,\mathcal{A}_i\,\Big(\mathcal{A}_i\,\eta_i^2\,(2\,y_i^2-6\,y_i\,\xi_i+3\,\xi_i^2)+B_i^2\,\xi_i\,\big(4\,a_i\,x_i+2\,x_i^2\,\mathcal{A}_i\,\xi_i+2\,x_i\,(2\,\beta_i+\mathcal{A}_i\,\eta_i\,\xi_i)+\eta_i\,(-4+4\,\beta_i+\mathcal{A}_i\,\eta_i\,\xi_i)\big)+2\,B_i\,\eta_i\,\big(y_i\,(-2+2\,\beta_i+2\,x_i\,\mathcal{A}_i\,\xi_i+\mathcal{A}_i\,\eta_i\,\xi_i)-\xi_i\,(-2+2\,a_i+2\,\beta_i+3\,x_i\,\mathcal{A}_i\,\xi_i+2\,\mathcal{A}_i\,\eta_i\,\xi_i)\big)\Big)$$

$$R_{i,j}\to a_j\,b_i+x_j\,y_i-\frac{1}{4}\,\epsilon\,x_j^2\,y_i^2$$

$$P_{i,j}\to \alpha_j\,\beta_i+\eta_i\,\xi_j+\frac{1}{4}\,\epsilon\,\eta_i^2\,\xi_j^2$$

A Quantum Algebra Example.

ωεβ/qa

Proto-Proposition[†0] (with Jesse Frohlich and Roland van der Veen, near [Ma, Proposition 1.7.3]). Let H be a finite dimensional Hopf algebra and let $U = H^{*cop} \otimes H$ be its Drinfel'd double, with R-matrix $R \in H^* \otimes H \subset U \otimes U$. Write $R^{\dagger 1} = \sum \rho_a \otimes r_a$, and let $\langle \cdot \mid \cdot \rangle \colon H^* \otimes H \to \mathbb{F}$ be the duality pairing. Then the functional $\int \in U^*$ defined by

$$\int \phi \otimes x := \sum \langle \phi \rho_a^{\dagger 2} \mid x r_a^{\dagger 3} \rangle$$

is a right[†4] integral in U^*. (Meaning $\Delta^i_{jk} /\!\!/ \int_j = \int_i /\!\!/ \epsilon_k$ in $\mathrm{Hom}(U^{\otimes\{i\}} \to U^{\otimes\{k\}})$).

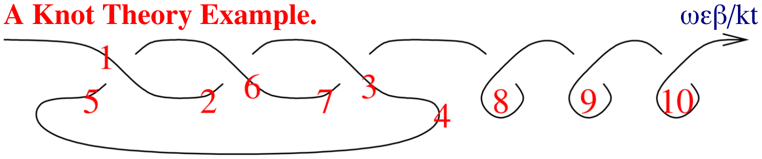
†0 A "proto-proposition" is something that will become a proposition once you figure out the correct statement. †1 Or did we want it to be $R /\!\!/ S_1^2$? Or $R /\!\!/ S_2^2$? †2 Or is it $\rho_a \phi$? †3 Or is it $r_a x$? †4 Or maybe "left"?

```
inp = 𝔼{}→{1}[3 a1 b1, 5 x1 y1, 1] // dmi,1→i;
Table[
  HL@TrueQ[
    (inp // (sϒi→1,1,2,2 RR) // BM // AM // P1,2 ) dϵj ≡
     (inp // ΔΔ // (sϒi→1,1,2,2 RR) // BM // AM // P1,2)],
  {ΔΔ, {dΔi→i,j, dΔi→j,i}}, {AM, {dm2,4→2, dm4,2→2}},
  {BM, {dm1,3→1, dm3,1→1}},
  {RR, {R3,4, R3,4 // dS3 // dS3, R3,4 // dS4 // dS4}}
 ] // MatrixForm
```

$$\begin{pmatrix} \begin{pmatrix} \text{False} & \text{False} & \text{False} \\ \text{False} & \text{False} & \text{False} \end{pmatrix} & \begin{pmatrix} \text{False} & \text{False} & \text{True} \\ \text{False} & \text{False} & \text{False} \end{pmatrix} \\ \begin{pmatrix} \text{False} & \text{False} & \text{False} \\ \text{False} & \text{False} & \text{True} \end{pmatrix} & \begin{pmatrix} \text{False} & \text{False} & \text{False} \\ \text{False} & \text{False} & \text{False} \end{pmatrix} \end{pmatrix}$$

A Knot Theory Example.

ωεβ/kt




```
$k = 2;
Simplify[
  R1,5 R6,2 R3,7 C̄4 K̄ink8 K̄ink9 K̄ink10 // dm1,2→1 // dm1,3→1 //
      dm1,4→1 // dm1,5→1 // dm1,6→1 // dm1,7→1 // dm1,8→1 //
   dm1,9→1 // dm1,10→1] /. v_1 ⇸ v
```

$$\mathbb{E}_{\{\}\to\{1\}}\Big[0,\,0,\,\frac{B}{1-B+B^2}+\frac{B\,\big(-B+2\,B^2+2\,B^4+a\,(-1+B-B^3+B^4)-2\,x\,y-B^3\,(3+2\,x\,y)\big)\,\epsilon}{(1-B+B^2)^3}+\frac{1}{2\,(1-B+B^2)^5}\,B\,\Big(4\,B^8+a^2\,(1-B+B^2)^2\,(1+B-6\,B^2+B^3+B^4)+6\,B^5\,x^2\,y^2+2\,x\,y\,(-2+3\,x\,y)-B^7\,(11+4\,x\,y)-2\,B^2\,(1+6\,x^2\,y^2)-2\,B^4\,(1-2\,x\,y+6\,x^2\,y^2)+B\,(1+8\,x\,y+6\,x^2\,y^2)+B^6\,(6+8\,x\,y+6\,x^2\,y^2)+B^3\,(4+4\,x\,y+30\,x^2\,y^2)+2\,a\,(1-B+B^2)\,\big(2\,B^6+2\,x\,y+8\,B^3\,(1+x\,y)-5\,B^2\,(1+2\,x\,y)-2\,B^5\,(1+2\,x\,y)-B^4\,(7+2\,x\,y)+B\,(2+4\,x\,y)\big)\Big)\,\epsilon^2+O[\epsilon]^3\Big]$$

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[Ro3] L. Rozansky, *A Universal U(1)-RCC Invariant of Links and Rationality Conjecture,* arXiv:math/0201139.

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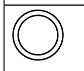
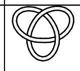
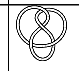
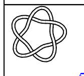
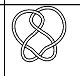
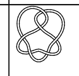
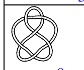
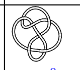
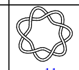
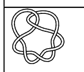
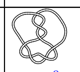
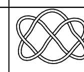
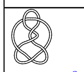
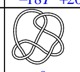
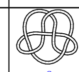
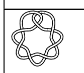
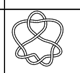
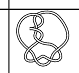
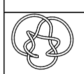
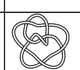
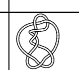
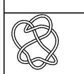
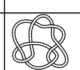
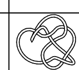
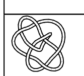
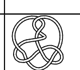
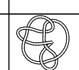
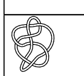
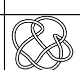
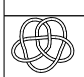
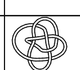
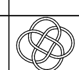
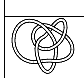
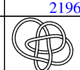
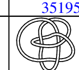
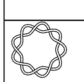
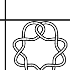
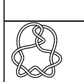
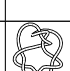
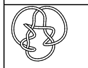
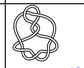
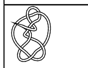
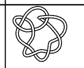
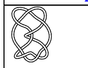
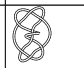
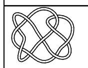
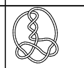
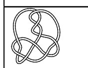
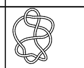
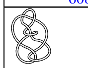
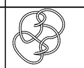
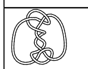
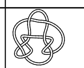
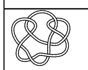
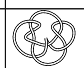
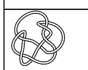
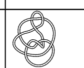
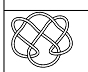
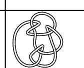
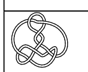
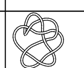
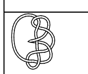
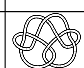
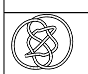
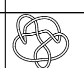
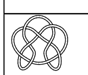
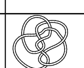
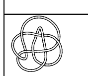
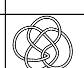
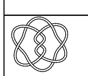
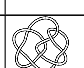
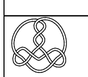
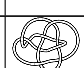
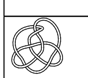
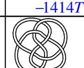
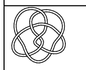
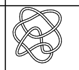
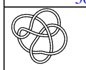
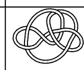
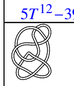
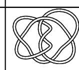
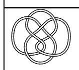
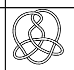
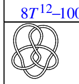
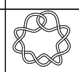
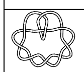
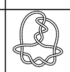
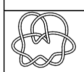
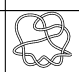
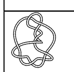
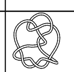
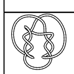
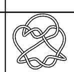
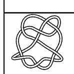
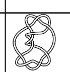
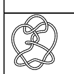
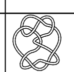
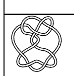
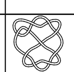
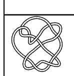
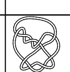
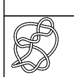
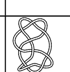
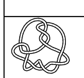
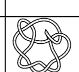
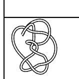
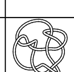
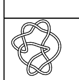
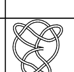
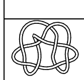
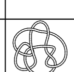
KiW 43 Abstract (ωεβ/kiw). Whether or not you like the formulas on this page, they describe the strongest truly computable knot invariant we know.

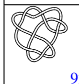
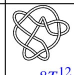
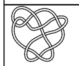
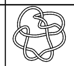
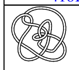
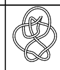
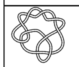
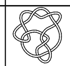
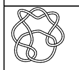
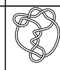
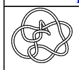
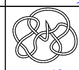
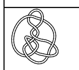
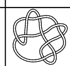
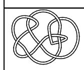
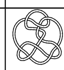
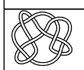
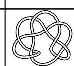
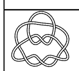
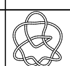
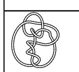
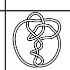
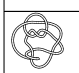
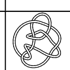
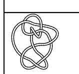
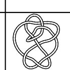
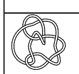
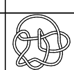
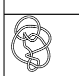
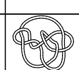
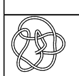
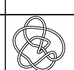
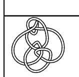
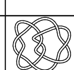
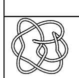
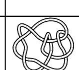
Observations. • Separates the Rolfsen table; does better than Khovanov plus HOMFLY-PT on knots with up to 12 crossings (not tested beyond). • The degrees are bounded by the genus! • ρ_1 vanishes for amphichiral knots. • Has a chance of detecting non-ribbonness (ωεβ/akt)!

