


Pensieve header: Making table1.tex and table2.tex.

```
In[ ]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\QMUL-1908"];
Once[<< KnotTheory`];
Ribbons = {Knot[0, 1], Knot[6, 1], Knot[8, 8], Knot[8, 9], Knot[8, 20], Knot[9, 27],
  Knot[9, 41], Knot[9, 46], Knot[10, 3], Knot[10, 22], Knot[10, 35], Knot[10, 42],
  Knot[10, 48], Knot[10, 75], Knot[10, 87], Knot[10, 99], Knot[10, 123],
  Knot[10, 129], Knot[10, 137], Knot[10, 140], Knot[10, 153], Knot[10, 155]};
```

ParentDirectory: Argument File should be a positive machine-size integer, a nonempty string, or a File specification. 

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ToFileName: String or list of strings expected at position 1 in ToFileName[{File, WikiLink, mathematica}]. 

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Loading KnotTheory` version of January 20, 2015, 10:42:19.1122.

Read more at <http://katlas.org/wiki/KnotTheory>.

```
In[ ]:= CF = Factor;
E[L_, Q_, P_]$_k := E[L, Q, Series[Normal@P, {ε, 0, $k}]];
E_d→r[L_, Q_, P_]$_k := E_d→r@@E[L, Q, P]$_k;
E3@E[ω_, L_, Q_, PS_] := CF /@ E[L, ω⁻¹ Q, ω⁻¹ (ω⁻⁴ ε)⁻¹+Range@Length@PS.PS]$_k;
E4@E[L_, Q_, P_] := Module[
  {ω = Normal[P]⁻¹ /. ε → 0, PS = CoefficientList[P, ε]},
  CF /@ E[ω, L, ω Q, ω⁻³+4 Range@Length@PS PS]];
E3@E_sp__[as___] := E3@E[as] /. E → E_sp;
E4@E_sp__[as___] := E4@E[as] /. E → E_sp;
```

Defining $\{\omega, \rho_{1p}, \rho_{2p}\}$.

```
In[ ]:= $k = 2;
Clear[QP, ω];
QP[Knot[n_, k_]] := QP[Knot[n, k]] = Collect[Module[{fname},
  fname = ".../Projects/SL2Invariant/k=2/Data/" <>
  ToString[n] <> "_" <> ToString[k] <> ".m";
  Collect[E3[Get[fname][[2, 2]][[3]] // Normal, ε, Simplify]
], ε, CF];
ω[K_Knot] := ω[K] = Factor[(QP@K /. ε → 0)⁻¹];
C_{k,d}[K_Knot] :=
  Factor[SeriesCoefficient[QP[K], {y, 0, 0}, {ε, 0, k}, {a, 0, d}] ω[K]¹+²k-d]
```

```
In[ ]:= ρ1p[K_Knot] := ρ1p[K] = Factor[
$$\frac{T(-c_{1,0}[K] + \omega[K] T \partial_T \omega[K])}{(T-1)^2}$$
];
ρ2p[K_Knot] := ρ2p[K] = CF[-2 c_{2,0}[K] + ω[K] c_{2,1}[K]];
plus[ε_] := Expand[ε] /. T^n -> /; n < 0 => 0;
```

```
In[ ]:= line = {ω[#], ρ1p[#], ρ2p[#]} & /@ {Knot[3, 1]}
plus /@ line
Out[ ]:= {{ 1 - T + T^2, 1 + T^2, 3 - 12 T + 26 T^2 - 38 T^3 + 26 T^4 - 12 T^5 + 3 T^6 } / T^3}
Out[ ]:= {{ -1 + T, T, -38 + 26 T - 12 T^2 + 3 T^3 } }
```

