

Pensieve header: Solving the equations for  $\rho_d$  degree by degree.

## Preliminaries

(Alt) In[ ]:=

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\Oaxaca-2210"];
Once[<< KnotTheory` ; << Rot.m];
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

Loading Rot.m from <http://drorbn.net/la22/ap> to compute rotation numbers.

## The Old Program

(Alt) In[ ]:=

```
R1[s_, i_, j_] := s (g_{j,i} (g_{j^+,j} + g_{j,j^+} - g_{ij}) - g_{ii} (g_{j,j^+} - 1) - 1 / 2);
rho[K_] := rho[K] = Module[{Cs, phi, n, A, s, i, j, k, Delta, G, rho1},
  {Cs, phi} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  Cases[Cs, {s_, i_, j_} -> (A[[{i, j}, {i + 1, j + 1}]] += ( -T^s T^s - 1 ))];
  Delta = T^(-Total[phi] - Total[Cs[[All, 1]]) / 2) Det[A];
  G = Inverse[A];
  rho1 = Sum_{k=1}^n R1 @@ Cs[[k]] - Sum_{k=1}^{2^n} phi[[k]] (g_{kk} - 1 / 2);
  Factor@{Delta, Delta^2 rho1 /. alpha_+ -> alpha + 1 /. g_{alpha,beta} -> G[[alpha, beta]]};
```

## The g-Rules

(Alt) In[ ]:=

```
delta_{i,j} := If[i === j, 1, 0];
gRules_{s_, i_, j_} := {g_{i,beta} -> delta_{i,beta} + T^s g_{i^+,beta} + (1 - T^s) g_{j^+,beta}, g_{j,beta} -> delta_{j,beta} + g_{j^+,beta},
  g_{alpha,i} -> T^{-s} (g_{alpha,i^+} - delta_{alpha,i^+}), g_{alpha,j} -> g_{alpha,j^+} - (1 - T^s) g_{alpha,i} - delta_{alpha,j^+}}
(alpha_+)^+ := alpha^{++}; (* this is for cosmetic reasons only *)
```

## CF

(Alt) In[ ]:=

```
CF[E_] := Module[{vs = Union[{e}, Cases[E, (g | p | x)_, Infinity]]}, Total[
  CoefficientRules[Expand@E, vs] /. (ps_ -> c_) -> Factor[c] (Times @@ vs^{ps})
]]
```

## g2px and px2g

(Alt) In[ ]:=

```
g2px[ε_] := Module[{λ}, Expand[ε /. gα,β → λ pα xβ] /. λk → 1 / k!]
```

(Alt) In[ ]:=

```
{p*, x*, π*, ξ*} = {π, ξ, p, x}; (u-i)* := (u*)i;
```

(Alt) In[ ]:=

```
Zip{}[ε_] := ε;
Zip{ε, εs...}[ε_] := (Collect[ε // Zip{εs}, ε] /. f- . εd → (D[f, {ε*, d}])) /. ε* → 0
```

(Alt) In[ ]:=

```
px2g[ε_] := Module[{ps, xs, Q},
  ps = Union[Cases[ε, pα, ∞]];
  xs = Union[Cases[ε, xα, ∞]];
  Q = Sum[p0* x0* gp0,x0, {p0, ps}, {x0, xs}];
  Zipps∪xs[ε eQ] // Expand
]
```

## Generic Perturbations

(Alt) In[ ]:=

```
Module[{i, j, k},
  AllMonomials[{}, 0] = {1};
  AllMonomials[{}, d_Integer] /; d > 0 := {};
  AllMonomials[{v_, vs___}, d_Integer] :=
    Join@@Table[vd-k AllMonomials[{vs}, k], {k, 0, d}];
  AllMonomials[vs_List, {d_}] := Join@@Table[AllMonomials[vs, k], {k, 0, d}];
  Basis[js_List, m_] := Flatten@Outer[Times,
    AllMonomials[Table[pj, {j, js}], m], AllMonomials[Table[xj, {j, js}], m]];
  Basis[js_List, {m_}] := Flatten@Table[Basis[js, k], {k, 0, m}];
  GenericCombination[bas_, c_] := bas.Table[cj, {j, Length@bas}];
  GenericCombination[bas_, c-k] := bas.Table[ck,j, {j, Length@bas}];
]
```

(Alt) In[ ]:=

```

Module[{k, x1, x2, p1, p2},
  r_d[1, i_, j_] :=
    Expand@Together@Sum[ $e^k$  GenericCombination[Basis[{i, j}, {k + 1}], ca_k], {k, d}];
  r_d[-1, i_, j_] :=
    Expand@Together@Sum[ $e^k$  GenericCombination[Basis[{i, j}, {k + 1}], cb_k], {k, d}];
   $\gamma_d[0, j_] := 0$ ;
   $\gamma_d[1, j_] :=$ 
    Expand@Together@Sum[ $e^k$  GenericCombination[Basis[{j}, {k + 1}], cc_k], {k, d}];
   $\gamma_d[-1, j_] :=$ 
    Expand@Together@Sum[ $e^k$  GenericCombination[Basis[{j}, {k + 1}], cd_k], {k, d}];
  {x1*, x2*, p1*, p2*} = {p1, p2, x1, x2};
  r_d[s_,  $\phi i$ _,  $\phi j$ _, i_, j_] := Normal[Log[0[ $e$ ]d+1 + Zip[{x1, x2}][Exp[0[ $e$ ]d+1 +
    ( $\gamma_d[\phi i, i]$  /.  $x_i \rightarrow x_i + x1$ ) +
    ( $\gamma_d[\phi j, j]$  /.  $x_j \rightarrow x_j + x2$ ) + ( $r_d[s, i, j]$  /. { $p_i \rightarrow p_i - p1$ ,  $p_j \rightarrow p_j - p2$ })
    ]]]];
];

```

(Alt) In[ ]:=

**r<sub>2</sub>[1, j, k] // CF**

(Alt) Out[ ]:=

$$\begin{aligned}
& \in ca_{1,1} + \epsilon p_j x_j ca_{1,2} + \epsilon p_j x_k ca_{1,3} + \epsilon p_k x_j ca_{1,4} + \epsilon p_k x_k ca_{1,5} + \epsilon p_j^2 x_j^2 ca_{1,6} + \epsilon p_j^2 x_j x_k ca_{1,7} + \\
& \epsilon p_j^2 x_k^2 ca_{1,8} + \epsilon p_j p_k x_j^2 ca_{1,9} + \epsilon p_j p_k x_j x_k ca_{1,10} + \epsilon p_j p_k x_k^2 ca_{1,11} + \epsilon p_k^2 x_j^2 ca_{1,12} + \epsilon p_k^2 x_j x_k ca_{1,13} + \\
& \epsilon p_k^2 x_k^2 ca_{1,14} + \epsilon^2 ca_{2,1} + \epsilon^2 p_j x_j ca_{2,2} + \epsilon^2 p_j x_k ca_{2,3} + \epsilon^2 p_k x_j ca_{2,4} + \epsilon^2 p_k x_k ca_{2,5} + \\
& \epsilon^2 p_j^2 x_j^2 ca_{2,6} + \epsilon^2 p_j^2 x_j x_k ca_{2,7} + \epsilon^2 p_j^2 x_k^2 ca_{2,8} + \epsilon^2 p_j p_k x_j^2 ca_{2,9} + \epsilon^2 p_j p_k x_j x_k ca_{2,10} + \\
& \epsilon^2 p_j p_k x_k^2 ca_{2,11} + \epsilon^2 p_k^2 x_j^2 ca_{2,12} + \epsilon^2 p_k^2 x_j x_k ca_{2,13} + \epsilon^2 p_k^2 x_k^2 ca_{2,14} + \epsilon^2 p_j^3 x_j^3 ca_{2,15} + \\
& \epsilon^2 p_j^3 x_j^2 x_k ca_{2,16} + \epsilon^2 p_j^3 x_j x_k^2 ca_{2,17} + \epsilon^2 p_j^3 x_k^3 ca_{2,18} + \epsilon^2 p_j^2 p_k x_j^3 ca_{2,19} + \epsilon^2 p_j^2 p_k x_j^2 x_k ca_{2,20} + \\
& \epsilon^2 p_j^2 p_k x_j x_k^2 ca_{2,21} + \epsilon^2 p_j^2 p_k x_k^3 ca_{2,22} + \epsilon^2 p_j p_k^2 x_j^3 ca_{2,23} + \epsilon^2 p_j p_k^2 x_j^2 x_k ca_{2,24} + \epsilon^2 p_j p_k^2 x_j x_k^2 ca_{2,25} + \\
& \epsilon^2 p_j p_k^2 x_k^3 ca_{2,26} + \epsilon^2 p_k^3 x_j^3 ca_{2,27} + \epsilon^2 p_k^3 x_j^2 x_k ca_{2,28} + \epsilon^2 p_k^3 x_j x_k^2 ca_{2,29} + \epsilon^2 p_k^3 x_k^3 ca_{2,30}
\end{aligned}$$

(Alt) In[ ]:=

**r<sub>2</sub>[-1, j, k] // CF**

(Alt) Out[ ]:=

$$\begin{aligned}
& \in cb_{1,1} + \epsilon p_j x_j cb_{1,2} + \epsilon p_j x_k cb_{1,3} + \epsilon p_k x_j cb_{1,4} + \epsilon p_k x_k cb_{1,5} + \epsilon p_j^2 x_j^2 cb_{1,6} + \epsilon p_j^2 x_j x_k cb_{1,7} + \\
& \epsilon p_j^2 x_k^2 cb_{1,8} + \epsilon p_j p_k x_j^2 cb_{1,9} + \epsilon p_j p_k x_j x_k cb_{1,10} + \epsilon p_j p_k x_k^2 cb_{1,11} + \epsilon p_k^2 x_j^2 cb_{1,12} + \epsilon p_k^2 x_j x_k cb_{1,13} + \\
& \epsilon p_k^2 x_k^2 cb_{1,14} + \epsilon^2 cb_{2,1} + \epsilon^2 p_j x_j cb_{2,2} + \epsilon^2 p_j x_k cb_{2,3} + \epsilon^2 p_k x_j cb_{2,4} + \epsilon^2 p_k x_k cb_{2,5} + \\
& \epsilon^2 p_j^2 x_j^2 cb_{2,6} + \epsilon^2 p_j^2 x_j x_k cb_{2,7} + \epsilon^2 p_j^2 x_k^2 cb_{2,8} + \epsilon^2 p_j p_k x_j^2 cb_{2,9} + \epsilon^2 p_j p_k x_j x_k cb_{2,10} + \\
& \epsilon^2 p_j p_k x_k^2 cb_{2,11} + \epsilon^2 p_k^2 x_j^2 cb_{2,12} + \epsilon^2 p_k^2 x_j x_k cb_{2,13} + \epsilon^2 p_k^2 x_k^2 cb_{2,14} + \epsilon^2 p_j^3 x_j^3 cb_{2,15} + \\
& \epsilon^2 p_j^3 x_j^2 x_k cb_{2,16} + \epsilon^2 p_j^3 x_j x_k^2 cb_{2,17} + \epsilon^2 p_j^3 x_k^3 cb_{2,18} + \epsilon^2 p_j^2 p_k x_j^3 cb_{2,19} + \epsilon^2 p_j^2 p_k x_j^2 x_k cb_{2,20} + \\
& \epsilon^2 p_j^2 p_k x_j x_k^2 cb_{2,21} + \epsilon^2 p_j^2 p_k x_k^3 cb_{2,22} + \epsilon^2 p_j p_k^2 x_j^3 cb_{2,23} + \epsilon^2 p_j p_k^2 x_j^2 x_k cb_{2,24} + \epsilon^2 p_j p_k^2 x_j x_k^2 cb_{2,25} + \\
& \epsilon^2 p_j p_k^2 x_k^3 cb_{2,26} + \epsilon^2 p_k^3 x_j^3 cb_{2,27} + \epsilon^2 p_k^3 x_j^2 x_k cb_{2,28} + \epsilon^2 p_k^3 x_j x_k^2 cb_{2,29} + \epsilon^2 p_k^3 x_k^3 cb_{2,30}
\end{aligned}$$

(Alt) In[ ]:=

$\gamma_2[1, i]$  // CF

(Alt) Out[ ]:=

$$\in cc_{1,1} + \in p_i x_i cc_{1,2} + \in p_i^2 x_i^2 cc_{1,3} + \in^2 cc_{2,1} + \in^2 p_i x_i cc_{2,2} + \in^2 p_i^2 x_i^2 cc_{2,3} + \in^2 p_i^3 x_i^3 cc_{2,4}$$

(Alt) In[ ]:=

$r_2[-1, \phi_j, \phi_k, j, k]$  // CF

(Alt) Out[ ]:=

$$\begin{aligned} & \text{Log} \left[ e^{\gamma_2[\phi_j, j] + \gamma_2[\phi_k, k]} \right] + \in cb_{1,1} + \in p_j x_j cb_{1,2} + \in p_j x_k cb_{1,3} + \in p_k x_j cb_{1,4} + \in p_k x_k cb_{1,5} + \in p_j^2 x_j^2 cb_{1,6} + \\ & \in p_j^2 x_j x_k cb_{1,7} + \in p_j^2 x_k^2 cb_{1,8} + \in p_j p_k x_j^2 cb_{1,9} + \in p_j p_k x_j x_k cb_{1,10} + \in p_j p_k x_k^2 cb_{1,11} + \in p_k^2 x_j^2 cb_{1,12} + \\ & \in p_k^2 x_j x_k cb_{1,13} + \in p_k^2 x_k^2 cb_{1,14} + \in^2 cb_{2,1} + \in^2 p_j x_j cb_{2,2} + \in^2 p_j x_k cb_{2,3} + \in^2 p_k x_j cb_{2,4} + \\ & \in^2 p_k x_k cb_{2,5} + \in^2 p_j^2 x_j^2 cb_{2,6} + \in^2 p_j^2 x_j x_k cb_{2,7} + \in^2 p_j^2 x_k^2 cb_{2,8} + \in^2 p_j p_k x_j^2 cb_{2,9} + \in^2 p_j p_k x_j x_k cb_{2,10} + \\ & \in^2 p_j p_k x_k^2 cb_{2,11} + \in^2 p_k^2 x_j^2 cb_{2,12} + \in^2 p_k^2 x_j x_k cb_{2,13} + \in^2 p_k^2 x_k^2 cb_{2,14} + \in^2 p_j^3 x_j^3 cb_{2,15} + \\ & \in^2 p_j^3 x_j^2 x_k cb_{2,16} + \in^2 p_j^3 x_j x_k^2 cb_{2,17} + \in^2 p_j^3 x_k^3 cb_{2,18} + \in^2 p_j^2 p_k x_j^3 cb_{2,19} + \in^2 p_j^2 p_k x_j^2 x_k cb_{2,20} + \\ & \in^2 p_j^2 p_k x_j x_k^2 cb_{2,21} + \in^2 p_j^2 p_k x_k^3 cb_{2,22} + \in^2 p_j p_k^2 x_j^3 cb_{2,23} + \in^2 p_j p_k^2 x_j^2 x_k cb_{2,24} + \in^2 p_j p_k^2 x_j x_k^2 cb_{2,25} + \\ & \in^2 p_j p_k^2 x_k^3 cb_{2,26} + \in^2 p_k^3 x_j^3 cb_{2,27} + \in^2 p_k^3 x_j^2 x_k cb_{2,28} + \in^2 p_k^3 x_j x_k^2 cb_{2,29} + \in^2 p_k^3 x_k^3 cb_{2,30} \end{aligned}$$

## Non-Universally Solving at d=1

(Alt) In[ ]:=

$d = 1;$

vars =

Cases[Variables[ $r_d[1, i1, j1] + r_d[-1, i2, j2] + \gamma_d[1, k1] + \gamma_d[-1, k2]$ ], (ca | cb | cc | cd) \_\_]

(Alt) Out[ ]:=

{ca<sub>1,1</sub>, ca<sub>1,2</sub>, ca<sub>1,3</sub>, ca<sub>1,4</sub>, ca<sub>1,5</sub>, ca<sub>1,6</sub>, ca<sub>1,7</sub>, ca<sub>1,8</sub>, ca<sub>1,9</sub>, ca<sub>1,10</sub>, ca<sub>1,11</sub>,  
ca<sub>1,12</sub>, ca<sub>1,13</sub>, ca<sub>1,14</sub>, cb<sub>1,1</sub>, cb<sub>1,2</sub>, cb<sub>1,3</sub>, cb<sub>1,4</sub>, cb<sub>1,5</sub>, cb<sub>1,6</sub>, cb<sub>1,7</sub>, cb<sub>1,8</sub>, cb<sub>1,9</sub>,  
cb<sub>1,10</sub>, cb<sub>1,11</sub>, cb<sub>1,12</sub>, cb<sub>1,13</sub>, cb<sub>1,14</sub>, cc<sub>1,1</sub>, cc<sub>1,2</sub>, cc<sub>1,3</sub>, cd<sub>1,1</sub>, cd<sub>1,2</sub>, cd<sub>1,3</sub>}

$c\bar{c}$

(Alt) In[ ]:=

lhs = Module[{x1, p1},

{x1\*, p1\*} = {p1, x1};

Normal[

Log[0[ $\in$ ]<sup>d+1</sup> + Zip<sub>{x1}</sub>[Exp[0[ $\in$ ]<sup>d+1</sup> + ( $\gamma_d[1, i] / \cdot x_i \rightarrow x_i + x1$ ) + ( $\gamma_d[-1, i] / \cdot p_i \rightarrow p_i - p1$ )]]]]]

]

rhs = 0

(Alt) Out[ ]:=

$$\in (cc_{1,1} + p_i x_i cc_{1,2} + p_i^2 x_i^2 cc_{1,3} + cd_{1,1} + p_i x_i cd_{1,2} + p_i^2 x_i^2 cd_{1,3})$$

(Alt) Out[ ]:=

0

```
(Alt) In[ ]:=
covars = DeleteCases[Variables[lhs - rhs], T | (ca | cb | cc | cd) __]

(Alt) Out[ ]:=
{ϵ, pi, xi}

(Alt) In[ ]:=
eqnsCCbar = (# == 0) & /@ Union[Last /@ CoefficientRules[Expand[lhs - rhs], covars]]

(Alt) Out[ ]:=
{cc1,1 + cd1,1 == 0, cc1,2 + cd1,2 == 0, cc1,3 + cd1,3 == 0}

(Alt) In[ ]:=
vars =
Cases[Variables[rd[1, i1, j1] + rd[-1, i2, j2] + γd[1, k1] + γd[-1, k2]], (ca | cb | cc | cd) __]
{sol} = Solve[eqnsCCbar, vars]
sol /. Rule -> Set;
γd[1, k]
γd[-1, k]

(Alt) Out[ ]:=
{ca1,1, ca1,2, ca1,3, ca1,4, ca1,5, ca1,6, ca1,7, ca1,8, ca1,9, ca1,10, ca1,11,
ca1,12, ca1,13, ca1,14, cb1,1, cb1,2, cb1,3, cb1,4, cb1,5, cb1,6, cb1,7, cb1,8, cb1,9,
cb1,10, cb1,11, cb1,12, cb1,13, cb1,14, cc1,1, cc1,2, cc1,3, cd1,1, cd1,2, cd1,3}
```

Solve: Equations may not give solutions for all "solve" variables.

```
(Alt) Out[ ]:=
{{cd1,1 -> -cc1,1, cd1,2 -> -cc1,2, cd1,3 -> -cc1,3}}

(Alt) Out[ ]:=
ϵ cc1,1 + ϵ pk xk cc1,2 + ϵ pk2 xk2 cc1,3

(Alt) Out[ ]:=
-ϵ cc1,1 - ϵ pk xk cc1,2 - ϵ pk2 xk2 cc1,3
```

### R3

```
(Alt) In[ ]:=
Short[lhs = CF[Module[{es = {i, j, k, i+, j+, k+}},
Times[
Normal@Series[Exp[rd[1, j, k] + rd[1, i, k+] + rd[1, i+, j+]], {ϵ, 0, d}],
Exp[Sum[gα,β πα ξβ, {α, es}, {β, es}]]
] // Zip[(pα&/@es) ∪ (xα&/@es) // Expand
] // . gRules1,j,k ∪ gRules1,i,k+ ∪ gRules1,i+,j+]]
```

```
(Alt) Out[ ]//Short=
1 + <<64>> + 4 ϵ (ca1,8 - 2 T ca1,8 + T2 ca<<1>> + ca<<1>> - T ca1,11 + ca1,14) gk+,k+2
```

```

(Alt) In[ ]:=
Short[rhs = CF[Module[{es = {i, j, k, i+, j+, k+}},
  Times[
    Normal@Series[Exp[r_d[1, i, j] + r_d[1, i+, k] + r_d[1, j+, k+]], {e, 0, d}],
    Exp[Sum[g_{\alpha, \beta} \pi_{\alpha} \xi_{\beta}, {\alpha, es}, {\beta, es}]]
  ] // Zip(p_x & /@ es) \cup (x_x & /@ es) // Expand
] // . gRules_{1, i, j} \cup gRules_{1, i+, k} \cup gRules_{1, j+, k+}]]

(Alt) Out[ ]//Short=
1 + <<61>> + 4 \in (ca_{1,8} - 2 T ca_{1,8} + T^2 ca_{<<1>>} + ca_{<<1>>} - T ca_{1,11} + ca_{1,14}) g_{k+, k+}^2

(Alt) In[ ]:=
me = Exponent[lhs - rhs, T, Min]

(Alt) Out[ ]:=
-4

(Alt) In[ ]:=
covars = DeleteCases[Variables[lhs - rhs], T | (ca | cb | cc | cd) __]

(Alt) Out[ ]:=
{\epsilon, g_{i+, i+}, g_{i+, j+}, g_{i+, k+}, g_{j+, i+}, g_{j+, j+}, g_{j+, k+}, g_{k+, i+}, g_{k+, j+}, g_{k+, k+}}

(Alt) In[ ]:=
Short[
  eqnsR3 = (# == 0) & /@ Union[Last /@ CoefficientRules[Expand[T^{-me} (lhs - rhs)], covars]]]

(Alt) Out[ ]//Short=
{-T^3 ca_{1,3} + T^4 ca_{1,3} == 0, <<31>>, -4 T^3 ca_{1,8} + <<10>> + 4 T^4 ca_{1,14} == 0}

```

## R2b

(Alt) In[ ]:=

```

lhs = CF[Module[{es = {i, j, i+, j+}},
  Times[
    Normal@Series[Exp[r_d[1, i, j] + r_d[-1, i+, j+]], {e, 0, d}],
    Exp[Sum[g_{\alpha,\beta} \pi_{\alpha} \xi_{\beta}, {\alpha, es}, {\beta, es}]]
  ] // Zip_{(p_{\alpha}&/@es)\cup(x_{\beta}&/@es)} // Expand
] // . gRules_{1,i,j} \cup gRules_{-1,i+,j+}

```

(Alt) Out[ ]:=

$$\begin{aligned}
 & 1 + \in (ca_{1,1} + cb_{1,1}) + \frac{\in (T ca_{1,2} + T cb_{1,2} + cb_{1,3} - T cb_{1,3}) g_{i^{++}, i^{++}}}{T} + \\
 & \frac{2 \in (T^2 ca_{1,6} + T^2 cb_{1,6} + T cb_{1,7} - T^2 cb_{1,7} + cb_{1,8} - 2 T cb_{1,8} + T^2 cb_{1,8}) g_{i^{++}, i^{++}}^2}{T^2} + \\
 & \frac{\in (T ca_{1,3} + cb_{1,3}) g_{i^{++}, j^{++}}}{T} + \frac{2 \in (T^2 ca_{1,7} + T cb_{1,7} + 2 cb_{1,8} - 2 T cb_{1,8}) g_{i^{++}, i^{++}} g_{i^{++}, j^{++}}}{T^2} + \\
 & \frac{2 \in (T^2 ca_{1,8} + cb_{1,8}) g_{i^{++}, j^{++}}^2}{T^2} + \\
 & \frac{\in (T ca_{1,4} - T cb_{1,2} + T^2 cb_{1,2} - cb_{1,3} + 2 T cb_{1,3} - T^2 cb_{1,3} + T^2 cb_{1,4} + T cb_{1,5} - T^2 cb_{1,5}) g_{j^{++}, i^{++}}}{T} + \frac{1}{T^2} \\
 & 2 \in (T^2 ca_{1,9} - 2 T^2 cb_{1,6} + 2 T^3 cb_{1,6} - 2 T cb_{1,7} + 4 T^2 cb_{1,7} - 2 T^3 cb_{1,7} - 2 cb_{1,8} + 6 T cb_{1,8} - 6 T^2 cb_{1,8} + \\
 & 2 T^3 cb_{1,8} + T^3 cb_{1,9} + T^2 cb_{1,10} - T^3 cb_{1,10} + T cb_{1,11} - 2 T^2 cb_{1,11} + T^3 cb_{1,11}) g_{i^{++}, i^{++}} g_{j^{++}, i^{++}} + \\
 & \frac{1}{T^2} \in (T^2 ca_{1,10} - 2 T cb_{1,7} + 2 T^2 cb_{1,7} - 4 cb_{1,8} + 8 T cb_{1,8} - 4 T^2 cb_{1,8} + T^2 cb_{1,10} + 2 T cb_{1,11} - 2 T^2 cb_{1,11}) \\
 & g_{i^{++}, j^{++}} g_{j^{++}, i^{++}} + \frac{1}{T^2} 2 \in (T^2 ca_{1,12} + T^2 cb_{1,6} - 2 T^3 cb_{1,6} + T^4 cb_{1,6} + T cb_{1,7} - 3 T^2 cb_{1,7} + 3 T^3 cb_{1,7} - \\
 & T^4 cb_{1,7} + cb_{1,8} - 4 T cb_{1,8} + 6 T^2 cb_{1,8} - 4 T^3 cb_{1,8} + T^4 cb_{1,8} - T^3 cb_{1,9} + T^4 cb_{1,9} - T^2 cb_{1,10} + \\
 & 2 T^3 cb_{1,10} - T^4 cb_{1,10} - T cb_{1,11} + 3 T^2 cb_{1,11} - 3 T^3 cb_{1,11} + T^4 cb_{1,11} + T^4 cb_{1,12} + T^3 cb_{1,13} - \\
 & T^4 cb_{1,13} + T^2 cb_{1,14} - 2 T^3 cb_{1,14} + T^4 cb_{1,14}) g_{j^{++}, i^{++}} + \frac{\in (T ca_{1,5} - cb_{1,3} + T cb_{1,3} + T cb_{1,5}) g_{j^{++}, j^{++}}}{T} + \\
 & \frac{1}{T^2} \in (T^2 ca_{1,10} - 2 T cb_{1,7} + 2 T^2 cb_{1,7} - 4 cb_{1,8} + 8 T cb_{1,8} - 4 T^2 cb_{1,8} + T^2 cb_{1,10} + 2 T cb_{1,11} - 2 T^2 cb_{1,11}) \\
 & g_{i^{++}, i^{++}} g_{j^{++}, j^{++}} + \frac{2 \in (T^2 ca_{1,11} - 2 cb_{1,8} + 2 T cb_{1,8} + T cb_{1,11}) g_{i^{++}, j^{++}} g_{j^{++}, j^{++}}}{T^2} + \\
 & \frac{1}{T^2} 2 \in (T^2 ca_{1,13} + T cb_{1,7} - 2 T^2 cb_{1,7} + T^3 cb_{1,7} + 2 cb_{1,8} - 6 T cb_{1,8} + 6 T^2 cb_{1,8} - 2 T^3 cb_{1,8} - T^2 cb_{1,10} + \\
 & T^3 cb_{1,10} - 2 T cb_{1,11} + 4 T^2 cb_{1,11} - 2 T^3 cb_{1,11} + T^3 cb_{1,13} + 2 T^2 cb_{1,14} - 2 T^3 cb_{1,14}) g_{j^{++}, i^{++}} g_{j^{++}, j^{++}} + \\
 & \frac{2 \in (T^2 ca_{1,14} + cb_{1,8} - 2 T cb_{1,8} + T^2 cb_{1,8} - T cb_{1,11} + T^2 cb_{1,11} + T^2 cb_{1,14}) g_{j^{++}, j^{++}}^2}{T^2}
 \end{aligned}$$

```

(Alt) In[ ]:=
  rhs = 1
(Alt) Out[ ]:=
  1
(Alt) In[ ]:=
  me = Exponent[lhs - rhs, T, Min]
(Alt) Out[ ]:=
  -2
(Alt) In[ ]:=
  covars = DeleteCases[Variables[lhs - rhs], T | (ca | cb | cc | cd) _]
(Alt) Out[ ]:=
  {ϵ, gi+,i+, gi+,j+, gj+,i+, gj+,j+}
(Alt) In[ ]:=
  eqnsR2b =
    (Factor[#] == 0) & /@ Union[Last /@ CoefficientRules[Expand[T-me (lhs - rhs)], covars]]
(Alt) Out[ ]:=
  {T2 (ca1,1 + cb1,1) == 0, T (T ca1,3 + cb1,3) == 0,
  T (T ca1,2 + T cb1,2 + cb1,3 - T cb1,3) == 0, T (T ca1,5 - cb1,3 + T cb1,3 + T cb1,5) == 0,
  T (T ca1,4 - T cb1,2 + T2 cb1,2 - cb1,3 + 2 T cb1,3 - T2 cb1,3 + T2 cb1,4 + T cb1,5 - T2 cb1,5) == 0,
  2 (T2 ca1,8 + cb1,8) == 0, 2 (T2 ca1,7 + T cb1,7 + 2 cb1,8 - 2 T cb1,8) == 0,
  2 (T2 ca1,6 + T2 cb1,6 + T cb1,7 - T2 cb1,7 + cb1,8 - 2 T cb1,8 + T2 cb1,8) == 0,
  2 (T2 ca1,11 - 2 cb1,8 + 2 T cb1,8 + T cb1,11) == 0,
  T2 ca1,10 - 2 T cb1,7 + 2 T2 cb1,7 - 4 cb1,8 + 8 T cb1,8 - 4 T2 cb1,8 + T2 cb1,10 + 2 T cb1,11 - 2 T2 cb1,11 == 0,
  2 (T2 ca1,9 - 2 T2 cb1,6 + 2 T3 cb1,6 - 2 T cb1,7 + 4 T2 cb1,7 - 2 T3 cb1,7 - 2 cb1,8 + 6 T cb1,8 -
    6 T2 cb1,8 + 2 T3 cb1,8 + T3 cb1,9 + T2 cb1,10 - T3 cb1,10 + T cb1,11 - 2 T2 cb1,11 + T3 cb1,11) == 0,
  2 (T2 ca1,14 + cb1,8 - 2 T cb1,8 + T2 cb1,8 - T cb1,11 + T2 cb1,11 + T2 cb1,14) == 0,
  2 (T2 ca1,13 + T cb1,7 - 2 T2 cb1,7 + T3 cb1,7 + 2 cb1,8 - 6 T cb1,8 + 6 T2 cb1,8 - 2 T3 cb1,8 - T2 cb1,10 +
    T3 cb1,10 - 2 T cb1,11 + 4 T2 cb1,11 - 2 T3 cb1,11 + T3 cb1,13 + 2 T2 cb1,14 - 2 T3 cb1,14) == 0,
  2 (T2 ca1,12 + T2 cb1,6 - 2 T3 cb1,6 + T4 cb1,6 + T cb1,7 - 3 T2 cb1,7 + 3 T3 cb1,7 - T4 cb1,7 +
    cb1,8 - 4 T cb1,8 + 6 T2 cb1,8 - 4 T3 cb1,8 + T4 cb1,8 - T3 cb1,9 + T4 cb1,9 -
    T2 cb1,10 + 2 T3 cb1,10 - T4 cb1,10 - T cb1,11 + 3 T2 cb1,11 - 3 T3 cb1,11 + T4 cb1,11 +
    T4 cb1,12 + T3 cb1,13 - T4 cb1,13 + T2 cb1,14 - 2 T3 cb1,14 + T4 cb1,14) == 0}

```

## R2c

(Alt) In[ ]:=

```

lhs = CF[Module[{es = {i, j, i+, j+}},
  Times[
    Normal@Series[Exp[r_d[-1, i, j+] + r_d[1, i+, j] + r_d[1, j+]], {e, 0, d}],
    Exp[Sum[g_{\alpha, \beta} \pi_{\alpha} \xi_{\beta}, {\alpha, es}, {\beta, es}]]
  ] // Zip_{(p, &/@es) \cup (x, &/@es)} // Expand
] // . gRules_{-1, i, j+} \cup gRules_{1, i+, j}

```

(Alt) Out[ ]:=

$$\begin{aligned}
& 1 + \in \left( ca_{1,1} - ca_{1,3} + T ca_{1,3} + 2 ca_{1,8} - 4 T ca_{1,8} + 2 T^2 ca_{1,8} + cb_{1,1} + cc_{1,1} \right) + \\
& \frac{\in \left( T ca_{1,2} - 2 T ca_{1,7} + 2 T^2 ca_{1,7} + T cb_{1,2} + cb_{1,3} - T cb_{1,3} \right) g_{i^{++}, i^{++}}}{T} + \\
& \frac{2 \in \left( T^2 ca_{1,6} + T^2 cb_{1,6} + T cb_{1,7} - T^2 cb_{1,7} + cb_{1,8} - 2 T cb_{1,8} + T^2 cb_{1,8} \right) g_{i^{++}, i^{++}}^2}{T^2} + \\
& \in \left( T ca_{1,3} - 4 T ca_{1,8} + 4 T^2 ca_{1,8} + cb_{1,3} \right) g_{i^{++}, j^{++}} + \\
& \frac{2 \in \left( T^2 ca_{1,7} + T cb_{1,7} + 2 cb_{1,8} - 2 T cb_{1,8} \right) g_{i^{++}, i^{++}} g_{i^{++}, j^{++}}}{T} + \\
& 2 \in \left( T^2 ca_{1,8} + cb_{1,8} \right) g_{i^{++}, j^{++}}^2 - \frac{1}{T} \in \left( -ca_{1,2} + T ca_{1,2} - ca_{1,4} + 2 ca_{1,7} - 4 T ca_{1,7} + \right. \\
& \quad \left. 2 T^2 ca_{1,7} + ca_{1,10} - T ca_{1,10} - T cb_{1,4} - cb_{1,5} + T cb_{1,5} - cc_{1,2} + T cc_{1,2} \right) g_{j^{++}, i^{++}} - \frac{1}{T^2} \\
& 2 \in \left( -2 T ca_{1,6} + 2 T^2 ca_{1,6} - T ca_{1,9} - T^2 cb_{1,9} - T cb_{1,10} + T^2 cb_{1,10} - cb_{1,11} + 2 T cb_{1,11} - T^2 cb_{1,11} \right) \\
& \quad \in \left( -2 T ca_{1,7} + 2 T^2 ca_{1,7} - T ca_{1,10} - T cb_{1,10} - 2 cb_{1,11} + 2 T cb_{1,11} \right) g_{i^{++}, j^{++}} g_{j^{++}, i^{++}} + \\
& \frac{1}{T^2} 2 \in \left( ca_{1,6} - 2 T ca_{1,6} + T^2 ca_{1,6} + ca_{1,9} - T ca_{1,9} + ca_{1,12} + T^2 cb_{1,12} + T cb_{1,13} - \right. \\
& \quad \left. T^2 cb_{1,13} + cb_{1,14} - 2 T cb_{1,14} + T^2 cb_{1,14} + cc_{1,3} - 2 T cc_{1,3} + T^2 cc_{1,3} \right) g_{j^{++}, i^{++}}^2 + \\
& \in \left( ca_{1,3} - T ca_{1,3} + ca_{1,5} - 4 ca_{1,8} + 8 T ca_{1,8} - 4 T^2 ca_{1,8} - 2 ca_{1,11} + 2 T ca_{1,11} + cb_{1,5} + cc_{1,2} \right) g_{j^{++}, j^{++}} - \\
& \in \left( -2 T ca_{1,7} + 2 T^2 ca_{1,7} - T ca_{1,10} - T cb_{1,10} - 2 cb_{1,11} + 2 T cb_{1,11} \right) g_{i^{++}, i^{++}} g_{j^{++}, j^{++}} - \\
& \frac{1}{T} 2 \in \left( ca_{1,7} - 2 T ca_{1,7} + T^2 ca_{1,7} + ca_{1,10} - T ca_{1,10} + ca_{1,13} + \right. \\
& \quad \left. T cb_{1,13} + 2 cb_{1,14} - 2 T cb_{1,14} + 2 cc_{1,3} - 2 T cc_{1,3} \right) g_{j^{++}, i^{++}} g_{j^{++}, j^{++}} + \\
& 2 \in \left( ca_{1,8} - 2 T ca_{1,8} + T^2 ca_{1,8} + ca_{1,11} - T ca_{1,11} + ca_{1,14} + cb_{1,14} + cc_{1,3} \right) g_{j^{++}, j^{++}}^2
\end{aligned}$$