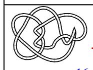
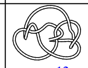
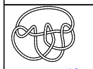
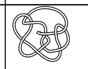
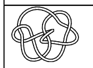
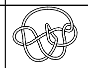
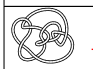
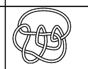
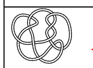
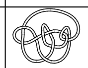
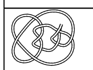
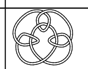
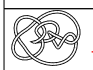
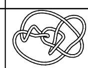
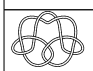
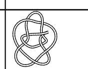
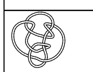
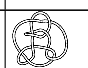
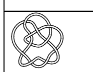
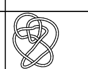
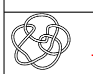
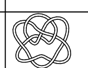
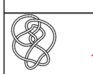
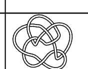
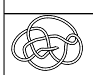
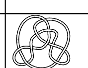
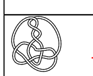
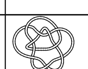
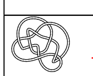
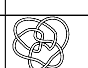
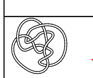
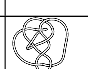
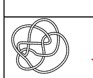
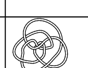
knot diag	n_k^t	Alexander's ω^+	$(\rho_1')^+$	$(\rho_2')^+$	genus / ribbon	unknotting # / amphi?
	0_1^a	1	0	0	0 / ✔	0 / ✔
	3_1^a	$T-1$	T	$3T^3-12T^2+26T-38$	1 / ✘	1 / ✘
	4_1^a	$3-T$	0	$T^4-3T^3-15T^2+74T-110$	1 / ✘	1 / ✔
	5_1^a	T^2-T+1	$2T^3+3T$	$5T^7-20T^6+55T^5-120T^4+217T^3-338T^2+450T-510$	2 / ✘	2 / ✘
	5_2^a	$2T-3$	$5T-4$	$-10T^4+120T^3-487T^2+1054T-1362$	1 / ✘	1 / ✘
	6_1^a	$5-2T$	$T-4$	$14T^4-16T^3-293T^2+1098T-1598$	1 / ✔	1 / ✘
	6_2^a	$-T^2+3T-3$	T^3-4T^2+4T-4	$3T^8-21T^7+49T^6+15T^5-433T^4+1543T^3-3431T^2+5482T-6410$	2 / ✘	1 / ✘
	6_3^a	T^2-3T+5	0	$4T^8-33T^7+121T^6-203T^5-111T^4+1499T^3-4210T^2+7186T-8510$	2 / ✘	1 / ✔
	7_1^a	T^3-T^2+T-1	$3T^5+5T^3+6T$	$7T^{11}-28T^{10}+77T^9-168T^8+322T^7-560T^6+891T^5-1310T^4+1777T^3-2238T^2+2604T-2772$	3 / ✘	3 / ✘
	7_2^a	$3T-5$	$14T-16$	$-129T^4+1177T^3-4421T^2+9226T-11718$	1 / ✘	1 / ✘
	7_3^a	$2T^2-3T+3$	$-9T^3+8T^2-16T+12$	$-18T^8+208T^7-917T^6+2666T^5-6049T^4+11283T^3-17671T^2+23356T-25736$	2 / ✘	2 / ✘
	7_4^a	$4T-7$	$32-24T$	$-352T^4+3616T^3-14378T^2+30700T-39188$	1 / ✘	2 / ✘
	7_5^a	$2T^2-4T+5$	$9T^3-16T^2+29T-28$	$-18T^8+264T^7-1548T^6+5680T^5-15107T^4+31152T^3-51476T^2+69252T-76414$	2 / ✘	2 / ✘
	7_6^a	$-T^2+5T-7$	$T^3-8T^2+19T-20$	$3T^8-35T^7+128T^6+105T^5-2610T^4+11225T^3-28031T^2+47186T-55946$	2 / ✘	1 / ✘
	7_7^a	T^2-5T+9	$8-3T$	$4T^8-55T^7+310T^6-805T^5+86T^4+6349T^3-22686T^2+43610T-53622$	2 / ✘	1 / ✘
	8_1^a	$7-3T$	$5T-16$	$42T^4+215T^3-2542T^2+7562T-10542$	1 / ✘	1 / ✘
	8_2^a	$-T^3+3T^2-3T+3$	$2T^5-8T^4+10T^3-12T^2+13T-12$	$5T^{12}-39T^{11}+119T^{10}-139T^9-249T^8+1660T^7-4959T^6+11131T^5-20813T^4+33595T^3-47521T^2+58988T-63556$	3 / ✘	2 / ✘
	8_3^a	$9-4T$	0	$224T^4-224T^3-3910T^2+14100T-20364$	1 / ✘	2 / ✔
	8_4^a	$-2T^2+5T-5$	$3T^3-8T^2+6T-4$	$54T^8-344T^7+865T^6-650T^5-2723T^4+12243T^3-28461T^2+45792T-53540$	2 / ✘	2 / ✘
	8_5^a	$-T^3+3T^2-4T+5$	$-2T^5+8T^4-13T^3+20T^2-22T+24$	$5T^{12}-39T^{11}+128T^{10}-182T^9-274T^8+2476T^7-8642T^6+21517T^5-42924T^4+71719T^3-102448T^2+126480T-135628$	3 / ✘	2 / ✘
	8_6^a	$-2T^2+6T-7$	$5T^3-20T^2+28T-32$	$38T^8-216T^7+112T^6+2880T^5-14787T^4+42444T^3-85415T^2+128406T-146916$	2 / ✘	2 / ✘
	8_7^a	T^3-3T^2+5T-5	$-T^5+4T^4-10T^3+12T^2-13T+12$	$8T^{12}-75T^{11}+343T^{10}-979T^9+1821T^8-1782T^7-1623T^6+12083T^5-33001T^4+64599T^3-101194T^2+131404T-143216$	3 / ✘	1 / ✘
	8_8^a	$2T^2-6T+9$	$-T^3+4T^2-12T+16$	$62T^8-504T^7+1736T^6-2408T^5-3717T^4+26492T^3-68493T^2+113418T-133180$	2 / ✔	2 / ✘
	8_9^a	$-T^3+3T^2-5T+7$	0	$9T^{12}-87T^{11}+417T^{10}-1305T^9+2858T^8-4134T^7+2114T^6+8285T^5-31925T^4+69235T^3-112773T^2+148508T-162396$	3 / ✔	1 / ✔
	8_{10}^a	T^3-3T^2+6T-7	$-T^5+4T^4-11T^3+16T^2-21T+20$	$8T^{12}-75T^{11}+362T^{10}-1122T^9+2306T^8-2540T^7-2198T^6+18817T^5-54380T^4+110103T^3-175694T^2+230080T-251346$	3 / ✘	2 / ✘
	8_{11}^a	$-2T^2+7T-9$	$5T^3-24T^2+39T-44$	$38T^8-264T^7+301T^6+3514T^5-21716T^4+68785T^3-146898T^2+227828T-263172$	2 / ✘	1 / ✘
	8_{12}^a	$T^2-7T+13$	0	$4T^8-77T^7+583T^6-1991T^5+987T^4+17311T^3-71802T^2+147914T-185846$	2 / ✘	2 / ✔
	8_{13}^a	$2T^2-7T+11$	$-T^3+4T^2-14T+20$	$62T^8-592T^7+2351T^6-3918T^5-4235T^4+40079T^3-111533T^2+191500T-227432$	2 / ✘	1 / ✘
	8_{14}^a	$-2T^2+8T-11$	$5T^3-28T^2+57T-68$	$38T^8-312T^7+444T^6+5096T^5-34777T^4+116368T^3-255750T^2+401632T-465478$	2 / ✘	1 / ✘
	8_{15}^a	$3T^2-8T+11$	$21T^3-64T^2+120T-140$	$-123T^8+2128T^7-15241T^6+66120T^5-199999T^4+451912T^3-792414T^2+1101720T-1228222$	2 / ✘	2 / ✘
	8_{16}^a	T^3-4T^2+8T-9	$T^5-6T^4+17T^3-28T^2+35T-36$	$8T^{12}-100T^{11}+598T^{10}-2205T^9+5292T^8-7164T^7-2380T^6+43100T^5-137314T^4+291750T^3-478742T^2+636488T-698666$	3 / ✘	2 / ✘
	8_{17}^a	$-T^3+4T^2-8T+11$	0	$9T^{12}-116T^{11}+722T^{10}-2843T^9+7656T^8-13668T^7+11117T^6+21968T^5-113086T^4+273778T^3-475622T^2+649064T-717954$	3 / ✘	1 / ✔
	8_{18}^a	$-T^3+5T^2-10T+13$	0	$9T^{12}-145T^{11}+1075T^{10}-4842T^9+14504T^8-28560T^7+27957T^6+35195T^5-225204T^4+573797T^3-1021641T^2+1411484T-1567262$	3 / ✘	2 / ✔
	8_{19}^n	T^3-T^2+1	$-3T^5-4T^2-3T$	$7T^{11}-19T^{10}+6T^9+48T^8-52T^7-91T^6+211T^5+16T^4-431T^3+289T^2+536T-1060$	3 / ✘	3 / ✘
	8_{20}^n	T^2-2T+3	$4T-4$	$4T^8-22T^7+66T^6-124T^5+52T^4+478T^3-1652T^2+3014T-3640$	2 / ✔	1 / ✘
	8_{21}^n	$-T^2+4T-5$	$T^3-8T^2+16T-20$	$3T^8-28T^7+49T^6+352T^5-2489T^4+8164T^3-17530T^2+27092T-31226$	2 / ✘	1 / ✘

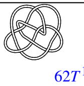
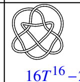
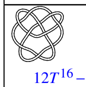
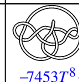
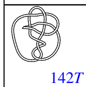
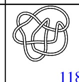
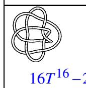
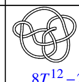
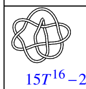
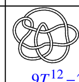
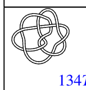
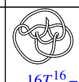
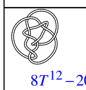
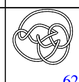
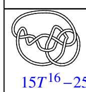
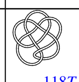
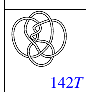
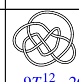
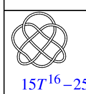
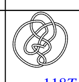
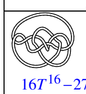
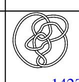
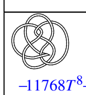
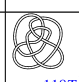
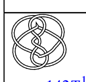
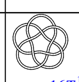
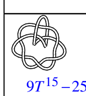
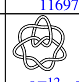
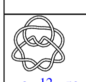
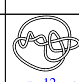
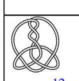
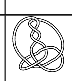
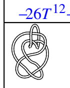
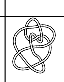
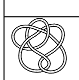
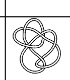
knot diag	n_k^t	Alexander's ω^+	$(\rho_1')^+$	$(\rho_2')^+$	genus / ribbon	unknotting # / amphi?
	9_1^a	$T^4-T^3+T^2-T+1$	$4T^7+7T^5+9T^3+10T$	$9T^{15}-36T^{14}+99T^{13}-216T^{12}+414T^{11}-720T^{10}+1170T^9-1800T^8+2630T^7-3662T^6+4853T^5-6142T^4+7423T^3-8572T^2+9420T-9780$	4 / ✘	4 / ✘
	9_2^a	$4T-7$	$30T-40$	$-728T^4+6088T^3-21946T^2+44788T-56420$	1 / ✘	1 / ✘
	9_3^a	$2T^3-3T^2+3T-3$	$-13T^5+12T^4-25T^3+20T^2-32T+24$	$-26T^{12}+296T^{11}-1311T^{10}+3838T^9-8867T^8+17613T^7-31407T^6+51061T^5-76085T^4+104297T^3-131779T^2+152840T-160976$	3 / ✘	3 / ✘
	9_4^a	$3T^2-5T+5$	$23T^3-28T^2+46T-44$	$-219T^8+1999T^7-8389T^6+23799T^5-52835T^4+96723T^3-149121T^2+194698T-213338$	2 / ✘	2 / ✘