Recovering Z from {ω, ρ1p, ρ2p}.

```
In[ ]:= Clear[LogP];
LogP[Knot[n_, k_]] := LogP[Knot[n, k]] = Collect[Module[{fname},
  fname = "../Projects/SL2Invariant/k=2/Data/" <>
  ToString[n] <> "_" <> ToString[k] <> ".m";
  Collect[ω[Knot[n, k]] * E3[Get[fname][2, 2]][3] // Log // Normal, ε, Simplify]
], ε, CF];
```

```
In[ ]:= LogP[Knot[3, 1]]
Out[ ]:= (-2 - 2 a + 3 T + 2 a T - 2 T^2 + T^3 - 2 a T^3 + 2 a T^4 - 2 x y - 2 T^3 x y) ε / (1 - T + T^2)^2
```

$$\frac{1}{2(1 - T + T^2)^4} (-T - 4 a T - 4 a^2 T + 11 T^2 + 32 a T^2 + 24 a^2 T^2 - 16 T^3 - 56 a T^3 - 48 a^2 T^3 + 12 T^4 + 64 a T^4 + 64 a^2 T^4 - 8 T^5 - 40 a T^5 - 48 a^2 T^5 + 3 T^6 + 16 a T^6 + 24 a^2 T^6 - T^7 - 4 a T^7 - 4 a^2 T^7 + 4 x y - 4 T x y - 16 a T x y + 8 T^2 x y + 48 a T^2 x y + 8 T^3 x y - 72 a T^3 x y - 8 T^4 x y + 64 a T^4 x y + 8 T^5 x y - 24 a T^5 x y + 4 T^6 x y - 4 T^7 x y + 8 a T^7 x y - 2 x^2 y^2 - 6 T x^2 y^2 + 12 T^2 x^2 y^2 - 22 T^3 x^2 y^2 + 12 T^4 x^2 y^2 - 6 T^5 x^2 y^2 - 2 T^6 x^2 y^2) \epsilon^2$$

Generating table1.tex and table2.tex

```
In[ ]:= pmrule = {"+" -> "\\!+\\!", "-" -> "\\!-\\!"};
KnotLine[K_] :=
StringReplace["\\rolcell{n_k}{n^t_{k}}{ω}{ρ1pp}{ρ2pp}{G}{U}{R}{C}", {
  "n" -> ToString@K[[1]],
  "k" -> ToString@K[[2]],
  "t" -> If[AlternatingQ[K], "a", "n"],
  "ω" -> StringReplace[ToString[plus@ω[K]], FormatType -> TeXForm], pmrule,
  "ρ1pp" -> StringReplace[ToString[plus@ρ1p[K]], FormatType -> TeXForm], pmrule,
  "ρ2pp" -> StringReplace[ToString[plus@ρ2p[K]], FormatType -> TeXForm], pmrule,
  "G" -> ToString@ThreeGenus@K,
  "U" -> ToString@UnknottingNumber@K,
  "R" -> If[MemberQ[Ribbons, K], "\\gY", "\\N"],
  "C" -> If[MemberQ[
    {FullyAmphicheiral, NegativeAmphicheiral, ""}, SymmetryType@K], "\\oY", "\\N"]
}]
```

In[*]:= **KnotLine** /@ **AllKnots**[{0, 6}]

```
Out[*]= {\rolcell{0_1}{0^a_1}{1}{0}{0}{0}{0}{\gY}{\oY},
\rolcell{3_1}{3^a_1}{T\!-\!1}{T}{3 T^3\!-\!12 T^2\!+\!26 T\!-\!38}{1}{1}{\N}{\N},
\rolcell{4_1}{4^a_1}{3\!-\!T}{0}{T^4\!-\!3
T^3\!-\!15 T^2\!+\!74 T\!-\!110}{1}{1}{\N}{\oY},
\rolcell{5_1}{5^a_1}{T^2\!-\!T\!+\!1}{2 T^3\!+\!3 T}{5 T^7\!-\!20 T^6\!+\!55
T^5\!-\!120 T^4\!+\!217 T^3\!-\!338 T^2\!+\!450 T\!-\!510}{2}{2}{\N}{\N},
\rolcell{5_2}{5^a_2}{2 T\!-\!3}{5 T\!-\!4}{\!-\!10 T^4\!+\!120
T^3\!-\!487 T^2\!+\!1054 T\!-\!1362}{1}{1}{\N}{\N},
\rolcell{6_1}{6^a_1}{5\!-\!2 T}{T\!-\!4}{14 T^4\!-\!16
T^3\!-\!293 T^2\!+\!1098 T\!-\!1598}{1}{1}{\gY}{\N},
\rolcell{6_2}{6^a_2}{\!-\!T^2\!+\!3 T\!-\!3}{T^3\!-\!4 T^2\!+\!4
T\!-\!4}{3 T^8\!-\!21 T^7\!+\!49 T^6\!+\!15 T^5\!-\!433
T^4\!+\!1543 T^3\!-\!3431 T^2\!+\!5482 T\!-\!6410}{2}{1}{\N}{\N},
\rolcell{6_3}{6^a_3}{T^2\!-\!3 T\!+\!5}{0}{4 T^8\!-\!33 T^7\!+\!121 T^6\!-\!203
T^5\!-\!111 T^4\!+\!1499 T^3\!-\!4210 T^2\!+\!7186 T\!-\!8510}{2}{1}{\N}{\oY}}
```