(Alt) In[ ]:=

```

rhs = CF[Module[{es = {(j+)+}},
  Times[
    Normal@Series[Exp[γd[1, (j+)+]], {ε, 0, d}],
    Exp[Sum[gα,β πα ξβ, {α, es}, {β, es}]]
  ] // Zip(p#/@es) ∪ (x#/@es) // Expand
]]

```

(Alt) Out[ ]:=

$$1 + \epsilon \text{cc}_{1,1} + \epsilon \text{cc}_{1,2} \mathbf{g}_{j^{++}, j^{++}} + 2 \epsilon \text{cc}_{1,3} \mathbf{g}_{j^{++}, j^{++}}^2$$

(Alt) In[ ]:=

```
me = Exponent[lhs - rhs, T, Min]
```

(Alt) Out[ ]:=

-2

(Alt) In[ ]:=

```
covars = DeleteCases[Variables[lhs - rhs], T | (ca | cb | cc | cd) ___]
```

(Alt) Out[ ]:=

$$\{\epsilon, \mathbf{g}_{i^{++}, i^{++}}, \mathbf{g}_{i^{++}, j^{++}}, \mathbf{g}_{j^{++}, i^{++}}, \mathbf{g}_{j^{++}, j^{++}}\}$$

(Alt) In[ ]:=

```
eqnsR2c =
  (Factor[#] == 0) & /@ Union[Last /@ CoefficientRules[Expand[T-me (lhs - rhs)], covars]]
```

(Alt) Out[ ]:=

$$\begin{aligned}
& \{T^2 (ca_{1,1} - ca_{1,3} + T ca_{1,3} + 2 ca_{1,8} - 4 T ca_{1,8} + 2 T^2 ca_{1,8} + cb_{1,1}) = 0, \\
& T (T ca_{1,2} - 2 T ca_{1,7} + 2 T^2 ca_{1,7} + T cb_{1,2} + cb_{1,3} - T cb_{1,3}) = 0, \\
& T^2 (T ca_{1,3} - 4 T ca_{1,8} + 4 T^2 ca_{1,8} + cb_{1,3}) = 0, \\
& -T^2 (-ca_{1,3} + T ca_{1,3} - ca_{1,5} + 4 ca_{1,8} - 8 T ca_{1,8} + 4 T^2 ca_{1,8} + 2 ca_{1,11} - 2 T ca_{1,11} - cb_{1,5}) = 0, \\
& 2 T (T^2 ca_{1,7} + T cb_{1,7} + 2 cb_{1,8} - 2 T cb_{1,8}) = 0, 2 T^2 (T^2 ca_{1,8} + cb_{1,8}) = 0, \\
& 2 (T^2 ca_{1,6} + T^2 cb_{1,6} + T cb_{1,7} - T^2 cb_{1,7} + cb_{1,8} - 2 T cb_{1,8} + T^2 cb_{1,8}) = 0, \\
& -T (-2 T ca_{1,7} + 2 T^2 ca_{1,7} - T ca_{1,10} - T cb_{1,10} - 2 cb_{1,11} + 2 T cb_{1,11}) = 0, \\
& -2 T^2 (-2 T ca_{1,8} + 2 T^2 ca_{1,8} - T ca_{1,11} - cb_{1,11}) = 0, \\
& -2 (-2 T ca_{1,6} + 2 T^2 ca_{1,6} - T ca_{1,9} - T^2 cb_{1,9} - T cb_{1,10} + T^2 cb_{1,10} - cb_{1,11} + 2 T cb_{1,11} - T^2 cb_{1,11}) = 0, \\
& 2 T^2 (ca_{1,8} - 2 T ca_{1,8} + T^2 ca_{1,8} + ca_{1,11} - T ca_{1,11} + ca_{1,14} + cb_{1,14}) = 0, \\
& -T (-ca_{1,2} + T ca_{1,2} - ca_{1,4} + 2 ca_{1,7} - 4 T ca_{1,7} + 2 T^2 ca_{1,7} + ca_{1,10} - T ca_{1,10} - \\
& \quad T cb_{1,4} - cb_{1,5} + T cb_{1,5} - cc_{1,2} + T cc_{1,2}) = 0, 2 T (ca_{1,7} - 2 T ca_{1,7} + T^2 ca_{1,7} + \\
& \quad ca_{1,10} - T ca_{1,10} + ca_{1,13} + T cb_{1,13} + 2 cb_{1,14} - 2 T cb_{1,14} + 2 cc_{1,3} - 2 T cc_{1,3}) = 0, \\
& 2 (ca_{1,6} - 2 T ca_{1,6} + T^2 ca_{1,6} + ca_{1,9} - T ca_{1,9} + ca_{1,12} + T^2 cb_{1,12} + T cb_{1,13} - T^2 cb_{1,13} + \\
& \quad cb_{1,14} - 2 T cb_{1,14} + T^2 cb_{1,14} + cc_{1,3} - 2 T cc_{1,3} + T^2 cc_{1,3}) = 0 \}
\end{aligned}$$

## R1l

(Alt) In[ ]:=

```
lhs = CF[Module[{es = {i, i+}},
  Times[
    Normal@Series[Exp[rd[1, i+, i] + γd[1, i+]], {ε, 0, d}],
    Exp[Sum[gα,β πα ξβ, {α, es}, {β, es}]]
  ] // Zip(p#&/@es) ∪ (x#&/@es) // Expand
] // . {gi+,β- → T-1 δi+,β- + gi+,β-, gi,β- → δi,β- + gi,β-}
```

(Alt) Out[ ]:=

$$1 + \frac{1}{T^2} \in \left( T^2 ca_{1,1} + T ca_{1,2} + T ca_{1,4} + T^2 ca_{1,5} + 2 ca_{1,6} + \right. \\ \left. 2 ca_{1,9} + T ca_{1,10} + 2 ca_{1,12} + 2 T ca_{1,13} + 2 T^2 ca_{1,14} + T^2 cc_{1,1} + T cc_{1,2} + 2 cc_{1,3} \right) + \\ \in \frac{(T ca_{1,3} + T ca_{1,5} + 2 ca_{1,7} + 2 ca_{1,10} + 2 T ca_{1,11} + 2 ca_{1,13} + 4 T ca_{1,14}) g_{i^{++},i} +}{T} \\ 2 \in (ca_{1,8} + ca_{1,11} + ca_{1,14}) g_{i^{++},i}^2 + \\ \in \frac{(T ca_{1,2} + T ca_{1,4} + 4 ca_{1,6} + 4 ca_{1,9} + T ca_{1,10} + 4 ca_{1,12} + 2 T ca_{1,13} + T cc_{1,2} + 4 cc_{1,3}) g_{i^{++},i^+} +}{T} \\ 2 \in (ca_{1,7} + ca_{1,10} + ca_{1,13}) g_{i^{++},i} g_{i^{++},i^+} + 2 \in (ca_{1,6} + ca_{1,9} + ca_{1,12} + cc_{1,3}) g_{i^{++},i^+}^2$$

(Alt) In[ ]:=

rhs = 1

(Alt) Out[ ]:=

1

(Alt) In[ ]:=

me = Exponent[lhs - rhs, T, Min]

(Alt) Out[ ]:=

-2

(Alt) In[ ]:=

covars = DeleteCases[Variables[lhs - rhs], T | (ca | cb | cc | cd) \_\_\_]

(Alt) Out[ ]:=

{ε, g<sub>i<sup>+</sup>,i, g<sub>i<sup>+</sup>,i<sup>+</sup></sub>}</sub>

(Alt) In[ ]:=

eqnsR1l =

(Factor[#] == 0) & /@ Union[Last /@ CoefficientRules[Expand[T<sup>-me</sup> (lhs - rhs)], covars]]

(Alt) Out[ ]:=

$$\{ 2 T^2 (ca_{1,7} + ca_{1,10} + ca_{1,13}) == 0, 2 T^2 (ca_{1,8} + ca_{1,11} + ca_{1,14}) == 0, \\ T (T ca_{1,3} + T ca_{1,5} + 2 ca_{1,7} + 2 ca_{1,10} + 2 T ca_{1,11} + 2 ca_{1,13} + 4 T ca_{1,14}) == 0, \\ T^2 ca_{1,1} + T ca_{1,2} + T ca_{1,4} + T^2 ca_{1,5} + 2 ca_{1,6} + 2 ca_{1,9} + \\ T ca_{1,10} + 2 ca_{1,12} + 2 T ca_{1,13} + 2 T^2 ca_{1,14} + T^2 cc_{1,1} + T cc_{1,2} + 2 cc_{1,3} == 0, \\ T (T ca_{1,2} + T ca_{1,4} + 4 ca_{1,6} + 4 ca_{1,9} + T ca_{1,10} + 4 ca_{1,12} + 2 T ca_{1,13} + T cc_{1,2} + 4 cc_{1,3}) == 0, \\ 2 T^2 (ca_{1,6} + ca_{1,9} + ca_{1,12} + cc_{1,3}) == 0 \}$$

## R1r

(Alt) In[ ]:=

```
lhs = CF[Module[{es = {i, i+}},
  Times[
    Normal@Series[Exp[r_d[1, i, i+] + \gamma_d[-1, i+]], {e, 0, d}],
    Exp[Sum[g_{\alpha, \beta} \pi_{\alpha} \xi_{\beta}, {\alpha, es}, {\beta, es}]]
  ] // Zip((p_#&/@es) \cup (x_#&/@es) // Expand
] // . {
  g_{i, \beta} \mapsto \delta_{i, \beta} + T g_{i+, \beta} + (1 - T) g_{i++, \beta}, g_{i+, \beta} \mapsto \delta_{i+, \beta} + g_{i++, \beta},
  g_{\alpha, i} \mapsto T^{-1} (g_{\alpha, i+} - \delta_{\alpha, i+}), g_{\alpha, i+} \mapsto T g_{\alpha, i++} - (1 - T) \delta_{\alpha, i+} - T \delta_{\alpha, i++}
}
```

(Alt) Out[ ]:=

$$1 + e \left( ca_{1,1} - ca_{1,4} + ca_{1,5} - T ca_{1,5} + 2 ca_{1,12} - 2 ca_{1,13} + 2 T ca_{1,13} + 2 ca_{1,14} - 4 T ca_{1,14} + 2 T^2 ca_{1,14} - cc_{1,1} - cc_{1,2} + T cc_{1,2} - 2 cc_{1,3} + 4 T cc_{1,3} - 2 T^2 cc_{1,3} \right) +$$

$$e \left( ca_{1,2} + T ca_{1,3} + ca_{1,4} + T ca_{1,5} - 2 ca_{1,9} + ca_{1,10} - 2 T ca_{1,10} + 2 T ca_{1,11} - 2 T^2 ca_{1,11} - 4 ca_{1,12} + 2 ca_{1,13} - 4 T ca_{1,13} + 4 T ca_{1,14} - 4 T^2 ca_{1,14} - T cc_{1,2} - 4 T cc_{1,3} + 4 T^2 cc_{1,3} \right) g_{i++, i++} +$$

$$2e \left( ca_{1,6} + T ca_{1,7} + T^2 ca_{1,8} + ca_{1,9} + T ca_{1,10} + T^2 ca_{1,11} + ca_{1,12} + T ca_{1,13} + T^2 ca_{1,14} - T^2 cc_{1,3} \right) g_{i++, i++}^2$$

(Alt) In[ ]:=

rhs = 1

(Alt) Out[ ]:=

1

(Alt) In[ ]:=

me = Exponent[lhs - rhs, T, Min]

(Alt) Out[ ]:=

0

(Alt) In[ ]:=

covars = DeleteCases[Variables[lhs - rhs], T | (ca | cb | cc | cd) \_]

(Alt) Out[ ]:=

{e, g\_{i++, i++}}

(Alt) In[ ]:=

```
eqnsR1r =
  (Factor[#] == 0) & /@ Union[Last /@ CoefficientRules[Expand[T^{-me} (lhs - rhs)], covars]]
```

(Alt) Out[ ]:=

$$\left\{ 2 \left( ca_{1,6} + T ca_{1,7} + T^2 ca_{1,8} + ca_{1,9} + T ca_{1,10} + T^2 ca_{1,11} + ca_{1,12} + T ca_{1,13} + T^2 ca_{1,14} - T^2 cc_{1,3} \right) = 0, \right.$$

$$ca_{1,1} - ca_{1,4} + ca_{1,5} - T ca_{1,5} + 2 ca_{1,12} - 2 ca_{1,13} + 2 T ca_{1,13} + 2 ca_{1,14} - 4 T ca_{1,14} + 2 T^2 ca_{1,14} - cc_{1,1} - cc_{1,2} + T cc_{1,2} - 2 cc_{1,3} + 4 T cc_{1,3} - 2 T^2 cc_{1,3} = 0,$$

$$ca_{1,2} + T ca_{1,3} + ca_{1,4} + T ca_{1,5} - 2 ca_{1,9} + ca_{1,10} - 2 T ca_{1,10} + 2 T ca_{1,11} - 2 T^2 ca_{1,11} - 4 ca_{1,12} + 2 ca_{1,13} - 4 T ca_{1,13} + 4 T ca_{1,14} - 4 T^2 ca_{1,14} - T cc_{1,2} - 4 T cc_{1,3} + 4 T^2 cc_{1,3} = 0 \left. \right\}$$

## Sw<sup>+</sup>

(Alt) In[ ]:=

```

lhs = CF[Module[{es = {i, j, i+, j+}},
  Times[
    Normal@
      Series[Exp[rd[1, i, j] + γd[-1, i] + γd[-1, j] + γd[1, i+] + γd[1, j+]], {ε, 0, d}],
      Exp[Sum[gα,β πα ξβ, {α, es}, {β, es}]]
    ] // Zip(pε&/@es)U(xε&/@es) // Expand
  ] // . gRules1,i,j
]

```

(Alt) Out[ ]:=

$$\begin{aligned}
 & 1 + \epsilon \text{ca}_{1,1} + \epsilon (\text{ca}_{1,2} - \text{ca}_{1,3} + T \text{ca}_{1,3}) g_{i^+,i^+} + \\
 & 2 \epsilon (\text{ca}_{1,6} - \text{ca}_{1,7} + T \text{ca}_{1,7} + \text{ca}_{1,8} - 2 T \text{ca}_{1,8} + T^2 \text{ca}_{1,8}) g_{i^+,i^+}^2 + T \epsilon \text{ca}_{1,3} g_{i^+,j^+} + \\
 & 2 T \epsilon (\text{ca}_{1,7} - 2 \text{ca}_{1,8} + 2 T \text{ca}_{1,8}) g_{i^+,i^+} g_{i^+,j^+} + 2 T^2 \epsilon \text{ca}_{1,8} g_{i^+,j^+}^2 - \\
 & \frac{\epsilon (-\text{ca}_{1,2} + T \text{ca}_{1,2} + \text{ca}_{1,3} - 2 T \text{ca}_{1,3} + T^2 \text{ca}_{1,3} - \text{ca}_{1,4} + \text{ca}_{1,5} - T \text{ca}_{1,5}) g_{j^+,i^+}}{T} - \\
 & \frac{1}{T} 2 \epsilon (-2 \text{ca}_{1,6} + 2 T \text{ca}_{1,6} + 2 \text{ca}_{1,7} - 4 T \text{ca}_{1,7} + 2 T^2 \text{ca}_{1,7} - 2 \text{ca}_{1,8} + 6 T \text{ca}_{1,8} - 6 T^2 \text{ca}_{1,8} + 2 T^3 \text{ca}_{1,8} - \\
 & \quad \text{ca}_{1,9} + \text{ca}_{1,10} - T \text{ca}_{1,10} - \text{ca}_{1,11} + 2 T \text{ca}_{1,11} - T^2 \text{ca}_{1,11} + 2 \text{cc}_{1,3} - 2 T \text{cc}_{1,3}) g_{i^+,i^+} g_{j^+,i^+} + \\
 & \epsilon (2 \text{ca}_{1,7} - 2 T \text{ca}_{1,7} - 4 \text{ca}_{1,8} + 8 T \text{ca}_{1,8} - 4 T^2 \text{ca}_{1,8} + \text{ca}_{1,10} - 2 \text{ca}_{1,11} + 2 T \text{ca}_{1,11}) g_{i^+,j^+} g_{j^+,i^+} + \\
 & \frac{1}{T^2} 2 \epsilon (\text{ca}_{1,6} - 2 T \text{ca}_{1,6} + T^2 \text{ca}_{1,6} - \text{ca}_{1,7} + 3 T \text{ca}_{1,7} - 3 T^2 \text{ca}_{1,7} + T^3 \text{ca}_{1,7} + \text{ca}_{1,8} - 4 T \text{ca}_{1,8} + \\
 & \quad 6 T^2 \text{ca}_{1,8} - 4 T^3 \text{ca}_{1,8} + T^4 \text{ca}_{1,8} + \text{ca}_{1,9} - T \text{ca}_{1,9} - \text{ca}_{1,10} + 2 T \text{ca}_{1,10} - T^2 \text{ca}_{1,10} + \text{ca}_{1,11} - \\
 & \quad 3 T \text{ca}_{1,11} + 3 T^2 \text{ca}_{1,11} - T^3 \text{ca}_{1,11} + \text{ca}_{1,12} - \text{ca}_{1,13} + T \text{ca}_{1,13} + \text{ca}_{1,14} - 2 T \text{ca}_{1,14} + \\
 & \quad T^2 \text{ca}_{1,14} - 2 \text{cc}_{1,3} + 4 T \text{cc}_{1,3} - 2 T^2 \text{cc}_{1,3}) g_{j^+,i^+}^2 + \epsilon (\text{ca}_{1,3} - T \text{ca}_{1,3} + \text{ca}_{1,5}) g_{j^+,j^+} + \\
 & \epsilon (2 \text{ca}_{1,7} - 2 T \text{ca}_{1,7} - 4 \text{ca}_{1,8} + 8 T \text{ca}_{1,8} - 4 T^2 \text{ca}_{1,8} + \text{ca}_{1,10} - 2 \text{ca}_{1,11} + 2 T \text{ca}_{1,11}) g_{i^+,i^+} g_{j^+,j^+} - \\
 & 2 T \epsilon (-2 \text{ca}_{1,8} + 2 T \text{ca}_{1,8} - \text{ca}_{1,11}) g_{i^+,j^+} g_{j^+,j^+} + \\
 & \frac{1}{T} 2 \epsilon (\text{ca}_{1,7} - 2 T \text{ca}_{1,7} + T^2 \text{ca}_{1,7} - 2 \text{ca}_{1,8} + 6 T \text{ca}_{1,8} - 6 T^2 \text{ca}_{1,8} + 2 T^3 \text{ca}_{1,8} + \text{ca}_{1,10} - T \text{ca}_{1,10} - 2 \text{ca}_{1,11} + \\
 & \quad 4 T \text{ca}_{1,11} - 2 T^2 \text{ca}_{1,11} + \text{ca}_{1,13} - 2 \text{ca}_{1,14} + 2 T \text{ca}_{1,14} + 2 \text{cc}_{1,3} - 2 T \text{cc}_{1,3}) g_{j^+,i^+} g_{j^+,j^+} + \\
 & 2 \epsilon (\text{ca}_{1,8} - 2 T \text{ca}_{1,8} + T^2 \text{ca}_{1,8} + \text{ca}_{1,11} - T \text{ca}_{1,11} + \text{ca}_{1,14}) g_{j^+,j^+}^2
 \end{aligned}$$

(Alt) In[ ]:=

```
rhs = CF[Module[{es = {i, j, i+, j+}},
  Times[
    Normal@Series[Exp[r_d[1, i, j]], {e, 0, d}],
    Exp[Sum[g_{\alpha,\beta} \pi_\alpha \xi_\beta, {\alpha, es}, {\beta, es}]]
  ] // Zip(p_{\alpha}&/@es) \cup (x_{\alpha}&/@es) // Expand
] // . gRules_{1,i,j}
]
```

(Alt) Out[ ]:=

$$\begin{aligned}
 & 1 + \epsilon \text{ca}_{1,1} + \epsilon (\text{ca}_{1,2} - \text{ca}_{1,3} + T \text{ca}_{1,3}) \text{g}_{i^+,i^+} + \\
 & 2 \epsilon (\text{ca}_{1,6} - \text{ca}_{1,7} + T \text{ca}_{1,7} + \text{ca}_{1,8} - 2 T \text{ca}_{1,8} + T^2 \text{ca}_{1,8}) \text{g}_{i^+,i^+}^2 + T \epsilon \text{ca}_{1,3} \text{g}_{i^+,j^+} + \\
 & 2 T \epsilon (\text{ca}_{1,7} - 2 \text{ca}_{1,8} + 2 T \text{ca}_{1,8}) \text{g}_{i^+,i^+} \text{g}_{i^+,j^+} + 2 T^2 \epsilon \text{ca}_{1,8} \text{g}_{i^+,j^+}^2 - \\
 & \frac{\epsilon (-\text{ca}_{1,2} + T \text{ca}_{1,2} + \text{ca}_{1,3} - 2 T \text{ca}_{1,3} + T^2 \text{ca}_{1,3} - \text{ca}_{1,4} + \text{ca}_{1,5} - T \text{ca}_{1,5}) \text{g}_{j^+,i^+}}{T} \\
 & \frac{1}{T} 2 \epsilon (-2 \text{ca}_{1,6} + 2 T \text{ca}_{1,6} + 2 \text{ca}_{1,7} - 4 T \text{ca}_{1,7} + 2 T^2 \text{ca}_{1,7} - 2 \text{ca}_{1,8} + 6 T \text{ca}_{1,8} - 6 T^2 \text{ca}_{1,8} + \\
 & 2 T^3 \text{ca}_{1,8} - \text{ca}_{1,9} + \text{ca}_{1,10} - T \text{ca}_{1,10} - \text{ca}_{1,11} + 2 T \text{ca}_{1,11} - T^2 \text{ca}_{1,11}) \text{g}_{i^+,i^+} \text{g}_{j^+,i^+} + \\
 & \epsilon (2 \text{ca}_{1,7} - 2 T \text{ca}_{1,7} - 4 \text{ca}_{1,8} + 8 T \text{ca}_{1,8} - 4 T^2 \text{ca}_{1,8} + \text{ca}_{1,10} - 2 \text{ca}_{1,11} + 2 T \text{ca}_{1,11}) \text{g}_{i^+,j^+} \text{g}_{j^+,i^+} + \\
 & \frac{1}{T^2} 2 \epsilon (\text{ca}_{1,6} - 2 T \text{ca}_{1,6} + T^2 \text{ca}_{1,6} - \text{ca}_{1,7} + 3 T \text{ca}_{1,7} - 3 T^2 \text{ca}_{1,7} + T^3 \text{ca}_{1,7} + \text{ca}_{1,8} - 4 T \text{ca}_{1,8} + 6 T^2 \text{ca}_{1,8} - \\
 & 4 T^3 \text{ca}_{1,8} + T^4 \text{ca}_{1,8} + \text{ca}_{1,9} - T \text{ca}_{1,9} - \text{ca}_{1,10} + 2 T \text{ca}_{1,10} - T^2 \text{ca}_{1,10} + \text{ca}_{1,11} - 3 T \text{ca}_{1,11} + 3 T^2 \text{ca}_{1,11} - \\
 & T^3 \text{ca}_{1,11} + \text{ca}_{1,12} - \text{ca}_{1,13} + T \text{ca}_{1,13} + \text{ca}_{1,14} - 2 T \text{ca}_{1,14} + T^2 \text{ca}_{1,14}) \text{g}_{j^+,i^+}^2 + \epsilon (\text{ca}_{1,3} - T \text{ca}_{1,3} + \text{ca}_{1,5}) \\
 & \text{g}_{j^+,j^+} + \epsilon (2 \text{ca}_{1,7} - 2 T \text{ca}_{1,7} - 4 \text{ca}_{1,8} + 8 T \text{ca}_{1,8} - 4 T^2 \text{ca}_{1,8} + \text{ca}_{1,10} - 2 \text{ca}_{1,11} + 2 T \text{ca}_{1,11}) \text{g}_{i^+,i^+} \text{g}_{j^+,j^+} - \\
 & 2 T \epsilon (-2 \text{ca}_{1,8} + 2 T \text{ca}_{1,8} - \text{ca}_{1,11}) \text{g}_{i^+,j^+} \text{g}_{j^+,j^+} + \\
 & \frac{1}{T} 2 \epsilon (\text{ca}_{1,7} - 2 T \text{ca}_{1,7} + T^2 \text{ca}_{1,7} - 2 \text{ca}_{1,8} + 6 T \text{ca}_{1,8} - 6 T^2 \text{ca}_{1,8} + 2 T^3 \text{ca}_{1,8} + \text{ca}_{1,10} - \\
 & T \text{ca}_{1,10} - 2 \text{ca}_{1,11} + 4 T \text{ca}_{1,11} - 2 T^2 \text{ca}_{1,11} + \text{ca}_{1,13} - 2 \text{ca}_{1,14} + 2 T \text{ca}_{1,14}) \text{g}_{j^+,i^+} \text{g}_{j^+,j^+} + \\
 & 2 \epsilon (\text{ca}_{1,8} - 2 T \text{ca}_{1,8} + T^2 \text{ca}_{1,8} + \text{ca}_{1,11} - T \text{ca}_{1,11} + \text{ca}_{1,14}) \text{g}_{j^+,j^+}^2
 \end{aligned}$$

(Alt) In[ ]:=

```
me = Exponent[lhs - rhs, T, Min]
```

(Alt) Out[ ]:=

-2

(Alt) In[ ]:=

```
covars = DeleteCases[Variables[lhs - rhs], T | (ca | cb | cc | cd) _]
```

(Alt) Out[ ]:=

{\epsilon, g\_{i^+,i^+}, g\_{j^+,i^+}, g\_{j^+,j^+}}

(Alt) In[ ]:=

```
eqnsSwp = {}
```

(Alt) Out[ ]:=

{}

## Solution

(Alt) In[ ]:=

```
vars = Cases[Variables[r_d[1, i1, j1] + r_d[-1, i2, j2] + \gamma_d[1, k]], (ca | cb | cc | cd) ___]
{sol} = Solve[eqnsR3 \cup eqnsR2b \cup eqnsR2c \cup eqnsR1l \cup eqnsR1r \cup eqnsSwp, vars]
sol /. Rule -> Set;
r_d[1, i, j]
r_d[-1, i, j]
\gamma_d[1, k]
\gamma_d[-1, k]
```

(Alt) Out[ ]:=

```
{ca1,1, ca1,2, ca1,3, ca1,4, ca1,5, ca1,6, ca1,7, ca1,8, ca1,9,
ca1,10, ca1,11, ca1,12, ca1,13, ca1,14, cb1,1, cb1,2, cb1,3, cb1,4, cb1,5, cb1,6,
cb1,7, cb1,8, cb1,9, cb1,10, cb1,11, cb1,12, cb1,13, cb1,14, cc1,1, cc1,2, cc1,3}
```

**Solve:** Equations may not give solutions for all "solve" variables.

(Alt) Out[ ]:=

$$\left\{ \left\{ \begin{aligned} ca_{1,1} &\rightarrow -\frac{ca_{1,2}}{2}, ca_{1,3} \rightarrow 0, ca_{1,4} \rightarrow -ca_{1,2}, ca_{1,5} \rightarrow 0, ca_{1,6} \rightarrow 0, ca_{1,7} \rightarrow 0, ca_{1,8} \rightarrow 0, \\ ca_{1,9} &\rightarrow -\frac{1}{2}(-1+T)ca_{1,10}, ca_{1,11} \rightarrow 0, ca_{1,12} \rightarrow -\frac{1}{2}(1-T)ca_{1,10}, ca_{1,13} \rightarrow -ca_{1,10}, \\ ca_{1,14} &\rightarrow 0, cb_{1,1} \rightarrow \frac{ca_{1,2}}{2}, cb_{1,2} \rightarrow -ca_{1,2}, cb_{1,3} \rightarrow 0, cb_{1,4} \rightarrow ca_{1,2}, cb_{1,5} \rightarrow 0, \\ cb_{1,6} &\rightarrow 0, cb_{1,7} \rightarrow 0, cb_{1,8} \rightarrow 0, cb_{1,9} \rightarrow -\frac{(-1+T)ca_{1,10}}{2T}, cb_{1,10} \rightarrow -ca_{1,10}, cb_{1,11} \rightarrow 0, \\ cb_{1,12} &\rightarrow -\frac{(1-T)ca_{1,10}}{2T}, cb_{1,13} \rightarrow ca_{1,10}, cb_{1,14} \rightarrow 0, cc_{1,1} \rightarrow \frac{ca_{1,2}}{2}, cc_{1,2} \rightarrow ca_{1,10}, cc_{1,3} \rightarrow 0 \end{aligned} \right\} \right\}$$

(Alt) Out[ ]:=

$$-\frac{1}{2} \in ca_{1,2} + \in p_i x_i ca_{1,2} - \in p_j x_j ca_{1,2} + \frac{1}{2} \in p_i p_j x_i^2 ca_{1,10} - \frac{1}{2} T \in p_i p_j x_i^2 ca_{1,10} -$$

$$\frac{1}{2} \in p_j^2 x_i^2 ca_{1,10} + \frac{1}{2} T \in p_j^2 x_i^2 ca_{1,10} + \in p_i p_j x_i x_j ca_{1,10} - \in p_j^2 x_i x_j ca_{1,10}$$

(Alt) Out[ ]:=

$$\frac{1}{2} \in ca_{1,2} - \in p_i x_i ca_{1,2} + \in p_j x_j ca_{1,2} - \frac{1}{2} \in p_i p_j x_i^2 ca_{1,10} + \frac{\in p_i p_j x_i^2 ca_{1,10}}{2T} +$$

$$\frac{1}{2} \in p_j^2 x_i^2 ca_{1,10} - \frac{\in p_j^2 x_i^2 ca_{1,10}}{2T} - \in p_i p_j x_i x_j ca_{1,10} + \in p_j^2 x_i x_j ca_{1,10}$$

(Alt) Out[ ]:=

$$\frac{1}{2} \in ca_{1,2} + \in p_k x_k ca_{1,10}$$

(Alt) Out[ ]:=

$$-\frac{1}{2} \in ca_{1,2} - \in p_k x_k ca_{1,10}$$

(Alt) In[ ]:=

```
lhs = CF[(r1[1, i, j] // px2g) /. gRules1,i,j /. {ϵ → 1, ca1,2 → 1, ca1,10 → -1}]
rhs = CF[R1[1, i, j] /. gRules1,i,j]
Simplify[lhs == rhs]
```

(Alt) Out[ ]:=

$$-\frac{1}{2} + g_{i^+,i^+} - g_{j^+,i^+} - \frac{(-1+T) g_{i^+,i^+} g_{j^+,i^+}}{T} - g_{i^+,j^+} g_{j^+,i^+} + \frac{(-1+T) g_{j^+,i^+}^2}{T} - g_{i^+,i^+} g_{j^+,j^+} + 2 g_{j^+,i^+} g_{j^+,j^+}$$

(Alt) Out[ ]:=

$$-\frac{1}{2} + g_{i^+,i^+} - g_{j^+,i^+} - \frac{(-1+T) g_{i^+,i^+} g_{j^+,i^+}}{T} - g_{i^+,j^+} g_{j^+,i^+} + \frac{(-1+T) g_{j^+,i^+}^2}{T} - g_{i^+,i^+} g_{j^+,j^+} + 2 g_{j^+,i^+} g_{j^+,j^+}$$

(Alt) Out[ ]:=

True

## Non-Universally Solving at d=2

(Alt) In[ ]:=

```
d = 2;
vars =
Cases[Variables[rd[1, i1, j1] + rd[-1, i2, j2] + γd[1, k1] + γd[-1, k2]], (ca | cb | cc | cd) __]
```

(Alt) Out[ ]:=

```
{ca1,2, ca1,10, ca2,1, ca2,2, ca2,3, ca2,4, ca2,5, ca2,6, ca2,7, ca2,8, ca2,9, ca2,10,
ca2,11, ca2,12, ca2,13, ca2,14, ca2,15, ca2,16, ca2,17, ca2,18, ca2,19, ca2,20, ca2,21,
ca2,22, ca2,23, ca2,24, ca2,25, ca2,26, ca2,27, ca2,28, ca2,29, ca2,30, cb2,1, cb2,2, cb2,3,
cb2,4, cb2,5, cb2,6, cb2,7, cb2,8, cb2,9, cb2,10, cb2,11, cb2,12, cb2,13, cb2,14, cb2,15,
cb2,16, cb2,17, cb2,18, cb2,19, cb2,20, cb2,21, cb2,22, cb2,23, cb2,24, cb2,25, cb2,26,
cb2,27, cb2,28, cb2,29, cb2,30, cc2,1, cc2,2, cc2,3, cc2,4, cd2,1, cd2,2, cd2,3, cd2,4}
```

### $c\bar{c}$

(Alt) In[ ]:=

```
lhs = Module[{x1, p1},
  {x1*, p1*} = {p1, x1};
  Normal[
    Log[0[ϵ]d+1 + Zip{x1}[Exp[0[ϵ]d+1 + (γd[1, i] /. xi → xi + x1) + (γd[-1, i] /. pi → pi - p1)]]]]
  ]
rhs = 0
```

(Alt) Out[ ]:=

$$\epsilon^2 (p_i x_i ca_{1,10}^2 + cc_{2,1} + p_i x_i cc_{2,2} + p_i^2 x_i^2 cc_{2,3} + p_i^3 x_i^3 cc_{2,4} + cd_{2,1} + p_i x_i cd_{2,2} + p_i^2 x_i^2 cd_{2,3} + p_i^3 x_i^3 cd_{2,4})$$

(Alt) Out[ ]:=

0

(Alt) In[ ]:=

```
covars = DeleteCases[Variables[lhs - rhs], T | (ca | cb | cc | cd) __]
```

(Alt) Out[ ]:=

```
{ϵ, pi, xi}
```

```
(Alt) In[ ]:=
eqnsCCbar = (# == 0) & /@ Union[Last /@ CoefficientRules[Expand[lhs - rhs], covars]]
```

```
(Alt) Out[ ]:=
{cc2,1 + cd2,1 == 0, ca1,10^2 + cc2,2 + cd2,2 == 0, cc2,3 + cd2,3 == 0, cc2,4 + cd2,4 == 0}
```

```
(Alt) In[ ]:=
vars =
Cases[Variables[r_d[1, i1, j1] + r_d[-1, i2, j2] + \gamma_d[1, k1] + \gamma_d[-1, k2]], (ca | cb | cc | cd) __]
{sol} = Solve[eqnsCCbar, vars]
sol /. Rule -> Set;
\gamma_d[1, k]
\gamma_d[-1, k]
```

```
(Alt) Out[ ]:=
{ca1,2, ca1,10, ca2,1, ca2,2, ca2,3, ca2,4, ca2,5, ca2,6, ca2,7, ca2,8, ca2,9, ca2,10,
ca2,11, ca2,12, ca2,13, ca2,14, ca2,15, ca2,16, ca2,17, ca2,18, ca2,19, ca2,20, ca2,21,
ca2,22, ca2,23, ca2,24, ca2,25, ca2,26, ca2,27, ca2,28, ca2,29, ca2,30, cb2,1, cb2,2, cb2,3,
cb2,4, cb2,5, cb2,6, cb2,7, cb2,8, cb2,9, cb2,10, cb2,11, cb2,12, cb2,13, cb2,14, cb2,15,
cb2,16, cb2,17, cb2,18, cb2,19, cb2,20, cb2,21, cb2,22, cb2,23, cb2,24, cb2,25, cb2,26,
cb2,27, cb2,28, cb2,29, cb2,30, cc2,1, cc2,2, cc2,3, cc2,4, cd2,1, cd2,2, cd2,3, cd2,4}
```

Solve: Equations may not give solutions for all "solve" variables.

