Pensieve header: Examples for the Da-Nang talk: Double Integration and the trefoil.

Startup

In[∘]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\DaNang-1905"];
<< "Engine-Speedy.m";
<< "Objects.m";
```

cm

In[∘]:=

$$\Lambda0 = \text{HoldForm}\left[\left(\eta_i + \frac{e^{-\alpha_i - \epsilon\,\beta_i}\,\eta_j}{1+\epsilon\,\eta_j\,\xi_i}\right) y_k + \left(\beta_i + \beta_j + \frac{\text{Log}[1+\epsilon\,\eta_j\,\xi_i]}{\epsilon}\right) b_k + \left(\alpha_i + \alpha_j + \text{Log}[1+\epsilon\,\eta_j\,\xi_i]\right) a_k + \left(\frac{e^{-\alpha_j - \epsilon\,\beta_j}\,\xi_i}{1+\epsilon\,\eta_j\,\xi_i} + \xi_j\right) x_k\right];$$

```
TeXForm[Λ0]
Λ = ReleaseHold[Λ0]
```

Out[∘]=

$$a_k\,(\text{Log}[1+\epsilon\,\eta_j\,\xi_i] + \alpha_i + \alpha_j) + b_k\left(\frac{\text{Log}[1+\epsilon\,\eta_j\,\xi_i]}{\epsilon} + \beta_i + \beta_j\right) + y_k\left(\eta_i + \frac{e^{-\alpha_i-\epsilon\,\beta_i}\,\eta_j}{1+\epsilon\,\eta_j\,\xi_i}\right) + x_k\left(\frac{e^{-\alpha_j-\epsilon\,\beta_j}\,\xi_i}{1+\epsilon\,\eta_j\,\xi_i} + \xi_j\right)$$

```
\left(\eta _i+\frac{e^{-\alpha _i-\epsilon  \beta _i} \eta _j}{1+\epsilon  \eta _j \xi _i}
  \left(1+\epsilon  \eta _j \xi _i\right)}{\epsilon }\right) b_k+\left(\alpha _i+\alpha _
  _i\right)\right) a_k+\left(\frac{e^{-\alpha _j-\epsilon  \beta _j} \xi _i}{1+\epsilon
```

rho

In[∘]:=
```
HL[ℰ_] := Style[ℰ, Background → If[TrueQ@ℰ, ■, ■]];
```

$$\left\{\rho y = \begin{pmatrix} 0 & 0 \\ \epsilon & 0 \end{pmatrix}, \rho b = \begin{pmatrix} 0 & 0 \\ 0 & -\epsilon \end{pmatrix}, \rho a = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \rho x = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\right\};$$