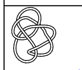
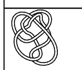
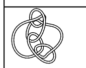
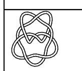
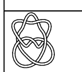
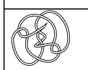
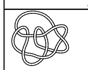
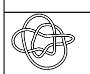
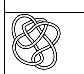
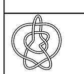
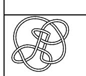
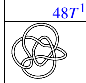
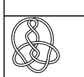
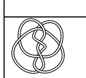
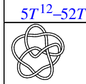
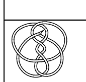
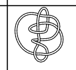
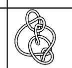
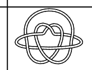
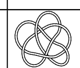
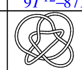
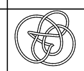
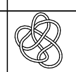
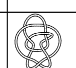
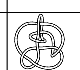
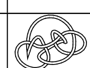
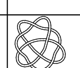
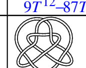
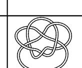
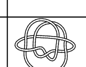
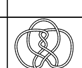
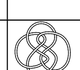
knot diag	n_k^t	Alexander's ω^+ $(\rho_1')^+$ $(\rho_2')^+$	genus / ribbon unknotting # / amphi?
	9^a_5	$6T-11$ $100-65T$ $-3234T^4+29792T^3-113241T^2+236818T-300294$	1 / ✘ 2 / ✘
	9^a_6	$2T^3-4T^2+5T-5$ $13T^5-24T^4+45T^3-52T^2+68T-64$ $-26T^{12}+376T^{11}-2212T^{10}+8280T^9-23249T^8+53488T^7-106013T^6+185990T^5-292853T^4+416673T^3-537062T^2+626488T-659788$	3 / ✘ 3 / ✘
	9^a_7	$3T^2-7T+9$ $23T^3-56T^2+99T-108$ $-219T^8+2717T^7-15720T^6+58389T^5-157698T^4+329265T^3-548657T^2+741610T-819394$	2 / ✘ 2 / ✘
	9^a_8	$-2T^2+8T-11$ $3T^3-16T^2+29T-28$ $54T^8-552T^7+2124T^6-2216T^5-12641T^4+67112T^3-172118T^2+289304T-342134$	2 / ✘ 2 / ✘
	9^a_9	$2T^3-4T^2+6T-7$ $13T^5-24T^4+55T^3-72T^2+98T-96$ $-26T^{12}+376T^{11}-2296T^{10}+9328T^9-28988T^8+73584T^7-158399T^6+295928T^5-486916T^4+712094T^3-930993T^2+1092074T-1151564$	3 / ✘ 3 / ✘
	9^a_{10}	$4T^2-8T+9$ $-40T^3+72T^2-114T+120$ $-608T^8+6720T^7-33776T^6+110928T^5-273462T^4+537040T^3-862768T^2+1145784T-1259748$	2 / ✘ 2, 3 / ✘
	9^a_{11}	$-T^3+5T^2-7T+7$ $-2T^5+16T^4-41T^3+52T^2-66T+64$ $5T^{12}-65T^{11}+312T^{10}-463T^9-2042T^8+14588T^7-50444T^6+126967T^5-258750T^4+444545T^3-654213T^2+827220T-895336$	3 / ✘ 2 / ✘
	9^a_{12}	$-2T^2+9T-13$ $5T^3-36T^2+84T-100$ $38T^8-312T^7+45T^6+9790T^5-60473T^4+202775T^3-453255T^2+722176T-841572$	2 / ✘ 1 / ✘
	9^a_{13}	$4T^2-9T+11$ $-40T^3+92T^2-154T+168$ $-608T^8+7680T^7-43650T^6+158004T^5-417129T^4+856533T^3-1412461T^2+1899222T-2095210$	2 / ✘ 2, 3 / ✘
	9^a_{14}	$2T^2-9T+15$ $-T^3+8T^2-35T+60$ $62T^8-752T^7+3655T^6-7178T^5-9502T^4+97737T^3-294656T^2+531720T-642168$	2 / ✘ 1 / ✘
	9^a_{15}	$-2T^2+10T-15$ $-5T^3+40T^2-108T+136$ $38T^8-360T^7+208T^6+12328T^5-84103T^4+298764T^3-691161T^2+1121034T-1313504$	2 / ✘ 2 / ✘
	9^a_{16}	$2T^3-5T^2+8T-9$ $-13T^5+36T^4-80T^3+120T^2-161T+168$ $-26T^{12}+456T^{11}-3331T^{10}+15554T^9-53941T^8+149494T^7-345106T^6+680900T^5-1167591T^4+1759576T^3-2347749T^2+2786466T-2949428$	3 / ✘ 3 / ✘
	9^a_{17}	T^3-5T^2+9T-9 $T^5-8T^4+23T^3-32T^2+28T-24$ $8T^{12}-125T^{11}+874T^{10}-3595T^9+9462T^8-15166T^7+6162T^6+47027T^5-181220T^4+415509T^3-716070T^2+982036T-1089796$	3 / ✘ 2 / ✘
	9^a_{18}	$4T^2-10T+13$ $40T^3-108T^2+193T-220$ $-608T^8+8224T^7-51208T^6+201904T^5-570516T^4+1228920T^3-2087725T^2+2850858T-3159722$	2 / ✘ 2 / ✘
	9^a_{19}	$2T^2-10T+17$ $T^3-8T^2+20T-24$ $62T^8-840T^7+4536T^6-10352T^5-7041T^4+116428T^3-372683T^2+688198T-836608$	2 / ✘ 1 / ✘
	9^a_{20}	$-T^3+5T^2-9T+11$ $2T^5-16T^4+47T^3-84T^2+117T-124$ $5T^{12}-65T^{11}+330T^{10}-577T^9-2439T^8+21482T^7-86959T^6+247237T^5-548658T^4+993841T^3-1502637T^2+1918532T-2080192$	3 / ✘ 2 / ✘
	9^a_{21}	$-2T^2+11T-17$ $-5T^3+44T^2-127T+164$ $38T^8-408T^7+493T^6+13802T^5-105014T^4+396685T^3-954552T^2+1583140T-1868380$	2 / ✘ 1 / ✘
	9^a_{22}	$T^3-5T^2+10T-11$ $-T^5+8T^4-24T^3+38T^2-40T+36$ $8T^{12}-125T^{11}+893T^{10}-3824T^9+10605T^8-17902T^7+6990T^6+64299T^5-251573T^4+584313T^3-1012133T^2+1388650T-1540398$	3 / ✘ 1 / ✘
	9^a_{23}	$4T^2-11T+15$ $40T^3-128T^2+243T-288$ $-608T^8+9184T^7-62698T^6+265980T^5-794496T^4+1781111T^3-3107204T^2+4307350T-4797258$	2 / ✘ 2 / ✘
	9^a_{24}	$-T^3+5T^2-10T+13$ $-4T^2+16T-20$ $9T^{12}-145T^{11}+1075T^{10}-4850T^9+14600T^8-29112T^7+29921T^6+30667T^5-218916T^4+570933T^3-1029833T^2+1433476T-1595654$	3 / ✘ 1 / ✘
	9^a_{25}	$-3T^2+12T-17$ $12T^3-70T^2+153T-188$ $174T^8-1200T^7-1027T^6+42696T^5-235512T^4+740956T^3-1585864T^2+2460360T-2841166$	2 / ✘ 2 / ✘
	9^a_{26}	$T^3-5T^2+11T-13$ $-T^5+8T^4-31T^3+64T^2-85T+92$ $8T^{12}-125T^{11}+900T^{10}-3861T^9+10351T^8-14356T^7-12391T^6+132473T^5-427732T^4+939309T^3-1588046T^2+2154028T-2381116$	3 / ✘ 1 / ✘
	9^a_{27}	$-T^3+5T^2-11T+15$ $T^3-8T^2+24T-32$ $9T^{12}-145T^{11}+1096T^{10}-5115T^9+16088T^8-33784T^7+37362T^6+34075T^5-273854T^4+743153T^3-1374545T^2+1941332T-2171344$	3 / ✔ 1 / ✘
	9^a_{28}	$T^3-5T^2+12T-15$ $T^5-8T^4+30T^3-68T^2+105T-120$ $8T^{12}-125T^{11}+923T^{10}-4138T^9+11800T^8-18092T^7-11101T^6+159415T^5-543916T^4+1228781T^3-2107809T^2+2877256T-3186008$	3 / ✘ 1 / ✘
	9^a_{29}	$T^3-5T^2+12T-15$ $T^5-8T^4+26T^3-48T^2+59T-56$ $8T^{12}-125T^{11}+931T^{10}-4290T^9+13096T^8-24848T^7+13335T^6+94047T^5-409576T^4+1010237T^3-1816557T^2+2543836T-2840192$	3 / ✘ 2 / ✘
	9^a_{30}	$-T^3+5T^2-12T+17$ $2T^3-10T^2+25T-32$ $9T^{12}-145T^{11}+1117T^{10}-5376T^9+17533T^8-38170T^7+43292T^6+43619T^5-347397T^4+957881T^3-1794189T^2+2553442T-2863228$	3 / ✘ 1 / ✘
	9^a_{31}	$T^3-5T^2+13T-17$ $T^5-8T^4+33T^3-80T^2+132T-152$ $8T^{12}-125T^{11}+938T^{10}-4303T^9+12544T^8-19138T^7-17200T^6+204143T^5-703180T^4+1617365T^3-2818190T^2+3886636T-4319004$	3 / ✘ 2 / ✘
	9^a_{32}	$T^3-6T^2+14T-17$ $-T^5+10T^4-42T^3+94T^2-133T+148$ $8T^{12}-150T^{11}+1269T^{10}-6297T^9+19455T^8-32720T^7-11156T^6+260282T^5-930836T^4+2153618T^3-3750358T^2+5165114T-5736454$	3 / ✘ 2 / ✘
	9^a_{33}	$-T^3+6T^2-14T+19$ $T^3-10T^2+30T-40$ $9T^{12}-174T^{11}+1539T^{10}-8207T^9+28913T^8-67184T^7+84077T^6+55866T^5-581640T^4+1664798T^3-3166838T^2+4539202T-5100726$	3 / ✘ 1 / ✘
	9^a_{34}	$-T^3+6T^2-16T+23$ $3T^3-18T^2+43T-56$ $9T^{12}-174T^{11}+1581T^{10}-8831T^9+32988T^8-81774T^7+109631T^6+73248T^5-829341T^4+2480938T^3-4869197T^2+7112552T-8043256$	3 / ✘ 1 / ✘
	9^a_{35}	$7T-13$ $90T-144$ $-6355T^4+58861T^3-224539T^2+470386T-596734$	1 / ✘ 2, 3 / ✘
	9^a_{36}	$-T^3+5T^2-8T+9$ $-2T^5+16T^4-44T^3+66T^2-87T+88$ $5T^{12}-65T^{11}+321T^{10}-532T^9-2081T^8+17066T^7-64846T^6+175611T^5-376739T^4+668001T^3-998037T^2+1267342T-1372104$	3 / ✘ 2 / ✘
	9^a_{37}	$2T^2-11T+19$ $T^3-8T^2+22T-28$ $62T^8-928T^7+5487T^6-13814T^5-6681T^4+154867T^3-520239T^2+983348T-1204192$	2 / ✘ 2 / ✘
	9^a_{38}	$5T^2-14T+19$ $62T^3-204T^2+382T-452$ $-1414T^8+22122T^7-153560T^6+657340T^5-1976110T^4+4454362T^3-7806448T^2+10855582T-12103772$	2 / ✘ 2, 3 / ✘
	9^a_{39}	$-3T^2+14T-21$ $-12T^3+84T^2-210T+268$ $174T^8-1442T^7-690T^6+59068T^5-366222T^4+1247214T^3-2815796T^2+4505578T-5255776$	2 / ✘ 1 / ✘
	9^a_{40}	$T^3-7T^2+18T-23$ $T^5-12T^4+57T^3-144T^2+229T-264$ $8T^{12}-175T^{11}+1712T^{10}-9738T^9+34250T^8-66108T^7-11148T^6+553509T^5-2149560T^4+5230963T^3-9406248T^2+13187800T-14730526$	3 / ✘ 2 / ✘