Solve: Equations may not give solutions for all "solve" variables.

```
(Alt) Out[ ]:=
{{cd2,1 -> -cc2,1, cd2,2 -> -ca1,10^2 - cc2,2, cd2,3 -> -cc2,3, cd2,4 -> -cc2,4}}
```

```
(Alt) Out[ ]:=
1
- \in ca1,2 + \in p_k x_k ca1,10 + \in^2 cc2,1 + \in^2 p_k x_k cc2,2 + \in^2 p_k^2 x_k^2 cc2,3 + \in^2 p_k^3 x_k^3 cc2,4
```

```
(Alt) Out[ ]:=
1
- \in ca1,2 - \in p_k x_k ca1,10 - \in^2 p_k x_k ca1,10^2 - \in^2 cc2,1 - \in^2 p_k x_k cc2,2 - \in^2 p_k^2 x_k^2 cc2,3 - \in^2 p_k^3 x_k^3 cc2,4
```

### R3 @ d = 2

```
(Alt) In[ ]:=
Short[lhs = CF[Module[{es = {i, j, k, i+, j+, k+}},
Times[
Normal@Series[Exp[r_d[1, j, k] + r_d[1, i, k+] + r_d[1, i+, j+]], {e, 0, d}],
Exp[Sum[g_{\alpha,\beta} \pi_\alpha \xi_\beta, {\alpha, es}, {\beta, es}]]
] // Zip[(p_&/@es) \cup (x_&/@es) // Expand
] // . gRules_{1,j,k} \cup gRules_{1,i,k+} \cup gRules_{1,i+,j+}], 5]
```

```
(Alt) Out[ ]//Short=
1 - \frac{3}{2} \in ca1,2 + \frac{3}{8} \in^2 (3 ca1,2^2 + 8 ca2,1) + <<560>> +
24 \in^2 ca1,10^2 g_{k+,i+} g_{k+,j+} g_{k+,k+}^2 + 12 \in^2 ca1,10^2 g_{k+,j+}^2 g_{k+,k+}^2 - 12 \in^2
(-ca2,18 + 3 T ca2,18 - 3 T^2 ca2,18 + T^3 ca2,18 - ca2,22 + 2 T ca2,22 - T^2 ca2,22 - ca2,26 + T ca2,26 - ca2,30)
g_{k+,k+}^3
```

(Alt) In[ ]:=

```
Short[rhs = CF[Module[{es = {i, j, k, i+, j+, k+}},
  Times[
    Normal@Series[Exp[r_d[1, i, j] + r_d[1, i+, k] + r_d[1, j+, k+]], {e, 0, d}],
    Exp[Sum[g_{\alpha, \beta} \pi_{\alpha} \xi_{\beta}, {\alpha, es}, {\beta, es}]]
  ] // Zip_{(p_x & /@ es) \cup (x_x & /@ es)} // Expand
] // . gRules_{1, i, j} \cup gRules_{1, i+, k} \cup gRules_{1, j+, k+}, 5]
```

(Alt) Out[ ]//Short=

$$1 - \frac{3}{2} \epsilon \text{ca}_{1,2} + \frac{3}{8} \epsilon^2 (3 \text{ca}_{1,2}^2 + 8 \text{ca}_{2,1}) + \ll 557 \gg +$$

$$24 \epsilon^2 \text{ca}_{1,10}^2 \mathfrak{g}_{k^{++}, i^{++}} \mathfrak{g}_{k^{++}, j^{++}} \mathfrak{g}_{k^{++}, k^{++}}^2 + 12 \epsilon^2 \text{ca}_{1,10}^2 \mathfrak{g}_{k^{++}, j^{++}}^2 \mathfrak{g}_{k^{++}, k^{++}}^2 - 12 \epsilon^2$$

$$(-\text{ca}_{2,18} + 3 T \text{ca}_{2,18} - 3 T^2 \text{ca}_{2,18} + T^3 \text{ca}_{2,18} - \text{ca}_{2,22} + 2 T \text{ca}_{2,22} - T^2 \text{ca}_{2,22} - \text{ca}_{2,26} + T \text{ca}_{2,26} - \text{ca}_{2,30})$$

$$\mathfrak{g}_{k^{++}, k^{++}}^3$$

(Alt) In[ ]:=

```
me = Exponent[lhs - rhs, T, Min]
```

(Alt) Out[ ]:=

-6

(Alt) In[ ]:=

```
covars = DeleteCases[Variables[lhs - rhs], T | (ca | cb | cc | cd) _]
```

(Alt) Out[ ]:=

```
{\epsilon, \mathfrak{g}_{i^{++}, i^{++}}, \mathfrak{g}_{i^{++}, j^{++}}, \mathfrak{g}_{i^{++}, k^{++}}, \mathfrak{g}_{j^{++}, i^{++}}, \mathfrak{g}_{j^{++}, j^{++}}, \mathfrak{g}_{j^{++}, k^{++}}, \mathfrak{g}_{k^{++}, i^{++}}, \mathfrak{g}_{k^{++}, j^{++}}, \mathfrak{g}_{k^{++}, k^{++}}}
```

(Alt) In[ ]:=

```
Short[
  eqnsR3 = (# == 0) & /@ Union[Last /@ CoefficientRules[Expand[T^{-me} (lhs - rhs)], covars]]]
```

(Alt) Out[ ]//Short=

$$\{-T^5 \text{ca}_{2,3} + T^6 \text{ca}_{2,3} == 0, T^6 \text{ca}_{2,3} - T^7 \text{ca}_{2,3} \ll 1 \gg \ll 1 \gg == 0,$$

$$\ll 125 \gg, \ll 1 \gg == 0, \ll 68 \gg + 36 T^6 \text{ca}_{2,30} == 0\}$$

## R2b @ d = 2

(Alt) In[ ]:=

```
Short[lhs = CF[Module[{es = {i, j, i+, j+}},
  Times[
    Normal@Series[Exp[r_d[1, i, j] + r_d[-1, i+, j+]], {e, 0, d}],
    Exp[Sum[g_{\alpha, \beta} \pi_{\alpha} \xi_{\beta}, {\alpha, es}, {\beta, es}]]
  ] // Zip_{(p_x & /@ es) \cup (x_x & /@ es)} // Expand
] // . gRules_{1, i, j} \cup gRules_{-1, i+, j+}]
```

(Alt) Out[ ]//Short=

$$1 + \epsilon^2 (\text{ca}_{2,1} + \text{cb}_{2,1}) + \frac{\ll 1 \gg}{T} + \ll 37 \gg + \frac{\ll 1 \gg}{\ll 1 \gg} - \frac{6 \ll 3 \gg \mathfrak{g}_{\ll 1 \gg}^2}{T^3} + \frac{6 \epsilon^2 (\ll 1 \gg) \mathfrak{g}_{\ll 1 \gg, \ll 1 \gg}^3}{T^3}$$

```

(Alt) In[ ]:=
  rhs = 1
(Alt) Out[ ]:=
  1
(Alt) In[ ]:=
  me = Exponent[lhs - rhs, T, Min]
(Alt) Out[ ]:=
  -3
(Alt) In[ ]:=
  covars = DeleteCases[Variables[lhs - rhs], T | (ca | cb | cc | cd) __]
(Alt) Out[ ]:=
  {ϵ, gi++,i++, gi++,j++, gj++,i++, gj++,j++}
(Alt) In[ ]:=
  Short[eqnsR2b =
    (Factor[#] == 0) & /@ Union[Last /@ CoefficientRules[Expand[Tme (lhs - rhs)], covars]]]
(Alt) Out[ ]//Short=
  {T3 (ca2,1 + cb2,1) == 0, T2 (T ca2,3 + cb2,3) == 0,
    <<30>>, 3 (T3 <<1>> + <<63>> + <<1>>) == 0, 3 (<<1>>) == 0}
(Alt) In[ ]:=
  vars = Cases[Variables[rd[1, i1, j1] + rd[-1, i2, j2] + γd[1, k]], (ca | cb | cc | cd) __]
  {sol} = Solve[eqnsR3 ∪ eqnsR2b ∪ eqnsR2c ∪ eqnsR1l ∪ eqnsR1r ∪ eqnsSwp, vars]
  sol /. Rule → Set;
  rd[1, i, j]
  γd[1, k]
(Alt) Out[ ]:=
  {ca1,2, ca1,10, ca2,1, ca2,2, ca2,3, ca2,4, ca2,5, ca2,6, ca2,7, ca2,8, ca2,9, ca2,10, ca2,11, ca2,12,
  ca2,13, ca2,14, ca2,15, ca2,16, ca2,17, ca2,18, ca2,19, ca2,20, ca2,21, ca2,22, ca2,23, ca2,24, ca2,25,
  ca2,26, ca2,27, ca2,28, ca2,29, ca2,30, cb2,1, cb2,2, cb2,3, cb2,4, cb2,5, cb2,6, cb2,7, cb2,8,
  cb2,9, cb2,10, cb2,11, cb2,12, cb2,13, cb2,14, cb2,15, cb2,16, cb2,17, cb2,18, cb2,19, cb2,20, cb2,21,
  cb2,22, cb2,23, cb2,24, cb2,25, cb2,26, cb2,27, cb2,28, cb2,29, cb2,30, cc2,1, cc2,2, cc2,3, cc2,4}

```

**Solve:** Equations may not give solutions for all "solve" variables.

(Alt) Out[\*]=

$$\left\{ \left\{ \begin{aligned}
& \text{ca}_{2,2} \rightarrow -\text{ca}_{1,2}^2 - \text{cb}_{2,2}, \text{ca}_{2,3} \rightarrow 0, \\
& \text{ca}_{2,4} \rightarrow \text{ca}_{1,2}^2 + \text{cb}_{2,2} + T \text{cb}_{2,5}, \text{ca}_{2,5} \rightarrow -\text{cb}_{2,5}, \text{ca}_{2,6} \rightarrow 0, \text{ca}_{2,7} \rightarrow 0, \text{ca}_{2,8} \rightarrow 0, \\
& \text{ca}_{2,9} \rightarrow \frac{1}{2} \left( -\text{ca}_{1,2} \text{ca}_{1,10} + 3 T \text{ca}_{1,2} \text{ca}_{1,10} + \text{ca}_{1,10}^2 - T \text{ca}_{1,10}^2 - 2 T \text{cb}_{2,9} - 2 \text{cb}_{2,10} + 2 T \text{cb}_{2,10} \right), \\
& \text{ca}_{2,10} \rightarrow -2 \text{ca}_{1,2} \text{ca}_{1,10} + \text{ca}_{1,10}^2 - \text{cb}_{2,10}, \text{ca}_{2,11} \rightarrow 0, \\
& \text{ca}_{2,12} \rightarrow \frac{1}{2} \left( \text{ca}_{1,2} \text{ca}_{1,10} - 3 T \text{ca}_{1,2} \text{ca}_{1,10} + T^2 \text{ca}_{1,2} \text{ca}_{1,10} - \right. \\
& \quad \left. \text{ca}_{1,10}^2 + T \text{ca}_{1,10}^2 + 2 T \text{cb}_{2,9} - 2 T^2 \text{cb}_{2,9} + 2 \text{cb}_{2,10} - 3 T \text{cb}_{2,10} + T^2 \text{cb}_{2,10} \right), \\
& \text{ca}_{2,13} \rightarrow \frac{1}{2} \left( 4 \text{ca}_{1,2} \text{ca}_{1,10} - T \text{ca}_{1,2} \text{ca}_{1,10} - 2 \text{ca}_{1,10}^2 + 2 T \text{cb}_{2,9} + 3 \text{cb}_{2,10} - T \text{cb}_{2,10} \right), \\
& \text{ca}_{2,14} \rightarrow 0, \text{ca}_{2,15} \rightarrow 0, \text{ca}_{2,16} \rightarrow 0, \text{ca}_{2,17} \rightarrow 0, \text{ca}_{2,18} \rightarrow 0, \\
& \text{ca}_{2,19} \rightarrow \frac{1}{6} \left( \text{ca}_{1,10}^2 + 2 T \text{ca}_{1,10}^2 - 6 \text{ca}_{2,29} \right), \text{ca}_{2,20} \rightarrow -\frac{1}{2} \text{ca}_{1,10}^2, \text{ca}_{2,21} \rightarrow 0, \text{ca}_{2,22} \rightarrow 0, \\
& \text{ca}_{2,23} \rightarrow -\frac{1}{6} (-1 + T) \left( 2 \text{ca}_{1,10}^2 + T \text{ca}_{1,10}^2 + 6 \text{ca}_{2,29} \right), \text{ca}_{2,24} \rightarrow \frac{1}{2} (2 + T) \text{ca}_{1,10}^2, \text{ca}_{2,25} \rightarrow -\frac{1}{2} \text{ca}_{1,10}^2, \\
& \text{ca}_{2,26} \rightarrow 0, \text{ca}_{2,27} \rightarrow \frac{1}{6} \left( -3 + 2 T + T^2 \right) \text{ca}_{1,10}^2, \text{ca}_{2,28} \rightarrow -\text{ca}_{1,10}^2 + \text{ca}_{2,29} - T \text{ca}_{2,29}, \\
& \text{ca}_{2,30} \rightarrow 0, \text{cb}_{2,1} \rightarrow -\text{ca}_{2,1}, \text{cb}_{2,3} \rightarrow 0, \text{cb}_{2,4} \rightarrow \frac{-T \text{cb}_{2,2} - \text{cb}_{2,5}}{T}, \text{cb}_{2,6} \rightarrow 0, \text{cb}_{2,7} \rightarrow 0, \\
& \text{cb}_{2,8} \rightarrow 0, \text{cb}_{2,11} \rightarrow 0, \text{cb}_{2,12} \rightarrow \frac{-T \text{ca}_{1,2} \text{ca}_{1,10} + 2 T \text{cb}_{2,9} - 2 T^2 \text{cb}_{2,9} + \text{cb}_{2,10} - T \text{cb}_{2,10}}{2 T^2}, \\
& \text{cb}_{2,13} \rightarrow \frac{T \text{ca}_{1,2} \text{ca}_{1,10} - 2 T \text{cb}_{2,9} - \text{cb}_{2,10} - T \text{cb}_{2,10}}{2 T}, \text{cb}_{2,14} \rightarrow 0, \text{cb}_{2,15} \rightarrow 0, \text{cb}_{2,16} \rightarrow 0, \\
& \text{cb}_{2,17} \rightarrow 0, \text{cb}_{2,18} \rightarrow 0, \text{cb}_{2,19} \rightarrow \frac{-\text{ca}_{1,10}^2 - 2 T \text{ca}_{1,10}^2 + 6 \text{ca}_{2,29}}{6 T}, \text{cb}_{2,20} \rightarrow -\frac{1}{2} \text{ca}_{1,10}^2, \\
& \text{cb}_{2,21} \rightarrow 0, \text{cb}_{2,22} \rightarrow 0, \text{cb}_{2,23} \rightarrow \frac{(-1 + T) \left( 4 \text{ca}_{1,10}^2 + 5 T \text{ca}_{1,10}^2 - 6 \text{ca}_{2,29} \right)}{6 T^2}, \\
& \text{cb}_{2,24} \rightarrow \frac{(1 + 2 T) \text{ca}_{1,10}^2}{2 T}, \text{cb}_{2,25} \rightarrow -\frac{1}{2} \text{ca}_{1,10}^2, \text{cb}_{2,26} \rightarrow 0, \text{cb}_{2,27} \rightarrow -\frac{(-1 - 2 T + 3 T^2) \text{ca}_{1,10}^2}{6 T^2}, \\
& \text{cb}_{2,28} \rightarrow \frac{-\text{ca}_{1,10}^2 - T^2 \text{ca}_{1,10}^2 + 2 \text{ca}_{2,29} - 2 T \text{ca}_{2,29}}{2 T^2}, \text{cb}_{2,29} \rightarrow \frac{\text{ca}_{1,10}^2 + T \text{ca}_{1,10}^2 - 2 \text{ca}_{2,29}}{2 T}, \text{cb}_{2,30} \rightarrow 0 \} \}
\end{aligned} \right.$$

(Alt) Out[\*]=

$$\begin{aligned}
& -\frac{1}{2} \in \text{ca}_{1,2} + \in p_i x_i \text{ca}_{1,2} - \in p_j x_i \text{ca}_{1,2} - \in^2 p_i x_i \text{ca}_{1,2}^2 + \in^2 p_j x_i \text{ca}_{1,2}^2 + \frac{1}{2} \in p_i p_j x_i^2 \text{ca}_{1,10} - \\
& \frac{1}{2} T \in p_i p_j x_i^2 \text{ca}_{1,10} - \frac{1}{2} \in p_j^2 x_i^2 \text{ca}_{1,10} + \frac{1}{2} T \in p_j^2 x_i^2 \text{ca}_{1,10} + \in p_i p_j x_i x_j \text{ca}_{1,10} - \\
& \in p_j^2 x_i x_j \text{ca}_{1,10} - \frac{1}{2} \in^2 p_i p_j x_i^2 \text{ca}_{1,2} \text{ca}_{1,10} + \frac{3}{2} T \in^2 p_i p_j x_i^2 \text{ca}_{1,2} \text{ca}_{1,10} + \frac{1}{2} \in^2 p_j^2 x_i^2 \text{ca}_{1,2} \text{ca}_{1,10} - \\
& \frac{3}{2} T \in^2 p_j^2 x_i^2 \text{ca}_{1,2} \text{ca}_{1,10} + \frac{1}{2} T^2 \in^2 p_j^2 x_i^2 \text{ca}_{1,2} \text{ca}_{1,10} - 2 \in^2 p_i p_j x_i x_j \text{ca}_{1,2} \text{ca}_{1,10} + \\
& 2 \in^2 p_j^2 x_i x_j \text{ca}_{1,2} \text{ca}_{1,10} - \frac{1}{2} T \in^2 p_j^2 x_i x_j \text{ca}_{1,2} \text{ca}_{1,10} + \frac{1}{2} \in^2 p_i p_j x_i^2 \text{ca}_{1,10}^2 - \frac{1}{2} T \in^2 p_i p_j x_i^2 \text{ca}_{1,10}^2 - \\
& \frac{1}{2} \in^2 p_j^2 x_i^2 \text{ca}_{1,10}^2 + \frac{1}{2} T \in^2 p_j^2 x_i^2 \text{ca}_{1,10}^2 + \frac{1}{6} \in^2 p_i^2 p_j x_i^3 \text{ca}_{1,10}^2 + \frac{1}{3} T \in^2 p_i^2 p_j x_i^3 \text{ca}_{1,10}^2 + \\
& \frac{1}{3} \in^2 p_i p_j^2 x_i^3 \text{ca}_{1,10}^2 - \frac{1}{6} T \in^2 p_i p_j^2 x_i^3 \text{ca}_{1,10}^2 - \frac{1}{6} T^2 \in^2 p_i p_j^2 x_i^3 \text{ca}_{1,10}^2 - \frac{1}{2} \in^2 p_j^3 x_i^3 \text{ca}_{1,10}^2 + \\
& \frac{1}{3} T \in^2 p_j^3 x_i^3 \text{ca}_{1,10}^2 + \frac{1}{6} T^2 \in^2 p_j^3 x_i^3 \text{ca}_{1,10}^2 + \in^2 p_i p_j x_i x_j \text{ca}_{1,10}^2 - \in^2 p_j^2 x_i x_j \text{ca}_{1,10}^2 - \\
& \frac{1}{2} \in^2 p_i^2 p_j x_i^2 x_j \text{ca}_{1,10}^2 + \in^2 p_i p_j^2 x_i^2 x_j \text{ca}_{1,10}^2 + \frac{1}{2} T \in^2 p_i p_j^2 x_i^2 x_j \text{ca}_{1,10}^2 - \in^2 p_j^3 x_i^2 x_j \text{ca}_{1,10}^2 - \\
& \frac{1}{2} \in^2 p_i p_j^2 x_i x_j^2 \text{ca}_{1,10}^2 + \in^2 \text{ca}_{2,1} - \in^2 p_i^2 p_j x_i^3 \text{ca}_{2,29} + \in^2 p_i p_j^2 x_i^3 \text{ca}_{2,29} - T \in^2 p_i p_j^2 x_i^3 \text{ca}_{2,29} + \\
& \in^2 p_j^3 x_i^2 x_j \text{ca}_{2,29} - T \in^2 p_j^3 x_i^2 x_j \text{ca}_{2,29} + \in^2 p_j^3 x_i x_j^2 \text{ca}_{2,29} - \in^2 p_i x_i \text{cb}_{2,2} + \in^2 p_j x_i \text{cb}_{2,2} + \\
& T \in^2 p_j x_i \text{cb}_{2,5} - \in^2 p_j x_j \text{cb}_{2,5} - T \in^2 p_i p_j x_i^2 \text{cb}_{2,9} + T \in^2 p_j^2 x_i^2 \text{cb}_{2,9} - T^2 \in^2 p_j^2 x_i^2 \text{cb}_{2,9} + \\
& T \in^2 p_j^2 x_i x_j \text{cb}_{2,9} - \in^2 p_i p_j x_i^2 \text{cb}_{2,10} + T \in^2 p_i p_j x_i^2 \text{cb}_{2,10} + \in^2 p_j^2 x_i^2 \text{cb}_{2,10} - \frac{3}{2} T \in^2 p_j^2 x_i^2 \text{cb}_{2,10} + \\
& \frac{1}{2} T^2 \in^2 p_j^2 x_i^2 \text{cb}_{2,10} - \in^2 p_i p_j x_i x_j \text{cb}_{2,10} + \frac{3}{2} \in^2 p_j^2 x_i x_j \text{cb}_{2,10} - \frac{1}{2} T \in^2 p_j^2 x_i x_j \text{cb}_{2,10}
\end{aligned}$$

(Alt) Out[\*]=

$$\frac{1}{2} \in \text{ca}_{1,2} + \in p_k x_k \text{ca}_{1,10} + \in^2 \text{cc}_{2,1} + \in^2 p_k x_k \text{cc}_{2,2} + \in^2 p_k^2 x_k^2 \text{cc}_{2,3} + \in^2 p_k^3 x_k^3 \text{cc}_{2,4}$$

## R2c @ d = 2

(Alt) In[ ]:=

```
lhs = CF[Module[{es = {i, j, i+, j+}},
  Times[
    Normal@Series[Exp[r_d[-1, 0, 1, i, j+] + r_d[1, i+, j]], {e, 0, d}],
    Exp[Sum[g_{\alpha,\beta} \pi_{\alpha} \xi_{\beta}, {\alpha, es}, {\beta, es}]]
  ] // Zip_{(p_{\alpha}&/@es)\cup(x_{\alpha}&/@es)} // Expand
] // . gRules_{-1,i,j+} \cup gRules_{1,i+,j}
```

(Alt) Out[ ]:=

$$\begin{aligned}
& 1 + \frac{1}{2} \epsilon \text{ca}_{1,2} + \frac{1}{8} \epsilon^2 (\text{ca}_{1,2}^2 + 8 \text{cc}_{2,1}) - \\
& \frac{(-1 + T) \epsilon^2 (\text{ca}_{1,2} \text{ca}_{1,10} + \text{cb}_{2,10} + \text{cc}_{2,2}) \text{g}_{j^{++}, i^{++}}}{T} + \frac{2 (-1 + T)^2 \epsilon^2 \text{cc}_{2,3} \text{g}_{j^{++}, i^{++}}^2}{T^2} - \\
& \frac{6 (-1 + T)^3 \epsilon^2 \text{cc}_{2,4} \text{g}_{j^{++}, i^{++}}^3}{T^3} + \epsilon \text{ca}_{1,10} \text{g}_{j^{++}, j^{++}} + \frac{1}{2} \epsilon^2 (\text{ca}_{1,2} \text{ca}_{1,10} + 2 \text{cc}_{2,2}) \text{g}_{j^{++}, j^{++}} - \\
& \frac{4 (-1 + T) \epsilon^2 \text{cc}_{2,3} \text{g}_{j^{++}, i^{++}} \text{g}_{j^{++}, j^{++}}}{T} + \frac{18 (-1 + T)^2 \epsilon^2 \text{cc}_{2,4} \text{g}_{j^{++}, i^{++}}^2 \text{g}_{j^{++}, j^{++}}}{T^2} + \\
& \epsilon^2 (\text{ca}_{1,10}^2 + 2 \text{cc}_{2,3}) \text{g}_{j^{++}, j^{++}}^2 - \frac{18 (-1 + T) \epsilon^2 \text{cc}_{2,4} \text{g}_{j^{++}, i^{++}} \text{g}_{j^{++}, j^{++}}^2}{T} + 6 \epsilon^2 \text{cc}_{2,4} \text{g}_{j^{++}, j^{++}}^3
\end{aligned}$$

(Alt) In[ ]:=

```
rhs = CF[Module[{es = {(j^+)^+}},
  Times[
    Normal@Series[Exp[\gamma_d[1, (j^+)^+]], {e, 0, d}],
    Exp[Sum[g_{\alpha,\beta} \pi_{\alpha} \xi_{\beta}, {\alpha, es}, {\beta, es}]]
  ] // Zip_{(p_{\alpha}&/@es)\cup(x_{\alpha}&/@es)} // Expand
]]
```

(Alt) Out[ ]:=

$$\begin{aligned}
& 1 + \frac{1}{2} \epsilon \text{ca}_{1,2} + \frac{1}{8} \epsilon^2 (\text{ca}_{1,2}^2 + 8 \text{cc}_{2,1}) + \epsilon \text{ca}_{1,10} \text{g}_{j^{++}, j^{++}} + \\
& \frac{1}{2} \epsilon^2 (\text{ca}_{1,2} \text{ca}_{1,10} + 2 \text{cc}_{2,2}) \text{g}_{j^{++}, j^{++}} + \epsilon^2 (\text{ca}_{1,10}^2 + 2 \text{cc}_{2,3}) \text{g}_{j^{++}, j^{++}}^2 + 6 \epsilon^2 \text{cc}_{2,4} \text{g}_{j^{++}, j^{++}}^3
\end{aligned}$$

(Alt) In[ ]:=

```
me = Exponent[lhs - rhs, T, Min]
```

(Alt) Out[ ]:=

```
-3
```

(Alt) In[ ]:=

```
covars = DeleteCases[Variables[lhs - rhs], T | (ca | cb | cc | cd) _]
```

(Alt) Out[ ]:=

```
{e, g_{j^{++}, i^{++}}, g_{j^{++}, j^{++}}}
```

```
(Alt) In[ ]:=
```

```
CoefficientRules[Expand[ $T^{-me}$  (lhs - rhs)], covars] // Column
```

```
(Alt) Out[ ]:=
```

```

{2, 3, 0} → 6 cC2,4 - 18 T cC2,4 + 18 T2 cC2,4 - 6 T3 cC2,4
{2, 2, 1} → 18 T cC2,4 - 36 T2 cC2,4 + 18 T3 cC2,4
{2, 2, 0} → 2 T cC2,3 - 4 T2 cC2,3 + 2 T3 cC2,3
{2, 1, 2} → 18 T2 cC2,4 - 18 T3 cC2,4
{2, 1, 1} → 4 T2 cC2,3 - 4 T3 cC2,3
{2, 1, 0} → T2 ca1,2 ca1,10 - T3 ca1,2 ca1,10 + T2 cb2,10 - T3 cb2,10 + T2 cC2,2 - T3 cC2,2

```

```
(Alt) In[ ]:=
```

```
eqnsR2c =
```

```
(Factor[#] == 0) & /@ Union[Last /@ CoefficientRules[Expand[ $T^{-me}$  (lhs - rhs)], covars]]
```

```
(Alt) Out[ ]:=
```

```

{- ((-1 + T) T2 (ca1,2 ca1,10 + cb2,10 + cC2,2)) == 0, -4 (-1 + T) T2 cC2,3 == 0, 2 (-1 + T)2 T cC2,3 == 0,
-18 (-1 + T) T2 cC2,4 == 0, -6 (-1 + T)3 cC2,4 == 0, 18 (-1 + T)2 T cC2,4 == 0}

```

(Alt) In[ ]:=

```
vars = Cases[Variables[r_d[1, i1, j1] + r_d[-1, i2, j2] + \gamma_d[1, k]], (ca | cb | cc | cd) __]
{sol} = Solve[eqnsR3 \cup eqnsR2b \cup eqnsR2c \cup eqnsR11 \cup eqnsR1r \cup eqnsSwp, vars]
sol /. Rule -> Set;
r_d[1, i, j]
\gamma_d[1, k]
```

(Alt) Out[ ]:=

{ca<sub>1,2</sub>, ca<sub>1,10</sub>, ca<sub>2,29</sub>, cb<sub>2,2</sub>, cb<sub>2,5</sub>, cb<sub>2,9</sub>, cb<sub>2,10</sub>, cc<sub>2,1</sub>, cc<sub>2,2</sub>, cc<sub>2,3</sub>, cc<sub>2,4</sub>}

Solve: Equations may not give solutions for all "solve" variables.

(Alt) Out[ ]:=

{ {cc<sub>2,2</sub> -> -ca<sub>1,2</sub> ca<sub>1,10</sub> - cb<sub>2,10</sub>, cc<sub>2,3</sub> -> 0, cc<sub>2,4</sub> -> 0 } }

(Alt) Out[ ]:=

$$\begin{aligned}
 & -\frac{1}{2} \epsilon ca_{1,2} + \epsilon p_i x_i ca_{1,2} - \epsilon p_j x_j ca_{1,2} - \epsilon^2 p_i x_i ca_{1,2}^2 + \epsilon^2 p_j x_j ca_{1,2}^2 + \frac{1}{2} \epsilon p_i p_j x_i^2 ca_{1,10} - \\
 & \frac{1}{2} T \epsilon p_i p_j x_i^2 ca_{1,10} - \frac{1}{2} \epsilon p_j^2 x_j^2 ca_{1,10} + \frac{1}{2} T \epsilon p_j^2 x_j^2 ca_{1,10} + \epsilon p_i p_j x_i x_j ca_{1,10} - \\
 & \epsilon p_j^2 x_j x_j ca_{1,10} - \frac{1}{2} \epsilon^2 p_i p_j x_i^2 ca_{1,2} ca_{1,10} + \frac{3}{2} T \epsilon^2 p_i p_j x_i^2 ca_{1,2} ca_{1,10} + \frac{1}{2} \epsilon^2 p_j^2 x_j^2 ca_{1,2} ca_{1,10} - \\
 & \frac{3}{2} T \epsilon^2 p_j^2 x_j^2 ca_{1,2} ca_{1,10} + \frac{1}{2} T^2 \epsilon^2 p_j^2 x_j^2 ca_{1,2} ca_{1,10} - 2 \epsilon^2 p_i p_j x_i x_j ca_{1,2} ca_{1,10} + \\
 & 2 \epsilon^2 p_j^2 x_j x_j ca_{1,2} ca_{1,10} - \frac{1}{2} T \epsilon^2 p_j^2 x_j x_j ca_{1,2} ca_{1,10} + \frac{1}{2} \epsilon^2 p_i p_j x_i^2 ca_{1,10}^2 - \frac{1}{2} T \epsilon^2 p_i p_j x_i^2 ca_{1,10}^2 - \\
 & \frac{1}{2} \epsilon^2 p_j^2 x_j^2 ca_{1,10}^2 + \frac{1}{2} T \epsilon^2 p_j^2 x_j^2 ca_{1,10}^2 + \frac{1}{6} \epsilon^2 p_i^2 p_j x_i^3 ca_{1,10}^2 + \frac{1}{3} T \epsilon^2 p_i^2 p_j x_i^3 ca_{1,10}^2 + \\
 & \frac{1}{3} \epsilon^2 p_i p_j^2 x_i^3 ca_{1,10}^2 - \frac{1}{6} T \epsilon^2 p_i p_j^2 x_i^3 ca_{1,10}^2 - \frac{1}{6} T^2 \epsilon^2 p_i p_j^2 x_i^3 ca_{1,10}^2 - \frac{1}{2} \epsilon^2 p_j^3 x_j^3 ca_{1,10}^2 + \\
 & \frac{1}{3} T \epsilon^2 p_j^3 x_j^3 ca_{1,10}^2 + \frac{1}{6} T^2 \epsilon^2 p_j^3 x_j^3 ca_{1,10}^2 + \epsilon^2 p_i p_j x_i x_j ca_{1,10}^2 - \epsilon^2 p_j^2 x_j x_j ca_{1,10}^2 - \\
 & \frac{1}{2} \epsilon^2 p_i^2 p_j x_i^2 x_j ca_{1,10}^2 + \epsilon^2 p_i p_j^2 x_i^2 x_j ca_{1,10}^2 + \frac{1}{2} T \epsilon^2 p_i p_j^2 x_i^2 x_j ca_{1,10}^2 - \epsilon^2 p_j^3 x_j^2 x_j ca_{1,10}^2 - \\
 & \frac{1}{2} \epsilon^2 p_i p_j^2 x_i x_j^2 ca_{1,10}^2 + \epsilon^2 ca_{2,1} - \epsilon^2 p_i^2 p_j x_i^3 ca_{2,29} + \epsilon^2 p_i p_j^2 x_i^3 ca_{2,29} - T \epsilon^2 p_i p_j^2 x_i^3 ca_{2,29} + \\
 & \epsilon^2 p_j^3 x_j^2 x_j ca_{2,29} - T \epsilon^2 p_j^3 x_j^2 x_j ca_{2,29} + \epsilon^2 p_j^3 x_i x_j^2 ca_{2,29} - \epsilon^2 p_i x_i cb_{2,2} + \epsilon^2 p_j x_i cb_{2,2} + \\
 & T \epsilon^2 p_j x_i cb_{2,5} - \epsilon^2 p_j x_j cb_{2,5} - T \epsilon^2 p_i p_j x_i^2 cb_{2,9} + T \epsilon^2 p_j^2 x_j^2 cb_{2,9} - T^2 \epsilon^2 p_j^2 x_j^2 cb_{2,9} + \\
 & T \epsilon^2 p_j^2 x_i x_j cb_{2,9} - \epsilon^2 p_i p_j x_i^2 cb_{2,10} + T \epsilon^2 p_i p_j x_i^2 cb_{2,10} + \epsilon^2 p_j^2 x_j^2 cb_{2,10} - \frac{3}{2} T \epsilon^2 p_j^2 x_j^2 cb_{2,10} + \\
 & \frac{1}{2} T^2 \epsilon^2 p_j^2 x_j^2 cb_{2,10} - \epsilon^2 p_i p_j x_i x_j cb_{2,10} + \frac{3}{2} \epsilon^2 p_j^2 x_i x_j cb_{2,10} - \frac{1}{2} T \epsilon^2 p_j^2 x_i x_j cb_{2,10}
 \end{aligned}$$

(Alt) Out[ ]:=

$$\frac{1}{2} \epsilon ca_{1,2} + \epsilon p_k x_k ca_{1,10} - \epsilon^2 p_k x_k ca_{1,2} ca_{1,10} - \epsilon^2 p_k x_k cb_{2,10} + \epsilon^2 cc_{2,1}$$

## R1l

(Alt) In[ ]:=

```
lhs = CF[Module[{es = {i, i^+}},
  Times[
    Normal@Series[Exp[r_d[1, 1, 0, i^+, i]], {e, 0, d}],
    Exp[Sum[g_{\alpha, \beta} \pi_{\alpha} \xi_{\beta}, {\alpha, es}, {\beta, es}]]
  ] // Zip((p_#&/@es) \cup (x_#&/@es) // Expand
] // . {g_{i^+, \beta_} \to T^{-1} \delta_{i^+, \beta} + g_{i^{++}, \beta}, g_{i, \beta_} \to \delta_{i, \beta} + g_{i^+, \beta}}]
```