```
HL /@ {ρa.ρx - ρx.ρa == ρx, ρa.ρy - ρy.ρa == -ρy,
  ρb.ρy - ρy.ρb == -ϵ ρy, ρb.ρx - ρx.ρb == ϵ ρx, ρx.ρy - ρy.ρx == ρb + ϵ ρa}
```

rho

Out[∘]= {True, True, True, True, True}

rho

In[∘]:=

$$\text{HL@Simplify@With}[\{\mathbb{E} = \text{MatrixExp}\},\ \mathbb{E}[\eta_i\,\rho y].\mathbb{E}[\beta_i\,\rho b].\mathbb{E}[\alpha_i\,\rho a].\mathbb{E}[\xi_i\,\rho x].\mathbb{E}[\eta_j\,\rho y].\mathbb{E}[\beta_j\,\rho b].\mathbb{E}[\alpha_j\,\rho a].\mathbb{E}[\xi_j\,\rho x] == \mathbb{E}[\partial_{y_k}\Lambda\,\rho y].\mathbb{E}[\partial_{b_k}\Lambda\,\rho b].\mathbb{E}[\partial_{a_k}\Lambda\,\rho a].\mathbb{E}[\partial_{x_k}\Lambda\,\rho x]]$$

rho

Out[∘]= True

rho

In[∘]:=
```
Series[Λ, {ϵ, 0, 1}]
```

rho

Out[∘]=

$$\left(a_k\,(\alpha_i + \alpha_j) + y_k\left(\eta_i + e^{-\alpha_i}\,\eta_j\right) + b_k\,(\beta_i + \beta_j + \eta_j\,\xi_i) + x_k\left(e^{-\alpha_j}\,\xi_i + \xi_j\right)\right) + \left(a_k\,\eta_j\,\xi_i - \frac{1}{2}\,b_k\,\eta_j^2\,\xi_i^2 - e^{-\alpha_i}\,y_k\,\eta_j\,(\beta_i + \eta_j\,\xi_i) - e^{-\alpha_j}\,x_k\,\xi_i\,(\beta_j + \eta_j\,\xi_i)\right)\epsilon + O[\epsilon]^2$$

Some Atoms

Atoms

In[∘]:=
```
PP_ := Identity; $k = 1; ℏ = γ = 1;
Column[(# → (ℰ = ToExpression[#]; Normal@Simplify[ℰ⟦1⟧ + ℰ⟦2⟧ + Log@ℰ⟦3⟧])) & /@
  {"dm_{i,j→k}", "dΔ_{i→j,k}", "dS_i", "R_{i,j}", "P_{i,j}"}]
```

Atoms

Out[∘]=

$$dm_{i,j\to k} \to a_k\,(\alpha_i+\alpha_j) + b_k\,(\beta_i+\beta_j) + y_k\,\eta_i + \frac{y_k\,\eta_j}{\mathcal{A}_i} + \frac{x_k\,\xi_i}{\mathcal{A}_j} + \eta_j\,\xi_i - B_k\,\eta_j\,\xi_i + \frac{\epsilon\left(2\,y_k\,\eta_j\,(2\,x_k\,\xi_i+\mathcal{A}_j\,(-2\,\beta_i+(1-3\,B_k)\,\eta_j\,\xi_i))+\mathcal{A}_i\,\xi_i\left(x_k\,(-4\,\beta_j+2\,(1-3\,B_k)\,\eta_j\,\xi_i)+\mathcal{A}_j\,\eta_j\left(4\,a_k\,B_k+\left(1-4\,B_k+3\,B_k^2\right)\eta_j\,\xi_i\right)\right)\right)}{4\,\mathcal{A}_i\,\mathcal{A}_j} + x_k\,\xi_j$$

$$d\triangle_{i\to j,k} \to a_j\,\alpha_i + a_k\,\alpha_i + b_j\,\beta_i + b_k\,\beta_i + y_j\,\eta_i + B_j\,y_k\,\eta_i + x_j\,\xi_i + x_k\,\xi_i + \frac{1}{2}\epsilon\left(B_j\,y_j\,y_k\,\eta_i^2 + x_k\,\xi_i\left(-2\,a_j + x_j\,\xi_i\right)\right)$$