knot diag	n_k^t	Alexander's ω^+ / $(\rho_1')^+$ / $(\rho_2')^+$	genus / ribbon	unknotting # / amphi?
	9^a_{41}	$3T^2-12T+19$ $3T^3-20T^2+70T-108$ $309T^8-3288T^7+13885T^6-20928T^5-55179T^4+378100T^3-1035810T^2+1787808T-2129794$	2 / ✔	2 / ✘
	9^n_{42}	$-T^2+2T-1$ $-T^3+2T^2+T-4$ $3T^8-14T^7+32T^6-96T^5+265T^4-294T^3-498T^2+2170T-3128$	2 / ✘	1 / ✘
	9^n_{43}	$-T^3+3T^2-2T+1$ $-2T^5+8T^4-7T^3+2T^2-5T+4$ $5T^{12}-39T^{11}+110T^{10}-108T^9-115T^8+570T^7-1477T^6+3453T^5-6651T^4+10951T^3-17188T^2+24718T-28462$	3 / ✘	2 / ✘
	9^n_{44}	T^2-4T+7 $-2T^2+9T-12$ $4T^8-48T^7+237T^6-496T^5-346T^4+4988T^3-15044T^2+26768T-32126$	2 / ✘	1 / ✘
	9^n_{45}	$-T^2+6T-9$ $T^3-14T^2+47T-60$ $3T^8-42T^7+78T^6+1376T^5-11135T^4+42574T^3-102522T^2+169806T-200284$	2 / ✘	1 / ✘
	9^n_{46}	$5-2T$ $3T-12$ $-2T^4+160T^3-1125T^2+3082T-4222$	1 / ✔	2 / ✘
	9^n_{47}	T^3-4T^2+6T-5 $-T^5+6T^4-15T^3+16T^2-10T+12$ $8T^{12}-100T^{11}+560T^{10}-1841T^9+3847T^8-4710T^7-42T^6+17494T^5-55447T^4+117058T^3-193749T^2+261386T-288924$	3 / ✘	2 / ✘
	9^n_{48}	$-T^2+7T-11$ $-T^3+12T^2-42T+52$ $3T^8-49T^7+243T^6+267T^5-8051T^4+40499T^3-112167T^2+199850T-241202$	2 / ✘	2 / ✘
	9^n_{49}	$3T^2-6T+7$ $-21T^3+38T^2-61T+60$ $-123T^8+1614T^7-8744T^6+29928T^5-75873T^4+152714T^3-250794T^2+338238T-373944$	2 / ✘	3 / ✘
	10^a_1	$9-4T$ $14T-40$ $-24T^4+2136T^3-13430T^2+34860T-47068$	1 / ✘	1 / ✘
	10^a_2	$-T^4+3T^3-3T^2+3T-3$ $3T^7-12T^6+16T^5-20T^4+24T^3-24T^2+27T-24$ $7T^{16}-57T^{15}+189T^{14}-293T^{13}-55T^{12}+1628T^{11}-5543T^{10}+13266T^9-26589T^8+47468T^7-77415T^6+116549T^5-162911T^4+212325T^3-258413T^2+292580T-305480$	4 / ✘	3 / ✘
	10^a_3	$13-6T$ $11T-28$ $870T^4+1288T^3-27795T^2+85718T-120138$	1 / ✔	2 / ✘
	10^a_4	$-3T^2+7T-7$ $4T^3-8T^2+T+8$ $294T^8-1807T^7+4570T^6-4305T^5-9550T^4+49581T^3-117456T^2+189330T-221294$	2 / ✘	2 / ✘
	10^a_5	$T^4-3T^3+5T^2-5T+5$ $-2T^7+8T^6-20T^5+28T^4-36T^3+36T^2-39T+36$ $12T^{16}-117T^{15}+565T^{14}-1757T^{13}+3847T^{12}-5960T^{11}+5381T^{10}+2968T^9-26625T^8+75008T^7-157415T^6+279173T^5-436999T^4+615297T^3-785328T^2+909916T-955948$	4 / ✘	2 / ✘
	10^a_6	$-2T^3+6T^2-7T+7$ $9T^5-36T^4+56T^3-72T^2+81T-84$ $62T^{12}-408T^{11}+712T^{10}+2280T^9-17493T^8+60652T^7-153492T^6+319048T^5-569584T^4+890397T^3-1228657T^2+1496150T-1599330$	3 / ✘	3 / ✘
	10^a_7	$-3T^2+11T-15$ $14T^3-72T^2+135T-160$ $114T^8-275T^7-5840T^6+51739T^5-222492T^4+626425T^3-1267348T^2+1914410T-2193462$	2 / ✘	1 / ✘
	10^a_8	$-2T^3+5T^2-5T+5$ $7T^5-20T^4+23T^3-28T^2+26T-24$ $94T^{12}-672T^{11}+2115T^{10}-3678T^9+2535T^8+6453T^7-30645T^6+78385T^5-154895T^4+256601T^3-367525T^2+458500T-494524$	3 / ✘	2 / ✘
	10^a_9	$-T^4+3T^3-5T^2+7T-7$ $-T^7+4T^6-10T^5+20T^4-25T^3+28T^2-28T+28$ $15T^{16}-153T^{15}+787T^{14}-2727T^{13}+7084T^{12}-14404T^{11}+22886T^{10}-26134T^9+11540T^8+39332T^7-146866T^6+325115T^5-571077T^4+856941T^3-1131013T^2+1330668T-1403980$	4 / ✘	1 / ✘
	10^a_{10}	$3T^2-11T+17$ $-5T^3+24T^2-71T+100$ $285T^8-2735T^7+10078T^6-9479T^5-64000T^4+327253T^3-827377T^2+1378130T-1624314$	2 / ✘	1 / ✘
	10^a_{11}	$-4T^2+11T-13$ $16T^3-52T^2+68T-72$ $736T^8-4672T^7+9634T^6+11132T^5-125367T^4+413121T^3-873095T^2+1336974T-1536906$	2 / ✘	2, 3 / ✘
	10^a_{12}	$2T^3-6T^2+10T-11$ $-5T^5+20T^4-50T^3+72T^2-89T+92$ $118T^{12}-1080T^{11}+4748T^{10}-12624T^9+19414T^8-2072T^7-88507T^6+320836T^5-750453T^4+1366922T^3-2053481T^2+2604638T-2816934$	3 / ✘	2 / ✘
	10^a_{13}	$2T^2-13T+23$ $T^3-12T^2+51T-84$ $62T^8-1088T^7+7367T^6-20586T^5-13356T^4+286509T^3-1005098T^2+1954280T-2416160$	2 / ✘	2 / ✘
	10^a_{14}	$-2T^3+8T^2-12T+13$ $9T^5-52T^4+119T^3-180T^2+225T-236$ $62T^{12}-584T^{11}+1720T^{10}+2816T^9-42848T^8+195040T^7-594177T^6+1407688T^5-2753604T^4+4575154T^3-6545078T^2+8106820T-8706026$	3 / ✘	2 / ✘
	10^a_{15}	$2T^3-6T^2+9T-9$ $-3T^5+12T^4-24T^3+24T^2-17T+12$ $134T^{12}-1272T^{11}+5792T^{10}-16520T^9+31765T^8-37636T^7+2396T^6+120176T^5-371368T^4+752873T^3-1195043T^2+1560190T-1702986$	3 / ✘	2 / ✘
	10^a_{16}	$-4T^2+12T-15$ $-16T^3+56T^2-76T+80$ $736T^8-5248T^7+12944T^6+6528T^5-144162T^4+522200T^3-1155370T^2+1809228T-2093696$	2 / ✘	2 / ✘
	10^a_{17}	$T^4-3T^3+5T^2-7T+9$ 0 $16T^{16}-165T^{15}+861T^{14}-3043T^{13}+8173T^{12}-17514T^{11}+30162T^{10}-39958T^9+32666T^8+13998T^7-125081T^6+317743T^5-588481T^4+904569T^3-1207020T^2+1426556T-1506972$	4 / ✘	1 / ✔
	10^a_{18}	$-4T^2+14T-19$ $16T^3-68T^2+121T-140$ $736T^8-6240T^7+17736T^6+11088T^5-245648T^4+930168T^3-2109201T^2+3338706T-3874682$	2 / ✘	1 / ✘
	10^a_{19}	$2T^3-7T^2+11T-11$ $3T^5-16T^4+35T^3-40T^2+30T-24$ $134T^{12}-1480T^{11}+7641T^{10}-24194T^9+50855T^8-66007T^7+12323T^6+201357T^5-665287T^4+1397797T^3-2271085T^2+3006128T-3296368$	3 / ✘	2 / ✘
	10^a_{20}	$-3T^2+9T-11$ $14T^3-56T^2+88T-104$ $114T^8-153T^7-4783T^6+34425T^5-128711T^4+327435T^3-618704T^2+899066T-1017366$	2 / ✘	2 / ✘
	10^a_{21}	$-2T^3+7T^2-9T+9$ $9T^5-44T^4+80T^3-104T^2+121T-124$ $62T^{12}-496T^{11}+1203T^{10}+2078T^9-24456T^8+97163T^7-267878T^6+592041T^5-1106738T^4+1789591T^3-2525732T^2+3113752T-3341184$	3 / ✘	2 / ✘
	10^a_{22}	$-2T^3+6T^2-10T+13$ $-T^5+4T^4-10T^3+24T^2-37T+44$ $142T^{12}-1368T^{11}+6524T^{10}-20120T^9+42790T^8-57928T^7+16919T^6+158700T^5-540707T^4+1130294T^3-1809643T^2+2363114T-2577418$	3 / ✔	2 / ✘
	10^a_{23}	$2T^3-7T^2+13T-15$ $-5T^5+24T^4-67T^3+108T^2-137T+144$ $118T^{12}-1272T^{11}+6541T^{10}-20402T^9+38443T^8-21945T^7-132442T^6+594335T^5-1530420T^4+2960363T^3-4622193T^2+5992048T-6526360$	3 / ✘	1 / ✘
	10^a_{24}	$-4T^2+14T-19$ $24T^3-116T^2+221T-268$ $416T^8-1568T^7-13224T^6+136928T^5-604124T^4+1701008T^3-3414673T^2+5118714T-5846946$	2 / ✘	2 / ✘
	10^a_{25}	$-2T^3+8T^2-14T+17$ $9T^5-52T^4+131T^3-232T^2+314T-344$ $62T^{12}-584T^{11}+1856T^{10}+2264T^9-47052T^8+241288T^7-809541T^6+2068016T^5-4270010T^4+7347930T^3-10723331T^2+13406206T-14434208$	3 / ✘	2 / ✘
	10^a_{26}	$-2T^3+7T^2-13T+17$ $-T^5+4T^4-10T^3+28T^2-49T+60$ $142T^{12}-1600T^{11}+8823T^{10}-31058T^9+74964T^8-117897T^7+67064T^6+255997T^5-1047600T^4+2360395T^3-3947888T^2+5281288T-5805248$	3 / ✘	1 / ✘
	10^a_{27}	$2T^3-8T^2+16T-19$ $5T^5-28T^4+87T^3-164T^2+229T-252$ $118T^{12}-1464T^{11}+8536T^{10}-29792T^9+62096T^8-39696T^7-242195T^6+1151848T^5-3078140T^4+6098910T^3-9661940T^2+12621240T-13779050$	3 / ✘	1 / ✘