(Alt) Out[ ]:=

$$\begin{aligned}
& 1 + \epsilon^2 (ca_{2,1} + cc_{2,1}) - \frac{1}{T^2} \\
& \epsilon^2 (T^2 ca_{1,2} ca_{1,10} + 3 ca_{1,10}^2 + 3 T ca_{1,10}^2 - 6 ca_{2,29} - 6 T ca_{2,29} + T^2 cb_{2,5} - 2 T^2 cb_{2,9} - T cb_{2,10} + T^2 cb_{2,10}) \\
& g_{i^{++}, i} - \frac{3 \epsilon^2 (ca_{1,10}^2 - 2 ca_{2,29}) g_{i^{++}, i}^2}{T} + \frac{1}{T} \\
& \epsilon^2 (T^2 ca_{1,2} ca_{1,10} + 3 ca_{1,10}^2 + 3 T ca_{1,10}^2 - 6 ca_{2,29} - 6 T ca_{2,29} + T^2 cb_{2,5} - 2 T^2 cb_{2,9} - T cb_{2,10} + T^2 cb_{2,10}) \\
& g_{i^{++}, i^+} - \frac{\epsilon^2 (T^2 ca_{1,2} ca_{1,10} + 6 ca_{1,10}^2 - 12 ca_{2,29} - 2 T^2 cb_{2,9} - T cb_{2,10} + T^2 cb_{2,10}) g_{i^{++}, i} g_{i^{++}, i^+}}{T} - \\
& 3 \epsilon^2 (ca_{1,10}^2 - 2 ca_{2,29}) g_{i^{++}, i} g_{i^{++}, i^+} + \\
& \epsilon^2 (T^2 ca_{1,2} ca_{1,10} + 6 ca_{1,10}^2 + 3 T ca_{1,10}^2 - 12 ca_{2,29} - 6 T ca_{2,29} - 2 T^2 cb_{2,9} - T cb_{2,10} + T^2 cb_{2,10}) g_{i^{++}, i^+}^2 + \\
& 3 (-1 + T) \epsilon^2 (ca_{1,10}^2 - 2 ca_{2,29}) g_{i^{++}, i} g_{i^{++}, i^+}^2 + 3 T \epsilon^2 (ca_{1,10}^2 - 2 ca_{2,29}) g_{i^{++}, i^+}^3
\end{aligned}$$

(Alt) In[ ]:=

rhs = 1

(Alt) Out[ ]:=

1

(Alt) In[ ]:=

me = Exponent[lhs - rhs, T, Min]

(Alt) Out[ ]:=

- 2

(Alt) In[ ]:=

covars = DeleteCases[Variables[lhs - rhs], T | (ca | cb | cc | cd) \_\_\_]

(Alt) Out[ ]:=

{e, g\_{i^{++}, i}, g\_{i^{++}, i^+}}

(Alt) In[ ]:=

**eqnsR11 =**

**(Factor[#] == 0) & /@ Union[Last /@ CoefficientRules[Expand[T<sup>me</sup> (lhs - rhs)], covars]]**

(Alt) Out[ ]:=

$$\begin{aligned} & \{-3 T (ca_{1,10}^2 - 2 ca_{2,29}) = 0, -3 T^2 (ca_{1,10}^2 - 2 ca_{2,29}) = 0, \\ & 3 T^3 (ca_{1,10}^2 - 2 ca_{2,29}) = 0, 3 (-1 + T) T^2 (ca_{1,10}^2 - 2 ca_{2,29}) = 0, \\ & -T^2 ca_{1,2} ca_{1,10} - 3 ca_{1,10}^2 - 3 T ca_{1,10}^2 + 6 ca_{2,29} + 6 T ca_{2,29} - T^2 cb_{2,5} + 2 T^2 cb_{2,9} + T cb_{2,10} - T^2 cb_{2,10} = 0, \\ & -T (T^2 ca_{1,2} ca_{1,10} + 6 ca_{1,10}^2 - 12 ca_{2,29} - 2 T^2 cb_{2,9} - T cb_{2,10} + T^2 cb_{2,10}) = 0, \\ & T (T^2 ca_{1,2} ca_{1,10} + 3 ca_{1,10}^2 + 3 T ca_{1,10}^2 - 6 ca_{2,29} - 6 T ca_{2,29} + T^2 cb_{2,5} - 2 T^2 cb_{2,9} - T cb_{2,10} + T^2 cb_{2,10}) = \\ & 0, T^2 (T^2 ca_{1,2} ca_{1,10} + 6 ca_{1,10}^2 + 3 T ca_{1,10}^2 - 12 ca_{2,29} - 6 T ca_{2,29} - 2 T^2 cb_{2,9} - T cb_{2,10} + T^2 cb_{2,10}) = \\ & 0, T^2 (ca_{2,1} + cc_{2,1}) = 0 \} \end{aligned}$$

(Alt) In[ ]:=

```
vars = Cases[Variables[r_d[1, i1, j1] + r_d[-1, i2, j2] + \gamma_d[1, k]], (ca | cb | cc | cd) __]
{sol} = Solve[eqnsR3 \cup eqnsR2b \cup eqnsR2c \cup eqnsR11 \cup eqnsR1r \cup eqnsSwp, vars]
sol /. Rule -> Set;
r_d[1, i, j]
\gamma_d[1, k]
```

(Alt) Out[ ]:=

{ca<sub>1,2</sub>, ca<sub>1,10</sub>, ca<sub>2,29</sub>, cb<sub>2,2</sub>, cb<sub>2,5</sub>, cb<sub>2,9</sub>, cb<sub>2,10</sub>, cc<sub>2,1</sub>}

**Solve:** Equations may not give solutions for all "solve" variables.

(Alt) Out[ ]:=

$$\left\{ \left\{ ca_{2,29} \rightarrow \frac{ca_{1,10}^2}{2}, cb_{2,5} \rightarrow 0, cb_{2,9} \rightarrow \frac{T ca_{1,2} ca_{1,10} - cb_{2,10} + T cb_{2,10}}{2 T}, cc_{2,1} \rightarrow -ca_{2,1} \right\} \right\}$$

(Alt) Out[ ]:=

$$\begin{aligned} & -\frac{1}{2} \in ca_{1,2} + \in p_i x_i ca_{1,2} - \in p_j x_i ca_{1,2} - \in^2 p_i x_i ca_{1,2}^2 + \in^2 p_j x_i ca_{1,2}^2 + \frac{1}{2} \in p_i p_j x_i^2 ca_{1,10} - \\ & \frac{1}{2} T \in p_i p_j x_i^2 ca_{1,10} - \frac{1}{2} \in p_j^2 x_i^2 ca_{1,10} + \frac{1}{2} T \in p_j^2 x_i^2 ca_{1,10} + \in p_i p_j x_i x_j ca_{1,10} - \\ & \in p_j^2 x_i x_j ca_{1,10} - \frac{1}{2} \in^2 p_i p_j x_i^2 ca_{1,2} ca_{1,10} + T \in^2 p_i p_j x_i^2 ca_{1,2} ca_{1,10} + \frac{1}{2} \in^2 p_j^2 x_i^2 ca_{1,2} ca_{1,10} - \\ & T \in^2 p_j^2 x_i^2 ca_{1,2} ca_{1,10} - 2 \in^2 p_i p_j x_i x_j ca_{1,2} ca_{1,10} + 2 \in^2 p_j^2 x_i x_j ca_{1,2} ca_{1,10} + \frac{1}{2} \in^2 p_i p_j x_i^2 ca_{1,10}^2 - \\ & \frac{1}{2} T \in^2 p_i p_j x_i^2 ca_{1,10}^2 - \frac{1}{2} \in^2 p_j^2 x_i^2 ca_{1,10}^2 + \frac{1}{2} T \in^2 p_j^2 x_i^2 ca_{1,10}^2 - \frac{1}{3} \in^2 p_i^2 p_j x_i^3 ca_{1,10}^2 + \\ & \frac{1}{3} T \in^2 p_i^2 p_j x_i^3 ca_{1,10}^2 + \frac{5}{6} \in^2 p_i p_j^2 x_i^3 ca_{1,10}^2 - \frac{2}{3} T \in^2 p_i p_j^2 x_i^3 ca_{1,10}^2 - \frac{1}{6} T^2 \in^2 p_i p_j^2 x_i^3 ca_{1,10}^2 - \\ & \frac{1}{2} \in^2 p_j^3 x_i^3 ca_{1,10}^2 + \frac{1}{3} T \in^2 p_j^3 x_i^3 ca_{1,10}^2 + \frac{1}{6} T^2 \in^2 p_j^3 x_i^3 ca_{1,10}^2 + \in^2 p_i p_j x_i x_j ca_{1,10}^2 - \\ & \in^2 p_j^2 x_i x_j ca_{1,10}^2 - \frac{1}{2} \in^2 p_i^2 p_j x_i^2 x_j ca_{1,10}^2 + \in^2 p_i p_j^2 x_i^2 x_j ca_{1,10}^2 + \frac{1}{2} T \in^2 p_i p_j^2 x_i^2 x_j ca_{1,10}^2 - \\ & \frac{1}{2} \in^2 p_j^3 x_i^2 x_j ca_{1,10}^2 - \frac{1}{2} T \in^2 p_j^3 x_i^2 x_j ca_{1,10}^2 - \frac{1}{2} \in^2 p_i p_j^2 x_i x_j^2 ca_{1,10}^2 + \frac{1}{2} \in^2 p_j^3 x_i x_j^2 ca_{1,10}^2 + \\ & \in^2 ca_{2,1} - \in^2 p_i x_i cb_{2,2} + \in^2 p_j x_i cb_{2,2} - \frac{1}{2} \in^2 p_i p_j x_i^2 cb_{2,10} + \frac{1}{2} T \in^2 p_i p_j x_i^2 cb_{2,10} + \\ & \frac{1}{2} \in^2 p_j^2 x_i^2 cb_{2,10} - \frac{1}{2} T \in^2 p_j^2 x_i^2 cb_{2,10} - \in^2 p_i p_j x_i x_j cb_{2,10} + \in^2 p_j^2 x_i x_j cb_{2,10} \end{aligned}$$

(Alt) Out[ ]:=

$$\frac{1}{2} \in ca_{1,2} + \in p_k x_k ca_{1,10} - \in^2 p_k x_k ca_{1,2} ca_{1,10} - \in^2 ca_{2,1} - \in^2 p_k x_k cb_{2,10}$$

## R1r

(Alt) In[ ]:=

```
lhs = CF[Module[{es = {i, i^+}},
  Times[
    Normal@Series[Exp[r_d[1, 0, -1, i, i^+]], {e, 0, d}],
    Exp[Sum[g_{\alpha, \beta} \pi_{\alpha} \xi_{\beta}, {\alpha, es}, {\beta, es}]]
  ] // Zip((p_#&/@es) \cup (x_#&/@es) // Expand
] // . {
  g_{i, \beta} \to \delta_{i, \beta} + T g_{i^+, \beta} + (1 - T) g_{i^+, \beta}, g_{i^+, \beta} \to \delta_{i^+, \beta} + g_{i^+, \beta},
  g_{\alpha, i} \to T^{-1} (g_{\alpha, i^+} - \delta_{\alpha, i^+}), g_{\alpha, i^+} \to T g_{\alpha, i^+} - (1 - T) \delta_{\alpha, i^+} - T \delta_{\alpha, i^+}
}
```

(Alt) Out[ ]:=

$$1 + \frac{1}{2} \epsilon^2 (-ca_{1,2}^2 + 4ca_{2,1} - 2cb_{2,2})$$

(Alt) In[ ]:=

rhs = 1

(Alt) Out[ ]:=

1

(Alt) In[ ]:=

me = Exponent[lhs - rhs, T, Min]

(Alt) Out[ ]:=

0

(Alt) In[ ]:=

covars = DeleteCases[Variables[lhs - rhs], T | (ca | cb | cc | cd) \_]

(Alt) Out[ ]:=

{\epsilon}

(Alt) In[ ]:=

```
eqnsR1r =
  (Factor[#] == 0) & /@ Union[Last /@ CoefficientRules[Expand[T^{-me} (lhs - rhs)], covars]]
```

(Alt) Out[ ]:=

$$\left\{ \frac{1}{2} (-ca_{1,2}^2 + 4ca_{2,1} - 2cb_{2,2}) = 0 \right\}$$

(Alt) In[ ]:=

```
vars = Cases[Variables[r_d[1, i1, j1] + r_d[-1, i2, j2] + \gamma_d[1, k]], (ca | cb | cc | cd) __]
{sol} = Solve[eqnsR3 \cup eqnsR2b \cup eqnsR2c \cup eqnsR11 \cup eqnsR1r \cup eqnsSwp, vars]
sol /. Rule -> Set;
r_d[1, i, j]
\gamma_d[1, k]
```

(Alt) Out[ ]:=

{ca<sub>1,2</sub>, ca<sub>1,10</sub>, ca<sub>2,1</sub>, cb<sub>2,2</sub>, cb<sub>2,10</sub>}

**Solve:** Equations may not give solutions for all "solve" variables.

(Alt) Out[ ]:=

$$\left\{ \left\{ cb_{2,2} \rightarrow \frac{1}{2} (-ca_{1,2}^2 + 4 ca_{2,1}) \right\} \right\}$$

(Alt) Out[ ]:=

$$\begin{aligned} & -\frac{1}{2} \epsilon ca_{1,2} + \epsilon p_i x_i ca_{1,2} - \epsilon p_j x_i ca_{1,2} - \frac{1}{2} \epsilon^2 p_i x_i ca_{1,2}^2 + \frac{1}{2} \epsilon^2 p_j x_i ca_{1,2}^2 + \frac{1}{2} \epsilon^2 p_i p_j x_i^2 ca_{1,10} - \\ & \frac{1}{2} T \epsilon p_i p_j x_i^2 ca_{1,10} - \frac{1}{2} \epsilon p_j^2 x_i^2 ca_{1,10} + \frac{1}{2} T \epsilon p_j^2 x_i^2 ca_{1,10} + \epsilon p_i p_j x_i x_j ca_{1,10} - \\ & \epsilon p_j^2 x_i x_j ca_{1,10} - \frac{1}{2} \epsilon^2 p_i p_j x_i^2 ca_{1,2} ca_{1,10} + T \epsilon^2 p_i p_j x_i^2 ca_{1,2} ca_{1,10} + \frac{1}{2} \epsilon^2 p_j^2 x_i^2 ca_{1,2} ca_{1,10} - \\ & T \epsilon^2 p_j^2 x_i^2 ca_{1,2} ca_{1,10} - 2 \epsilon^2 p_i p_j x_i x_j ca_{1,2} ca_{1,10} + 2 \epsilon^2 p_j^2 x_i x_j ca_{1,2} ca_{1,10} + \frac{1}{2} \epsilon^2 p_i p_j x_i^2 ca_{1,10}^2 - \\ & \frac{1}{2} T \epsilon^2 p_i p_j x_i^2 ca_{1,10}^2 - \frac{1}{2} \epsilon^2 p_j^2 x_i^2 ca_{1,10}^2 + \frac{1}{2} T \epsilon^2 p_j^2 x_i^2 ca_{1,10}^2 - \frac{1}{3} \epsilon^2 p_i^2 p_j x_i^3 ca_{1,10}^2 + \\ & \frac{1}{3} T \epsilon^2 p_i^2 p_j x_i^3 ca_{1,10}^2 + \frac{5}{6} \epsilon^2 p_i p_j^2 x_i^3 ca_{1,10}^2 - \frac{2}{3} T \epsilon^2 p_i p_j^2 x_i^3 ca_{1,10}^2 - \frac{1}{6} T^2 \epsilon^2 p_i p_j^2 x_i^3 ca_{1,10}^2 - \\ & \frac{1}{2} \epsilon^2 p_j^3 x_i^3 ca_{1,10}^2 + \frac{1}{3} T \epsilon^2 p_j^3 x_i^3 ca_{1,10}^2 + \frac{1}{6} T^2 \epsilon^2 p_j^3 x_i^3 ca_{1,10}^2 + \epsilon^2 p_i p_j x_i x_j ca_{1,10}^2 - \\ & \epsilon^2 p_j^2 x_i x_j ca_{1,10}^2 - \frac{1}{2} \epsilon^2 p_i^2 p_j x_i^2 x_j ca_{1,10}^2 + \epsilon^2 p_i p_j^2 x_i^2 x_j ca_{1,10}^2 + \frac{1}{2} T \epsilon^2 p_i p_j^2 x_i^2 x_j ca_{1,10}^2 - \\ & \frac{1}{2} \epsilon^2 p_j^3 x_i^2 x_j ca_{1,10}^2 - \frac{1}{2} T \epsilon^2 p_j^3 x_i^2 x_j ca_{1,10}^2 - \frac{1}{2} \epsilon^2 p_i p_j^2 x_i x_j^2 ca_{1,10}^2 + \frac{1}{2} \epsilon^2 p_j^3 x_i x_j^2 ca_{1,10}^2 + \\ & \epsilon^2 ca_{2,1} - 2 \epsilon^2 p_i x_i ca_{2,1} + 2 \epsilon^2 p_j x_i ca_{2,1} - \frac{1}{2} \epsilon^2 p_i p_j x_i^2 cb_{2,10} + \frac{1}{2} T \epsilon^2 p_i p_j x_i^2 cb_{2,10} + \\ & \frac{1}{2} \epsilon^2 p_j^2 x_i^2 cb_{2,10} - \frac{1}{2} T \epsilon^2 p_j^2 x_i^2 cb_{2,10} - \epsilon^2 p_i p_j x_i x_j cb_{2,10} + \epsilon^2 p_j^2 x_i x_j cb_{2,10} \end{aligned}$$

(Alt) Out[ ]:=

$$\frac{1}{2} \epsilon ca_{1,2} + \epsilon p_k x_k ca_{1,10} - \epsilon^2 p_k x_k ca_{1,2} ca_{1,10} - \epsilon^2 ca_{2,1} - \epsilon^2 p_k x_k cb_{2,10}$$

## Sw<sup>+</sup>

(Alt) In[ ]:=

```
lhs = CF[Module[{es = {i, j, i+, j+}},
  Times[
    Normal@Series[Exp[rd[1, -1, -1, i, j] + γd[1, i+] + γd[1, j+]], {ε, 0, d}],
    Exp[Sum[gα,β πα ξβ, {α, es}, {β, es}]]
  ] // Zip(pα&/@es) ∪ (xα&/@es) // Expand
] // . gRules1,i,j
]
```

(Alt) Out[\*]=

$$\begin{aligned}
 & 1 - \frac{1}{2} \in ca_{1,2} + \frac{1}{8} \in^2 (ca_{1,2}^2 + 8 ca_{2,1}) + \in ca_{1,2} g_{i^+,i^+} + \in^2 (-ca_{1,2}^2 - 2 ca_{2,1}) g_{i^+,i^+} + \\
 & \in^2 ca_{1,2}^2 g_{i^+,i^+}^2 - \in ca_{1,2} g_{j^+,i^+} + \in^2 (ca_{1,2}^2 + 2 ca_{2,1}) g_{j^+,i^+} + \frac{(-1 + T) \in ca_{1,10} g_{i^+,i^+} g_{j^+,i^+}}{T} - \\
 & \frac{\in^2 (4 T ca_{1,2}^2 - 7 ca_{1,2} ca_{1,10} + 5 T ca_{1,2} ca_{1,10} + 2 ca_{1,10}^2 - 2 T ca_{1,10}^2 - 2 cb_{2,10} + 2 T cb_{2,10}) g_{i^+,i^+} g_{j^+,i^+}}{2 T} + \\
 & \frac{(-1 + T) \in^2 (3 ca_{1,2} - ca_{1,10}) ca_{1,10} g_{i^+,i^+}^2 g_{j^+,i^+}}{T} + \in ca_{1,10} g_{i^+,j^+} g_{j^+,i^+} + \\
 & \frac{1}{2} \in^2 (-5 ca_{1,2} ca_{1,10} + 2 ca_{1,10}^2 - 2 cb_{2,10}) g_{i^+,j^+} g_{j^+,i^+} + \\
 & 2 \in^2 (2 ca_{1,2} - ca_{1,10}) ca_{1,10} g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} - \frac{(-1 + T) \in ca_{1,10} g_{j^+,i^+}^2}{T} + \\
 & \frac{\in^2 (2 T ca_{1,2}^2 - 7 ca_{1,2} ca_{1,10} + 5 T ca_{1,2} ca_{1,10} + 2 ca_{1,10}^2 - 2 T ca_{1,10}^2 - 2 cb_{2,10} + 2 T cb_{2,10}) g_{j^+,i^+}^2}{2 T} - \\
 & \frac{(-1 + T) \in^2 ca_{1,10} (6 T ca_{1,2} - 2 ca_{1,10} - T ca_{1,10}) g_{i^+,i^+} g_{j^+,i^+}^2}{T^2} + \\
 & \frac{3 (-1 + T)^2 \in^2 ca_{1,10}^2 g_{i^+,i^+}^2 g_{j^+,i^+}^2}{T^2} - \frac{\in^2 ca_{1,10} (4 T ca_{1,2} - 2 ca_{1,10} - T ca_{1,10}) g_{i^+,j^+} g_{j^+,i^+}^2}{T} + \\
 & \frac{6 (-1 + T) \in^2 ca_{1,10}^2 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^2}{T} + 2 \in^2 ca_{1,10}^2 g_{i^+,j^+}^2 g_{j^+,i^+}^2 + \\
 & \frac{(-1 + T) \in^2 (3 T ca_{1,2} - 2 ca_{1,10}) ca_{1,10} g_{j^+,i^+}^3}{T^2} - \frac{6 (-1 + T)^2 \in^2 ca_{1,10}^2 g_{i^+,i^+} g_{j^+,i^+}^3}{T^2} - \\
 & \frac{6 (-1 + T) \in^2 ca_{1,10}^2 g_{i^+,j^+} g_{j^+,i^+}^3}{T} + \frac{3 (-1 + T)^2 \in^2 ca_{1,10}^2 g_{j^+,i^+}^4}{T^2} + \in ca_{1,10} g_{i^+,i^+} g_{j^+,j^+} + \\
 & \frac{1}{2} \in^2 (-5 ca_{1,2} ca_{1,10} + 2 ca_{1,10}^2 - 2 cb_{2,10}) g_{i^+,i^+} g_{j^+,j^+} + \in^2 (2 ca_{1,2} - ca_{1,10}) ca_{1,10} g_{i^+,i^+}^2 g_{j^+,j^+} - \\
 & 2 \in ca_{1,10} g_{j^+,i^+} g_{j^+,j^+} + \in^2 (5 ca_{1,2} ca_{1,10} - 2 ca_{1,10}^2 + 2 cb_{2,10}) g_{j^+,i^+} g_{j^+,j^+} - \\
 & \frac{2 \in^2 ca_{1,10} (4 T ca_{1,2} - 2 ca_{1,10} - T ca_{1,10}) g_{i^+,i^+} g_{j^+,i^+} g_{j^+,j^+}}{T} + \frac{6 (-1 + T) \in^2 ca_{1,10}^2 g_{i^+,i^+}^2 g_{j^+,i^+} g_{j^+,j^+}}{T} - \\
 & 2 \in^2 ca_{1,10}^2 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} + 8 \in^2 ca_{1,10}^2 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} + \\
 & \frac{6 \in^2 (T ca_{1,2} - ca_{1,10}) ca_{1,10} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} - \frac{18 (-1 + T) \in^2 ca_{1,10}^2 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} - \\
 & 12 \in^2 ca_{1,10}^2 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+} + \frac{12 (-1 + T) \in^2 ca_{1,10}^2 g_{j^+,i^+}^3 g_{j^+,j^+}}{T} - \in^2 ca_{1,10}^2 g_{i^+,i^+} g_{j^+,j^+}^2 + \\
 & 2 \in^2 ca_{1,10}^2 g_{i^+,i^+}^2 g_{j^+,j^+}^2 + 3 \in^2 ca_{1,10}^2 g_{j^+,i^+} g_{j^+,j^+}^2 - 12 \in^2 ca_{1,10}^2 g_{i^+,i^+} g_{j^+,i^+} g_{j^+,j^+}^2 + 12 \in^2 ca_{1,10}^2 g_{j^+,i^+}^2 g_{j^+,j^+}^2
 \end{aligned}$$

*(Alt) In[ ]:=*

```

rhs = CF[Module[{es = {i, j, i+, j+}},
  Times[
    Normal@Series[Exp[rd[1, i, j]], {e, 0, d}],
    Exp[Sum[gα,β πα ξβ, {α, es}, {β, es}]]
  ] // Zip(pα&/@es)∪(xα&/@es) // Expand
] // . gRules1,i,j
]

```

(Alt) Out[\*]=

$$\begin{aligned}
 & 1 - \frac{1}{2} \in ca_{1,2} + \frac{1}{8} \in^2 (ca_{1,2}^2 + 8 ca_{2,1}) + \in ca_{1,2} g_{i^+,i^+} + \in^2 (-ca_{1,2}^2 - 2 ca_{2,1}) g_{i^+,i^+} + \\
 & \in^2 ca_{1,2}^2 g_{i^+,i^+}^2 - \in ca_{1,2} g_{j^+,i^+} + \in^2 (ca_{1,2}^2 + 2 ca_{2,1}) g_{j^+,i^+} + \frac{(-1 + T) \in ca_{1,10} g_{i^+,i^+} g_{j^+,i^+}}{T} - \\
 & \frac{\in^2 (4 T ca_{1,2}^2 - 7 ca_{1,2} ca_{1,10} + 5 T ca_{1,2} ca_{1,10} + 2 ca_{1,10}^2 - 2 T ca_{1,10}^2 - 2 cb_{2,10} + 2 T cb_{2,10}) g_{i^+,i^+} g_{j^+,i^+}}{2 T} + \\
 & \frac{(-1 + T) \in^2 (3 ca_{1,2} - ca_{1,10}) ca_{1,10} g_{i^+,i^+}^2 g_{j^+,i^+}}{T} + \in ca_{1,10} g_{i^+,j^+} g_{j^+,i^+} + \\
 & \frac{1}{2} \in^2 (-5 ca_{1,2} ca_{1,10} + 2 ca_{1,10}^2 - 2 cb_{2,10}) g_{i^+,j^+} g_{j^+,i^+} + \\
 & 2 \in^2 (2 ca_{1,2} - ca_{1,10}) ca_{1,10} g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} - \frac{(-1 + T) \in ca_{1,10} g_{j^+,i^+}^2}{T} + \\
 & \frac{\in^2 (2 T ca_{1,2}^2 - 7 ca_{1,2} ca_{1,10} + 5 T ca_{1,2} ca_{1,10} + 2 ca_{1,10}^2 - 2 T ca_{1,10}^2 - 2 cb_{2,10} + 2 T cb_{2,10}) g_{j^+,i^+}^2}{2 T} - \\
 & \frac{(-1 + T) \in^2 ca_{1,10} (6 T ca_{1,2} - 2 ca_{1,10} - T ca_{1,10}) g_{i^+,i^+} g_{j^+,i^+}^2}{T^2} + \\
 & \frac{3 (-1 + T)^2 \in^2 ca_{1,10}^2 g_{i^+,i^+}^2 g_{j^+,i^+}^2}{T^2} - \frac{\in^2 ca_{1,10} (4 T ca_{1,2} - 2 ca_{1,10} - T ca_{1,10}) g_{i^+,j^+} g_{j^+,i^+}^2}{T} + \\
 & \frac{6 (-1 + T) \in^2 ca_{1,10}^2 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^2}{T} + 2 \in^2 ca_{1,10}^2 g_{i^+,j^+}^2 g_{j^+,i^+}^2 + \\
 & \frac{(-1 + T) \in^2 (3 T ca_{1,2} - 2 ca_{1,10}) ca_{1,10} g_{j^+,i^+}^3}{T^2} - \frac{6 (-1 + T)^2 \in^2 ca_{1,10}^2 g_{i^+,i^+} g_{j^+,i^+}^3}{T^2} - \\
 & \frac{6 (-1 + T) \in^2 ca_{1,10}^2 g_{i^+,j^+} g_{j^+,i^+}^3}{T} + \frac{3 (-1 + T)^2 \in^2 ca_{1,10}^2 g_{j^+,i^+}^4}{T^2} + \in ca_{1,10} g_{i^+,i^+} g_{j^+,j^+} + \\
 & \frac{1}{2} \in^2 (-5 ca_{1,2} ca_{1,10} + 2 ca_{1,10}^2 - 2 cb_{2,10}) g_{i^+,i^+} g_{j^+,j^+} + \in^2 (2 ca_{1,2} - ca_{1,10}) ca_{1,10} g_{i^+,i^+}^2 g_{j^+,j^+} - \\
 & 2 \in ca_{1,10} g_{j^+,i^+} g_{j^+,j^+} + \in^2 (5 ca_{1,2} ca_{1,10} - 2 ca_{1,10}^2 + 2 cb_{2,10}) g_{j^+,i^+} g_{j^+,j^+} - \\
 & \frac{2 \in^2 ca_{1,10} (4 T ca_{1,2} - 2 ca_{1,10} - T ca_{1,10}) g_{i^+,i^+} g_{j^+,i^+} g_{j^+,j^+}}{T} + \frac{6 (-1 + T) \in^2 ca_{1,10}^2 g_{i^+,i^+}^2 g_{j^+,i^+} g_{j^+,j^+}}{T} - \\
 & 2 \in^2 ca_{1,10}^2 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} + 8 \in^2 ca_{1,10}^2 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} + \\
 & \frac{6 \in^2 (T ca_{1,2} - ca_{1,10}) ca_{1,10} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} - \frac{18 (-1 + T) \in^2 ca_{1,10}^2 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} - \\
 & 12 \in^2 ca_{1,10}^2 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+} + \frac{12 (-1 + T) \in^2 ca_{1,10}^2 g_{j^+,i^+}^3 g_{j^+,j^+}}{T} - \in^2 ca_{1,10}^2 g_{i^+,i^+} g_{j^+,j^+}^2 + \\
 & 2 \in^2 ca_{1,10}^2 g_{i^+,i^+}^2 g_{j^+,j^+}^2 + 3 \in^2 ca_{1,10}^2 g_{j^+,i^+} g_{j^+,j^+}^2 - 12 \in^2 ca_{1,10}^2 g_{i^+,i^+} g_{j^+,i^+} g_{j^+,j^+}^2 + 12 \in^2 ca_{1,10}^2 g_{j^+,i^+}^2 g_{j^+,j^+}^2
 \end{aligned}$$

```
(Alt) In[ ]:=
```

```
me = Exponent [lhs - rhs, T, Min]
```

```
(Alt) Out[ ]:=
```

```
 $\infty$ 
```

```
(Alt) In[ ]:=
```

```
covars = DeleteCases [Variables [lhs - rhs], T | (ca | cb | cc | cd) __]
```

```
(Alt) Out[ ]:=
```

```
{ }
```

```
(Alt) In[ ]:=
```

```
eqnsSwp = { }
```

```
(Alt) Out[ ]:=
```

```
{ }
```

## Solution

```
(Alt) In[ ]:=
```

```
vars =
```

```
Cases [Variables [rd[1, i1, j1] + rd[-1, i2, j2] +  $\gamma_d$ [1, k1] +  $\gamma_d$ [-1, k2]], (ca | cb | cc | cd) __]
```

```
(Alt) Out[ ]:=
```

```
{ ca1,2, ca1,10, ca2,1, cb2,10 }
```

```
(Alt) In[ ]:=
```

```
{sol} = Solve [eqnsR3  $\cup$  eqnsR2b  $\cup$  eqnsR2c  $\cup$  eqnsR11  $\cup$  eqnsR1r  $\cup$  eqnsSwp, vars]
```

**Solve:** The solution set contains a full-dimensional component; use Reduce for complete solution information.

```
(Alt) Out[ ]:=
```

```
{ { } }
```

```
(Alt) In[ ]:=
```

```
sol /. Rule  $\rightarrow$  Set
```

```
(Alt) Out[ ]:=
```

```
{ }
```

(Alt) In[ ]:=

**Column[Collect[#, ε, CF] & /@ {rd[1, i, j], rd[-1, i, j], γd[1, k], γd[-1, k]}]**

(Alt) Out[ ]:=

$$\begin{aligned}
& \in \left( -\frac{ca_{1,2}}{2} + p_i x_i ca_{1,2} - p_j x_j ca_{1,2} - \frac{1}{2} (-1+T) p_i p_j x_i^2 ca_{1,10} + \right. \\
& \quad \left. \frac{1}{2} (-1+T) p_j^2 x_i^2 ca_{1,10} + p_i p_j x_i x_j ca_{1,10} - p_j^2 x_i x_j ca_{1,10} \right) + \\
& \in^2 \left( \frac{1}{3} (-1+T) p_i^2 p_j x_i^3 ca_{1,10}^2 - \frac{1}{6} (-1+T) (5+T) p_i p_j^2 x_i^3 ca_{1,10}^2 + \frac{1}{6} (-1+T) (3+T) p_j^3 x_i^3 ca_{1,10}^2 - \right. \\
& \quad \frac{1}{2} p_i^2 p_j x_i^2 x_j ca_{1,10}^2 + \frac{1}{2} (2+T) p_i p_j^2 x_i^2 x_j ca_{1,10}^2 - \frac{1}{2} (1+T) p_j^3 x_i^2 x_j ca_{1,10}^2 - \frac{1}{2} p_i p_j^2 x_i x_j^2 ca_{1,10}^2 + \\
& \quad \frac{1}{2} p_j^3 x_i x_j^2 ca_{1,10}^2 + \frac{1}{2} p_i x_i (-ca_{1,2}^2 - 4 ca_{2,1}) + ca_{2,1} + \frac{1}{2} p_j x_j (ca_{1,2}^2 + 4 ca_{2,1}) + \\
& \quad p_i p_j x_i x_j (-2 ca_{1,2} ca_{1,10} + ca_{1,10}^2 - cb_{2,10}) + p_j^2 x_i x_j (2 ca_{1,2} ca_{1,10} - ca_{1,10}^2 + cb_{2,10}) + \\
& \quad \left. \frac{1}{2} p_j^2 x_i^2 (ca_{1,2} ca_{1,10} - 2 T ca_{1,2} ca_{1,10} - ca_{1,10}^2 + T ca_{1,10}^2 + cb_{2,10} - T cb_{2,10}) + \right. \\
& \quad \left. \frac{1}{2} p_i p_j x_i^2 (-ca_{1,2} ca_{1,10} + 2 T ca_{1,2} ca_{1,10} + ca_{1,10}^2 - T ca_{1,10}^2 - cb_{2,10} + T cb_{2,10}) \right) \\
& \in \left( \frac{ca_{1,2}}{2} - p_i x_i ca_{1,2} + p_j x_j ca_{1,2} - \frac{(-1+T) p_i p_j x_i^2 ca_{1,10}}{2 T} + \frac{(-1+T) p_j^2 x_i^2 ca_{1,10}}{2 T} - p_i p_j x_i x_j ca_{1,10} + p_j^2 x_i x_j ca_{1,10} \right) + \\
& \in^2 \left( -\frac{(-1+T) p_i^2 p_j x_i^3 ca_{1,10}^2}{3 T} + \frac{(-1+T) (1+5 T) p_i p_j^2 x_i^3 ca_{1,10}^2}{6 T^2} - \frac{(-1+T) (1+3 T) p_j^3 x_i^3 ca_{1,10}^2}{6 T^2} - \frac{1}{2} p_i^2 p_j x_i^2 x_j ca_{1,10}^2 + \right. \\
& \quad \frac{(1+2 T) p_i p_j^2 x_i^2 x_j ca_{1,10}^2}{2 T} - \frac{(1+T) p_j^3 x_i^2 x_j ca_{1,10}^2}{2 T} - \frac{1}{2} p_i p_j^2 x_i x_j^2 ca_{1,10}^2 + \frac{1}{2} p_j^3 x_i x_j^2 ca_{1,10}^2 + \\
& \quad \frac{1}{2} p_j x_j (ca_{1,2}^2 - 4 ca_{2,1}) - ca_{2,1} + \frac{1}{2} p_i x_i (-ca_{1,2}^2 + 4 ca_{2,1}) + p_i p_j x_i x_j cb_{2,10} - \\
& \quad \left. p_j^2 x_i x_j cb_{2,10} + \frac{p_i p_j x_i^2 (T ca_{1,2} ca_{1,10} - cb_{2,10} + T cb_{2,10})}{2 T} - \frac{p_j^2 x_i^2 (T ca_{1,2} ca_{1,10} - cb_{2,10} + T cb_{2,10})}{2 T} \right) \\
& \in \left( \frac{ca_{1,2}}{2} + p_k x_k ca_{1,10} \right) + \in^2 \left( -ca_{2,1} + p_k x_k (-ca_{1,2} ca_{1,10} - cb_{2,10}) \right) \\
& \in \left( -\frac{ca_{1,2}}{2} - p_k x_k ca_{1,10} \right) + \in^2 \left( ca_{2,1} + p_k x_k (ca_{1,2} ca_{1,10} - ca_{1,10}^2 + cb_{2,10}) \right)
\end{aligned}$$