In[]:=

```

Make[range_] := Make[range, 2];
Make[range_, col_] := StringJoin@@Table[
  StringJoin[StringJoin@@Riffle[L, " &\n"], " \\\n\\hline\n"],
  {L, Partition[KnotLine /@ AllKnots[range], UpTo@col]}
];
Make[{0, 6}, 3]

```

- Get:** ParentDirectory[File] in \$Path is not a string. +
- KnotTheory:** Loading precomputed data in PD4Knots`. +
- KnotTheory:** The 3-genus data known to KnotTheory` is taken from Charles Livingston's <http://www.indiana.edu/~knotinf> +
- Get:** ParentDirectory[File] in \$Path is not a string. +
- KnotTheory:** Loading precomputed data in IndianaData`. +

Out[]:=

```

\rolcell{0_1}{0^a_{1}}{1}{0}{0}{0}{\gY}{\oY} &
\rolcell{3_1}{3^a_{1}}{T\!-\!1}{T}{3
  T^3\!-\!12 T^2\!+\!26 T\!-\!38}{1}{1}{\N}{\N} &
\rolcell{4_1}{4^a_{1}}{3\!-\!T}{0}{T^4\!-\!3 T^3\!-\!15
  T^2\!+\!74 T\!-\!110}{1}{1}{\N}{\oY} \\
\hline
\rolcell{5_1}{5^a_{1}}{T^2\!-\!T\!+\!1}{2 T^3\!+\!3 T}{5 T^7\!-\!20 T^6\!+\!55
  T^5\!-\!120 T^4\!+\!217 T^3\!-\!338 T^2\!+\!450 T\!-\!510}{2}{2}{\N}{\N} &
\rolcell{5_2}{5^a_{2}}{2 T\!-\!3}{5 T\!-\!4}{\!-\!10 T^4\!+\!120
  T^3\!-\!487 T^2\!+\!1054 T\!-\!1362}{1}{1}{\N}{\N} &
\rolcell{6_1}{6^a_{1}}{5\!-\!2 T}{T\!-\!4}{14 T^4\!-\!16
  T^3\!-\!293 T^2\!+\!1098 T\!-\!1598}{1}{1}{\gY}{\N} \\
\hline
\rolcell{6_2}{6^a_{2}}{\!-\!T^2\!+\!3 T\!-\!3}{T^3\!-\!4
  T^2\!+\!4 T\!-\!4}{3 T^8\!-\!21 T^7\!+\!49 T^6\!+\!15 T^5\!-\!433
  T^4\!+\!1543 T^3\!-\!3431 T^2\!+\!5482 T\!-\!6410}{2}{1}{\N}{\N} &
\rolcell{6_3}{6^a_{3}}{T^2\!-\!3 T\!+\!5}{0}{4 T^8\!-\!33 T^7\!+\!121 T^6\!-\!203
  T^5\!-\!111 T^4\!+\!1499 T^3\!-\!4210 T^2\!+\!7186 T\!-\!8510}{2}{1}{\N}{\oY} \\
\hline

```

```

DeleteFile["table1.tex"];
WriteString["table1.tex", Make[{0, 8}, 3]];
Close["table1.tex"]

```

Out[]:= table1.tex

```

DeleteFile["table2.tex"];
WriteString["table2.tex", Make[{9, 10}, 2]];
Close["table2.tex"]

```

Out[]:= table2.tex