knot diag	n^t_k	Alexander's ω^+ / $(\rho'_1)^+$ / $(\rho'_2)^+$	genus / ribbon unknotting # / amphi?
	10^a_{28}	$4T^2-13T+19$ $-8T^3+36T^2-100T+136$ $928T^8-7872T^7+26174T^6-22588T^5-142295T^4+689113T^3-1676391T^2+2728998T-3192146$	2 / ✗ 2 / ✗
	10^a_{29}	$T^3-7T^2+15T-17$ $T^5-12T^4+52T^3-104T^2+124T-128$ $8T^{12}-175T^{11}+1659T^{10}-8913T^9+29252T^8-54292T^7+10686T^6+290989T^5-1126663T^4+2673211T^3-4723498T^2+6566572T-7317656$	3 / ✗ 2 / ✗
	10^a_{30}	$-4T^2+17T-25$ $24T^3-148T^2+345T-440$ $416T^8-2048T^7-17490T^6+219996T^5-1101894T^4+3396907T^3-7245510T^2+11243734T-12988226$	2 / ✗ 1 / ✗
	10^a_{31}	$4T^2-14T+21$ $-4T^2+9T-12$ $992T^8-9440T^7+36936T^6-59136T^5-72624T^4+623304T^3-1691899T^2+2867550T-3391374$	2 / ✗ 1 / ✗
	10^a_{32}	$-2T^3+8T^2-15T+19$ $T^5-4T^4+13T^3-40T^2+78T-96$ $142T^{12}-1832T^{11}+11204T^{10}-42688T^9+109909T^8-184384T^7+124831T^6+360782T^5-1615391T^4+3759585T^3-6404890T^2+8655360T-9545252$	3 / ✗ 1 / ✗
	10^a_{33}	$4T^2-16T+25$ 0 $992T^8-10816T^7+47856T^6-88336T^5-84402T^4+920320T^3-2655340T^2+4640912T-5542372$	2 / ✗ 1 / ✔
	10^a_{34}	$3T^2-9T+13$ $-5T^3+20T^2-52T+68$ $285T^8-2205T^7+6601T^6-3429T^5-43369T^4+185703T^3-431857T^2+687874T-799218$	2 / ✗ 2 / ✗
	10^a_{35}	$2T^2-12T+21$ $-T^3+12T^2-47T+76$ $62T^8-1000T^7+6244T^6-15744T^5-15707T^4+232680T^3-775840T^2+1474372T-1810118$	2 / ✔ 2 / ✗
	10^a_{36}	$-3T^2+13T-19$ $14T^3-88T^2+208T-264$ $114T^8-397T^7-7597T^6+81141T^5-393441T^4+1198967T^3-2544952T^2+3941362T-4550398$	2 / ✗ 2 / ✗
	10^a_{37}	$4T^2-13T+19$ 0 $992T^8-8736T^7+31914T^6-47212T^5-64499T^4+497921T^3-1308755T^2+2181630T-2566522$	2 / ✗ 2 / ✔
	10^a_{38}	$-4T^2+15T-21$ $24T^3-128T^2+270T-336$ $416T^8-1632T^7-16122T^6+172460T^5-788845T^4+2280037T^3-4653713T^2+7038342T-8061882$	2 / ✗ 2 / ✗
	10^a_{39}	$-2T^3+8T^2-13T+15$ $9T^5-52T^4+125T^3-204T^2+263T-280$ $62T^{12}-584T^{11}+1788T^{10}+2480T^9-44191T^8+213488T^7-683173T^6+1684054T^5-3393468T^4+5753447T^3-8330571T^2+10379080T-11164828$	3 / ✗ 2 / ✗
	10^a_{40}	$2T^3-8T^2+17T-21$ $-5T^5+28T^4-89T^3+176T^2-258T+288$ $118T^{12}-1464T^{11}+8692T^{10}-31256T^9+67987T^8-49624T^7-257955T^6+1301482T^5-3582545T^4+7240253T^3-11620382T^2+15292356T-16735336$	3 / ✗ 2 / ✗
	10^a_{41}	$T^3-7T^2+17T-21$ $T^5-12T^4+54T^3-120T^2+157T-164$ $8T^{12}-175T^{11}+1697T^{10}-9543T^9+33561T^8-69114T^7+29117T^6+354127T^5-1527139T^4+3836499T^3-7019042T^2+9942516T-11145016$	3 / ✗ 2 / ✗
	10^a_{42}	$-T^3+7T^2-19T+27$ $2T^3-8T^2+11T-12$ $9T^{12}-203T^{11}+2093T^{10}-12971T^9+52885T^8-142268T^7+214987T^6+60931T^5-1368859T^4+4365895T^3-8815357T^2+13058404T-14831092$	3 / ✔ 1 / ✗
	10^a_{43}	$-T^3+7T^2-17T+23$ 0 $9T^{12}-203T^{11}+2051T^{10}-12253T^9+47594T^8-120962T^7+170450T^6+61017T^5-1045911T^4+3175271T^3-6209661T^2+9025932T-10186676$	3 / ✗ 2 / ✔
	10^a_{44}	$T^3-7T^2+19T-25$ $T^5-12T^4+56T^3-140T^2+220T-248$ $8T^{12}-175T^{11}+1735T^{10}-10157T^9+37586T^8-81160T^7+29232T^6+500937T^5-2197451T^4+5635115T^3-10448058T^2+14900236T-16735696$	3 / ✗ 1 / ✗
	10^a_{45}	$-T^3+7T^2-21T+31$ 0 $9T^{12}-203T^{11}+2135T^{10}-13689T^9+58324T^8-165246T^7+266640T^6+52413T^5-1738539T^4+5821367T^3-12123077T^2+18290148T-20900556$	3 / ✗ 2 / ✔
	10^a_{46}	$-T^4+3T^3-4T^2+5T-5$ $-3T^7+12T^6-21T^5+34T^4-43T^3+52T^2-55T+56$ $7T^{16}-57T^{15}+204T^{14}-382T^{13}+69T^{12}+2247T^{11}-9674T^{10}+27287T^9-61957T^8+121378T^7-211961T^6+335438T^5-485235T^4+644818T^3-789365T^2+891215T-928064$	4 / ✗ 3 / ✗
	10^a_{47}	$T^4-3T^3+6T^2-7T+7$ $-2T^7+8T^6-23T^5+38T^4-56T^3+60T^2-68T+64$ $12T^{16}-117T^{15}+598T^{14}-2030T^{13}+4959T^{12}-8715T^{11}+9312T^{10}+2921T^9-44823T^8+139602T^7-312112T^6+579182T^5-936546T^4+1347538T^3-1741633T^2+2029805T-2135930$	4 / ✗ 2, 3 / ✗
	10^a_{48}	$T^4-3T^3+6T^2-9T+11$ $T^5-2T^4+2T^3-3T+4$ $16T^{16}-165T^{15}+906T^{14}-3452T^{13}+10069T^{12}-23423T^{11}+43765T^{10}-63343T^9+59588T^8+8232T^7-192505T^6+537134T^5-1048176T^4+1669528T^3-2281994T^2+2735109T-2902594$	4 / ✔ 2 / ✗
	10^a_{49}	$3T^3-8T^2+12T-13$ $30T^5-94T^4+196T^3-292T^2+372T-392$ $-177T^{12}+3028T^{11}-22080T^{10}+101361T^9-341354T^8+914348T^7-2044469T^6+3931812T^5-6622778T^4+9874270T^3-13105110T^2+15522532T-16422794$	3 / ✗ 3 / ✗
	10^a_{50}	$-2T^3+7T^2-11T+13$ $-9T^5+44T^4-94T^3+150T^2-186T+200$ $62T^{12}-496T^{11}+1283T^{10}+2094T^9-29732T^8+134301T^7-412809T^6+990903T^5-1959941T^4+3278621T^3-4702408T^2+5824956T-6253664$	3 / ✗ 2 / ✗
	10^a_{51}	$2T^3-7T^2+15T-19$ $-5T^5+24T^4-73T^3+134T^2-194T+212$ $118T^{12}-1272T^{11}+6813T^{10}-22602T^9+45771T^8-28275T^7-180411T^6+857569T^5-2306697T^4+4602641T^3-7332665T^2+9612128T-10506256$	3 / ✗ 2, 3 / ✗
	10^a_{52}	$2T^3-7T^2+13T-15$ $-3T^5+16T^4-37T^3+50T^2-49T+44$ $134T^{12}-1480T^{11}+7961T^{10}-27058T^9+62159T^8-88993T^7+22042T^6+296843T^5-1040240T^4+2254967T^3-3720017T^2+4952400T-5437448$	3 / ✗ 2 / ✗
	10^a_{53}	$6T^2-18T+25$ $93T^3-346T^2+680T-828$ $-3642T^8+58248T^7-417976T^6+1846212T^5-5694639T^4+13084936T^3-23231163T^2+32545278T-36374532$	2 / ✗ 2, 3 / ✗
	10^a_{54}	$2T^3-6T^2+10T-11$ $-3T^5+12T^4-24T^3+26T^2-21T+16$ $134T^{12}-1272T^{11}+5964T^{10}-17880T^9+36606T^8-46740T^7+6565T^6+150576T^5-487825T^4+1010638T^3-1619593T^2+2120978T-2316318$	3 / ✗ 2, 3 / ✗
	10^a_{55}	$5T^2-15T+21$ $66T^3-246T^2+488T-596$ $-1966T^8+30491T^7-215627T^6+945597T^5-2905831T^4+6662951T^3-11814712T^2+16540014T-18481854$	2 / ✗ 2 / ✗
	10^a_{56}	$-2T^3+8T^2-14T+17$ $-9T^5+52T^4-133T^3+234T^2-312T+340$ $62T^{12}-584T^{11}+1800T^{10}+2840T^9-49588T^8+247616T^7-819257T^6+2077408T^5-4277830T^4+7364010T^3-10765639T^2+13481990T-14525656$	3 / ✗ 2 / ✗
	10^a_{57}	$2T^3-8T^2+18T-23$ $-5T^5+28T^4-93T^3+194T^2-300T+340$ $118T^{12}-1464T^{11}+8808T^{10}-32264T^9+71276T^8-49320T^7-305843T^6+1537376T^5-4286854T^4+8774390T^3-14221383T^2+18829374T-20648444$	3 / ✗ 2 / ✗
	10^a_{58}	$3T^2-16T+27$ $3T^3-28T^2+94T-140$ $309T^8-4384T^7+24039T^6-49896T^5-90763T^4+864784T^3-2647834T^2+4837480T-5867454$	2 / ✗ 2 / ✗
	10^a_{59}	$T^3-7T^2+18T-23$ $-T^5+12T^4-55T^3+128T^2-181T+196$ $8T^{12}-175T^{11}+1716T^{10}-9858T^9+35706T^8-76124T^7+33704T^6+412653T^5-1824096T^4+4655939T^3-8596644T^2+12230816T-13727286$	3 / ✗ 1 / ✗
	10^a_{60}	$-T^3+7T^2-20T+29$ $5T^3-40T^2+122T-176$ $9T^{12}-203T^{11}+2114T^{10}-13338T^9+55732T^8-154496T^7+241898T^6+66137T^5-1621594T^4+5326603T^3-10989858T^2+16499428T-18824860$	3 / ✗ 1 / ✗
	10^a_{61}	$-2T^3+5T^2-6T+7$ $-7T^5+20T^4-27T^3+36T^2-35T+36$ $94T^{12}-672T^{11}+2231T^{10}-4382T^9+4108T^8+6320T^7-40187T^6+113296T^5-235714T^4+400470T^3-576529T^2+714816T-767686$	3 / ✗ 2, 3 / ✗
	10^a_{62}	$T^4-3T^3+6T^2-8T+9$ $-2T^7+8T^6-23T^5+40T^4-63T^3+76T^2-89T+88$ $12T^{16}-117T^{15}+598T^{14}-2057T^{13}+5172T^{12}-9509T^{11}+10856T^{10}+2734T^9-54502T^8+178917T^7-414312T^6+786766T^5-1289208T^4+1865866T^3-2414454T^2+2812025T-2957594$	4 / ✗ 2 / ✗
	10^a_{63}	$5T^2-14T+19$ $66T^3-220T^2+416T-496$ $-1966T^8+28318T^7-188080T^6+783388T^5-2311570T^4+5141906T^3-8929148T^2+12349082T-13743884$	2 / ✗ 2 / ✗