## Non-Universally Solving at d=3

(Alt) In[ ]:=

**d = 3;****vars =**

**Cases[Variables[rd[1, i1, j1] + rd[-1, i2, j2] + γd[1, k1] + γd[-1, k2]], (ca | cb | cc | cd) \_\_]**

(Alt) Out[ ]:=

{ca<sub>1,2</sub>, ca<sub>1,10</sub>, ca<sub>2,1</sub>, ca<sub>3,1</sub>, ca<sub>3,2</sub>, ca<sub>3,3</sub>, ca<sub>3,4</sub>, ca<sub>3,5</sub>, ca<sub>3,6</sub>, ca<sub>3,7</sub>, ca<sub>3,8</sub>, ca<sub>3,9</sub>, ca<sub>3,10</sub>, ca<sub>3,11</sub>,  
ca<sub>3,12</sub>, ca<sub>3,13</sub>, ca<sub>3,14</sub>, ca<sub>3,15</sub>, ca<sub>3,16</sub>, ca<sub>3,17</sub>, ca<sub>3,18</sub>, ca<sub>3,19</sub>, ca<sub>3,20</sub>, ca<sub>3,21</sub>, ca<sub>3,22</sub>, ca<sub>3,23</sub>,  
ca<sub>3,24</sub>, ca<sub>3,25</sub>, ca<sub>3,26</sub>, ca<sub>3,27</sub>, ca<sub>3,28</sub>, ca<sub>3,29</sub>, ca<sub>3,30</sub>, ca<sub>3,31</sub>, ca<sub>3,32</sub>, ca<sub>3,33</sub>, ca<sub>3,34</sub>, ca<sub>3,35</sub>,  
ca<sub>3,36</sub>, ca<sub>3,37</sub>, ca<sub>3,38</sub>, ca<sub>3,39</sub>, ca<sub>3,40</sub>, ca<sub>3,41</sub>, ca<sub>3,42</sub>, ca<sub>3,43</sub>, ca<sub>3,44</sub>, ca<sub>3,45</sub>, ca<sub>3,46</sub>, ca<sub>3,47</sub>,  
ca<sub>3,48</sub>, ca<sub>3,49</sub>, ca<sub>3,50</sub>, ca<sub>3,51</sub>, ca<sub>3,52</sub>, ca<sub>3,53</sub>, ca<sub>3,54</sub>, ca<sub>3,55</sub>, cb<sub>2,10</sub>, cb<sub>3,1</sub>, cb<sub>3,2</sub>, cb<sub>3,3</sub>,  
cb<sub>3,4</sub>, cb<sub>3,5</sub>, cb<sub>3,6</sub>, cb<sub>3,7</sub>, cb<sub>3,8</sub>, cb<sub>3,9</sub>, cb<sub>3,10</sub>, cb<sub>3,11</sub>, cb<sub>3,12</sub>, cb<sub>3,13</sub>, cb<sub>3,14</sub>, cb<sub>3,15</sub>, cb<sub>3,16</sub>,  
cb<sub>3,17</sub>, cb<sub>3,18</sub>, cb<sub>3,19</sub>, cb<sub>3,20</sub>, cb<sub>3,21</sub>, cb<sub>3,22</sub>, cb<sub>3,23</sub>, cb<sub>3,24</sub>, cb<sub>3,25</sub>, cb<sub>3,26</sub>, cb<sub>3,27</sub>, cb<sub>3,28</sub>,  
cb<sub>3,29</sub>, cb<sub>3,30</sub>, cb<sub>3,31</sub>, cb<sub>3,32</sub>, cb<sub>3,33</sub>, cb<sub>3,34</sub>, cb<sub>3,35</sub>, cb<sub>3,36</sub>, cb<sub>3,37</sub>, cb<sub>3,38</sub>, cb<sub>3,39</sub>, cb<sub>3,40</sub>,  
cb<sub>3,41</sub>, cb<sub>3,42</sub>, cb<sub>3,43</sub>, cb<sub>3,44</sub>, cb<sub>3,45</sub>, cb<sub>3,46</sub>, cb<sub>3,47</sub>, cb<sub>3,48</sub>, cb<sub>3,49</sub>, cb<sub>3,50</sub>, cb<sub>3,51</sub>, cb<sub>3,52</sub>,  
cb<sub>3,53</sub>, cb<sub>3,54</sub>, cb<sub>3,55</sub>, cc<sub>3,1</sub>, cc<sub>3,2</sub>, cc<sub>3,3</sub>, cc<sub>3,4</sub>, cc<sub>3,5</sub>, cd<sub>3,1</sub>, cd<sub>3,2</sub>, cd<sub>3,3</sub>, cd<sub>3,4</sub>, cd<sub>3,5</sub>}

## c̄

(Alt) In[ ]:=

```
lhs = Module[{x1, p1},
  {x1*, p1*} = {p1, x1};
  Normal[
    Log[0[ε]d+1 + Zip{x1}[Exp[0[ε]d+1 + (γd[1, i] /. xi → xi + x1) + (γd[-1, i] /. pi → pi - p1)]]]]]
  ]
rhs = 0
```

(Alt) Out[ ]:=

$$\epsilon^3 \left( -2 p_i x_i c_{a_{1,2}} c_{a_{1,10}}^2 + p_i x_i c_{a_{1,10}}^3 - 2 p_i x_i c_{a_{1,10}} c_{b_{2,10}} + c_{c_{3,1}} + p_i x_i c_{c_{3,2}} + p_i^2 x_i^2 c_{c_{3,3}} + p_i^3 x_i^3 c_{c_{3,4}} + p_i^4 x_i^4 c_{c_{3,5}} + c_{d_{3,1}} + p_i x_i c_{d_{3,2}} + p_i^2 x_i^2 c_{d_{3,3}} + p_i^3 x_i^3 c_{d_{3,4}} + p_i^4 x_i^4 c_{d_{3,5}} \right)$$

(Alt) Out[ ]:=

$$0$$

(Alt) In[ ]:=

```
covars = DeleteCases[Variables[lhs - rhs], T | (ca | cb | cc | cd) _]
```

(Alt) Out[ ]:=

$$\{\epsilon, p_i, x_i\}$$

(Alt) In[ ]:=

```
eqnsCCbar = (# == 0) & /@ Union[Last /@ CoefficientRules[Expand[lhs - rhs], covars]]
```

(Alt) Out[ ]:=

$$\{c_{c_{3,1}} + c_{d_{3,1}} == 0, -2 c_{a_{1,2}} c_{a_{1,10}}^2 + c_{a_{1,10}}^3 - 2 c_{a_{1,10}} c_{b_{2,10}} + c_{c_{3,2}} + c_{d_{3,2}} == 0, c_{c_{3,3}} + c_{d_{3,3}} == 0, c_{c_{3,4}} + c_{d_{3,4}} == 0, c_{c_{3,5}} + c_{d_{3,5}} == 0\}$$

(Alt) In[ ]:=

```
vars =
Cases[Variables[r_d[1, i1, j1] + r_d[-1, i2, j2] + \gamma_d[1, k1] + \gamma_d[-1, k2]], (ca | cb | cc | cd) __]
{sol} = Solve[eqnsCCbar, vars]
sol /. Rule -> Set;
\gamma_d[1, k]
\gamma_d[-1, k]
```

(Alt) Out[ ]:=

```
{ca1,2, ca1,10, ca2,1, ca3,1, ca3,2, ca3,3, ca3,4, ca3,5, ca3,6, ca3,7, ca3,8, ca3,9, ca3,10, ca3,11,
ca3,12, ca3,13, ca3,14, ca3,15, ca3,16, ca3,17, ca3,18, ca3,19, ca3,20, ca3,21, ca3,22, ca3,23,
ca3,24, ca3,25, ca3,26, ca3,27, ca3,28, ca3,29, ca3,30, ca3,31, ca3,32, ca3,33, ca3,34, ca3,35,
ca3,36, ca3,37, ca3,38, ca3,39, ca3,40, ca3,41, ca3,42, ca3,43, ca3,44, ca3,45, ca3,46, ca3,47,
ca3,48, ca3,49, ca3,50, ca3,51, ca3,52, ca3,53, ca3,54, ca3,55, cb2,10, cb3,1, cb3,2, cb3,3,
cb3,4, cb3,5, cb3,6, cb3,7, cb3,8, cb3,9, cb3,10, cb3,11, cb3,12, cb3,13, cb3,14, cb3,15, cb3,16,
cb3,17, cb3,18, cb3,19, cb3,20, cb3,21, cb3,22, cb3,23, cb3,24, cb3,25, cb3,26, cb3,27, cb3,28,
cb3,29, cb3,30, cb3,31, cb3,32, cb3,33, cb3,34, cb3,35, cb3,36, cb3,37, cb3,38, cb3,39, cb3,40,
cb3,41, cb3,42, cb3,43, cb3,44, cb3,45, cb3,46, cb3,47, cb3,48, cb3,49, cb3,50, cb3,51, cb3,52,
cb3,53, cb3,54, cb3,55, cc3,1, cc3,2, cc3,3, cc3,4, cc3,5, cd3,1, cd3,2, cd3,3, cd3,4, cd3,5}
```

Solve: Equations may not give solutions for all "solve" variables.

Solve: Equations may not give solutions for all "solve" variables.

(Alt) Out[ ]:=

$$\left\{ \left\{ \begin{aligned} cd_{3,1} &\rightarrow -cc_{3,1}, & cd_{3,2} &\rightarrow 2ca_{1,2}ca_{1,10}^2 - ca_{1,10}^3 + 2ca_{1,10}cb_{2,10} - cc_{3,2}, \\ cd_{3,3} &\rightarrow -cc_{3,3}, & cd_{3,4} &\rightarrow -cc_{3,4}, & cd_{3,5} &\rightarrow -cc_{3,5} \end{aligned} \right\} \right\}$$

(Alt) Out[ ]:=

$$\frac{1}{2} \epsilon ca_{1,2} + \epsilon p_k x_k ca_{1,10} - \epsilon^2 p_k x_k ca_{1,2} ca_{1,10} - \epsilon^2 ca_{2,1} - \epsilon^2 p_k x_k cb_{2,10} + \epsilon^3 cc_{3,1} + \epsilon^3 p_k x_k cc_{3,2} + \epsilon^3 p_k^2 x_k^2 cc_{3,3} + \epsilon^3 p_k^3 x_k^3 cc_{3,4} + \epsilon^3 p_k^4 x_k^4 cc_{3,5}$$

(Alt) Out[ ]:=

$$-\frac{1}{2} \epsilon ca_{1,2} - \epsilon p_k x_k ca_{1,10} + \epsilon^2 p_k x_k ca_{1,2} ca_{1,10} - \epsilon^2 p_k x_k ca_{1,10}^2 + 2\epsilon^3 p_k x_k ca_{1,2} ca_{1,10}^2 - \epsilon^3 p_k x_k ca_{1,10}^3 + \epsilon^2 ca_{2,1} + \epsilon^2 p_k x_k cb_{2,10} + 2\epsilon^3 p_k x_k ca_{1,10} cb_{2,10} - \epsilon^3 cc_{3,1} - \epsilon^3 p_k x_k cc_{3,2} - \epsilon^3 p_k^2 x_k^2 cc_{3,3} - \epsilon^3 p_k^3 x_k^3 cc_{3,4} - \epsilon^3 p_k^4 x_k^4 cc_{3,5}$$

## R3 @ d = 3

(Alt) In[ ]:=

```
Short[lhs = CF[Module[{es = {i, j, k, i+, j+, k+}},
  Times[
    Normal@Series[Exp[r_d[1, j, k] + r_d[1, i, k+] + r_d[1, i+, j+]], {e, 0, d}],
    Exp[Sum[g_{\alpha, \beta} \pi_{\alpha} \xi_{\beta}, {\alpha, es}, {\beta, es}]]
  ] // Zip_{(p_{\alpha} &/@es) \cup (x_{\beta} &/@es)} // Expand
] // . gRules_{1, j, k} \cup gRules_{1, i, k+} \cup gRules_{1, i+, j+}, 5]
```

(Alt) Out[ ]:=

$$1 - \frac{3}{2} \in ca_{1,2} + \ll 3655 \gg + 48 \epsilon^3 \left( ca_{3,35} - 4 T ca_{3,35} + 6 T^2 ca_{3,35} - 4 T^3 ca_{3,35} + T^4 ca_{3,35} + ca_{3,40} - 3 T ca_{3,40} + 3 T^2 ca_{3,40} - T^3 ca_{3,40} + ca_{3,45} - 2 T ca_{3,45} + T^2 ca_{3,45} + ca_{3,50} - T ca_{3,50} + ca_{3,55} \right) g_{k^{++}, k^{++}}^4$$

(Alt) In[ ]:=

```
Short[rhs = CF[Module[{es = {i, j, k, i+, j+, k+}},
  Times[
    Normal@Series[Exp[r_d[1, i, j] + r_d[1, i+, k] + r_d[1, j+, k+]], {e, 0, d}],
    Exp[Sum[g_{\alpha, \beta} \pi_{\alpha} \xi_{\beta}, {\alpha, es}, {\beta, es}]]
  ] // Zip_{(p_{\alpha} &/@es) \cup (x_{\beta} &/@es)} // Expand
] // . gRules_{1, i, j} \cup gRules_{1, i+, k} \cup gRules_{1, j+, k+}, 5]
```

(Alt) Out[ ]:=

$$1 - \frac{3}{2} \in ca_{1,2} + \ll 3649 \gg + 48 \epsilon^3 \left( ca_{3,35} - 4 T ca_{3,35} + 6 T^2 ca_{3,35} - 4 T^3 ca_{3,35} + T^4 ca_{3,35} + ca_{3,40} - 3 T ca_{3,40} + 3 T^2 ca_{3,40} - T^3 ca_{3,40} + ca_{3,45} - 2 T ca_{3,45} + T^2 ca_{3,45} + ca_{3,50} - T ca_{3,50} + ca_{3,55} \right) g_{k^{++}, k^{++}}^4$$

(Alt) In[ ]:=

```
me = Exponent[lhs - rhs, T, Min]
```

(Alt) Out[ ]:=

```
-8
```

(Alt) In[ ]:=

```
covars = DeleteCases[Variables[lhs - rhs], T | (ca | cb | cc | cd) _]
```

(Alt) Out[ ]:=

```
{e, g_{i^{++}, i^{++}}, g_{i^{++}, j^{++}}, g_{i^{++}, k^{++}}, g_{j^{++}, i^{++}}, g_{j^{++}, j^{++}}, g_{j^{++}, k^{++}}, g_{k^{++}, i^{++}}, g_{k^{++}, j^{++}}, g_{k^{++}, k^{++}}}
```

(Alt) In[ ]:=

```
Short[eqnsR3 = (# == 0) & /@ Union[Last /@ CoefficientRules[Expand[T^{-me} (lhs - rhs)], covars]]]
```

(Alt) Out[ ]:=

$$\{-T^7 ca_{3,3} + T^8 ca_{3,3} == 0, T^8 ca_{\ll 1 \gg} - \ll 1 \gg \ll 1 \gg == 0, \ll 345 \gg, \ll 53 \gg + 8 T^8 ca_{1 \ll 1 \gg \ll 2 \gg} cb_{2,10} == 0\}$$

## R2b @ d = 3

```
(Alt) In[ ]:=
Short[lhs = CF[Module[{es = {i, j, i+, j+}},
  Times[
    Normal@Series[Exp[r_d[1, i, j] + r_d[-1, i+, j+]], {e, 0, d}],
    Exp[Sum[g_{\alpha, \beta} \pi_{\alpha} \xi_{\beta}, {\alpha, es}, {\beta, es}]]
  ] // Zip((p_#&/@es) \cup (x_#&/@es) // Expand
] // . gRules_{1, i, j} \cup gRules_{-1, i+, j+}]
```

(Alt) Out[ ]//Short=

$$1 + \ll 85 \gg + \frac{24 \ll 3 \gg \ll 1 \gg}{T^4} + \frac{24 e^3 (T^4 \ll 1 \gg + \ll 21 \gg) g_{\ll 1 \gg, \ll 1 \gg}^4}{T^4}$$

```
(Alt) In[ ]:=
rhs = 1
```

```
(Alt) Out[ ]:=
1
```

```
(Alt) In[ ]:=
me = Exponent[lhs - rhs, T, Min]
```

```
(Alt) Out[ ]:=
-4
```

```
(Alt) In[ ]:=
covars = DeleteCases[Variables[lhs - rhs], T | (ca | cb | cc | cd) _]
```

```
(Alt) Out[ ]:=
{e, g_{i+, i+}, g_{i+, j+}, g_{j+, i+}, g_{j+, j+}}
```

```
(Alt) In[ ]:=
Short[eqnsR2b =
  (Factor[#] == 0) & /@ Union[Last /@ CoefficientRules[Expand[T^{-me} (lhs - rhs)], covars]]]
```

(Alt) Out[ ]//Short=

$$\{T^4 (ca_{3,1} + cb_{3,1}) == 0, \ll 62 \gg, 7 T^4 ca_{1,10}^3 + \ll 189 \gg + 24 T^8 cb_{3,55} == 0\}$$

```
(Alt) In[ ]:=
vars = Cases[Variables[r_d[1, i1, j1] + r_d[-1, i2, j2] + \gamma_d[1, k]], (ca | cb | cc | cd) _]
{sol} = Solve[eqnsR3 \cup eqnsR2b \cup eqnsR2c \cup eqnsR1l \cup eqnsR1r \cup eqnsSwp, vars]
sol /. Rule -> Set;
r_d[1, i, j]
\gamma_d[1, k]
```

(Alt) Out[ ]=

{ ca<sub>1,2</sub>, ca<sub>1,10</sub>, ca<sub>2,1</sub>, ca<sub>3,1</sub>, ca<sub>3,2</sub>, ca<sub>3,3</sub>, ca<sub>3,4</sub>, ca<sub>3,5</sub>, ca<sub>3,6</sub>, ca<sub>3,7</sub>, ca<sub>3,8</sub>, ca<sub>3,9</sub>, ca<sub>3,10</sub>,  
ca<sub>3,11</sub>, ca<sub>3,12</sub>, ca<sub>3,13</sub>, ca<sub>3,14</sub>, ca<sub>3,15</sub>, ca<sub>3,16</sub>, ca<sub>3,17</sub>, ca<sub>3,18</sub>, ca<sub>3,19</sub>, ca<sub>3,20</sub>, ca<sub>3,21</sub>, ca<sub>3,22</sub>,  
ca<sub>3,23</sub>, ca<sub>3,24</sub>, ca<sub>3,25</sub>, ca<sub>3,26</sub>, ca<sub>3,27</sub>, ca<sub>3,28</sub>, ca<sub>3,29</sub>, ca<sub>3,30</sub>, ca<sub>3,31</sub>, ca<sub>3,32</sub>, ca<sub>3,33</sub>, ca<sub>3,34</sub>,  
ca<sub>3,35</sub>, ca<sub>3,36</sub>, ca<sub>3,37</sub>, ca<sub>3,38</sub>, ca<sub>3,39</sub>, ca<sub>3,40</sub>, ca<sub>3,41</sub>, ca<sub>3,42</sub>, ca<sub>3,43</sub>, ca<sub>3,44</sub>, ca<sub>3,45</sub>, ca<sub>3,46</sub>,  
ca<sub>3,47</sub>, ca<sub>3,48</sub>, ca<sub>3,49</sub>, ca<sub>3,50</sub>, ca<sub>3,51</sub>, ca<sub>3,52</sub>, ca<sub>3,53</sub>, ca<sub>3,54</sub>, ca<sub>3,55</sub>, cb<sub>2,10</sub>, cb<sub>3,1</sub>, cb<sub>3,2</sub>,  
cb<sub>3,3</sub>, cb<sub>3,4</sub>, cb<sub>3,5</sub>, cb<sub>3,6</sub>, cb<sub>3,7</sub>, cb<sub>3,8</sub>, cb<sub>3,9</sub>, cb<sub>3,10</sub>, cb<sub>3,11</sub>, cb<sub>3,12</sub>, cb<sub>3,13</sub>, cb<sub>3,14</sub>,  
cb<sub>3,15</sub>, cb<sub>3,16</sub>, cb<sub>3,17</sub>, cb<sub>3,18</sub>, cb<sub>3,19</sub>, cb<sub>3,20</sub>, cb<sub>3,21</sub>, cb<sub>3,22</sub>, cb<sub>3,23</sub>, cb<sub>3,24</sub>, cb<sub>3,25</sub>,  
cb<sub>3,26</sub>, cb<sub>3,27</sub>, cb<sub>3,28</sub>, cb<sub>3,29</sub>, cb<sub>3,30</sub>, cb<sub>3,31</sub>, cb<sub>3,32</sub>, cb<sub>3,33</sub>, cb<sub>3,34</sub>, cb<sub>3,35</sub>, cb<sub>3,36</sub>,  
cb<sub>3,37</sub>, cb<sub>3,38</sub>, cb<sub>3,39</sub>, cb<sub>3,40</sub>, cb<sub>3,41</sub>, cb<sub>3,42</sub>, cb<sub>3,43</sub>, cb<sub>3,44</sub>, cb<sub>3,45</sub>, cb<sub>3,46</sub>, cb<sub>3,47</sub>, cb<sub>3,48</sub>,  
cb<sub>3,49</sub>, cb<sub>3,50</sub>, cb<sub>3,51</sub>, cb<sub>3,52</sub>, cb<sub>3,53</sub>, cb<sub>3,54</sub>, cb<sub>3,55</sub>, cc<sub>3,1</sub>, cc<sub>3,2</sub>, cc<sub>3,3</sub>, cc<sub>3,4</sub>, cc<sub>3,5</sub> }

Solve: Equations may not give solutions for all "solve" variables.

(Alt) Out[ ]=

{ { ca<sub>3,2</sub> → 4 ca<sub>1,2</sub> ca<sub>2,1</sub> - cb<sub>3,2</sub>, ca<sub>3,3</sub> → 0,  
ca<sub>3,4</sub> → -4 ca<sub>1,2</sub> ca<sub>2,1</sub> + cb<sub>3,2</sub> + T cb<sub>3,5</sub>, ca<sub>3,5</sub> → -cb<sub>3,5</sub>, ca<sub>3,6</sub> → 0, ca<sub>3,7</sub> → 0,  
ca<sub>3,8</sub> → 0, ca<sub>3,9</sub> →  $\frac{1}{4}$  ( -ca<sub>1,2</sub><sup>2</sup> ca<sub>1,10</sub> - 3 T ca<sub>1,2</sub><sup>2</sup> ca<sub>1,10</sub> - 4 ca<sub>1,2</sub> ca<sub>1,10</sub><sup>2</sup> + 6 T ca<sub>1,2</sub> ca<sub>1,10</sub><sup>2</sup> +  
2 ca<sub>1,10</sub><sup>3</sup> - 2 T ca<sub>1,10</sub><sup>3</sup> + 4 ca<sub>1,10</sub> ca<sub>2,1</sub> - 12 T ca<sub>1,10</sub> ca<sub>2,1</sub> + 2 ca<sub>1,2</sub> cb<sub>2,10</sub> -  
6 T ca<sub>1,2</sub> cb<sub>2,10</sub> - 4 ca<sub>1,10</sub> cb<sub>2,10</sub> + 4 T ca<sub>1,10</sub> cb<sub>2,10</sub> - 4 T cb<sub>3,9</sub> - 4 cb<sub>3,10</sub> + 4 T cb<sub>3,10</sub> ),  
ca<sub>3,10</sub> → 2 ca<sub>1,2</sub><sup>2</sup> ca<sub>1,10</sub> - 3 ca<sub>1,2</sub> ca<sub>1,10</sub><sup>2</sup> + ca<sub>1,10</sub><sup>3</sup> + 4 ca<sub>1,10</sub> ca<sub>2,1</sub> + 2 ca<sub>1,2</sub> cb<sub>2,10</sub> - 2 ca<sub>1,10</sub> cb<sub>2,10</sub> - cb<sub>3,10</sub>,  
ca<sub>3,11</sub> → 0, ca<sub>3,12</sub> →  $\frac{1}{4}$  ( ca<sub>1,2</sub><sup>2</sup> ca<sub>1,10</sub> + 3 T ca<sub>1,2</sub><sup>2</sup> ca<sub>1,10</sub> + T<sup>2</sup> ca<sub>1,2</sub><sup>2</sup> ca<sub>1,10</sub> + 4 ca<sub>1,2</sub> ca<sub>1,10</sub><sup>2</sup> -  
6 T ca<sub>1,2</sub> ca<sub>1,10</sub><sup>2</sup> - 2 ca<sub>1,10</sub><sup>3</sup> + 2 T ca<sub>1,10</sub><sup>3</sup> - 4 ca<sub>1,10</sub> ca<sub>2,1</sub> + 12 T ca<sub>1,10</sub> ca<sub>2,1</sub> -  
4 T<sup>2</sup> ca<sub>1,10</sub> ca<sub>2,1</sub> - 2 ca<sub>1,2</sub> cb<sub>2,10</sub> + 6 T ca<sub>1,2</sub> cb<sub>2,10</sub> - 2 T<sup>2</sup> ca<sub>1,2</sub> cb<sub>2,10</sub> + 4 ca<sub>1,10</sub> cb<sub>2,10</sub> -  
4 T ca<sub>1,10</sub> cb<sub>2,10</sub> + 4 T cb<sub>3,9</sub> - 4 T<sup>2</sup> cb<sub>3,9</sub> + 4 cb<sub>3,10</sub> - 6 T cb<sub>3,10</sub> + 2 T<sup>2</sup> cb<sub>3,10</sub> ),  
ca<sub>3,13</sub> →  $\frac{1}{4}$  ( -8 ca<sub>1,2</sub><sup>2</sup> ca<sub>1,10</sub> - T ca<sub>1,2</sub><sup>2</sup> ca<sub>1,10</sub> + 12 ca<sub>1,2</sub> ca<sub>1,10</sub><sup>2</sup> - 4 ca<sub>1,10</sub><sup>3</sup> - 16 ca<sub>1,10</sub> ca<sub>2,1</sub> +  
4 T ca<sub>1,10</sub> ca<sub>2,1</sub> - 8 ca<sub>1,2</sub> cb<sub>2,10</sub> + 2 T ca<sub>1,2</sub> cb<sub>2,10</sub> + 8 ca<sub>1,10</sub> cb<sub>2,10</sub> + 4 T cb<sub>3,9</sub> + 6 cb<sub>3,10</sub> - 2 T cb<sub>3,10</sub> ),  
ca<sub>3,14</sub> → 0, ca<sub>3,15</sub> → 0, ca<sub>3,16</sub> → 0, ca<sub>3,17</sub> → 0, ca<sub>3,18</sub> → 0, ca<sub>3,19</sub> →  
 $\frac{1}{3}$  ( -ca<sub>1,2</sub> ca<sub>1,10</sub><sup>2</sup> - 4 T ca<sub>1,2</sub> ca<sub>1,10</sub><sup>2</sup> + ca<sub>1,10</sub><sup>3</sup> + 2 T ca<sub>1,10</sub><sup>3</sup> - 3 ca<sub>3,29</sub> - ca<sub>1,10</sub> cb<sub>2,10</sub> - 2 T ca<sub>1,10</sub> cb<sub>2,10</sub> ),  
ca<sub>3,20</sub> → -ca<sub>1,10</sub> ( -2 ca<sub>1,2</sub> ca<sub>1,10</sub> + ca<sub>1,10</sub><sup>2</sup> - cb<sub>2,10</sub> ), ca<sub>3,21</sub> → 0, ca<sub>3,22</sub> → 0,  
ca<sub>3,23</sub> →  $\frac{1}{6}$  ( -10 ca<sub>1,2</sub> ca<sub>1,10</sub><sup>2</sup> + 10 T ca<sub>1,2</sub> ca<sub>1,10</sub><sup>2</sup> + 3 T<sup>2</sup> ca<sub>1,2</sub> ca<sub>1,10</sub><sup>2</sup> + 6 ca<sub>1,10</sub><sup>3</sup> - 4 T ca<sub>1,10</sub><sup>3</sup> -  
2 T<sup>2</sup> ca<sub>1,10</sub><sup>3</sup> + 6 ca<sub>3,29</sub> - 6 T ca<sub>3,29</sub> - 4 ca<sub>1,10</sub> cb<sub>2,10</sub> + 2 T ca<sub>1,10</sub> cb<sub>2,10</sub> + 2 T<sup>2</sup> ca<sub>1,10</sub> cb<sub>2,10</sub> ),  
ca<sub>3,24</sub> →  $\frac{1}{2}$  ca<sub>1,10</sub> ( -10 ca<sub>1,2</sub> ca<sub>1,10</sub> - 3 T ca<sub>1,2</sub> ca<sub>1,10</sub> + 5 ca<sub>1,10</sub><sup>2</sup> + 2 T ca<sub>1,10</sub><sup>2</sup> - 4 cb<sub>2,10</sub> - 2 T cb<sub>2,10</sub> ),  
ca<sub>3,25</sub> → - $\frac{1}{2}$  ca<sub>1,10</sub> ( -3 ca<sub>1,2</sub> ca<sub>1,10</sub> + 2 ca<sub>1,10</sub><sup>2</sup> - 2 cb<sub>2,10</sub> ), ca<sub>3,26</sub> → 0,  
ca<sub>3,27</sub> →  $\frac{1}{6}$  ca<sub>1,10</sub> ( 12 ca<sub>1,2</sub> ca<sub>1,10</sub> - 11 T ca<sub>1,2</sub> ca<sub>1,10</sub> - 3 T<sup>2</sup> ca<sub>1,2</sub> ca<sub>1,10</sub> -  
8 ca<sub>1,10</sub><sup>2</sup> + 6 T ca<sub>1,10</sub><sup>2</sup> + 2 T<sup>2</sup> ca<sub>1,10</sub><sup>2</sup> + 6 cb<sub>2,10</sub> - 4 T cb<sub>2,10</sub> - 2 T<sup>2</sup> cb<sub>2,10</sub> ),  
ca<sub>3,28</sub> →  $\frac{1}{2}$  ( 9 ca<sub>1,2</sub> ca<sub>1,10</sub><sup>2</sup> - 5 ca<sub>1,10</sub><sup>3</sup> + 2 ca<sub>3,29</sub> - 2 T ca<sub>3,29</sub> + 4 ca<sub>1,10</sub> cb<sub>2,10</sub> ),

$$\begin{aligned}
 &ca_{3,30} \rightarrow 0, ca_{3,31} \rightarrow 0, ca_{3,32} \rightarrow 0, ca_{3,33} \rightarrow 0, ca_{3,34} \rightarrow 0, \\
 &ca_{3,35} \rightarrow 0, ca_{3,36} \rightarrow \frac{1}{24} \left( -ca_{1,10}^3 - 3Tca_{1,10}^3 - 24ca_{3,54} \right), \\
 &ca_{3,37} \rightarrow \frac{ca_{1,10}^3}{6}, ca_{3,38} \rightarrow 0, ca_{3,39} \rightarrow 0, ca_{3,40} \rightarrow 0, \\
 &ca_{3,41} \rightarrow \frac{1}{8} \left( -1 + T \right) \left( 2ca_{1,10}^3 + 3Tca_{1,10}^3 - 12ca_{3,54} \right), \\
 &ca_{3,42} \rightarrow -\frac{7}{6}Tca_{1,10}^3, ca_{3,43} \rightarrow ca_{1,10}^3, ca_{3,44} \rightarrow 0, ca_{3,45} \rightarrow 0, \\
 &ca_{3,46} \rightarrow -\frac{1}{24} \left( -1 + T \right) \left( 9ca_{1,10}^3 + 26Tca_{1,10}^3 + T^2ca_{1,10}^3 - 24ca_{3,54} + 24Tca_{3,54} \right), \\
 &ca_{3,47} \rightarrow \frac{1}{6} \left( -6 + 17T + T^2 \right) ca_{1,10}^3, \\
 &ca_{3,48} \rightarrow -\frac{1}{4} \left( 10 + T \right) ca_{1,10}^3, ca_{3,49} \rightarrow \frac{ca_{1,10}^3}{6}, ca_{3,50} \rightarrow 0, \\
 &ca_{3,51} \rightarrow -\frac{1}{24} \left( -1 + T \right) \left( -2ca_{1,10}^3 - 17Tca_{1,10}^3 + T^2ca_{1,10}^3 + 12ca_{3,54} - 24Tca_{3,54} + 12T^2ca_{3,54} \right), \\
 &ca_{3,52} \rightarrow ca_{1,10}^3 - 2Tca_{1,10}^3 + ca_{3,54} - 2Tca_{3,54} + T^2ca_{3,54}, \\
 &ca_{3,53} \rightarrow \frac{1}{4} \left( 7ca_{1,10}^3 + 6ca_{3,54} - 6Tca_{3,54} \right), ca_{3,55} \rightarrow 0, cb_{3,1} \rightarrow -ca_{3,1}, \\
 &cb_{3,3} \rightarrow 0, cb_{3,4} \rightarrow \frac{-Tcb_{3,2} - cb_{3,5}}{T}, cb_{3,6} \rightarrow 0, cb_{3,7} \rightarrow 0, cb_{3,8} \rightarrow 0, cb_{3,11} \rightarrow 0, \\
 &cb_{3,12} \rightarrow \frac{-Tca_{1,2}^2ca_{1,10} + 4Tca_{1,10}ca_{2,1} + 2Tca_{1,2}cb_{2,10} + 4Tcb_{3,9} - 4T^2cb_{3,9} + 2cb_{3,10} - 2Tcb_{3,10}}{4T^2}, \\
 &cb_{3,13} \rightarrow \frac{Tca_{1,2}^2ca_{1,10} - 4Tca_{1,10}ca_{2,1} - 2Tca_{1,2}cb_{2,10} - 4Tcb_{3,9} - 2cb_{3,10} - 2Tcb_{3,10}}{4T}, \\
 &cb_{3,14} \rightarrow 0, cb_{3,15} \rightarrow 0, cb_{3,16} \rightarrow 0, cb_{3,17} \rightarrow 0, cb_{3,18} \rightarrow 0, \\
 &cb_{3,19} \rightarrow \frac{9ca_{1,2}ca_{1,10}^2 + Tca_{1,2}ca_{1,10}^2 - 6ca_{1,10}^3 + 6ca_{3,29} + 2ca_{1,10}cb_{2,10} + 4Tca_{1,10}cb_{2,10}}{6T}, \\
 &cb_{3,20} \rightarrow ca_{1,10}cb_{2,10}, cb_{3,21} \rightarrow 0, cb_{3,22} \rightarrow 0, \\
 &cb_{3,23} \rightarrow \frac{1}{6T^2} \left( 10ca_{1,2}ca_{1,10}^2 - 12Tca_{1,2}ca_{1,10}^2 - T^2ca_{1,2}ca_{1,10}^2 - 6ca_{1,10}^3 + 8Tca_{1,10}^3 - \right. \\
 &\quad \left. 2T^2ca_{1,10}^3 + 6ca_{3,29} - 6Tca_{3,29} + 8ca_{1,10}cb_{2,10} + 2Tca_{1,10}cb_{2,10} - 10T^2ca_{1,10}cb_{2,10} \right), \\
 &cb_{3,24} \rightarrow -\frac{ca_{1,10} \left( ca_{1,2}ca_{1,10} - 2Tca_{1,2}ca_{1,10} + Tca_{1,10}^2 + 2cb_{2,10} + 4Tcb_{2,10} \right)}{2T}, \\
 &cb_{3,25} \rightarrow \frac{1}{2}ca_{1,10} \left( ca_{1,2}ca_{1,10} + 2cb_{2,10} \right), cb_{3,26} \rightarrow 0, cb_{3,27} \rightarrow \\
 &\quad \frac{ca_{1,10} \left( -ca_{1,2}ca_{1,10} + 3Tca_{1,2}ca_{1,10} - 2Tca_{1,10}^2 + 2T^2ca_{1,10}^2 - 2cb_{2,10} - 4Tcb_{2,10} + 6T^2cb_{2,10} \right)}{6T^2}, \\
 &cb_{3,28} \rightarrow \frac{1}{2T^2} \left( 3ca_{1,2}ca_{1,10}^2 - 2Tca_{1,2}ca_{1,10}^2 - 2T^2ca_{1,2}ca_{1,10}^2 - 2ca_{1,10}^3 + \right.
 \end{aligned}$$

$$\begin{aligned}
& 2 T ca_{1,10}^3 + T^2 ca_{1,10}^3 + 2 ca_{3,29} - 2 T ca_{3,29} + 2 ca_{1,10} cb_{2,10} + 2 T^2 ca_{1,10} cb_{2,10} \Big), \\
cb_{3,29} & \rightarrow \frac{-3 ca_{1,2} ca_{1,10}^2 - T ca_{1,2} ca_{1,10}^2 + 2 ca_{1,10}^3 - 2 ca_{3,29} - 2 ca_{1,10} cb_{2,10} - 2 T ca_{1,10} cb_{2,10}}{2 T}, \\
cb_{3,30} & \rightarrow \theta, cb_{3,31} \rightarrow \theta, cb_{3,32} \rightarrow \theta, cb_{3,33} \rightarrow \theta, cb_{3,34} \rightarrow \theta, cb_{3,35} \rightarrow \theta, \\
cb_{3,36} & \rightarrow \frac{7 ca_{1,10}^3 - 3 T ca_{1,10}^3 + 24 ca_{3,54}}{24 T}, cb_{3,37} \rightarrow -\frac{1}{6} ca_{1,10}^3, cb_{3,38} \rightarrow \theta, \\
cb_{3,39} & \rightarrow \theta, cb_{3,40} \rightarrow \theta, cb_{3,41} \rightarrow \frac{(-1 + T) (ca_{1,10}^3 + 4 T ca_{1,10}^3 - 12 ca_{3,54})}{8 T^2}, \\
cb_{3,42} & \rightarrow \frac{7 ca_{1,10}^3}{6 T}, cb_{3,43} \rightarrow -ca_{1,10}^3, cb_{3,44} \rightarrow \theta, cb_{3,45} \rightarrow \theta, \\
cb_{3,46} & \rightarrow -\frac{(-1 + T) (5 ca_{1,10}^3 + 18 T ca_{1,10}^3 + 13 T^2 ca_{1,10}^3 + 24 ca_{3,54} - 24 T ca_{3,54})}{24 T^3}, \\
cb_{3,47} & \rightarrow \frac{(-1 - 17 T + 6 T^2) ca_{1,10}^3}{6 T^2}, cb_{3,48} \rightarrow \frac{(1 + 10 T) ca_{1,10}^3}{4 T}, cb_{3,49} \rightarrow -\frac{1}{6} ca_{1,10}^3, cb_{3,50} \rightarrow \theta, cb_{3,51} \rightarrow \\
& \frac{(-1 + T) (-2 ca_{1,10}^3 + 5 T ca_{1,10}^3 + 11 T^2 ca_{1,10}^3 + 4 T^3 ca_{1,10}^3 - 12 ca_{3,54} + 24 T ca_{3,54} - 12 T^2 ca_{3,54})}{24 T^4}, \\
cb_{3,52} & \rightarrow \frac{-ca_{1,10}^3 + 3 T ca_{1,10}^3 + 9 T^2 ca_{1,10}^3 - 5 T^3 ca_{1,10}^3 - 6 ca_{3,54} + 12 T ca_{3,54} - 6 T^2 ca_{3,54}}{6 T^3}, \\
cb_{3,53} & \rightarrow \frac{ca_{1,10}^3 - 2 T ca_{1,10}^3 - 6 T^2 ca_{1,10}^3 + 6 ca_{3,54} - 6 T ca_{3,54}}{4 T^2}, \\
cb_{3,54} & \rightarrow \frac{-ca_{1,10}^3 + T ca_{1,10}^3 - 6 ca_{3,54}}{6 T}, cb_{3,55} \rightarrow \theta \} \}
\end{aligned}$$