$$dS_i \to -a_i\,\alpha_i - b_i\,\beta_i - \frac{\mathcal{A}_i\,(y_i\,\eta_i+(-\eta_i+B_i\,(x_i+\eta_i))\,\xi_i)}{B_i} - \frac{1}{4\,B_i^2}\epsilon\,\mathcal{A}_i\left(\mathcal{A}_i\,\eta_i^2\left(2\,y_i^2-6\,y_i\,\xi_i+3\,\xi_i^2\right) + B_i^2\,\xi_i\left(4\,a_i\,x_i+2\,x_i^2\,\mathcal{A}_i\,\xi_i+2\,x_i\left(2\,\beta_i+\mathcal{A}_i\,\eta_i\,\xi_i\right)+\eta_i\,(-4+4\,\beta_i+\mathcal{A}_i\,\eta_i\,\xi_i)\right) + 2\,B_i\,\eta_i\left(y_i\left(-2+2\,\beta_i+2\,x_i\,\mathcal{A}_i\,\xi_i+\mathcal{A}_i\,\eta_i\,\xi_i\right)-\xi_i\left(-2+2\,a_i+2\,\beta_i+3\,x_i\,\mathcal{A}_i\,\xi_i+2\,\mathcal{A}_i\,\eta_i\,\xi_i\right)\right)\right)$$

$$R_{i,j} \to a_j\,b_i + x_j\,y_i - \frac{1}{4}\epsilon\,x_j^2\,y_i^2$$

$$P_{i,j} \to \alpha_j\,\beta_i + \eta_i\,\xi_j + \frac{1}{4}\epsilon\,\eta_i^2\,\xi_j^2$$

Double Integration

Integrals

In[∘]:=
```
inp = 𝔼_{{}→{1}}[3 a₁ b₁, 5 x₁ y₁, 1] // dm_{i,1→i};
Table[
  HL@TrueQ[
    (inp // (sΥ_{i→1,1,2,2} RR) // BM // AM // P_{1,2}) dϵ_j ≡
     (inp // ΔΔ // (sΥ_{i→1,1,2,2} RR) // BM // AM // P_{1,2})],
  {ΔΔ, {dΔ_{i→i,j}, dΔ_{i→j,i}}}, {AM, {dm_{2,4→2}, dm_{4,2→2}}}, {BM, {dm_{1,3→1}, dm_{3,1→1}}},
  {RR, {R_{3,4}, R_{3,4} // dS_3 // dS_3, R_{3,4} // dS_4 // dS_4}}
] // MatrixForm
```

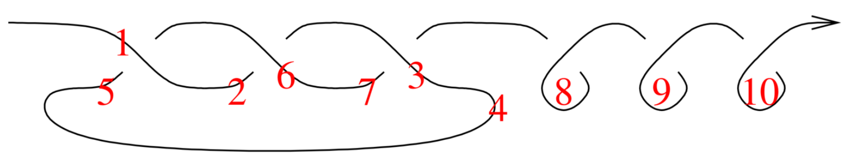
Out[∘]//MatrixForm=

Integrals

False	False	False	False	False	True
False	False	False	False	False	False
False	False	False	False	False	False
False	False	True	False	False	False

The Trefoil

Trefoil



Trefoil

In[∘]:=
```
$k = 2;
Simplify[R_{1,5} R_{6,2} R_{3,7} C̄_4 K̄ink_8 K̄ink_9 K̄ink_10 // dm_{1,2→1} // dm_{1,3→1} // dm_{1,4→1} // dm_{1,5→1} // dm_{1,6→1} //
    dm_{1,7→1} // dm_{1,8→1} // dm_{1,9→1} // dm_{1,10→1}] /. v_1 :→ v
```

Trefoil

Out[∘]=
$$\mathbb{E}_{\{\}\to\{1\}}\Big[0,\ 0,\ \frac{B}{1-B+B^2}+\frac{B\left(-B+2B^2+2B^4+a\left(-1+B-B^3+B^4\right)-2xy-B^3\left(3+2xy\right)\right)\in}{\left(1-B+B^2\right)^3}+\frac{1}{2\left(1-B+B^2\right)^5}B\Big(4B^8+a^2\left(1-B+B^2\right)^2\left(1+B-6B^2+B^3+B^4\right)+6B^5x^2y^2+2xy\left(-2+3xy\right)-B^7\left(11+4xy\right)-2B^2\left(1+6x^2y^2\right)-2B^4\left(1-2xy+6x^2y^2\right)+B\left(1+8xy+6x^2y^2\right)+B^6\left(6+8xy+6x^2y^2\right)+B^3\left(4+4xy+30x^2y^2\right)+2a\left(1-B+B^2\right)\left(2B^6+2xy+8B^3\left(1+xy\right)-5B^2\left(1+2xy\right)-2B^5\left(1+2xy\right)-B^4\left(7+2xy\right)+B\left(2+4xy\right)\right)\Big)\in^2+O[\in]^3\Big]$$