knot diag	n^t_k	Alexander's ω^+ / $(\rho'_1)^+$ / $(\rho'_2)^+$	genus / ribbon	unknotting # / amphi?
	10^a_{64}	$-T^4+3T^3-6T^2+10T-11$ $-T^7+4T^6-11T^5+24T^4-37T^3+52T^2-60T+64$ $15T^{16}-153T^{15}+830T^{14}-3147T^{13}+9133T^{12}-20983T^{11}+37963T^{10}-50164T^9+30642T^8+68741T^7-310036T^6+745430T^5-1381735T^4+2150560T^3-2906317T^2+3464829T-3671204$	4 / ✘	2 / ✘
	10^a_{65}	$2T^3-7T^2+14T-17$ $-5T^5+24T^4-71T^3+124T^2-169T+180$ $118T^{12}-1272T^{11}+6657T^{10}-21282T^9+40874T^8-20768T^7-166691T^6+742216T^5-1933704T^4+3781794T^3-5950947T^2+7749120T-8452246$	3 / ✘	2 / ✘
	10^a_{66}	$3T^3-9T^2+16T-19$ $30T^5-112T^4+279T^3-480T^2+662T-724$ $-177T^{12}+3321T^{11}-27536T^{10}+145346T^9-561614T^8+1706788T^7-4256134T^6+8946173T^5-16135424T^4+25271935T^3-34647456T^2+41790680T-44471832$	3 / ✘	3 / ✘
	10^a_{67}	$-4T^2+16T-23$ $24T^3-140T^2+312T-392$ $416T^8-1696T^7-18592T^6+205384T^5-971474T^4+2884880T^3-6004484T^2+9188872T-10566612$	2 / ✘	2 / ✘
	10^a_{68}	$4T^2-14T+21$ $8T^3-40T^2+117T-164$ $928T^8-8448T^7+29784T^6-26736T^5-178984T^4+891736T^3-2217147T^2+3657390T-4297054$	2 / ✘	2 / ✘
	10^a_{69}	$T^3-7T^2+21T-29$ $-T^5+12T^4-68T^3+212T^2-397T+476$ $8T^{12}-175T^{11}+1753T^{10}-10339T^9+37435T^8-68174T^7-78997T^6+1015635T^5-3880779T^4+9697491T^3-17937826T^2+25646300T-28844672$	3 / ✘	2 / ✘
	10^a_{70}	$T^3-7T^2+16T-19$ $-T^5+12T^4-53T^3+114T^2-146T+152$ $8T^{12}-175T^{11}+1678T^{10}-9220T^9+31251T^8-60450T^7+14335T^6+337593T^5-1351773T^4+3275803T^3-5864336T^2+8208654T-9166724$	3 / ✘	2 / ✘
	10^a_{71}	$-T^3+7T^2-18T+25$ T^3-2T^2-T+4 $9T^{12}-203T^{11}+2072T^{10}-12608T^9+50167T^8-131082T^7+190655T^6+64937T^5-1206917T^4+3745659T^3-7436102T^2+10906778T-12346734$	3 / ✘	1 / ✘
	10^a_{72}	$-2T^3+9T^2-16T+19$ $-9T^5+60T^4-167T^3+298T^2-410T+448$ $62T^{12}-672T^{11}+2407T^{10}+2846T^9-67046T^8+358714T^7-1237440T^6+3225136T^5-6760702T^4+11767984T^3-17315777T^2+21757146T-23465324$	3 / ✘	2 / ✘
	10^a_{73}	$T^3-7T^2+20T-27$ $T^5-12T^4+65T^3-194T^2+350T-416$ $8T^{12}-175T^{11}+1738T^{10}-10112T^9+36117T^8-66038T^7-61235T^6+869449T^5-3296603T^4+8133803T^3-14880880T^2+21122890T-23697928$	3 / ✘	1 / ✘
	10^a_{74}	$-4T^2+16T-23$ $24T^3-136T^2+290T-360$ $416T^8-1984T^7-14448T^6+178832T^5-870542T^4+2626104T^3-5521764T^2+8500760T-9794748$	2 / ✘	2 / ✘
	10^a_{75}	$-T^3+7T^2-19T+27$ $-4T^3+36T^2-117T+172$ $9T^{12}-203T^{11}+2093T^{10}-12979T^9+53085T^8-144060T^7+222795T^6+45939T^5-1382507T^4+4528919T^3-9302365T^2+13926940T-15875332$	3 / ✔	2 / ✘
	10^a_{76}	$-2T^3+7T^2-12T+15$ $-9T^5+44T^4-104T^3+184T^2-245T+272$ $62T^{12}-496T^{11}+1263T^{10}+2926T^9-37611T^8+174774T^7-553794T^6+1359740T^5-2727505T^4+4595668T^3-6610039T^2+8193314T-8796596$	3 / ✘	2, 3 / ✘
	10^a_{77}	$2T^3-7T^2+14T-17$ $-5T^5+24T^4-71T^3+132T^2-189T+208$ $118T^{12}-1272T^{11}+6657T^{10}-21170T^9+39602T^8-13480T^7-193563T^6+812568T^5-2072452T^4+3997538T^3-6227879T^2+8058912T-8771174$	3 / ✘	2, 3 / ✘
	10^a_{78}	$-T^3+7T^2-16T+21$ $2T^5-24T^4+105T^3-244T^2+390T-448$ $5T^{12}-91T^{11}+626T^{10}-1310T^9-9682T^8+98268T^7-472808T^6+1558897T^5-3892200T^4+7699107T^3-12365278T^2+16351352T-17933784$	3 / ✘	2 / ✘
	10^a_{79}	$T^4-3T^3+7T^2-12T+15$ 0 $16T^{16}-165T^{15}+951T^{14}-3892T^{13}+12327T^{12}-31301T^{11}+64047T^{10}-102088T^9+108942T^8-5172T^7-328635T^6+1013644T^5-2099318T^4+3486798T^3-4904824T^2+5979109T-6380898$	4 / ✘	2, 3 / ✔
	10^a_{80}	$3T^3-9T^2+15T-17$ $30T^5-112T^4+260T^3-426T^2+568T-616$ $-177T^{12}+3321T^{11}-26919T^{10}+137419T^9-511788T^8+1500906T^7-3625608T^6+7420093T^5-13101785T^4+20196767T^3-27388655T^2+32826444T-34860060$	3 / ✘	3 / ✘
	10^a_{81}	$-T^3+8T^2-20T+27$ 0 $9T^{12}-232T^{11}+2632T^{10}-17347T^9+73146T^8-199476T^7+303717T^6+63516T^5-1783222T^4+5636674T^3-11239918T^2+16501092T-18681194$	3 / ✘	2 / ✔
	10^a_{82}	$-T^4+4T^3-8T^2+12T-13$ $T^7-6T^6+19T^5-42T^4+64T^3-78T^2+84T-84$ $15T^{16}-204T^{15}+1362T^{14}-5956T^{13}+19067T^{12}-46940T^{11}+89646T^{10}-125984T^9+94379T^8+118488T^7-663600T^6+1675944T^5-3187626T^4+5046508T^3-6899632T^2+8282752T-8796438$	4 / ✘	1 / ✘
	10^a_{83}	$2T^3-9T^2+19T-23$ $-5T^5+34T^4-110T^3+214T^2-301T+332$ $118T^{12}-1632T^{11}+10501T^{10}-40166T^9+92154T^8-74661T^7-344938T^6+1829049T^5-5155786T^4+10589003T^3-17184002T^2+22763416T-24966116$	3 / ✘	2 / ✘
	10^a_{84}	$2T^3-9T^2+20T-25$ $-5T^5+34T^4-116T^3+246T^2-373T+424$ $118T^{12}-1632T^{11}+10601T^{10}-40970T^9+93361T^8-60130T^7-457712T^6+2276184T^5-6379977T^4+13131088T^3-21370125T^2+28363542T-31128704$	3 / ✘	1 / ✘
	10^a_{85}	$T^4-4T^3+8T^2-10T+11$ $2T^7-12T^6+36T^5-68T^4+101T^3-124T^2+138T-140$ $12T^{16}-156T^{15}+986T^{14}-3982T^{13}+11319T^{12}-23042T^{11}+29987T^{10}-3098T^9-116460T^8+418314T^7-1005425T^6+1953048T^5-3252398T^4+4764776T^3-6220611T^2+7285042T-7676632$	4 / ✘	2 / ✘
	10^a_{86}	$-2T^3+9T^2-19T+25$ $-T^5+6T^4-21T^3+58T^2-105T+128$ $142T^{12}-2056T^{11}+14135T^{10}-60346T^9+173073T^8-322457T^7+256132T^6+640839T^5-3192178T^4+7806511T^3-13712731T^2+18852080T-20906284$	3 / ✘	2 / ✘
	10^a_{87}	$-2T^3+9T^2-18T+23$ $-T^5+6T^4-23T^3+66T^2-125T+152$ $142T^{12}-2056T^{11}+13955T^{10}-58318T^9+162798T^8-293228T^7+214867T^6+612960T^5-2882460T^4+6902570T^3-11979669T^2+16361444T-18106010$	3 / ✔	2 / ✘
	10^a_{88}	$-T^3+8T^2-24T+35$ 0 $9T^{12}-232T^{11}+2716T^{10}-18955T^9+86300T^8-257664T^7+436281T^6+55760T^5-2823656T^4+9657962T^3-20306480T^2+30775472T-35215022$	3 / ✘	1 / ✔
	10^a_{89}	$T^3-8T^2+24T-33$ $T^5-14T^4+83T^3-264T^2+495T-596$ $8T^{12}-200T^{11}+2236T^{10}-14461T^9+56992T^8-117072T^7-76152T^6+1508604T^5-6093936T^4+15620030T^3-29286604T^2+42155400T-47509694$	3 / ✘	2 / ✘
	10^a_{90}	$-2T^3+8T^2-17T+23$ $-T^5+6T^4-21T^3+54T^2-93T+112$ $142T^{12}-1824T^{11}+11452T^{10}-45568T^9+123153T^8-214976T^7+138515T^6+523918T^5-2309034T^4+5458443T^3-9432309T^2+12861496T-14226804$	3 / ✘	2 / ✘
	10^a_{91}	$T^4-4T^3+9T^2-14T+17$ $T^5-2T^4+2T^3-3T+4$ $16T^{16}-220T^{15}+1535T^{14}-7166T^{13}+24885T^{12}-67476T^{11}+145070T^{10}-242014T^9+278753T^8-78212T^7-624329T^6+2091910T^5-4424108T^4+7397630T^3-10425418T^2+12711814T-13565348$	4 / ✘	1 / ✘
	10^a_{92}	$-2T^3+10T^2-20T+25$ $-9T^5+68T^4-216T^3+428T^2-622T+696$ $62T^{12}-760T^{11}+3228T^{10}+1776T^9-90686T^8+555772T^7-2114169T^6+5951964T^5-13251159T^4+24127850T^3-36624016T^2+46862460T-50844652$	3 / ✘	2 / ✘
	10^a_{93}	$2T^3-8T^2+15T-17$ $3T^5-18T^4+43T^3-58T^2+55T-48$ $134T^{12}-1696T^{11}+10180T^{10}-37880T^9+94183T^8-147272T^7+62729T^6+424866T^5-1618596T^4+3616743T^3-6059793T^2+8130868T-8948936$	3 / ✘	2 / ✘
	10^a_{94}	$-T^4+4T^3-9T^2+14T-15$ $-T^7+6T^6-20T^5+46T^4-76T^3+102T^2-115T+120$ $15T^{16}-204T^{15}+1405T^{14}-6454T^{13}+21907T^{12}-57432T^{11}+117080T^{10}-176754T^9+150405T^8+135972T^7-928717T^6+2460642T^5-4804019T^4+7729462T^3-10672990T^2+12881566T-13703760$	4 / ✘	2 / ✘
	10^a_{95}	$2T^3-9T^2+21T-27$ $-5T^5+32T^4-114T^3+248T^2-384T+436$ $118T^{12}-1656T^{11}+11045T^{10}-44462T^9+109118T^8-104035T^7-391583T^6+2298083T^5-6804711T^4+14456709T^3-24008082T^2+32236696T-35514492$	3 / ✘	1 / ✘
	10^a_{96}	$-T^3+7T^2-22T+33$ $-7T^3+50T^2-147T+212$ $9T^{12}-203T^{11}+2156T^{10}-14060T^9+61189T^8-177034T^7+287437T^6+96689T^5-2149699T^4+7231587T^3-15228082T^2+23163354T-26546674$	3 / ✘	2 / ✘
	10^a_{97}	$-5T^2+22T-33$ $-37T^3+242T^2-603T+788$ $1061T^8-5486T^7-47090T^6+615064T^5-3157165T^4+9904926T^3-21376446T^2+33395786T-38661308$	2 / ✘	2 / ✘