(Alt) Out[ ] =

$$\begin{aligned}
& -\frac{1}{2} \in ca_{1,2} + \in p_i x_i ca_{1,2} - \in p_j x_j ca_{1,2} - \frac{1}{2} \in^2 p_i x_i ca_{1,2} + \frac{1}{2} \in^2 p_j x_j ca_{1,2} + \frac{1}{2} \in p_i p_j x_i^2 ca_{1,10} - \\
& \frac{1}{2} T \in p_i p_j x_i^2 ca_{1,10} - \frac{1}{2} \in p_j^2 x_j^2 ca_{1,10} + \frac{1}{2} T \in p_j^2 x_j^2 ca_{1,10} + \in p_i p_j x_i x_j ca_{1,10} - \in p_j^2 x_j x_j ca_{1,10} - \\
& \frac{1}{2} \in^2 p_i p_j x_i^2 ca_{1,2} ca_{1,10} + T \in^2 p_i p_j x_i^2 ca_{1,2} ca_{1,10} + \frac{1}{2} \in^2 p_j^2 x_j^2 ca_{1,2} ca_{1,10} - T \in^2 p_j^2 x_j^2 ca_{1,2} ca_{1,10} - \\
& 2 \in^2 p_i p_j x_i x_j ca_{1,2} ca_{1,10} + 2 \in^2 p_j^2 x_j x_j ca_{1,2} ca_{1,10} - \frac{1}{4} \in^3 p_i p_j x_i^2 ca_{1,2} ca_{1,10} - \\
& \frac{3}{4} T \in^3 p_i p_j x_i^2 ca_{1,2} ca_{1,10} + \frac{1}{4} \in^3 p_j^2 x_j^2 ca_{1,2} ca_{1,10} + \frac{3}{4} T \in^3 p_j^2 x_j^2 ca_{1,2} ca_{1,10} + \frac{1}{4} T^2 \in^3 p_j^2 x_j^2 ca_{1,2} ca_{1,10} + \\
& 2 \in^3 p_i p_j x_i x_j ca_{1,2} ca_{1,10} - 2 \in^3 p_j^2 x_j x_j ca_{1,2} ca_{1,10} - \frac{1}{4} T \in^3 p_j^2 x_i x_j ca_{1,2} ca_{1,10} + \\
& \frac{1}{2} \in^2 p_i p_j x_i^2 ca_{1,10}^2 - \frac{1}{2} T \in^2 p_i p_j x_i^2 ca_{1,10}^2 - \frac{1}{2} \in^2 p_j^2 x_j^2 ca_{1,10}^2 + \frac{1}{2} T \in^2 p_j^2 x_j^2 ca_{1,10}^2 - \frac{1}{3} \in^2 p_i^2 p_j x_i^3 ca_{1,10}^2 + \\
& \frac{1}{3} T \in^2 p_i^2 p_j x_i^3 ca_{1,10}^2 + \frac{5}{6} \in^2 p_i p_j^2 x_i^3 ca_{1,10}^2 - \frac{2}{3} T \in^2 p_i p_j^2 x_i^3 ca_{1,10}^2 - \frac{1}{6} T^2 \in^2 p_i p_j^2 x_i^3 ca_{1,10}^2 - \\
& \frac{1}{2} \in^2 p_j^3 x_j^3 ca_{1,10}^2 + \frac{1}{3} T \in^2 p_j^3 x_j^3 ca_{1,10}^2 + \frac{1}{6} T^2 \in^2 p_j^3 x_j^3 ca_{1,10}^2 + \in^2 p_i p_j x_i x_j ca_{1,10}^2 - \in^2 p_j^2 x_i x_j ca_{1,10}^2 -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} \epsilon^2 p_i^2 p_j x_i^2 x_j ca_{1,10}^2 + \epsilon^2 p_i p_j^2 x_i^2 x_j ca_{1,10}^2 + \frac{1}{2} T \epsilon^2 p_i p_j^2 x_i^2 x_j ca_{1,10}^2 - \frac{1}{2} \epsilon^2 p_j^3 x_i^2 x_j ca_{1,10}^2 - \\
& \frac{1}{2} T \epsilon^2 p_j^3 x_i^2 x_j ca_{1,10}^2 - \frac{1}{2} \epsilon^2 p_i p_j^2 x_i x_j^2 ca_{1,10}^2 + \frac{1}{2} \epsilon^2 p_j^3 x_i x_j^2 ca_{1,10}^2 - \epsilon^3 p_i p_j x_i^2 ca_{1,2} ca_{1,10}^2 + \\
& \frac{3}{2} T \epsilon^3 p_i p_j x_i^2 ca_{1,2} ca_{1,10}^2 + \epsilon^3 p_j^2 x_i^2 ca_{1,2} ca_{1,10}^2 - \frac{3}{2} T \epsilon^3 p_j^2 x_i^2 ca_{1,2} ca_{1,10}^2 - \frac{1}{3} \epsilon^3 p_i^2 p_j x_i^3 ca_{1,2} ca_{1,10}^2 - \\
& \frac{4}{3} T \epsilon^3 p_i^2 p_j x_i^3 ca_{1,2} ca_{1,10}^2 - \frac{5}{3} \epsilon^3 p_i p_j^2 x_i^3 ca_{1,2} ca_{1,10}^2 + \frac{5}{3} T \epsilon^3 p_i p_j^2 x_i^3 ca_{1,2} ca_{1,10}^2 + \\
& \frac{1}{2} T^2 \epsilon^3 p_i p_j^2 x_i^3 ca_{1,2} ca_{1,10}^2 + 2 \epsilon^3 p_j^3 x_i^3 ca_{1,2} ca_{1,10}^2 - \frac{11}{6} T \epsilon^3 p_j^3 x_i^3 ca_{1,2} ca_{1,10}^2 - \\
& \frac{1}{2} T^2 \epsilon^3 p_j^3 x_i^3 ca_{1,2} ca_{1,10}^2 - 3 \epsilon^3 p_i p_j x_i x_j ca_{1,2} ca_{1,10}^2 + 3 \epsilon^3 p_j^2 x_i x_j ca_{1,2} ca_{1,10}^2 + \\
& 2 \epsilon^3 p_i^2 p_j x_i^2 x_j ca_{1,2} ca_{1,10}^2 - 5 \epsilon^3 p_i p_j^2 x_i^2 x_j ca_{1,2} ca_{1,10}^2 - \frac{3}{2} T \epsilon^3 p_i p_j^2 x_i^2 x_j ca_{1,2} ca_{1,10}^2 + \\
& \frac{9}{2} \epsilon^3 p_j^3 x_i^2 x_j ca_{1,2} ca_{1,10}^2 + \frac{3}{2} \epsilon^3 p_i p_j^2 x_i x_j^2 ca_{1,2} ca_{1,10}^2 + \frac{1}{2} \epsilon^3 p_i p_j x_i^2 ca_{1,10}^3 - \frac{1}{2} T \epsilon^3 p_i p_j x_i^2 ca_{1,10}^3 - \\
& \frac{1}{2} \epsilon^3 p_j^2 x_i^2 ca_{1,10}^3 + \frac{1}{2} T \epsilon^3 p_j^2 x_i^2 ca_{1,10}^3 + \frac{1}{3} \epsilon^3 p_i^2 p_j x_i^3 ca_{1,10}^3 + \frac{2}{3} T \epsilon^3 p_i^2 p_j x_i^3 ca_{1,10}^3 + \epsilon^3 p_i p_j^2 x_i^3 ca_{1,10}^3 - \\
& \frac{2}{3} T \epsilon^3 p_i p_j^2 x_i^3 ca_{1,10}^3 - \frac{1}{3} T^2 \epsilon^3 p_i p_j^2 x_i^3 ca_{1,10}^3 - \frac{4}{3} \epsilon^3 p_j^3 x_i^3 ca_{1,10}^3 + T \epsilon^3 p_j^3 x_i^3 ca_{1,10}^3 + \\
& \frac{1}{3} T^2 \epsilon^3 p_j^3 x_i^3 ca_{1,10}^3 - \frac{1}{24} \epsilon^3 p_i^3 p_j x_i^4 ca_{1,10}^3 - \frac{1}{8} T \epsilon^3 p_i^3 p_j x_i^4 ca_{1,10}^3 - \frac{1}{4} \epsilon^3 p_i^2 p_j^2 x_i^4 ca_{1,10}^3 - \\
& \frac{1}{8} T \epsilon^3 p_i^2 p_j^2 x_i^4 ca_{1,10}^3 + \frac{3}{8} T^2 \epsilon^3 p_i^2 p_j^2 x_i^4 ca_{1,10}^3 + \frac{3}{8} \epsilon^3 p_i p_j^3 x_i^4 ca_{1,10}^3 + \frac{17}{24} T \epsilon^3 p_i p_j^3 x_i^4 ca_{1,10}^3 - \\
& \frac{25}{24} T^2 \epsilon^3 p_i p_j^3 x_i^4 ca_{1,10}^3 - \frac{1}{24} T^3 \epsilon^3 p_i p_j^3 x_i^4 ca_{1,10}^3 - \frac{1}{12} \epsilon^3 p_j^4 x_i^4 ca_{1,10}^3 - \frac{5}{8} T \epsilon^3 p_j^4 x_i^4 ca_{1,10}^3 + \\
& \frac{3}{4} T^2 \epsilon^3 p_j^4 x_i^4 ca_{1,10}^3 - \frac{1}{24} T^3 \epsilon^3 p_j^4 x_i^4 ca_{1,10}^3 + \epsilon^3 p_i p_j x_i x_j ca_{1,10}^3 - \epsilon^3 p_j^2 x_i x_j ca_{1,10}^3 - \\
& \epsilon^3 p_i^2 p_j x_i^2 x_j ca_{1,10}^3 + \frac{5}{2} \epsilon^3 p_i p_j^2 x_i^2 x_j ca_{1,10}^3 + T \epsilon^3 p_i p_j^2 x_i^2 x_j ca_{1,10}^3 - \frac{5}{2} \epsilon^3 p_j^3 x_i^2 x_j ca_{1,10}^3 + \\
& \frac{1}{6} \epsilon^3 p_i^3 p_j x_i^3 x_j ca_{1,10}^3 - \frac{7}{6} T \epsilon^3 p_i^3 p_j x_i^3 x_j ca_{1,10}^3 - \epsilon^3 p_i p_j^3 x_i^3 x_j ca_{1,10}^3 + \frac{17}{6} T \epsilon^3 p_i p_j^3 x_i^3 x_j ca_{1,10}^3 + \\
& \frac{1}{6} T^2 \epsilon^3 p_i p_j^3 x_i^3 x_j ca_{1,10}^3 + \epsilon^3 p_j^4 x_i^3 x_j ca_{1,10}^3 - 2 T \epsilon^3 p_j^4 x_i^3 x_j ca_{1,10}^3 - \epsilon^3 p_i p_j^2 x_i x_j^2 ca_{1,10}^3 + \\
& \epsilon^3 p_i^2 p_j^2 x_i^2 x_j^2 ca_{1,10}^3 - \frac{5}{2} \epsilon^3 p_i p_j^3 x_i^2 x_j^2 ca_{1,10}^3 - \frac{1}{4} T \epsilon^3 p_i p_j^3 x_i^2 x_j^2 ca_{1,10}^3 + \frac{7}{4} \epsilon^3 p_j^4 x_i^2 x_j^2 ca_{1,10}^3 + \\
& \frac{1}{6} \epsilon^3 p_i p_j^3 x_i x_j^3 ca_{1,10}^3 + \epsilon^2 ca_{2,1} - 2 \epsilon^2 p_i x_i ca_{2,1} + 2 \epsilon^2 p_j x_i ca_{2,1} + 4 \epsilon^3 p_i x_i ca_{1,2} ca_{2,1} - \\
& 4 \epsilon^3 p_j x_i ca_{1,2} ca_{2,1} + \epsilon^3 p_i p_j x_i^2 ca_{1,10} ca_{2,1} - 3 T \epsilon^3 p_i p_j x_i^2 ca_{1,10} ca_{2,1} - \epsilon^3 p_j^2 x_i^2 ca_{1,10} ca_{2,1} + \\
& 3 T \epsilon^3 p_j^2 x_i^2 ca_{1,10} ca_{2,1} - T^2 \epsilon^3 p_j^2 x_i^2 ca_{1,10} ca_{2,1} + 4 \epsilon^3 p_i p_j x_i x_j ca_{1,10} ca_{2,1} - 4 \epsilon^3 p_j^2 x_i x_j ca_{1,10} ca_{2,1} + \\
& T \epsilon^3 p_j^2 x_i x_j ca_{1,10} ca_{2,1} + \epsilon^3 ca_{3,1} - \epsilon^3 p_i^2 p_j x_i^3 ca_{3,29} + \epsilon^3 p_i p_j^2 x_i^3 ca_{3,29} - T \epsilon^3 p_i p_j^2 x_i^3 ca_{3,29} + \\
& \epsilon^3 p_j^3 x_i^3 x_j ca_{3,29} - T \epsilon^3 p_j^3 x_i^3 x_j ca_{3,29} + \epsilon^3 p_j^3 x_i x_j^2 ca_{3,29} - \epsilon^3 p_i^3 p_j x_i^4 ca_{3,54} + \frac{3}{2} \epsilon^3 p_i^2 p_j^2 x_i^4 ca_{3,54} -
\end{aligned}$$

$$\begin{aligned}
 & \frac{3}{2} T \epsilon^3 p_i^2 p_j^2 x_i^4 ca_{3,54} - \epsilon^3 p_i p_j^3 x_i^4 ca_{3,54} + 2 T \epsilon^3 p_i p_j^3 x_i^4 ca_{3,54} - T^2 \epsilon^3 p_i p_j^3 x_i^4 ca_{3,54} + \frac{1}{2} \epsilon^3 p_j^4 x_i^4 ca_{3,54} - \\
 & \frac{3}{2} T \epsilon^3 p_j^4 x_i^4 ca_{3,54} + \frac{3}{2} T^2 \epsilon^3 p_j^4 x_i^4 ca_{3,54} - \frac{1}{2} T^3 \epsilon^3 p_j^4 x_i^4 ca_{3,54} + \epsilon^3 p_j^4 x_i^3 x_j ca_{3,54} - 2 T \epsilon^3 p_j^4 x_i^3 x_j ca_{3,54} + \\
 & T^2 \epsilon^3 p_j^4 x_i^3 x_j ca_{3,54} + \frac{3}{2} \epsilon^3 p_j^4 x_i^2 x_j^2 ca_{3,54} - \frac{3}{2} T \epsilon^3 p_j^4 x_i^2 x_j^2 ca_{3,54} + \epsilon^3 p_j^4 x_i x_j^3 ca_{3,54} - \frac{1}{2} \epsilon^2 p_i p_j x_i^2 cb_{2,10} + \\
 & \frac{1}{2} T \epsilon^2 p_i p_j x_i^2 cb_{2,10} + \frac{1}{2} \epsilon^2 p_j^2 x_i^2 cb_{2,10} - \frac{1}{2} T \epsilon^2 p_j^2 x_i^2 cb_{2,10} - \epsilon^2 p_i p_j x_i x_j cb_{2,10} + \epsilon^2 p_j^2 x_i x_j cb_{2,10} + \\
 & \frac{1}{2} \epsilon^3 p_i p_j x_i^2 ca_{1,2} cb_{2,10} - \frac{3}{2} T \epsilon^3 p_i p_j x_i^2 ca_{1,2} cb_{2,10} - \frac{1}{2} \epsilon^3 p_j^2 x_i^2 ca_{1,2} cb_{2,10} + \frac{3}{2} T \epsilon^3 p_j^2 x_i^2 ca_{1,2} cb_{2,10} - \\
 & \frac{1}{2} T^2 \epsilon^3 p_j^2 x_i^2 ca_{1,2} cb_{2,10} + 2 \epsilon^3 p_i p_j x_i x_j ca_{1,2} cb_{2,10} - 2 \epsilon^3 p_j^2 x_i x_j ca_{1,2} cb_{2,10} + \\
 & \frac{1}{2} T \epsilon^3 p_j^2 x_i x_j ca_{1,2} cb_{2,10} - \epsilon^3 p_i p_j x_i^2 ca_{1,10} cb_{2,10} + T \epsilon^3 p_i p_j x_i^2 ca_{1,10} cb_{2,10} + \epsilon^3 p_j^2 x_i^2 ca_{1,10} cb_{2,10} - \\
 & T \epsilon^3 p_j^2 x_i^2 ca_{1,10} cb_{2,10} - \frac{1}{3} \epsilon^3 p_i^2 p_j x_i^3 ca_{1,10} cb_{2,10} - \frac{2}{3} T \epsilon^3 p_i^2 p_j x_i^3 ca_{1,10} cb_{2,10} - \\
 & \frac{2}{3} \epsilon^3 p_i p_j^2 x_i^3 ca_{1,10} cb_{2,10} + \frac{1}{3} T \epsilon^3 p_i p_j^2 x_i^3 ca_{1,10} cb_{2,10} + \frac{1}{3} T^2 \epsilon^3 p_i p_j^2 x_i^3 ca_{1,10} cb_{2,10} + \\
 & \epsilon^3 p_j^3 x_i^3 ca_{1,10} cb_{2,10} - \frac{2}{3} T \epsilon^3 p_j^3 x_i^3 ca_{1,10} cb_{2,10} - \frac{1}{3} T^2 \epsilon^3 p_j^3 x_i^3 ca_{1,10} cb_{2,10} - 2 \epsilon^3 p_i p_j x_i x_j ca_{1,10} cb_{2,10} + \\
 & 2 \epsilon^3 p_j^2 x_i x_j ca_{1,10} cb_{2,10} + \epsilon^3 p_i^2 p_j x_i^2 x_j ca_{1,10} cb_{2,10} - 2 \epsilon^3 p_i p_j^2 x_i^2 x_j ca_{1,10} cb_{2,10} - \\
 & T \epsilon^3 p_i p_j^2 x_i^2 x_j ca_{1,10} cb_{2,10} + 2 \epsilon^3 p_j^3 x_i^2 x_j ca_{1,10} cb_{2,10} + \epsilon^3 p_i p_j^2 x_i x_j^2 ca_{1,10} cb_{2,10} - \epsilon^3 p_i x_i cb_{3,2} + \\
 & \epsilon^3 p_j x_i cb_{3,2} + T \epsilon^3 p_j x_i cb_{3,5} - \epsilon^3 p_j x_j cb_{3,5} - T \epsilon^3 p_i p_j x_i^2 cb_{3,9} + T \epsilon^3 p_j^2 x_i^2 cb_{3,9} - T^2 \epsilon^3 p_j^2 x_i^2 cb_{3,9} + \\
 & T \epsilon^3 p_j^2 x_i x_j cb_{3,9} - \epsilon^3 p_i p_j x_i^2 cb_{3,10} + T \epsilon^3 p_i p_j x_i^2 cb_{3,10} + \epsilon^3 p_j^2 x_i^2 cb_{3,10} - \frac{3}{2} T \epsilon^3 p_j^2 x_i^2 cb_{3,10} + \\
 & \frac{1}{2} T^2 \epsilon^3 p_j^2 x_i^2 cb_{3,10} - \epsilon^3 p_i p_j x_i x_j cb_{3,10} + \frac{3}{2} \epsilon^3 p_j^2 x_i x_j cb_{3,10} - \frac{1}{2} T \epsilon^3 p_j^2 x_i x_j cb_{3,10}
 \end{aligned}$$

(Alt) Out[\*]=

$$\begin{aligned}
 & \frac{1}{2} \epsilon ca_{1,2} + \epsilon p_k x_k ca_{1,10} - \epsilon^2 p_k x_k ca_{1,2} ca_{1,10} - \epsilon^2 ca_{2,1} - \epsilon^2 p_k x_k cb_{2,10} + \\
 & \epsilon^3 cc_{3,1} + \epsilon^3 p_k x_k cc_{3,2} + \epsilon^3 p_k^2 x_k^2 cc_{3,3} + \epsilon^3 p_k^3 x_k^3 cc_{3,4} + \epsilon^3 p_k^4 x_k^4 cc_{3,5}
 \end{aligned}$$

## R2c @ d = 3

(Alt) In[ ]:=

```

lhs = CF[Module[{es = {i, j, i+, j+}},
  Times[
    Normal@Series[Exp[r_d[-1, 0, 1, i, j+] + r_d[1, i+, j]], {e, 0, d}],
    Exp[Sum[g_{\alpha,\beta} \pi_{\alpha} \xi_{\beta}, {\alpha, es}, {\beta, es}]]
  ] // Zip((p_#&/@es) \cup (x_#&/@es) // Expand
] // . gRules_{-1,i,j+} \cup gRules_{1,i+,j+}

```

(Alt) Out[ ]:=

$$\begin{aligned}
 & 1 + \frac{1}{2} \epsilon \text{ca}_{1,2} + \frac{1}{8} \epsilon^2 (\text{ca}_{1,2}^2 - 8 \text{ca}_{2,1}) + \frac{1}{48} \epsilon^3 (\text{ca}_{1,2}^3 - 24 \text{ca}_{1,2} \text{ca}_{2,1} + 48 \text{cc}_{3,1}) + \\
 & \frac{(-1 + T) \epsilon^3 (\text{ca}_{1,2}^2 \text{ca}_{1,10} + 4 \text{ca}_{1,10} \text{ca}_{2,1} + 2 \text{ca}_{1,2} \text{cb}_{2,10} - 2 \text{cb}_{3,10} - 2 \text{cc}_{3,2}) \mathfrak{g}_{j^{++}, i^{++}}}{2 T} + \\
 & \frac{2 (-1 + T)^2 \epsilon^3 \text{cc}_{3,3} \mathfrak{g}_{j^{++}, i^{++}}^2}{T^2} - \frac{6 (-1 + T)^3 \epsilon^3 \text{cc}_{3,4} \mathfrak{g}_{j^{++}, i^{++}}^3}{T^3} + \\
 & \frac{24 (-1 + T)^4 \epsilon^3 \text{cc}_{3,5} \mathfrak{g}_{j^{++}, i^{++}}^4}{T^4} + \epsilon \text{ca}_{1,10} \mathfrak{g}_{j^{++}, j^{++}} + \frac{1}{2} \epsilon^2 (-\text{ca}_{1,2} \text{ca}_{1,10} - 2 \text{cb}_{2,10}) \mathfrak{g}_{j^{++}, j^{++}} + \\
 & \frac{1}{8} \epsilon^3 (-3 \text{ca}_{1,2}^2 \text{ca}_{1,10} - 8 \text{ca}_{1,10} \text{ca}_{2,1} - 4 \text{ca}_{1,2} \text{cb}_{2,10} + 8 \text{cc}_{3,2}) \mathfrak{g}_{j^{++}, j^{++}} - \\
 & \frac{4 (-1 + T) \epsilon^3 \text{cc}_{3,3} \mathfrak{g}_{j^{++}, i^{++}} \mathfrak{g}_{j^{++}, j^{++}}}{T} + \frac{18 (-1 + T)^2 \epsilon^3 \text{cc}_{3,4} \mathfrak{g}_{j^{++}, i^{++}}^2 \mathfrak{g}_{j^{++}, j^{++}}}{T^2} - \\
 & \frac{96 (-1 + T)^3 \epsilon^3 \text{cc}_{3,5} \mathfrak{g}_{j^{++}, i^{++}}^3 \mathfrak{g}_{j^{++}, j^{++}}}{T^3} + \epsilon^2 \text{ca}_{1,10}^2 \mathfrak{g}_{j^{++}, j^{++}}^2 + \\
 & \frac{1}{2} \epsilon^3 (-3 \text{ca}_{1,2} \text{ca}_{1,10}^2 - 4 \text{ca}_{1,10} \text{cb}_{2,10} + 4 \text{cc}_{3,3}) \mathfrak{g}_{j^{++}, j^{++}}^2 - \\
 & \frac{18 (-1 + T) \epsilon^3 \text{cc}_{3,4} \mathfrak{g}_{j^{++}, i^{++}} \mathfrak{g}_{j^{++}, j^{++}}^2}{T} + \frac{144 (-1 + T)^2 \epsilon^3 \text{cc}_{3,5} \mathfrak{g}_{j^{++}, i^{++}}^2 \mathfrak{g}_{j^{++}, j^{++}}^2}{T^2} + \\
 & \epsilon^3 (\text{ca}_{1,10}^3 + 6 \text{cc}_{3,4}) \mathfrak{g}_{j^{++}, j^{++}}^3 - \frac{96 (-1 + T) \epsilon^3 \text{cc}_{3,5} \mathfrak{g}_{j^{++}, i^{++}} \mathfrak{g}_{j^{++}, j^{++}}^3}{T} + 24 \epsilon^3 \text{cc}_{3,5} \mathfrak{g}_{j^{++}, j^{++}}^4
 \end{aligned}$$

(Alt) In[ ]:=

```

rhs = CF[Module[{es = {(j+)+}},
  Times[
    Normal@Series[Exp[γd[1, (j+)+]], {ε, 0, d}],
    Exp[Sum[gα,β πα ξβ, {α, es}, {β, es}]]
  ] // Zip (p#&/@es) ∪ (x#&/@es) // Expand
]]

```

(Alt) Out[ ]:=

$$\begin{aligned}
& 1 + \frac{1}{2} \epsilon \text{ca}_{1,2} + \frac{1}{8} \epsilon^2 (\text{ca}_{1,2}^2 - 8 \text{ca}_{2,1}) + \frac{1}{48} \epsilon^3 (\text{ca}_{1,2}^3 - 24 \text{ca}_{1,2} \text{ca}_{2,1} + 48 \text{cc}_{3,1}) + \\
& \epsilon \text{ca}_{1,10} \mathbf{g}_{j^{++}, j^{++}} + \frac{1}{2} \epsilon^2 (-\text{ca}_{1,2} \text{ca}_{1,10} - 2 \text{cb}_{2,10}) \mathbf{g}_{j^{++}, j^{++}} + \\
& \frac{1}{8} \epsilon^3 (-3 \text{ca}_{1,2}^2 \text{ca}_{1,10} - 8 \text{ca}_{1,10} \text{ca}_{2,1} - 4 \text{ca}_{1,2} \text{cb}_{2,10} + 8 \text{cc}_{3,2}) \mathbf{g}_{j^{++}, j^{++}} + \epsilon^2 \text{ca}_{1,10}^2 \mathbf{g}_{j^{++}, j^{++}}^2 + \\
& \frac{1}{2} \epsilon^3 (-3 \text{ca}_{1,2} \text{ca}_{1,10}^2 - 4 \text{ca}_{1,10} \text{cb}_{2,10} + 4 \text{cc}_{3,3}) \mathbf{g}_{j^{++}, j^{++}}^2 + \epsilon^3 (\text{ca}_{1,10}^3 + 6 \text{cc}_{3,4}) \mathbf{g}_{j^{++}, j^{++}}^3 + 24 \epsilon^3 \text{cc}_{3,5} \mathbf{g}_{j^{++}, j^{++}}^4
\end{aligned}$$

(Alt) In[ ]:=

```
me = Exponent[lhs - rhs, T, Min]
```

(Alt) Out[ ]:=

```
-4
```

(Alt) In[ ]:=

```
covars = DeleteCases[Variables[lhs - rhs], T | (ca | cb | cc | cd) ___]
```

(Alt) Out[ ]:=

```
{ε, gj++, i++, gj++, j++}
```

(Alt) In[ ]:=

```
CoefficientRules[Expand[T-me (lhs - rhs)], covars] // Column
```

(Alt) Out[ ]:=

$$\begin{aligned}
\{3, 4, 0\} & \rightarrow 24 \text{cc}_{3,5} - 96 T \text{cc}_{3,5} + 144 T^2 \text{cc}_{3,5} - 96 T^3 \text{cc}_{3,5} + 24 T^4 \text{cc}_{3,5} \\
\{3, 3, 1\} & \rightarrow 96 T \text{cc}_{3,5} - 288 T^2 \text{cc}_{3,5} + 288 T^3 \text{cc}_{3,5} - 96 T^4 \text{cc}_{3,5} \\
\{3, 3, 0\} & \rightarrow 6 T \text{cc}_{3,4} - 18 T^2 \text{cc}_{3,4} + 18 T^3 \text{cc}_{3,4} - 6 T^4 \text{cc}_{3,4} \\
\{3, 2, 2\} & \rightarrow 144 T^2 \text{cc}_{3,5} - 288 T^3 \text{cc}_{3,5} + 144 T^4 \text{cc}_{3,5} \\
\{3, 2, 1\} & \rightarrow 18 T^2 \text{cc}_{3,4} - 36 T^3 \text{cc}_{3,4} + 18 T^4 \text{cc}_{3,4} \\
\{3, 2, 0\} & \rightarrow 2 T^2 \text{cc}_{3,3} - 4 T^3 \text{cc}_{3,3} + 2 T^4 \text{cc}_{3,3} \\
\{3, 1, 3\} & \rightarrow 96 T^3 \text{cc}_{3,5} - 96 T^4 \text{cc}_{3,5} \\
\{3, 1, 2\} & \rightarrow 18 T^3 \text{cc}_{3,4} - 18 T^4 \text{cc}_{3,4} \\
\{3, 1, 1\} & \rightarrow 4 T^3 \text{cc}_{3,3} - 4 T^4 \text{cc}_{3,3} \\
\{3, 1, 0\} & \rightarrow -\frac{1}{2} T^3 \text{ca}_{1,2}^2 \text{ca}_{1,10} + \frac{1}{2} T^4 \text{ca}_{1,2}^2 \text{ca}_{1,10} - 2 T^3 \text{ca}_{1,10} \text{ca}_{2,1} + \\
& 2 T^4 \text{ca}_{1,10} \text{ca}_{2,1} - T^3 \text{ca}_{1,2} \text{cb}_{2,10} + T^4 \text{ca}_{1,2} \text{cb}_{2,10} + T^3 \text{cb}_{3,10} - T^4 \text{cb}_{3,10} + T^3 \text{cc}_{3,2} - T^4 \text{cc}_{3,2}
\end{aligned}$$

(Alt) In[ ]:=

**eqnsR2c =**

**(Factor[#] == 0) & /@Union[Last /@ CoefficientRules[Expand[T<sup>-me</sup> (lhs - rhs)], covars]]**

(Alt) Out[ ]:=

$$\left\{ \begin{aligned} &\frac{1}{2} (-1 + T) T^3 (ca_{1,2}^2 ca_{1,10} + 4 ca_{1,10} ca_{2,1} + 2 ca_{1,2} cb_{2,10} - 2 cb_{3,10} - 2 cc_{3,2}) = 0, \\ &-4 (-1 + T) T^3 cc_{3,3} = 0, 2 (-1 + T)^2 T^2 cc_{3,3} = 0, -18 (-1 + T) T^3 cc_{3,4} = 0, \\ &-6 (-1 + T)^3 T cc_{3,4} = 0, 18 (-1 + T)^2 T^2 cc_{3,4} = 0, -96 (-1 + T) T^3 cc_{3,5} = 0, \\ &-96 (-1 + T)^3 T cc_{3,5} = 0, 24 (-1 + T)^4 cc_{3,5} = 0, 144 (-1 + T)^2 T^2 cc_{3,5} = 0 \end{aligned} \right\}$$

(Alt) In[ ]:=

**vars = Cases[Variables[r<sub>d</sub>[1, i1, j1] + r<sub>d</sub>[-1, i2, j2] + γ<sub>d</sub>[1, k]], (ca | cb | cc | cd) \_]**  
**{sol} = Solve[eqnsR3 ∪ eqnsR2b ∪ eqnsR2c ∪ eqnsR11 ∪ eqnsR1r ∪ eqnsSwp, vars]**  
**sol /. Rule → Set;**  
**r<sub>d</sub>[1, i, j]**  
**γ<sub>d</sub>[1, k]**

(Alt) Out[ ]:=

$$\{ca_{1,2}, ca_{1,10}, ca_{2,1}, ca_{3,29}, ca_{3,54}, cb_{2,10}, cb_{3,2}, cb_{3,5}, cb_{3,9}, cb_{3,10}, cc_{3,1}, cc_{3,2}, cc_{3,3}, cc_{3,4}, cc_{3,5}\}$$

**Solve:** Equations may not give solutions for all "solve" variables.

(Alt) Out[ ]:=

$$\left\{ \left\{ cc_{3,2} \rightarrow \frac{1}{2} (ca_{1,2}^2 ca_{1,10} + 4 ca_{1,10} ca_{2,1} + 2 ca_{1,2} cb_{2,10} - 2 cb_{3,10}), cc_{3,3} \rightarrow 0, cc_{3,4} \rightarrow 0, cc_{3,5} \rightarrow 0 \right\} \right\}$$

(Alt) Out[ ]:=

$$\begin{aligned} &-\frac{1}{2} \in ca_{1,2} + \in p_i x_i ca_{1,2} - \in p_j x_j ca_{1,2} - \frac{1}{2} \in^2 p_i x_i ca_{1,2}^2 + \frac{1}{2} \in^2 p_j x_j ca_{1,2}^2 + \frac{1}{2} \in p_i p_j x_i^2 ca_{1,10} - \\ &\frac{1}{2} T \in p_i p_j x_i^2 ca_{1,10} - \frac{1}{2} \in p_j^2 x_j^2 ca_{1,10} + \frac{1}{2} T \in p_j^2 x_j^2 ca_{1,10} + \in p_i p_j x_i x_j ca_{1,10} - \in p_j^2 x_j x_j ca_{1,10} - \\ &\frac{1}{2} \in^2 p_i p_j x_i^2 ca_{1,2} ca_{1,10} + T \in^2 p_i p_j x_i^2 ca_{1,2} ca_{1,10} + \frac{1}{2} \in^2 p_j^2 x_j^2 ca_{1,2} ca_{1,10} - T \in^2 p_j^2 x_j^2 ca_{1,2} ca_{1,10} - \\ &2 \in^2 p_i p_j x_i x_j ca_{1,2} ca_{1,10} + 2 \in^2 p_j^2 x_j x_j ca_{1,2} ca_{1,10} - \frac{1}{4} \in^3 p_i p_j x_i^2 ca_{1,2}^2 ca_{1,10} - \\ &\frac{3}{4} T \in^3 p_i p_j x_i^2 ca_{1,2}^2 ca_{1,10} + \frac{1}{4} \in^3 p_j^2 x_j^2 ca_{1,2}^2 ca_{1,10} + \frac{3}{4} T \in^3 p_j^2 x_j^2 ca_{1,2}^2 ca_{1,10} + \frac{1}{4} T^2 \in^3 p_j^2 x_j^2 ca_{1,2}^2 ca_{1,10} + \\ &2 \in^3 p_i p_j x_i x_j ca_{1,2}^2 ca_{1,10} - 2 \in^3 p_j^2 x_j x_j ca_{1,2}^2 ca_{1,10} - \frac{1}{4} T \in^3 p_j^2 x_j x_j ca_{1,2}^2 ca_{1,10} + \\ &\frac{1}{2} \in^2 p_i p_j x_i^2 ca_{1,10}^2 - \frac{1}{2} T \in^2 p_i p_j x_i^2 ca_{1,10}^2 - \frac{1}{2} \in^2 p_j^2 x_j^2 ca_{1,10}^2 + \frac{1}{2} T \in^2 p_j^2 x_j^2 ca_{1,10}^2 - \frac{1}{3} \in^2 p_i^2 p_j x_i^3 ca_{1,10}^2 + \\ &\frac{1}{3} T \in^2 p_i^2 p_j x_i^3 ca_{1,10}^2 + \frac{5}{6} \in^2 p_i p_j^2 x_i^3 ca_{1,10}^2 - \frac{2}{3} T \in^2 p_i p_j^2 x_i^3 ca_{1,10}^2 - \frac{1}{6} T^2 \in^2 p_i p_j^2 x_i^3 ca_{1,10}^2 - \\ &\frac{1}{2} \in^2 p_j^3 x_j^3 ca_{1,10}^2 + \frac{1}{3} T \in^2 p_j^3 x_j^3 ca_{1,10}^2 + \frac{1}{6} T^2 \in^2 p_j^3 x_j^3 ca_{1,10}^2 + \in^2 p_i p_j x_i x_j ca_{1,10}^2 - \in^2 p_j^2 x_i x_j ca_{1,10}^2 - \\ &\frac{1}{2} \in^2 p_i^2 p_j x_i^2 x_j ca_{1,10}^2 + \in^2 p_i p_j^2 x_i^2 x_j ca_{1,10}^2 + \frac{1}{2} T \in^2 p_i p_j^2 x_i^2 x_j ca_{1,10}^2 - \frac{1}{2} \in^2 p_j^3 x_i^2 x_j ca_{1,10}^2 - \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{2} T \epsilon^2 p_j^3 x_i^2 x_j ca_{1,10}^2 - \frac{1}{2} \epsilon^2 p_i p_j^2 x_i x_j^2 ca_{1,10}^2 + \frac{1}{2} \epsilon^2 p_j^3 x_i x_j^2 ca_{1,10}^2 - \epsilon^3 p_i p_j x_i^2 ca_{1,2} ca_{1,10}^2 + \\
 & \frac{3}{2} T \epsilon^3 p_i p_j x_i^2 ca_{1,2} ca_{1,10}^2 + \epsilon^3 p_j^2 x_i^2 ca_{1,2} ca_{1,10}^2 - \frac{3}{2} T \epsilon^3 p_j^2 x_i^2 ca_{1,2} ca_{1,10}^2 - \frac{1}{3} \epsilon^3 p_i^2 p_j x_i^3 ca_{1,2} ca_{1,10}^2 - \\
 & \frac{4}{3} T \epsilon^3 p_i^2 p_j x_i^3 ca_{1,2} ca_{1,10}^2 - \frac{5}{3} \epsilon^3 p_i p_j^2 x_i^3 ca_{1,2} ca_{1,10}^2 + \frac{5}{3} T \epsilon^3 p_i p_j^2 x_i^3 ca_{1,2} ca_{1,10}^2 + \\
 & \frac{1}{2} T^2 \epsilon^3 p_i p_j^2 x_i^3 ca_{1,2} ca_{1,10}^2 + 2 \epsilon^3 p_j^3 x_i^3 ca_{1,2} ca_{1,10}^2 - \frac{11}{6} T \epsilon^3 p_j^3 x_i^3 ca_{1,2} ca_{1,10}^2 - \\
 & \frac{1}{2} T^2 \epsilon^3 p_j^3 x_i^3 ca_{1,2} ca_{1,10}^2 - 3 \epsilon^3 p_i p_j x_i x_j ca_{1,2} ca_{1,10}^2 + 3 \epsilon^3 p_j^2 x_i x_j ca_{1,2} ca_{1,10}^2 + \\
 & 2 \epsilon^3 p_i^2 p_j x_i^2 x_j ca_{1,2} ca_{1,10}^2 - 5 \epsilon^3 p_i p_j^2 x_i^2 x_j ca_{1,2} ca_{1,10}^2 - \frac{3}{2} T \epsilon^3 p_i p_j^2 x_i^2 x_j ca_{1,2} ca_{1,10}^2 + \\
 & \frac{9}{2} \epsilon^3 p_j^3 x_i^2 x_j ca_{1,2} ca_{1,10}^2 + \frac{3}{2} \epsilon^3 p_i p_j^2 x_i x_j^2 ca_{1,2} ca_{1,10}^2 + \frac{1}{2} \epsilon^3 p_i p_j x_i^2 ca_{1,10}^3 - \frac{1}{2} T \epsilon^3 p_i p_j x_i^2 ca_{1,10}^3 - \\
 & \frac{1}{2} \epsilon^3 p_j^2 x_i^2 ca_{1,10}^3 + \frac{1}{2} T \epsilon^3 p_j^2 x_i^2 ca_{1,10}^3 + \frac{1}{3} \epsilon^3 p_i^2 p_j x_i^3 ca_{1,10}^3 + \frac{2}{3} T \epsilon^3 p_i^2 p_j x_i^3 ca_{1,10}^3 + \epsilon^3 p_i p_j^2 x_i^3 ca_{1,10}^3 - \\
 & \frac{2}{3} T \epsilon^3 p_i p_j^2 x_i^3 ca_{1,10}^3 - \frac{1}{3} T^2 \epsilon^3 p_i p_j^2 x_i^3 ca_{1,10}^3 - \frac{4}{3} \epsilon^3 p_j^3 x_i^3 ca_{1,10}^3 + T \epsilon^3 p_j^3 x_i^3 ca_{1,10}^3 + \\
 & \frac{1}{3} T^2 \epsilon^3 p_j^3 x_i^3 ca_{1,10}^3 - \frac{1}{24} \epsilon^3 p_i^3 p_j x_i^4 ca_{1,10}^3 - \frac{1}{8} T \epsilon^3 p_i^3 p_j x_i^4 ca_{1,10}^3 - \frac{1}{4} \epsilon^3 p_i^2 p_j^2 x_i^4 ca_{1,10}^3 - \\
 & \frac{1}{8} T \epsilon^3 p_i^2 p_j^2 x_i^4 ca_{1,10}^3 + \frac{3}{8} T^2 \epsilon^3 p_i^2 p_j^2 x_i^4 ca_{1,10}^3 + \frac{3}{8} \epsilon^3 p_i p_j^3 x_i^4 ca_{1,10}^3 + \frac{17}{24} T \epsilon^3 p_i p_j^3 x_i^4 ca_{1,10}^3 - \\
 & \frac{25}{24} T^2 \epsilon^3 p_i p_j^3 x_i^4 ca_{1,10}^3 - \frac{1}{24} T^3 \epsilon^3 p_i p_j^3 x_i^4 ca_{1,10}^3 - \frac{1}{12} \epsilon^3 p_j^4 x_i^4 ca_{1,10}^3 - \frac{5}{8} T \epsilon^3 p_j^4 x_i^4 ca_{1,10}^3 + \\
 & \frac{3}{4} T^2 \epsilon^3 p_j^4 x_i^4 ca_{1,10}^3 - \frac{1}{24} T^3 \epsilon^3 p_j^4 x_i^4 ca_{1,10}^3 + \epsilon^3 p_i p_j x_i x_j ca_{1,10}^3 - \epsilon^3 p_j^2 x_i x_j ca_{1,10}^3 - \\
 & \epsilon^3 p_i^2 p_j x_i^2 x_j ca_{1,10}^3 + \frac{5}{2} \epsilon^3 p_i p_j^2 x_i^2 x_j ca_{1,10}^3 + T \epsilon^3 p_i p_j^2 x_i^2 x_j ca_{1,10}^3 - \frac{5}{2} \epsilon^3 p_j^3 x_i^2 x_j ca_{1,10}^3 + \\
 & \frac{1}{6} \epsilon^3 p_i^3 p_j x_i^3 x_j ca_{1,10}^3 - \frac{7}{6} T \epsilon^3 p_i^3 p_j x_i^3 x_j ca_{1,10}^3 - \epsilon^3 p_i p_j^3 x_i^3 x_j ca_{1,10}^3 + \frac{17}{6} T \epsilon^3 p_i p_j^3 x_i^3 x_j ca_{1,10}^3 + \\
 & \frac{1}{6} T^2 \epsilon^3 p_i p_j^3 x_i^3 x_j ca_{1,10}^3 + \epsilon^3 p_j^4 x_i^3 x_j ca_{1,10}^3 - 2 T \epsilon^3 p_j^4 x_i^3 x_j ca_{1,10}^3 - \epsilon^3 p_i p_j^2 x_i x_j^2 ca_{1,10}^3 + \\
 & \epsilon^3 p_i^2 p_j^2 x_i^2 x_j^2 ca_{1,10}^3 - \frac{5}{2} \epsilon^3 p_i p_j^3 x_i^2 x_j^2 ca_{1,10}^3 - \frac{1}{4} T \epsilon^3 p_i p_j^3 x_i^2 x_j^2 ca_{1,10}^3 + \frac{7}{4} \epsilon^3 p_j^4 x_i^2 x_j^2 ca_{1,10}^3 + \\
 & \frac{1}{6} \epsilon^3 p_i p_j^3 x_i x_j^3 ca_{1,10}^3 + \epsilon^2 ca_{2,1} - 2 \epsilon^2 p_i x_i ca_{2,1} + 2 \epsilon^2 p_j x_i ca_{2,1} + 4 \epsilon^3 p_i x_i ca_{1,2} ca_{2,1} - \\
 & 4 \epsilon^3 p_j x_i ca_{1,2} ca_{2,1} + \epsilon^3 p_i p_j x_i^2 ca_{1,10} ca_{2,1} - 3 T \epsilon^3 p_i p_j x_i^2 ca_{1,10} ca_{2,1} - \epsilon^3 p_j^2 x_i^2 ca_{1,10} ca_{2,1} + \\
 & 3 T \epsilon^3 p_j^2 x_i^2 ca_{1,10} ca_{2,1} - T^2 \epsilon^3 p_j^2 x_i^2 ca_{1,10} ca_{2,1} + 4 \epsilon^3 p_i p_j x_i x_j ca_{1,10} ca_{2,1} - 4 \epsilon^3 p_j^2 x_i x_j ca_{1,10} ca_{2,1} + \\
 & T \epsilon^3 p_j^2 x_i x_j ca_{1,10} ca_{2,1} + \epsilon^3 ca_{3,1} - \epsilon^3 p_i^2 p_j x_i^3 ca_{3,29} + \epsilon^3 p_i p_j^2 x_i^3 ca_{3,29} - T \epsilon^3 p_i p_j^2 x_i^3 ca_{3,29} + \\
 & \epsilon^3 p_j^3 x_i^2 x_j ca_{3,29} - T \epsilon^3 p_j^3 x_i^2 x_j ca_{3,29} + \epsilon^3 p_j^3 x_i x_j^2 ca_{3,29} - \epsilon^3 p_i^3 p_j x_i^4 ca_{3,54} + \frac{3}{2} \epsilon^3 p_i^2 p_j^2 x_i^4 ca_{3,54} - \\
 & \frac{3}{2} T \epsilon^3 p_i^2 p_j^2 x_i^4 ca_{3,54} - \epsilon^3 p_i p_j^3 x_i^4 ca_{3,54} + 2 T \epsilon^3 p_i p_j^3 x_i^4 ca_{3,54} - T^2 \epsilon^3 p_i p_j^3 x_i^4 ca_{3,54} + \frac{1}{2} \epsilon^3 p_j^4 x_i^4 ca_{3,54} -
 \end{aligned}$$

$$\begin{aligned}
 & \frac{3}{2} T \epsilon^3 p_j^4 x_i^4 ca_{3,54} + \frac{3}{2} T^2 \epsilon^3 p_j^4 x_i^4 ca_{3,54} - \frac{1}{2} T^3 \epsilon^3 p_j^4 x_i^4 ca_{3,54} + \epsilon^3 p_j^4 x_i^3 x_j ca_{3,54} - 2 T \epsilon^3 p_j^4 x_i^3 x_j ca_{3,54} + \\
 & T^2 \epsilon^3 p_j^4 x_i^3 x_j ca_{3,54} + \frac{3}{2} \epsilon^3 p_j^4 x_i^2 x_j^2 ca_{3,54} - \frac{3}{2} T \epsilon^3 p_j^4 x_i^2 x_j^2 ca_{3,54} + \epsilon^3 p_j^4 x_i x_j^3 ca_{3,54} - \frac{1}{2} \epsilon^2 p_i p_j x_i^2 cb_{2,10} + \\
 & \frac{1}{2} T \epsilon^2 p_i p_j x_i^2 cb_{2,10} + \frac{1}{2} \epsilon^2 p_j^2 x_i^2 cb_{2,10} - \frac{1}{2} T \epsilon^2 p_j^2 x_i^2 cb_{2,10} - \epsilon^2 p_i p_j x_i x_j cb_{2,10} + \epsilon^2 p_j^2 x_i x_j cb_{2,10} + \\
 & \frac{1}{2} \epsilon^3 p_i p_j x_i^2 ca_{1,2} cb_{2,10} - \frac{3}{2} T \epsilon^3 p_i p_j x_i^2 ca_{1,2} cb_{2,10} - \frac{1}{2} \epsilon^3 p_j^2 x_i^2 ca_{1,2} cb_{2,10} + \frac{3}{2} T \epsilon^3 p_j^2 x_i^2 ca_{1,2} cb_{2,10} - \\
 & \frac{1}{2} T^2 \epsilon^3 p_j^2 x_i^2 ca_{1,2} cb_{2,10} + 2 \epsilon^3 p_i p_j x_i x_j ca_{1,2} cb_{2,10} - 2 \epsilon^3 p_j^2 x_i x_j ca_{1,2} cb_{2,10} + \\
 & \frac{1}{2} T \epsilon^3 p_j^2 x_i x_j ca_{1,2} cb_{2,10} - \epsilon^3 p_i p_j x_i^2 ca_{1,10} cb_{2,10} + T \epsilon^3 p_i p_j x_i^2 ca_{1,10} cb_{2,10} + \epsilon^3 p_j^2 x_i^2 ca_{1,10} cb_{2,10} - \\
 & T \epsilon^3 p_j^2 x_i^2 ca_{1,10} cb_{2,10} - \frac{1}{3} \epsilon^3 p_i^2 p_j x_i^3 ca_{1,10} cb_{2,10} - \frac{2}{3} T \epsilon^3 p_i^2 p_j x_i^3 ca_{1,10} cb_{2,10} - \\
 & \frac{2}{3} \epsilon^3 p_i p_j^2 x_i^3 ca_{1,10} cb_{2,10} + \frac{1}{3} T \epsilon^3 p_i p_j^2 x_i^3 ca_{1,10} cb_{2,10} + \frac{1}{3} T^2 \epsilon^3 p_i p_j^2 x_i^3 ca_{1,10} cb_{2,10} + \\
 & \epsilon^3 p_j^3 x_i^3 ca_{1,10} cb_{2,10} - \frac{2}{3} T \epsilon^3 p_j^3 x_i^3 ca_{1,10} cb_{2,10} - \frac{1}{3} T^2 \epsilon^3 p_j^3 x_i^3 ca_{1,10} cb_{2,10} - 2 \epsilon^3 p_i p_j x_i x_j ca_{1,10} cb_{2,10} + \\
 & 2 \epsilon^3 p_j^2 x_i x_j ca_{1,10} cb_{2,10} + \epsilon^3 p_i^2 p_j x_i^2 x_j ca_{1,10} cb_{2,10} - 2 \epsilon^3 p_i p_j^2 x_i^2 x_j ca_{1,10} cb_{2,10} - \\
 & T \epsilon^3 p_i p_j^2 x_i^2 x_j ca_{1,10} cb_{2,10} + 2 \epsilon^3 p_j^3 x_i^2 x_j ca_{1,10} cb_{2,10} + \epsilon^3 p_i p_j^2 x_i x_j^2 ca_{1,10} cb_{2,10} - \epsilon^3 p_i x_i cb_{3,2} + \\
 & \epsilon^3 p_j x_i cb_{3,2} + T \epsilon^3 p_j x_i cb_{3,5} - \epsilon^3 p_j x_j cb_{3,5} - T \epsilon^3 p_i p_j x_i^2 cb_{3,9} + T \epsilon^3 p_j^2 x_i^2 cb_{3,9} - T^2 \epsilon^3 p_j^2 x_i^2 cb_{3,9} + \\
 & T \epsilon^3 p_j^2 x_i x_j cb_{3,9} - \epsilon^3 p_i p_j x_i^2 cb_{3,10} + T \epsilon^3 p_i p_j x_i^2 cb_{3,10} + \epsilon^3 p_j^2 x_i^2 cb_{3,10} - \frac{3}{2} T \epsilon^3 p_j^2 x_i^2 cb_{3,10} + \\
 & \frac{1}{2} T^2 \epsilon^3 p_j^2 x_i^2 cb_{3,10} - \epsilon^3 p_i p_j x_i x_j cb_{3,10} + \frac{3}{2} \epsilon^3 p_j^2 x_i x_j cb_{3,10} - \frac{1}{2} T \epsilon^3 p_j^2 x_i x_j cb_{3,10}
 \end{aligned}$$

(Alt) Out[\*]=

$$\begin{aligned}
 & \frac{1}{2} \epsilon ca_{1,2} + \epsilon p_k x_k ca_{1,10} - \epsilon^2 p_k x_k ca_{1,2} ca_{1,10} + \frac{1}{2} \epsilon^3 p_k x_k ca_{1,2}^2 ca_{1,10} - \epsilon^2 ca_{2,1} + \\
 & 2 \epsilon^3 p_k x_k ca_{1,10} ca_{2,1} - \epsilon^2 p_k x_k cb_{2,10} + \epsilon^3 p_k x_k ca_{1,2} cb_{2,10} - \epsilon^3 p_k x_k cb_{3,10} + \epsilon^3 cc_{3,1}
 \end{aligned}$$

## R1l

(Alt) In[ ]:=

```

lhs = CF[Module[{es = {i, i^}},
  Times[
    Normal@Series[Exp[r_d[1, 1, 0, i^, i]], {e, 0, d}],
    Exp[Sum[g_{\alpha,\beta} \pi_{\alpha} \xi_{\beta}, {\alpha, es}, {\beta, es}]]
  ] // Zip((p_#&/@es) \cup (x_#&/@es) // Expand
] // . {g_{i^+,\beta_} \to T^{-1} \delta_{i^+,\beta} + g_{i^{++},\beta}, g_{i,\beta_} \to \delta_{i,\beta} + g_{i^+,\beta}}]

```

(Alt) Out[ ]:=

$$\begin{aligned}
 & 1 + \epsilon^3 (ca_{3,1} + cc_{3,1}) - \\
 & \frac{1}{2 T^3} \epsilon^3 (T^3 ca_{1,2}^2 ca_{1,10} - 18 T ca_{1,2} ca_{1,10}^2 - 18 T^2 ca_{1,2} ca_{1,10}^2 - 8 ca_{1,10}^3 + 4 T ca_{1,10}^3 + 4 T^2 ca_{1,10}^3 - \\
 & \quad 4 T^3 ca_{1,10} ca_{2,1} - 12 T ca_{3,29} - 12 T^2 ca_{3,29} - 48 ca_{3,54} - 48 T ca_{3,54} - 48 T^2 ca_{3,54} - 2 T^3 ca_{1,2} cb_{2,10} - \\
 & \quad 12 T ca_{1,10} cb_{2,10} - 12 T^2 ca_{1,10} cb_{2,10} + 2 T^3 cb_{3,5} - 4 T^3 cb_{3,9} - 2 T^2 cb_{3,10} + 2 T^3 cb_{3,10}) g_{i^{++},i} + \\
 & \frac{3 \epsilon^3 (3 T ca_{1,2} ca_{1,10}^2 + 2 ca_{1,10}^3 + 2 T ca_{3,29} + 12 ca_{3,54} + 12 T ca_{3,54} + 2 T ca_{1,10} cb_{2,10}) g_{i^{++},i}^2}{T^2} + \\
 & \frac{4 \epsilon^3 (ca_{1,10}^3 + 6 ca_{3,54}) g_{i^{++},i}^3}{T} + \\
 & \frac{1}{2 T^2} \epsilon^3 (T^3 ca_{1,2}^2 ca_{1,10} - 18 T ca_{1,2} ca_{1,10}^2 - 18 T^2 ca_{1,2} ca_{1,10}^2 - 8 ca_{1,10}^3 + 4 T ca_{1,10}^3 + 4 T^2 ca_{1,10}^3 - \\
 & \quad 4 T^3 ca_{1,10} ca_{2,1} - 12 T ca_{3,29} - 12 T^2 ca_{3,29} - 48 ca_{3,54} - 48 T ca_{3,54} - 48 T^2 ca_{3,54} - 2 T^3 ca_{1,2} cb_{2,10} - \\
 & \quad 12 T ca_{1,10} cb_{2,10} - 12 T^2 ca_{1,10} cb_{2,10} + 2 T^3 cb_{3,5} - 4 T^3 cb_{3,9} - 2 T^2 cb_{3,10} + 2 T^3 cb_{3,10}) g_{i^{++},i^+} - \\
 & \frac{1}{2 T^2} \epsilon^3 (T^3 ca_{1,2}^2 ca_{1,10} - 36 T ca_{1,2} ca_{1,10}^2 - 24 ca_{1,10}^3 + 24 T ca_{1,10}^3 - 4 T^3 ca_{1,10} ca_{2,1} - 24 T ca_{3,29} - \\
 & \quad 144 ca_{3,54} - 2 T^3 ca_{1,2} cb_{2,10} - 24 T ca_{1,10} cb_{2,10} - 4 T^3 cb_{3,9} - 2 T^2 cb_{3,10} + 2 T^3 cb_{3,10}) g_{i^{++},i} g_{i^{++},i^+} + \\
 & \frac{3 \epsilon^3 (3 T ca_{1,2} ca_{1,10}^2 + 4 ca_{1,10}^3 - 2 T ca_{1,10}^3 + 2 T ca_{3,29} + 24 ca_{3,54} + 2 T ca_{1,10} cb_{2,10}) g_{i^{++},i}^2 g_{i^{++},i^+}}{T} + \\
 & \frac{4 \epsilon^3 (ca_{1,10}^3 + 6 ca_{3,54}) g_{i^{++},i}^3 g_{i^{++},i^+}}{T} + \\
 & \frac{1}{2 T} \epsilon^3 (T^3 ca_{1,2}^2 ca_{1,10} - 36 T ca_{1,2} ca_{1,10}^2 - 18 T^2 ca_{1,2} ca_{1,10}^2 - 24 ca_{1,10}^3 + 12 T ca_{1,10}^3 - 4 T^3 ca_{1,10} ca_{2,1} - \\
 & \quad 24 T ca_{3,29} - 12 T^2 ca_{3,29} - 144 ca_{3,54} - 72 T ca_{3,54} - 72 T^2 ca_{3,54} - 2 T^3 ca_{1,2} cb_{2,10} - \\
 & \quad 24 T ca_{1,10} cb_{2,10} - 12 T^2 ca_{1,10} cb_{2,10} - 4 T^3 cb_{3,9} - 2 T^2 cb_{3,10} + 2 T^3 cb_{3,10}) g_{i^{++},i^+}^2 - \\
 & \frac{3 (-1 + T) \epsilon^3 (3 T ca_{1,2} ca_{1,10}^2 + 4 ca_{1,10}^3 - 2 T ca_{1,10}^3 + 2 T ca_{3,29} + 24 ca_{3,54} + 2 T ca_{1,10} cb_{2,10}) g_{i^{++},i} g_{i^{++},i^+}^2}{T} \\
 & - 6 (-1 + T) \epsilon^3 (ca_{1,10}^3 + 6 ca_{3,54}) g_{i^{++},i}^2 g_{i^{++},i^+}^2 + \\
 & \epsilon^3 (-9 T ca_{1,2} ca_{1,10}^2 - 12 ca_{1,10}^3 + 6 T ca_{1,10}^3 - 4 T^2 ca_{1,10}^3 - \\
 & \quad 6 T ca_{3,29} - 72 ca_{3,54} - 24 T^2 ca_{3,54} - 6 T ca_{1,10} cb_{2,10}) g_{i^{++},i^+}^3 + \\
 & 4 (-1 + T)^2 \epsilon^3 (ca_{1,10}^3 + 6 ca_{3,54}) g_{i^{++},i} g_{i^{++},i^+}^3 - 2 T (2 - T + T^2) \epsilon^3 (ca_{1,10}^3 + 6 ca_{3,54}) g_{i^{++},i^+}^4
 \end{aligned}$$

(Alt) In[ ]:=

**rhs = 1**

(Alt) Out[ ]:=

1

(Alt) In[ ]:=

**me = Exponent[lhs - rhs, T, Min]**

(Alt) Out[ ]:=

-3

(Alt) In[ ]:=

**covars = DeleteCases[Variables[lhs - rhs], T | (ca | cb | cc | cd) \_]**

(Alt) Out[ ]:=

{ $\epsilon$ ,  $g_{i^{++},i}$ ,  $g_{i^{++},i^+}$ }

(Alt) In[ ]:=

**eqnsR11 =****(Factor[#] == 0) & /@ Union[Last /@ CoefficientRules[Expand[T<sup>-me</sup> (lhs - rhs)], covars]]**

(Alt) Out[ ]:=

$$\left\{ \begin{aligned} &4 T^2 (ca_{1,10}^3 + 6 ca_{3,54}) = 0, \quad 4 T^3 (ca_{1,10}^3 + 6 ca_{3,54}) = 0, \quad -6 (-1 + T) T^3 (ca_{1,10}^3 + 6 ca_{3,54}) = 0, \\ &4 (-1 + T)^2 T^3 (ca_{1,10}^3 + 6 ca_{3,54}) = 0, \quad -2 T^4 (2 - T + T^2) (ca_{1,10}^3 + 6 ca_{3,54}) = 0, \\ &3 T (3 T ca_{1,2} ca_{1,10}^2 + 2 ca_{1,10}^3 + 2 T ca_{3,29} + 12 ca_{3,54} + 12 T ca_{3,54} + 2 T ca_{1,10} cb_{2,10}) = 0, \\ &3 T^2 (3 T ca_{1,2} ca_{1,10}^2 + 4 ca_{1,10}^3 - 2 T ca_{1,10}^3 + 2 T ca_{3,29} + 24 ca_{3,54} + 2 T ca_{1,10} cb_{2,10}) = 0, \\ &-T^3 (9 T ca_{1,2} ca_{1,10}^2 + 12 ca_{1,10}^3 - 6 T ca_{1,10}^3 + \\ &\quad 4 T^2 ca_{1,10}^3 + 6 T ca_{3,29} + 72 ca_{3,54} + 24 T^2 ca_{3,54} + 6 T ca_{1,10} cb_{2,10}) = 0, \\ &-3 (-1 + T) T^2 (3 T ca_{1,2} ca_{1,10}^2 + 4 ca_{1,10}^3 - 2 T ca_{1,10}^3 + 2 T ca_{3,29} + 24 ca_{3,54} + 2 T ca_{1,10} cb_{2,10}) = 0, \\ &\frac{1}{2} (-T^3 ca_{1,2}^2 ca_{1,10} + 18 T ca_{1,2} ca_{1,10}^2 + 18 T^2 ca_{1,2} ca_{1,10}^2 + 8 ca_{1,10}^3 - 4 T ca_{1,10}^3 - 4 T^2 ca_{1,10}^3 + \\ &\quad 4 T^3 ca_{1,10} ca_{2,1} + 12 T ca_{3,29} + 12 T^2 ca_{3,29} + 48 ca_{3,54} + 48 T ca_{3,54} + 48 T^2 ca_{3,54} + 2 T^3 ca_{1,2} cb_{2,10} + \\ &\quad 12 T ca_{1,10} cb_{2,10} + 12 T^2 ca_{1,10} cb_{2,10} - 2 T^3 cb_{3,5} + 4 T^3 cb_{3,9} + 2 T^2 cb_{3,10} - 2 T^3 cb_{3,10}) = 0, \\ &-\frac{1}{2} T (T^3 ca_{1,2}^2 ca_{1,10} - 36 T ca_{1,2} ca_{1,10}^2 - 24 ca_{1,10}^3 + 24 T ca_{1,10}^3 - 4 T^3 ca_{1,10} ca_{2,1} - 24 T ca_{3,29} - \\ &\quad 144 ca_{3,54} - 2 T^3 ca_{1,2} cb_{2,10} - 24 T ca_{1,10} cb_{2,10} - 4 T^3 cb_{3,9} - 2 T^2 cb_{3,10} + 2 T^3 cb_{3,10}) = 0, \\ &\frac{1}{2} T (T^3 ca_{1,2}^2 ca_{1,10} - 18 T ca_{1,2} ca_{1,10}^2 - 18 T^2 ca_{1,2} ca_{1,10}^2 - 8 ca_{1,10}^3 + 4 T ca_{1,10}^3 + 4 T^2 ca_{1,10}^3 - \\ &\quad 4 T^3 ca_{1,10} ca_{2,1} - 12 T ca_{3,29} - 12 T^2 ca_{3,29} - 48 ca_{3,54} - 48 T ca_{3,54} - 48 T^2 ca_{3,54} - 2 T^3 ca_{1,2} cb_{2,10} - \\ &\quad 12 T ca_{1,10} cb_{2,10} - 12 T^2 ca_{1,10} cb_{2,10} + 2 T^3 cb_{3,5} - 4 T^3 cb_{3,9} - 2 T^2 cb_{3,10} + 2 T^3 cb_{3,10}) = 0, \\ &\frac{1}{2} T^2 (T^3 ca_{1,2}^2 ca_{1,10} - 36 T ca_{1,2} ca_{1,10}^2 - 18 T^2 ca_{1,2} ca_{1,10}^2 - 24 ca_{1,10}^3 + 12 T ca_{1,10}^3 - 4 T^3 ca_{1,10} ca_{2,1} - \\ &\quad 24 T ca_{3,29} - 12 T^2 ca_{3,29} - 144 ca_{3,54} - 72 T ca_{3,54} - 72 T^2 ca_{3,54} - 2 T^3 ca_{1,2} cb_{2,10} - 24 T ca_{1,10} \\ &\quad cb_{2,10} - 12 T^2 ca_{1,10} cb_{2,10} - 4 T^3 cb_{3,9} - 2 T^2 cb_{3,10} + 2 T^3 cb_{3,10}) = 0, \quad T^3 (ca_{3,1} + cc_{3,1}) = 0 \end{aligned} \right\}$$

(Alt) In[ ]:=

```
vars = Cases[Variables[r_d[1, i1, j1] + r_d[-1, i2, j2] + \gamma_d[1, k]], (ca | cb | cc | cd) __]
{sol} = Solve[eqnsR3 \cup eqnsR2b \cup eqnsR2c \cup eqnsR11 \cup eqnsR1r \cup eqnsSwp, vars]
sol /. Rule -> Set;
r_d[1, i, j]
\gamma_d[1, k]
```

(Alt) Out[ ]:=

```
{ca_{1,2}, ca_{1,10}, ca_{2,1}, ca_{3,29}, ca_{3,54}, cb_{2,10}, cb_{3,2}, cb_{3,5}, cb_{3,9}, cb_{3,10}, cc_{3,1}}
```

**Solve:** Equations may not give solutions for all "solve" variables.

(Alt) Out[ ]:=

$$\left\{ \left\{ ca_{3,29} \rightarrow \frac{1}{2} \left( -3 ca_{1,2} ca_{1,10}^2 + 2 ca_{1,10}^3 - 2 ca_{1,10} cb_{2,10} \right), ca_{3,54} \rightarrow -\frac{1}{6} ca_{1,10}^3, cb_{3,5} \rightarrow 0, \right. \right. \\ \left. \left. cb_{3,9} \rightarrow \frac{T ca_{1,2}^2 ca_{1,10} - 4 T ca_{1,10} ca_{2,1} - 2 T ca_{1,2} cb_{2,10} - 2 cb_{3,10} + 2 T cb_{3,10}}{4 T}, cc_{3,1} \rightarrow -ca_{3,1} \right\} \right\}$$