knot diag	n^t_k	Alexander's ω^+ / $(\rho'_1)^+$ / $(\rho'_2)^+$	genus / ribbon, unknotting # / amphi?
	10^a_{98}	$-2T^3+9T^2-18T+23$ $9T^5-60T^4+177T^3-348T^2+501T-564$ $62T^{12}-672T^{11}+2575T^{10}+1666T^9-67602T^8+398948T^7-1483813T^6+4115776T^5-9069800T^4+16396378T^3-24767965T^2+31602148T-34255402$	3 / ✘ 2 / ✘
	10^a_{99}	$T^4-4T^3+10T^2-16T+19$ 0 $16T^{16}-220T^{15}+1580T^{14}-7688T^{13}+27976T^{12}-79612T^{11}+179656T^{10}-315060T^9+386272T^8-148160T^7-792172T^6+2854748T^5-6237824T^4+10649644T^3-15214156T^2+18696608T-20003232$	4 / ✔ 2 / ✔
	10^a_{100}	$T^4-4T^3+9T^2-12T+13$ $2T^7-12T^6+39T^5-80T^4+128T^3-164T^2+192T-196$ $12T^{16}-156T^{15}+1019T^{14}-4340T^{13}+13189T^{12}-29012T^{11}+41715T^{10}-11232T^9-153611T^8+603116T^7-1520513T^6+3049452T^5-5190414T^4+7715304T^3-10164234T^2+11961684T-12623974$	4 / ✘ 2, 3 / ✘
	10^a_{101}	$7T^2-21T+29$ $-129T^3+480T^2-942T+1148$ $-7453T^8+115979T^7-819947T^6+3586847T^5-10987573T^4+25120359T^3-44436695T^2+62133778T-69396618$	2 / ✘ 2, 3 / ✘
	10^a_{102}	$-2T^3+8T^2-16T+21$ $-T^5+6T^4-19T^3+50T^2-89T+108$ $142T^{12}-1824T^{11}+11296T^{10}-44000T^9+115984T^8-197200T^7+123203T^6+462512T^5-1996064T^4+4649298T^3-7951840T^2+10771160T-11897326$	3 / ✘ 1 / ✘
	10^a_{103}	$2T^3-8T^2+17T-21$ $5T^5-30T^4+93T^3-178T^2+254T-280$ $118T^{12}-1440T^{11}+8404T^{10}-29584T^9+61863T^8-33736T^7-289763T^6+1355186T^5-3666373T^4+7367413T^3-11802974T^2+15525908T-16990056$	3 / ✘ 3 / ✘
	10^a_{104}	$T^4-4T^3+9T^2-15T+19$ $T^5-2T^4+2T^3-3T+4$ $16T^{16}-220T^{15}+1535T^{14}-7197T^{13}+25227T^{12}-69332T^{11}+151513T^{10}-257279T^9+301366T^8-83393T^7-710402T^6+2409469T^5-5162297T^4+8726478T^3-12397663T^2+15191203T-16238052$	4 / ✘ 1 / ✘
	10^a_{105}	$T^3-8T^2+22T-29$ $-T^5+14T^4-71T^3+184T^2-292T+332$ $8T^{12}-200T^{11}+2218T^{10}-14261T^9+57123T^8-132986T^7+65302T^6+805306T^5-3722841T^4+9784430T^3-18400587T^2+26441286T-29769592$	3 / ✘ 2 / ✘
	10^a_{106}	$-T^4+4T^3-9T^2+15T-17$ $-T^7+6T^6-20T^5+48T^4-82T^3+114T^2-134T+140$ $15T^{16}-204T^{15}+1405T^{14}-6481T^{13}+22197T^{12}-58948T^{11}+122017T^{10}-186937T^9+159252T^8+161653T^7-1073190T^6+2872671T^5-5674479T^4+9221494T^3-12827310T^2+15551003T-16568312$	4 / ✘ 2 / ✘
	10^a_{107}	$-T^3+8T^2-22T+31$ $2T^3-8T^2+13T-16$ $9T^{12}-232T^{11}+2674T^{10}-18155T^9+79705T^8-227986T^7+366663T^6+65430T^5-2285283T^4+7518398T^3-15408513T^2+22997470T-26180364$	3 / ✘ 1 / ✘
	10^a_{108}	$2T^3-8T^2+14T-15$ $-3T^5+18T^4-41T^3+50T^2-40T+32$ $134T^{12}-1696T^{11}+10032T^{10}-36416T^9+87916T^8-133860T^7+58617T^6+353392T^5-1337642T^4+2961006T^3-4930449T^2+6594854T-7251776$	3 / ✘ 2 / ✘
	10^a_{109}	$T^4-4T^3+10T^2-17T+21$ 0 $16T^{16}-220T^{15}+1580T^{14}-7719T^{13}+28318T^{12}-81525T^{11}+186591T^{10}-332351T^9+413696T^8-158284T^7-889129T^6+3239371T^5-7165411T^4+12361738T^3-17799197T^2+21979657T-23554274$	4 / ✘ 2 / ✔
	10^a_{110}	$T^3-8T^2+20T-25$ $T^5-14T^4+69T^3-160T^2+219T-236$ $8T^{12}-200T^{11}+2180T^{10}-13569T^9+52114T^8-116472T^7+61616T^6+604668T^5-2747906T^4+7072274T^3-13103918T^2+18672836T-20967250$	3 / ✘ 2 / ✘
	10^a_{111}	$-2T^3+9T^2-17T+21$ $-9T^5+60T^4-171T^3+316T^2-436T+480$ $62T^{12}-672T^{11}+2507T^{10}+1894T^9-64067T^8+361705T^7-1299145T^6+3506889T^5-7575591T^4+13510069T^3-20234835T^2+25700228T-27818092$	3 / ✘ 2 / ✘
	10^a_{112}	$-T^4+5T^3-11T^2+17T-19$ $T^7-8T^6+29T^5-68T^4+115T^3-152T^2+175T-180$ $15T^{16}-255T^{15}+2068T^{14}-10699T^{13}+39650T^{12}-111160T^{11}+239401T^{10}-381338T^9+357595T^8+215240T^7-1900590T^6+5252099T^5-10470652T^4+17062683T^3-23747257T^2+28786648T-30666904$	4 / ✘ 2 / ✘
	10^a_{113}	$2T^3-11T^2+26T-33$ $-5T^5+42T^4-167T^3+394T^2-623T+720$ $118T^{12}-2016T^{11}+15681T^{10}-71126T^9+190712T^8-187416T^7-827053T^6+4935892T^5-14986146T^4+32456282T^3-54606535T^2+73872380T-81581546$	3 / ✘ 1 / ✘
	10^a_{114}	$-2T^3+10T^2-21T+27$ $T^5-8T^4+30T^3-78T^2+140T-168$ $142T^{12}-2280T^{11}+16976T^{10}-76976T^9+230999T^8-445876T^7+369450T^6+890044T^5-4554487T^4+11256519T^3-19890736T^2+27431686T-30450926$	3 / ✘ 1 / ✘
	10^a_{115}	$-T^3+9T^2-26T+37$ 0 $9T^{12}-261T^{11}+3345T^{10}-24942T^9+118870T^8-365932T^7+636497T^6+31527T^5-3907730T^4+13472649T^3-28298039T^2+42798944T-48929878$	3 / ✘ 2 / ✔
	10^a_{116}	$-T^4+5T^3-12T^2+19T-21$ $T^7-8T^6+30T^5-74T^4+132T^3-184T^2+217T-228$ $15T^{16}-255T^{15}+2111T^{14}-11302T^{13}+43668T^{12}-128023T^{11}+288575T^{10}-482307T^9+485985T^8+215018T^7-2416711T^6+6942030T^5-14142246T^4+23374622T^3-32832655T^2+40008697T-42694444$	4 / ✘ 2 / ✘
	10^a_{117}	$2T^3-10T^2+24T-31$ $-5T^5+38T^4-144T^3+330T^2-522T+600$ $118T^{12}-1824T^{11}+13156T^{10}-56312T^9+143746T^8-128212T^7-648731T^6+3701012T^5-11080717T^4+23844230T^3-39994730T^2+54033352T-59650184$	3 / ✘ 2 / ✘
	10^a_{118}	$T^4-5T^3+12T^2-19T+23$ 0 $16T^{16}-275T^{15}+2305T^{14}-12526T^{13}+49379T^{12}-149077T^{11}+352067T^{10}-641987T^9+825146T^8-399494T^7-1458086T^6+5641784T^5-12589879T^4+21712756T^3-31187934T^2+38432195T-41152780$	4 / ✘ 1 / ✔
	10^a_{119}	$-2T^3+10T^2-23T+31$ $-T^5+6T^4-26T^3+86T^2-175T+220$ $142T^{12}-2288T^{11}+17392T^{10}-81560T^9+255719T^8-521820T^7+483354T^6+990524T^5-5618050T^4+14499405T^3-26339835T^2+36916418T-41198798$	3 / ✘ 1 / ✘
	10^a_{120}	$8T^2-26T+37$ $166T^3-692T^2+1433T-1788$ $-11768T^8+201320T^7-1541132T^6+7193960T^5-23193562T^4+55098408T^3-100101157T^2+142136186T-159564534$	2 / ✘ 2, 3 / ✘
	10^a_{121}	$2T^3-11T^2+27T-35$ $5T^5-42T^4+167T^3-396T^2+634T-732$ $118T^{12}-2016T^{11}+15853T^{10}-73450T^9+204605T^8-232351T^7-764251T^6+5054205T^5-15890853T^4+35160633T^3-59996079T^2+81831748T-90616328$	3 / ✘ 2 / ✘
	10^a_{122}	$-2T^3+11T^2-24T+31$ $-T^5+8T^4-34T^3+104T^2-211T+264$ $142T^{12}-2512T^{11}+20355T^{10}-99362T^9+318535T^8-657014T^7+617040T^6+1199636T^5-6869579T^4+17663208T^3-31953091T^2+44656222T-49787168$	3 / ✘ 2 / ✘
	10^a_{123}	$T^4-6T^3+15T^2-24T+29$ 0 $16T^{16}-330T^{15}+3216T^{14}-19770T^{13}+86170T^{12}-282500T^{11}+715162T^{10}-1388790T^9+1917350T^8-1169720T^7-2832520T^6+12363784T^5-28689660T^4+50560110T^3-73579700T^2+91325158T-98015944$	4 / ✔ 2 / ✔
	10^n_{124}	T^4-T^3+T-1 $-4T^7-6T^4-4T^2-6T$ $9T^{15}-25T^{14}+10T^{13}+75T^{12}-177T^{11}+155T^{10}+113T^9-570T^8+850T^7-428T^6-824T^5+2167T^4-2340T^3+510T^2+2375T-3832$	4 / ✘ 4 / ✘
	10^n_{125}	T^3-2T^2+2T-1 $-T^5+2T^4-2T^3+3T-4$ $8T^{12}-50T^{11}+151T^{10}-289T^9+417T^8-524T^7+536T^6-150T^5-1168T^4+3942T^3-8130T^2+12314T-14126$	3 / ✘ 2 / ✘
	10^n_{126}	T^3-2T^2+4T-5 $T^5-2T^4+10T^3-12T^2+22T-20$ $8T^{12}-50T^{11}+185T^{10}-457T^9+666T^8-18T^7-3074T^6+10724T^5-24495T^4+43738T^3-64631T^2+81072T-87356$	3 / ✘ 2 / ✘
	10^n_{127}	$-T^3+4T^2-6T+7$ $2T^5-14T^4+32T^3-52T^2+67T-72$ $5T^{12}-48T^{11}+128T^{10}+289T^9-3551T^8+15554T^7-46589T^6+109206T^5-211625T^4+348370T^3-494107T^2+608154T-651576$	3 / ✘ 2 / ✘
	10^n_{128}	$2T^3-3T^2+T+1$ $-13T^5+12T^4-3T^3-10T^2-9T+12$ $-26T^{12}+296T^{11}-1071T^{10}+1750T^9-1107T^8+287T^7-2938T^6+7959T^5-7820T^4+3175T^3-8727T^2+28392T-40368$	3 / ✘ 3 / ✘
	10^n_{129}	$2T^2-6T+9$ $-T^3-2T^2+14T-20$ $62T^8-568T^7+2280T^6-4308T^5-553T^4+25616T^3-76125T^2+132258T-157332$	2 / ✔ 1 / ✘
	10^n_{130}	$2T^2-4T+5$ $T^3-2T^2+19T-24$ $62T^8-336T^7+924T^6-1568T^5+253T^4+8384T^3-28668T^2+53628T-65374$	2 / ✘ 2 / ✘
	10^n_{131}	$-2T^2+8T-11$ $5T^3-38T^2+87T-112$ $38T^8-272T^7-580T^6+12792T^5-66417T^4+202096T^3-422662T^2+646440T-742870$	2 / ✘ 1 / ✘
	10^n_{132}	T^2-T+1 $2T^2+5T-4$ $4T^8-7T^7+12T^6-145T^5+508T^4-631T^3-322T^2+2150T-3150$	2 / ✘ 1 / ✘
	10^n_{133}	$-T^2+5T-7$ $T^3-14T^2+37T-48$ $3T^8-43T^7+16T^6+1489T^5-9322T^4+30945T^3-68047T^2+106954T-123994$	2 / ✘ 1 / ✘

knot diag	n_k^t	Alexander's ω^+ / $(\rho'_1)^+$ / $(\rho'_2)^+$	genus / ribbon / unknotting # / amphi?
	10^n_{134}	$2T^3-4T^2+4T-3$ $-13T^5+24T^4-33T^3+30T^2-41T+40$ $-26T^{12}+376T^{11}-2056T^{10}+6760T^9-16248T^8+32568T^7-58951T^6+98316T^5-150194T^4+210738T^3-273246T^2+324124T-344346$	3 / ✘ 3 / ✘
	10^n_{136}	$-T^2+4T-5$ $-T^3+4T^2-2T-4$ $3T^8-36T^7+189T^6-512T^5+347T^4+2660T^3-11142T^2+22668T-28354$	2 / ✘ 1 / ✘
	10^n_{138}	T^3-5T^2+8T-7 $-T^5+8T^4-22T^3+24T^2-11T+8$ $8T^{12}-125T^{11}+855T^{10}-3374T^9+8458T^8-13328T^7+8173T^6+25863T^5-114602T^4+277037T^3-497313T^2+702260T-787812$	3 / ✘ 2 / ✘
	10^n_{140}	T^2-2T+3 $8T-8$ $4T^8-22T^7+90T^6-292T^5+424T^4+430T^3-3056T^2+6470T-8104$	2 / ✔ 2 / ✘
	10^n_{142}	$2T^3-3T^2+2T-1$ $-13T^5+12T^4-13T^3+4T^2-17T+12$ $-26T^{12}+296T^{11}-1155T^{10}+2582T^9-4276T^8+6812T^7-11749T^6+19392T^5-27878T^4+36798T^3-48891T^2+62932T-69706$	3 / ✘ 3 / ✘
	10^n_{144}	$-3T^2+10T-13$ $10T^3-44T^2+80T-96$ $222T^8-1642T^7+3140T^6+12252T^5-94326T^4+307146T^3-651636T^2+998418T-1147140$	2 / ✘ 2 / ✘
	10^n_{146}	$2T^2-8T+13$ $T^3-8T^2+21T-28$ $62T^8-664T^7+2844T^6-4544T^5-9663T^4+71376T^3-197106T^2+340392T-405394$	2 / ✘ 1 / ✘
	10^n_{148}	T^3-3T^2+7T-9 $T^5-4T^4+18T^3-36T^2+62T-68$ $8T^{12}-75T^{11}+377T^{10}-1209T^9+2330T^8-864T^7-11900T^6+51677T^5-135261T^4+266207T^3-420746T^2+549160T-599424$	3 / ✘ 2 / ✘
	10^n_{150}	$-T^3+4T^2-6T+7$ $-2T^5+12T^4-26T^3+38T^2-45T+44$ $5T^{12}-52T^{11}+216T^{10}-355T^9-719T^8+6578T^7-24361T^6+64526T^5-137117T^4+243126T^3-364723T^2+464942T-504136$	3 / ✘ 2 / ✘
	10^n_{152}	$T^4-T^3-T^2+4T-5$ $4T^7-7T^5+18T^4-7T^3-12T^2+45T-52$ $9T^{15}-14T^{14}-92T^{13}+396T^{12}-419T^{11}-1212T^{10}+5444T^9-9692T^8+6412T^7+11488T^6-39344T^5+55244T^4-33234T^3-30168T^2+102115T-133894$	4 / ✘ 4 / ✘
	10^n_{154}	T^3-4T+7 $-3T^5-6T^4+13T^3-47T+68$ $48T^{10}-93T^9-546T^8+2396T^7-1956T^6-8376T^5+25906T^4-23802T^3-25690T^2+102540T-140874$	3 / ✘ 3 / ✘
	10^n_{156}	T^3-4T^2+8T-9 $T^5-6T^4+19T^3-30T^2+33T-32$ $8T^{12}-100T^{11}+594T^{10}-2165T^9+5120T^8-6852T^7-2208T^6+41208T^5-134214T^4+293026T^3-493422T^2+668112T-738218$	3 / ✘ 1 / ✘
	10^n_{158}	$-T^3+4T^2-10T+15$ $2T^2-7T+12$ $9T^{12}-116T^{11}+764T^{10}-3275T^9+9743T^8-19422T^7+18439T^6+32898T^5-196271T^4+513374T^3-940025T^2+1323614T-1479452$	3 / ✘ 2 / ✘
	10^n_{160}	$-T^3+4T^2-4T+3$ $-2T^5+12T^4-20T^3+14T^2-16T+12$ $5T^{12}-52T^{11}+198T^{10}-255T^9-522T^8+3092T^7-8443T^6+18756T^5-37588T^4+67858T^3-108568T^2+148444T-165862$	3 / ✘ 2 / ✘
	10^n_{162}	$-3T^2+9T-11$ $10T^3-38T^2+58T-68$ $222T^8-1473T^7+2609T^6+8829T^5-65543T^4+206079T^3-427536T^2+647498T-741358$	2 / ✘ 2 / ✘
	10^n_{164}	$3T^2-11T+17$ $T^3-10T^2+29T-40$ $321T^8-3179T^7+12782T^6-20103T^5-32876T^4+254013T^3-688337T^2+1170838T-1386922$	2 / ✘ 1 / ✘
	10^n_{135}	$3T^2-9T+13$ $T^3-6T^2+18T-24$ $321T^8-2613T^7+8905T^6-12033T^5-19329T^4+132451T^3-337025T^2+553002T-647370$	2 / ✘ 2 / ✘
	10^n_{137}	$T^2-6T+11$ $-4T^2+24T-44$ $4T^8-74T^7+512T^6-1420T^5-1160T^4+21074T^3-72904T^2+140922T-173900$	2 / ✔ 1 / ✘
	10^n_{139}	T^4-T^3+2T-3 $-4T^7-12T^4+5T^3-4T^2-16T+12$ $9T^{15}-25T^{14}-3T^{13}+172T^{12}-425T^{11}+290T^{10}+924T^9-3099T^8+4327T^7-1756T^6-5200T^5+12117T^4-11846T^3+1547T^2+12451T-19002$	4 / ✘ 4 / ✘
	10^n_{141}	$-T^3+3T^2-4T+5$ $T^3-8T^2+16T-20$ $9T^{12}-87T^{11}+396T^{10}-1150T^9+2382T^8-3516T^7+2746T^6+3397T^5-19148T^4+46359T^3-80476T^2+109936T-121692$	3 / ✘ 1 / ✘
	10^n_{143}	T^3-3T^2+6T-7 $T^5-4T^4+15T^3-28T^2+45T-48$ $8T^{12}-75T^{11}+362T^{10}-1106T^9+2070T^8-1092T^7-7698T^6+33841T^5-86216T^4+164927T^3-254838T^2+327896T-356170$	3 / ✘ 1 / ✘
	10^n_{145}	T^2+T-3 $2T^3+8T^2+6T-8$ $-5T^7+7T^6+113T^5-141T^4-465T^3+730T^2+850T-2198$	2 / ✘ 2 / ✘
	10^n_{147}	$-2T^2+7T-9$ $-3T^3+12T^2-15T+12$ $54T^8-488T^7+1697T^6-1694T^5-8312T^4+42905T^3-107222T^2+177492T-208860$	2 / ✘ 1 / ✘
	10^n_{149}	$-T^3+5T^2-9T+11$ $2T^5-18T^4+55T^3-104T^2+149T-164$ $5T^{12}-61T^{11}+226T^{10}+339T^9-7195T^8+38874T^7-135727T^6+357173T^5-753890T^4+1318245T^3-1945105T^2+2447584T-2640944$	3 / ✘ 2 / ✘
	10^n_{151}	$T^3-4T^2+10T-13$ $-T^5+6T^4-21T^3+42T^2-66T+72$ $8T^{12}-100T^{11}+632T^{10}-2529T^9+6645T^8-9606T^7-5854T^6+80466T^5-270269T^4+605378T^3-1033839T^2+1408362T-1558600$	3 / ✘ 2 / ✘
	10^n_{153}	T^3-T^2-T+3 $T^5-2T^4+T^3+2T^2-T$ $8T^{12}-17T^{11}-46T^{10}+231T^9-381T^8+364T^7-367T^6+157T^5+1142T^4-2815T^3+1874T^2+2128T-4572$	3 / ✔ 2 / ✘
	10^n_{155}	$-T^3+3T^2-5T+7$ $-2T^3+12T^2-22T+28$ $9T^{12}-87T^{11}+417T^{10}-1321T^9+3014T^8-4806T^7+3646T^6+6917T^5-34773T^4+82963T^3-142781T^2+193836T-214060$	3 / ✔ 2 / ✘
	10^n_{157}	$-T^3+6T^2-11T+13$ $-2T^5+22T^4-78T^3+148T^2-218T+240$ $5T^{12}-74T^{11}+340T^{10}+321T^9-11314T^8+67637T^7-250977T^6+688036T^5-1493487T^4+2661131T^3-3974091T^2+5034465T-5444000$	3 / ✘ 2 / ✘
	10^n_{159}	$T^3-4T^2+9T-11$ $T^5-6T^4+26T^3-60T^2+98T-112$ $8T^{12}-100T^{11}+609T^{10}-2267T^9+5047T^8-3237T^7-23513T^6+115362T^5-318739T^4+648093T^3-1045247T^2+1379659T-1511358$	3 / ✘ 1 / ✘
	10^n_{161}	T^3-2T+3 $3T^5+6T^4-3T^3+4T^2+14T-12$ $30T^{10}-53T^9-145T^8+630T^7-674T^6-870T^5+3591T^4-4450T^3+581T^2+6166T-9640$	3 / ✘ 3 / ✘
	10^n_{163}	$T^3-5T^2+12T-15$ $-T^5+8T^4-30T^3+62T^2-89T+96$ $8T^{12}-125T^{11}+923T^{10}-4154T^9+12040T^8-19732T^7-4345T^6+140575T^5-506052T^4+1171653T^3-2040193T^2+2809224T-3119648$	3 / ✘ 1, 2 / ✘
	10^n_{165}	$-2T^2+10T-15$ $-5T^3+50T^2-146T+196$ $38T^8-344T^7-848T^6+23020T^5-137555T^4+465256T^3-1047705T^2+1673914T-1951560$	2 / ✘ 2 / ✘