(Alt) Out[ ]:=

$$\begin{aligned} & -\frac{1}{2} \in ca_{1,2} + \in p_i x_i ca_{1,2} - \in p_j x_i ca_{1,2} - \frac{1}{2} \in^2 p_i x_i ca_{1,2}^2 + \frac{1}{2} \in^2 p_j x_i ca_{1,2}^2 + \frac{1}{2} \in p_i p_j x_i^2 ca_{1,10} - \\ & \frac{1}{2} T \in p_i p_j x_i^2 ca_{1,10} - \frac{1}{2} \in p_j^2 x_i^2 ca_{1,10} + \frac{1}{2} T \in p_j^2 x_i^2 ca_{1,10} + \in p_i p_j x_i x_j ca_{1,10} - \in p_j^2 x_i x_j ca_{1,10} - \\ & \frac{1}{2} \in^2 p_i p_j x_i^2 ca_{1,2} ca_{1,10} + T \in^2 p_i p_j x_i^2 ca_{1,2} ca_{1,10} + \frac{1}{2} \in^2 p_j^2 x_i^2 ca_{1,2} ca_{1,10} - T \in^2 p_j^2 x_i^2 ca_{1,2} ca_{1,10} - \\ & 2 \in^2 p_i p_j x_i x_j ca_{1,2} ca_{1,10} + 2 \in^2 p_j^2 x_i x_j ca_{1,2} ca_{1,10} - \frac{1}{4} \in^3 p_i p_j x_i^2 ca_{1,2}^2 ca_{1,10} - \\ & T \in^3 p_i p_j x_i^2 ca_{1,2}^2 ca_{1,10} + \frac{1}{4} \in^3 p_j^2 x_i^2 ca_{1,2}^2 ca_{1,10} + T \in^3 p_j^2 x_i^2 ca_{1,2}^2 ca_{1,10} + 2 \in^3 p_i p_j x_i x_j ca_{1,2}^2 ca_{1,10} - \\ & 2 \in^3 p_j^2 x_i x_j ca_{1,2}^2 ca_{1,10} + \frac{1}{2} \in^2 p_i p_j x_i^2 ca_{1,10}^2 - \frac{1}{2} T \in^2 p_i p_j x_i^2 ca_{1,10}^2 - \frac{1}{2} \in^2 p_j^2 x_i^2 ca_{1,10}^2 + \\ & \frac{1}{2} T \in^2 p_j^2 x_i^2 ca_{1,10}^2 - \frac{1}{3} \in^2 p_i^2 p_j x_i^3 ca_{1,10}^2 + \frac{1}{3} T \in^2 p_i^2 p_j x_i^3 ca_{1,10}^2 + \frac{5}{6} \in^2 p_i p_j^2 x_i^3 ca_{1,10}^2 - \\ & \frac{2}{3} T \in^2 p_i p_j^2 x_i^3 ca_{1,10}^2 - \frac{1}{6} T^2 \in^2 p_i p_j^2 x_i^3 ca_{1,10}^2 - \frac{1}{2} \in^2 p_j^3 x_i^3 ca_{1,10}^2 + \frac{1}{3} T \in^2 p_j^3 x_i^3 ca_{1,10}^2 + \\ & \frac{1}{6} T^2 \in^2 p_j^3 x_i^3 ca_{1,10}^2 + \in^2 p_i p_j x_i x_j ca_{1,10}^2 - \in^2 p_j^2 x_i x_j ca_{1,10}^2 - \frac{1}{2} \in^2 p_i^2 p_j x_i^2 x_j ca_{1,10}^2 + \\ & \in^2 p_i p_j^2 x_i^2 x_j ca_{1,10}^2 + \frac{1}{2} T \in^2 p_i p_j^2 x_i^2 x_j ca_{1,10}^2 - \frac{1}{2} \in^2 p_j^3 x_i^2 x_j ca_{1,10}^2 - \frac{1}{2} T \in^2 p_j^3 x_i^2 x_j ca_{1,10}^2 - \\ & \frac{1}{2} \in^2 p_i p_j^2 x_i x_j^2 ca_{1,10}^2 + \frac{1}{2} \in^2 p_j^3 x_i x_j^2 ca_{1,10}^2 - \in^3 p_i p_j x_i^2 ca_{1,2} ca_{1,10}^2 + \frac{3}{2} T \in^3 p_i p_j x_i^2 ca_{1,2} ca_{1,10}^2 + \\ & \in^3 p_j^2 x_i^2 ca_{1,2} ca_{1,10}^2 - \frac{3}{2} T \in^3 p_j^2 x_i^2 ca_{1,2} ca_{1,10}^2 + \frac{7}{6} \in^3 p_i^2 p_j x_i^3 ca_{1,2} ca_{1,10}^2 - \frac{4}{3} T \in^3 p_i^2 p_j x_i^3 ca_{1,2} ca_{1,10}^2 - \\ & \frac{19}{6} \in^3 p_i p_j^2 x_i^3 ca_{1,2} ca_{1,10}^2 + \frac{19}{6} T \in^3 p_i p_j^2 x_i^3 ca_{1,2} ca_{1,10}^2 + \frac{1}{2} T^2 \in^3 p_i p_j^2 x_i^3 ca_{1,2} ca_{1,10}^2 + \\ & 2 \in^3 p_j^3 x_i^3 ca_{1,2} ca_{1,10}^2 - \frac{11}{6} T \in^3 p_j^3 x_i^3 ca_{1,2} ca_{1,10}^2 - \frac{1}{2} T^2 \in^3 p_j^3 x_i^3 ca_{1,2} ca_{1,10}^2 - 3 \in^3 p_i p_j x_i x_j ca_{1,2} ca_{1,10}^2 + \\ & 3 \in^3 p_j^2 x_i x_j ca_{1,2} ca_{1,10}^2 + 2 \in^3 p_i^2 p_j x_i^2 x_j ca_{1,2} ca_{1,10}^2 - 5 \in^3 p_i p_j^2 x_i^2 x_j ca_{1,2} ca_{1,10}^2 - \end{aligned}$$

$$\begin{aligned}
& \frac{3}{2} T \epsilon^3 p_i p_j^2 x_i^2 x_j ca_{1,2} ca_{1,10}^2 + 3 \epsilon^3 p_j^3 x_i^2 x_j ca_{1,2} ca_{1,10}^2 + \frac{3}{2} T \epsilon^3 p_j^3 x_i^2 x_j ca_{1,2} ca_{1,10}^2 + \\
& \frac{3}{2} \epsilon^3 p_i p_j^2 x_i x_j^2 ca_{1,2} ca_{1,10}^2 - \frac{3}{2} \epsilon^3 p_j^3 x_i x_j^2 ca_{1,2} ca_{1,10}^2 + \frac{1}{2} \epsilon^3 p_i p_j x_i^2 ca_{1,10}^3 - \frac{1}{2} T \epsilon^3 p_i p_j x_i^2 ca_{1,10}^3 - \\
& \frac{1}{2} \epsilon^3 p_j^2 x_i^2 ca_{1,10}^3 + \frac{1}{2} T \epsilon^3 p_j^2 x_i^2 ca_{1,10}^3 - \frac{2}{3} \epsilon^3 p_i^2 p_j x_i^3 ca_{1,10}^3 + \frac{2}{3} T \epsilon^3 p_i^2 p_j x_i^3 ca_{1,10}^3 + 2 \epsilon^3 p_i p_j^2 x_i^3 ca_{1,10}^3 - \\
& \frac{5}{3} T \epsilon^3 p_i p_j^2 x_i^3 ca_{1,10}^3 - \frac{1}{3} T^2 \epsilon^3 p_i p_j^2 x_i^3 ca_{1,10}^3 - \frac{4}{3} \epsilon^3 p_j^3 x_i^3 ca_{1,10}^3 + T \epsilon^3 p_j^3 x_i^3 ca_{1,10}^3 + \\
& \frac{1}{3} T^2 \epsilon^3 p_j^3 x_i^3 ca_{1,10}^3 + \frac{1}{8} \epsilon^3 p_i^3 p_j x_i^4 ca_{1,10}^3 - \frac{1}{8} T \epsilon^3 p_i^3 p_j x_i^4 ca_{1,10}^3 - \frac{1}{2} \epsilon^3 p_i^2 p_j^2 x_i^4 ca_{1,10}^3 + \\
& \frac{1}{8} T \epsilon^3 p_i^2 p_j^2 x_i^4 ca_{1,10}^3 + \frac{3}{8} T^2 \epsilon^3 p_i^2 p_j^2 x_i^4 ca_{1,10}^3 + \frac{13}{24} \epsilon^3 p_i p_j^3 x_i^4 ca_{1,10}^3 + \frac{3}{8} T \epsilon^3 p_i p_j^3 x_i^4 ca_{1,10}^3 - \\
& \frac{7}{8} T^2 \epsilon^3 p_i p_j^3 x_i^4 ca_{1,10}^3 - \frac{1}{24} T^3 \epsilon^3 p_i p_j^3 x_i^4 ca_{1,10}^3 - \frac{1}{6} \epsilon^3 p_j^4 x_i^4 ca_{1,10}^3 - \frac{3}{8} T \epsilon^3 p_j^4 x_i^4 ca_{1,10}^3 + \\
& \frac{1}{2} T^2 \epsilon^3 p_j^4 x_i^4 ca_{1,10}^3 + \frac{1}{24} T^3 \epsilon^3 p_j^4 x_i^4 ca_{1,10}^3 + \epsilon^3 p_i p_j x_i x_j ca_{1,10}^3 - \epsilon^3 p_j^2 x_i x_j ca_{1,10}^3 - \\
& \epsilon^3 p_i^2 p_j x_i^2 x_j ca_{1,10}^3 + \frac{5}{2} \epsilon^3 p_i p_j^2 x_i^2 x_j ca_{1,10}^3 + T \epsilon^3 p_i p_j^2 x_i^2 x_j ca_{1,10}^3 - \frac{3}{2} \epsilon^3 p_j^3 x_i^2 x_j ca_{1,10}^3 - \\
& T \epsilon^3 p_j^3 x_i^2 x_j ca_{1,10}^3 + \frac{1}{6} \epsilon^3 p_i^3 p_j x_i^3 x_j ca_{1,10}^3 - \frac{7}{6} T \epsilon^3 p_i^3 p_j x_i^3 x_j ca_{1,10}^3 - \epsilon^3 p_i p_j^3 x_i^3 x_j ca_{1,10}^3 + \\
& \frac{17}{6} T \epsilon^3 p_i p_j^3 x_i^3 x_j ca_{1,10}^3 + \frac{1}{6} T^2 \epsilon^3 p_i p_j^3 x_i^3 x_j ca_{1,10}^3 + \frac{5}{6} \epsilon^3 p_j^4 x_i^3 x_j ca_{1,10}^3 - \frac{5}{3} T \epsilon^3 p_j^4 x_i^3 x_j ca_{1,10}^3 - \\
& \frac{1}{6} T^2 \epsilon^3 p_j^4 x_i^3 x_j ca_{1,10}^3 - \epsilon^3 p_i p_j^2 x_i x_j^2 ca_{1,10}^3 + \epsilon^3 p_j^3 x_i x_j^2 ca_{1,10}^3 + \epsilon^3 p_i^2 p_j^2 x_i^2 x_j^2 ca_{1,10}^3 - \\
& \frac{5}{2} \epsilon^3 p_i p_j^3 x_i^2 x_j^2 ca_{1,10}^3 - \frac{1}{4} T \epsilon^3 p_i p_j^3 x_i^2 x_j^2 ca_{1,10}^3 + \frac{3}{2} \epsilon^3 p_j^4 x_i^2 x_j^2 ca_{1,10}^3 + \frac{1}{4} T \epsilon^3 p_j^4 x_i^2 x_j^2 ca_{1,10}^3 + \\
& \frac{1}{6} \epsilon^3 p_i p_j^3 x_i x_j^3 ca_{1,10}^3 - \frac{1}{6} \epsilon^3 p_j^4 x_i x_j^3 ca_{1,10}^3 + \epsilon^2 ca_{2,1} - 2 \epsilon^2 p_i x_i ca_{2,1} + 2 \epsilon^2 p_j x_i ca_{2,1} + \\
& 4 \epsilon^3 p_i x_i ca_{1,2} ca_{2,1} - 4 \epsilon^3 p_j x_i ca_{1,2} ca_{2,1} + \epsilon^3 p_i p_j x_i^2 ca_{1,10} ca_{2,1} - 2 T \epsilon^3 p_i p_j x_i^2 ca_{1,10} ca_{2,1} - \\
& \epsilon^3 p_j^2 x_i^2 ca_{1,10} ca_{2,1} + 2 T \epsilon^3 p_j^2 x_i^2 ca_{1,10} ca_{2,1} + 4 \epsilon^3 p_i p_j x_i x_j ca_{1,10} ca_{2,1} - 4 \epsilon^3 p_j^2 x_i x_j ca_{1,10} ca_{2,1} + \\
& \epsilon^3 ca_{3,1} - \frac{1}{2} \epsilon^2 p_i p_j x_i^2 cb_{2,10} + \frac{1}{2} T \epsilon^2 p_i p_j x_i^2 cb_{2,10} + \frac{1}{2} \epsilon^2 p_j^2 x_i^2 cb_{2,10} - \frac{1}{2} T \epsilon^2 p_j^2 x_i^2 cb_{2,10} - \\
& \epsilon^2 p_i p_j x_i x_j cb_{2,10} + \epsilon^2 p_j^2 x_i x_j cb_{2,10} + \frac{1}{2} \epsilon^3 p_i p_j x_i^2 ca_{1,2} cb_{2,10} - T \epsilon^3 p_i p_j x_i^2 ca_{1,2} cb_{2,10} - \\
& \frac{1}{2} \epsilon^3 p_j^2 x_i^2 ca_{1,2} cb_{2,10} + T \epsilon^3 p_j^2 x_i^2 ca_{1,2} cb_{2,10} + 2 \epsilon^3 p_i p_j x_i x_j ca_{1,2} cb_{2,10} - 2 \epsilon^3 p_j^2 x_i x_j ca_{1,2} cb_{2,10} - \\
& \epsilon^3 p_i p_j x_i^2 ca_{1,10} cb_{2,10} + T \epsilon^3 p_i p_j x_i^2 ca_{1,10} cb_{2,10} + \epsilon^3 p_j^2 x_i^2 ca_{1,10} cb_{2,10} - T \epsilon^3 p_j^2 x_i^2 ca_{1,10} cb_{2,10} + \\
& \frac{2}{3} \epsilon^3 p_i^2 p_j x_i^3 ca_{1,10} cb_{2,10} - \frac{2}{3} T \epsilon^3 p_i^2 p_j x_i^3 ca_{1,10} cb_{2,10} - \frac{5}{3} \epsilon^3 p_i p_j^2 x_i^3 ca_{1,10} cb_{2,10} + \\
& \frac{4}{3} T \epsilon^3 p_i p_j^2 x_i^3 ca_{1,10} cb_{2,10} + \frac{1}{3} T^2 \epsilon^3 p_i p_j^2 x_i^3 ca_{1,10} cb_{2,10} + \epsilon^3 p_j^3 x_i^3 ca_{1,10} cb_{2,10} - \\
& \frac{2}{3} T \epsilon^3 p_j^3 x_i^3 ca_{1,10} cb_{2,10} - \frac{1}{3} T^2 \epsilon^3 p_j^3 x_i^3 ca_{1,10} cb_{2,10} - 2 \epsilon^3 p_i p_j x_i x_j ca_{1,10} cb_{2,10} + \\
& 2 \epsilon^3 p_j^2 x_i x_j ca_{1,10} cb_{2,10} + \epsilon^3 p_i^2 p_j x_i^2 x_j ca_{1,10} cb_{2,10} - 2 \epsilon^3 p_i p_j^2 x_i^2 x_j ca_{1,10} cb_{2,10} -
\end{aligned}$$

$$\begin{aligned} & T \epsilon^3 p_i p_j^2 x_i^2 x_j ca_{1,10} cb_{2,10} + \epsilon^3 p_j^3 x_i^2 x_j ca_{1,10} cb_{2,10} + T \epsilon^3 p_j^3 x_i^2 x_j ca_{1,10} cb_{2,10} + \\ & \epsilon^3 p_i p_j^2 x_i x_j^2 ca_{1,10} cb_{2,10} - \epsilon^3 p_j^3 x_i x_j^2 ca_{1,10} cb_{2,10} - \epsilon^3 p_i x_i cb_{3,2} + \epsilon^3 p_j x_i cb_{3,2} - \frac{1}{2} \epsilon^3 p_i p_j x_i^2 cb_{3,10} + \\ & \frac{1}{2} T \epsilon^3 p_i p_j x_i^2 cb_{3,10} + \frac{1}{2} \epsilon^3 p_j^2 x_i^2 cb_{3,10} - \frac{1}{2} T \epsilon^3 p_j^2 x_i^2 cb_{3,10} - \epsilon^3 p_i p_j x_i x_j cb_{3,10} + \epsilon^3 p_j^2 x_i x_j cb_{3,10} \end{aligned}$$

(Alt) Out[ ]:=

$$\begin{aligned} & \frac{1}{2} \epsilon ca_{1,2} + \epsilon p_k x_k ca_{1,10} - \epsilon^2 p_k x_k ca_{1,2} ca_{1,10} + \frac{1}{2} \epsilon^3 p_k x_k ca_{1,2}^2 ca_{1,10} - \epsilon^2 ca_{2,1} + \\ & 2 \epsilon^3 p_k x_k ca_{1,10} ca_{2,1} - \epsilon^3 ca_{3,1} - \epsilon^2 p_k x_k cb_{2,10} + \epsilon^3 p_k x_k ca_{1,2} cb_{2,10} - \epsilon^3 p_k x_k cb_{3,10} \end{aligned}$$

## R1r

(Alt) In[ ]:=

```
lhs = CF[Module[{es = {i, i^+}},
  Times[
    Normal@Series[Exp[r_d[1, 0, -1, i, i^+]], {epsilon, 0, d}],
    Exp[Sum[g_alpha_beta pi_alpha xi_beta, {alpha, es}, {beta, es}]]
  ] // Zip((p_#&/@es) | (x_#&/@es) // Expand
] // . {
  g_i_beta -> delta_i_beta + T g_i^+_beta + (1 - T) g_i^+_beta, g_i^+_beta -> delta_i^+_beta + g_i^+_beta,
  g_alpha_i -> T^-1 (g_alpha_i^+ - delta_alpha_i^+), g_alpha_i^+ -> T g_alpha_i^+ - (1 - T) delta_alpha_i^+ - T delta_alpha_i^+
}
```

(Alt) Out[ ]:=

$$1 + \frac{1}{6} \epsilon^3 (-ca_{1,2}^3 + 12 ca_{1,2} ca_{2,1} + 12 ca_{3,1} - 6 cb_{3,2})$$

(Alt) In[ ]:=

$$rhs = 1$$

(Alt) Out[ ]:=

$$1$$

(Alt) In[ ]:=

```
me = Exponent[lhs - rhs, T, Min]
```

(Alt) Out[ ]:=

$$0$$

(Alt) In[ ]:=

```
covars = DeleteCases[Variables[lhs - rhs], T | (ca | cb | cc | cd) _]
```

(Alt) Out[ ]:=

$$\{\epsilon\}$$

(Alt) In[ ]:=

```
eqnsR1r =
  (Factor[#] == 0) & /@ Union[Last /@ CoefficientRules[Expand[T^-me (lhs - rhs)], covars]]
```

(Alt) Out[ ]:=

$$\left\{ \frac{1}{6} (-ca_{1,2}^3 + 12 ca_{1,2} ca_{2,1} + 12 ca_{3,1} - 6 cb_{3,2}) == 0 \right\}$$

(Alt) In[ ]:=

```
vars = Cases[Variables[r_d[1, i1, j1] + r_d[-1, i2, j2] + \gamma_d[1, k]], (ca | cb | cc | cd) __]
{sol} = Solve[eqnsR3 \cup eqnsR2b \cup eqnsR2c \cup eqnsR11 \cup eqnsR1r \cup eqnsSwp, vars]
sol /. Rule -> Set;
r_d[1, i, j]
\gamma_d[1, k]
```

(Alt) Out[ ]:=

```
{ca_{1,2}, ca_{1,10}, ca_{2,1}, ca_{3,1}, cb_{2,10}, cb_{3,2}, cb_{3,10}}
```

**Solve:** Equations may not give solutions for all "solve" variables.

(Alt) Out[ ]:=

```
{ {cb_{3,2} -> \frac{1}{6} (-ca_{1,2}^3 + 12 ca_{1,2} ca_{2,1} + 12 ca_{3,1}) } }
```

(Alt) Out[ ]:=

$$\begin{aligned}
& -\frac{1}{2} \in ca_{1,2} + \in p_i x_i ca_{1,2} - \in p_j x_i ca_{1,2} - \frac{1}{2} \in^2 p_i x_i ca_{1,2}^2 + \frac{1}{2} \in^2 p_j x_i ca_{1,2}^2 + \frac{1}{6} \in^3 p_i x_i ca_{1,2}^3 - \\
& \frac{1}{6} \in^3 p_j x_i ca_{1,2}^3 + \frac{1}{2} \in p_i p_j x_i^2 ca_{1,10} - \frac{1}{2} T \in p_i p_j x_i^2 ca_{1,10} - \frac{1}{2} \in p_j^2 x_i^2 ca_{1,10} + \frac{1}{2} T \in p_j^2 x_i^2 ca_{1,10} + \\
& \in p_i p_j x_i x_j ca_{1,10} - \in p_j^2 x_i x_j ca_{1,10} - \frac{1}{2} \in^2 p_i p_j x_i^2 ca_{1,2} ca_{1,10} + T \in^2 p_i p_j x_i^2 ca_{1,2} ca_{1,10} + \\
& \frac{1}{2} \in^2 p_j^2 x_i^2 ca_{1,2} ca_{1,10} - T \in^2 p_j^2 x_i^2 ca_{1,2} ca_{1,10} - 2 \in^2 p_i p_j x_i x_j ca_{1,2} ca_{1,10} + 2 \in^2 p_j^2 x_i x_j ca_{1,2} ca_{1,10} - \\
& \frac{1}{4} \in^3 p_i p_j x_i^2 ca_{1,2} ca_{1,10} - T \in^3 p_i p_j x_i^2 ca_{1,2} ca_{1,10} + \frac{1}{4} \in^3 p_j^2 x_i^2 ca_{1,2} ca_{1,10} + T \in^3 p_j^2 x_i^2 ca_{1,2} ca_{1,10} + \\
& 2 \in^3 p_i p_j x_i x_j ca_{1,2} ca_{1,10} - 2 \in^3 p_j^2 x_i x_j ca_{1,2} ca_{1,10} + \frac{1}{2} \in^2 p_i p_j x_i^2 ca_{1,10}^2 - \frac{1}{2} T \in^2 p_i p_j x_i^2 ca_{1,10}^2 - \\
& \frac{1}{2} \in^2 p_j^2 x_i^2 ca_{1,10}^2 + \frac{1}{2} T \in^2 p_j^2 x_i^2 ca_{1,10}^2 - \frac{1}{3} \in^2 p_i^2 p_j x_i^3 ca_{1,10} + \frac{1}{3} T \in^2 p_i^2 p_j x_i^3 ca_{1,10} + \frac{5}{6} \in^2 p_i p_j^2 x_i^3 ca_{1,10}^2 - \\
& \frac{2}{3} T \in^2 p_i p_j^2 x_i^3 ca_{1,10}^2 - \frac{1}{6} T^2 \in^2 p_i p_j^2 x_i^3 ca_{1,10}^2 - \frac{1}{2} \in^2 p_j^3 x_i^3 ca_{1,10}^2 + \frac{1}{3} T \in^2 p_j^3 x_i^3 ca_{1,10}^2 + \\
& \frac{1}{6} T^2 \in^2 p_j^3 x_i^3 ca_{1,10}^2 + \in^2 p_i p_j x_i x_j ca_{1,10}^2 - \in^2 p_j^2 x_i x_j ca_{1,10}^2 - \frac{1}{2} \in^2 p_i^2 p_j x_i^2 x_j ca_{1,10}^2 + \\
& \in^2 p_i p_j^2 x_i^2 x_j ca_{1,10}^2 + \frac{1}{2} T \in^2 p_i p_j^2 x_i^2 x_j ca_{1,10}^2 - \frac{1}{2} \in^2 p_j^3 x_i^2 x_j ca_{1,10}^2 - \frac{1}{2} T \in^2 p_j^3 x_i^2 x_j ca_{1,10}^2 - \\
& \frac{1}{2} \in^2 p_i p_j^2 x_i x_j^2 ca_{1,10}^2 + \frac{1}{2} \in^2 p_j^3 x_i x_j^2 ca_{1,10}^2 - \in^3 p_i p_j x_i^2 ca_{1,2} ca_{1,10}^2 + \frac{3}{2} T \in^3 p_i p_j x_i^2 ca_{1,2} ca_{1,10}^2 + \\
& \in^3 p_j^2 x_i^2 ca_{1,2} ca_{1,10}^2 - \frac{3}{2} T \in^3 p_j^2 x_i^2 ca_{1,2} ca_{1,10}^2 + \frac{7}{6} \in^3 p_i^2 p_j x_i^3 ca_{1,2} ca_{1,10}^2 - \frac{4}{3} T \in^3 p_i^2 p_j x_i^3 ca_{1,2} ca_{1,10}^2 - \\
& \frac{19}{6} \in^3 p_i p_j^2 x_i^3 ca_{1,2} ca_{1,10}^2 + \frac{19}{6} T \in^3 p_i p_j^2 x_i^3 ca_{1,2} ca_{1,10}^2 + \frac{1}{2} T^2 \in^3 p_i p_j^2 x_i^3 ca_{1,2} ca_{1,10}^2 + \\
& 2 \in^3 p_j^3 x_i^3 ca_{1,2} ca_{1,10}^2 - \frac{11}{6} T \in^3 p_j^3 x_i^3 ca_{1,2} ca_{1,10}^2 - \frac{1}{2} T^2 \in^3 p_j^3 x_i^3 ca_{1,2} ca_{1,10}^2 - 3 \in^3 p_i p_j x_i x_j ca_{1,2} ca_{1,10}^2 + \\
& 3 \in^3 p_j^2 x_i x_j ca_{1,2} ca_{1,10}^2 + 2 \in^3 p_i^2 p_j x_i^2 x_j ca_{1,2} ca_{1,10}^2 - 5 \in^3 p_i p_j^2 x_i^2 x_j ca_{1,2} ca_{1,10}^2 - \\
& \frac{3}{2} T \in^3 p_i p_j^2 x_i^2 x_j ca_{1,2} ca_{1,10}^2 + 3 \in^3 p_j^3 x_i^2 x_j ca_{1,2} ca_{1,10}^2 + \frac{3}{2} T \in^3 p_j^3 x_i^2 x_j ca_{1,2} ca_{1,10}^2 +
\end{aligned}$$

$$\begin{aligned}
 & \frac{3}{2} \epsilon^3 p_i p_j^2 x_i x_j^2 ca_{1,2} ca_{1,10}^2 - \frac{3}{2} \epsilon^3 p_j^3 x_i x_j^2 ca_{1,2} ca_{1,10}^2 + \frac{1}{2} \epsilon^3 p_i p_j x_i^2 ca_{1,10}^3 - \frac{1}{2} T \epsilon^3 p_i p_j x_i^2 ca_{1,10}^3 - \\
 & \frac{1}{2} \epsilon^3 p_j^2 x_i^2 ca_{1,10}^3 + \frac{1}{2} T \epsilon^3 p_j^2 x_i^2 ca_{1,10}^3 - \frac{2}{3} \epsilon^3 p_i^2 p_j x_i^3 ca_{1,10}^3 + \frac{2}{3} T \epsilon^3 p_i^2 p_j x_i^3 ca_{1,10}^3 + 2 \epsilon^3 p_i p_j^2 x_i^3 ca_{1,10}^3 - \\
 & \frac{5}{3} T \epsilon^3 p_i p_j^2 x_i^3 ca_{1,10}^3 - \frac{1}{3} T^2 \epsilon^3 p_i p_j^2 x_i^3 ca_{1,10}^3 - \frac{4}{3} \epsilon^3 p_j^3 x_i^3 ca_{1,10}^3 + T \epsilon^3 p_j^3 x_i^3 ca_{1,10}^3 + \frac{1}{3} T^2 \epsilon^3 p_j^3 x_i^3 ca_{1,10}^3 + \\
 & \frac{1}{8} \epsilon^3 p_i^3 p_j x_i^4 ca_{1,10}^3 - \frac{1}{8} T \epsilon^3 p_i^3 p_j x_i^4 ca_{1,10}^3 - \frac{1}{2} \epsilon^3 p_i^2 p_j^2 x_i^4 ca_{1,10}^3 + \frac{1}{8} T \epsilon^3 p_i^2 p_j^2 x_i^4 ca_{1,10}^3 + \\
 & \frac{3}{8} T^2 \epsilon^3 p_i^2 p_j^2 x_i^4 ca_{1,10}^3 + \frac{13}{24} \epsilon^3 p_i p_j^3 x_i^4 ca_{1,10}^3 + \frac{3}{8} T \epsilon^3 p_i p_j^3 x_i^4 ca_{1,10}^3 - \frac{7}{8} T^2 \epsilon^3 p_i p_j^3 x_i^4 ca_{1,10}^3 - \\
 & \frac{1}{24} T^3 \epsilon^3 p_i p_j^3 x_i^4 ca_{1,10}^3 - \frac{1}{6} \epsilon^3 p_j^4 x_i^4 ca_{1,10}^3 - \frac{3}{8} T \epsilon^3 p_j^4 x_i^4 ca_{1,10}^3 + \frac{1}{2} T^2 \epsilon^3 p_j^4 x_i^4 ca_{1,10}^3 + \\
 & \frac{1}{24} T^3 \epsilon^3 p_j^4 x_i^4 ca_{1,10}^3 + \epsilon^3 p_i p_j x_i x_j ca_{1,10}^3 - \epsilon^3 p_j^2 x_i x_j ca_{1,10}^3 - \epsilon^3 p_i^2 p_j x_i^2 x_j ca_{1,10}^3 + \\
 & \frac{5}{2} \epsilon^3 p_i p_j^2 x_i^2 x_j ca_{1,10}^3 + T \epsilon^3 p_i p_j^2 x_i^2 x_j ca_{1,10}^3 - \frac{3}{2} \epsilon^3 p_j^3 x_i^2 x_j ca_{1,10}^3 - T \epsilon^3 p_j^3 x_i^2 x_j ca_{1,10}^3 + \\
 & \frac{1}{6} \epsilon^3 p_i^3 p_j x_i^3 x_j ca_{1,10}^3 - \frac{7}{6} T \epsilon^3 p_i^3 p_j x_i^3 x_j ca_{1,10}^3 - \epsilon^3 p_i p_j^3 x_i^3 x_j ca_{1,10}^3 + \frac{17}{6} T \epsilon^3 p_i p_j^3 x_i^3 x_j ca_{1,10}^3 + \\
 & \frac{1}{6} T^2 \epsilon^3 p_i p_j^3 x_i^3 x_j ca_{1,10}^3 + \frac{5}{6} \epsilon^3 p_j^4 x_i^3 x_j ca_{1,10}^3 - \frac{5}{3} T \epsilon^3 p_j^4 x_i^3 x_j ca_{1,10}^3 - \frac{1}{6} T^2 \epsilon^3 p_j^4 x_i^3 x_j ca_{1,10}^3 - \\
 & \epsilon^3 p_i p_j^2 x_i x_j^2 ca_{1,10}^3 + \epsilon^3 p_j^3 x_i x_j^2 ca_{1,10}^3 + \epsilon^3 p_i^2 p_j^2 x_i^2 x_j^2 ca_{1,10}^3 - \frac{5}{2} \epsilon^3 p_i p_j^2 x_i^2 x_j^2 ca_{1,10}^3 - \\
 & \frac{1}{4} T \epsilon^3 p_i p_j^3 x_i^2 x_j^2 ca_{1,10}^3 + \frac{3}{2} \epsilon^3 p_j^4 x_i^2 x_j^2 ca_{1,10}^3 + \frac{1}{4} T \epsilon^3 p_j^4 x_i^2 x_j^2 ca_{1,10}^3 + \frac{1}{6} \epsilon^3 p_i p_j^3 x_i x_j^3 ca_{1,10}^3 - \\
 & \frac{1}{6} \epsilon^3 p_j^4 x_i x_j^3 ca_{1,10}^3 + \epsilon^2 ca_{2,1} - 2 \epsilon^2 p_i x_i ca_{2,1} + 2 \epsilon^2 p_j x_i ca_{2,1} + 2 \epsilon^3 p_i x_i ca_{1,2} ca_{2,1} - \\
 & 2 \epsilon^3 p_j x_i ca_{1,2} ca_{2,1} + \epsilon^3 p_i p_j x_i^2 ca_{1,10} ca_{2,1} - 2 T \epsilon^3 p_i p_j x_i^2 ca_{1,10} ca_{2,1} - \epsilon^3 p_j^2 x_i^2 ca_{1,10} ca_{2,1} + \\
 & 2 T \epsilon^3 p_j^2 x_i^2 ca_{1,10} ca_{2,1} + 4 \epsilon^3 p_i p_j x_i x_j ca_{1,10} ca_{2,1} - 4 \epsilon^3 p_j^2 x_i x_j ca_{1,10} ca_{2,1} + \epsilon^3 ca_{3,1} - \\
 & 2 \epsilon^3 p_i x_i ca_{3,1} + 2 \epsilon^3 p_j x_i ca_{3,1} - \frac{1}{2} \epsilon^2 p_i p_j x_i^2 cb_{2,10} + \frac{1}{2} T \epsilon^2 p_i p_j x_i^2 cb_{2,10} + \frac{1}{2} \epsilon^2 p_j^2 x_i^2 cb_{2,10} - \\
 & \frac{1}{2} T \epsilon^2 p_j^2 x_i^2 cb_{2,10} - \epsilon^2 p_i p_j x_i x_j cb_{2,10} + \epsilon^2 p_j^2 x_i x_j cb_{2,10} + \frac{1}{2} \epsilon^3 p_i p_j x_i^2 ca_{1,2} cb_{2,10} - \\
 & T \epsilon^3 p_i p_j x_i^2 ca_{1,2} cb_{2,10} - \frac{1}{2} \epsilon^3 p_j^2 x_i^2 ca_{1,2} cb_{2,10} + T \epsilon^3 p_j^2 x_i^2 ca_{1,2} cb_{2,10} + 2 \epsilon^3 p_i p_j x_i x_j ca_{1,2} cb_{2,10} - \\
 & 2 \epsilon^3 p_j^2 x_i x_j ca_{1,2} cb_{2,10} - \epsilon^3 p_i p_j x_i^2 ca_{1,10} cb_{2,10} + T \epsilon^3 p_i p_j x_i^2 ca_{1,10} cb_{2,10} + \epsilon^3 p_j^2 x_i^2 ca_{1,10} cb_{2,10} - \\
 & T \epsilon^3 p_j^2 x_i^2 ca_{1,10} cb_{2,10} + \frac{2}{3} \epsilon^3 p_i^2 p_j x_i^3 ca_{1,10} cb_{2,10} - \frac{2}{3} T \epsilon^3 p_i^2 p_j x_i^3 ca_{1,10} cb_{2,10} - \\
 & \frac{5}{3} \epsilon^3 p_i p_j^2 x_i^3 ca_{1,10} cb_{2,10} + \frac{4}{3} T \epsilon^3 p_i p_j^2 x_i^3 ca_{1,10} cb_{2,10} + \frac{1}{3} T^2 \epsilon^3 p_i p_j^2 x_i^3 ca_{1,10} cb_{2,10} + \\
 & \epsilon^3 p_j^3 x_i^3 ca_{1,10} cb_{2,10} - \frac{2}{3} T \epsilon^3 p_j^3 x_i^3 ca_{1,10} cb_{2,10} - \frac{1}{3} T^2 \epsilon^3 p_j^3 x_i^3 ca_{1,10} cb_{2,10} - \\
 & 2 \epsilon^3 p_i p_j x_i x_j ca_{1,10} cb_{2,10} + 2 \epsilon^3 p_j^2 x_i x_j ca_{1,10} cb_{2,10} + \epsilon^3 p_i^2 p_j x_i^2 x_j ca_{1,10} cb_{2,10} - \\
 & 2 \epsilon^3 p_i p_j^2 x_i^2 x_j ca_{1,10} cb_{2,10} - T \epsilon^3 p_i p_j^2 x_i^2 x_j ca_{1,10} cb_{2,10} + \epsilon^3 p_j^3 x_i^2 x_j ca_{1,10} cb_{2,10} +
 \end{aligned}$$

$$T \epsilon^3 p_j^3 x_i^2 x_j ca_{1,10} cb_{2,10} + \epsilon^3 p_i p_j^2 x_i x_j^2 ca_{1,10} cb_{2,10} - \epsilon^3 p_j^3 x_i x_j^2 ca_{1,10} cb_{2,10} - \frac{1}{2} \epsilon^3 p_i p_j x_i^2 cb_{3,10} +$$

$$\frac{1}{2} T \epsilon^3 p_i p_j x_i^2 cb_{3,10} + \frac{1}{2} \epsilon^3 p_j^2 x_i^2 cb_{3,10} - \frac{1}{2} T \epsilon^3 p_j^2 x_i^2 cb_{3,10} - \epsilon^3 p_i p_j x_i x_j cb_{3,10} + \epsilon^3 p_j^2 x_i x_j cb_{3,10}$$

(Alt) Out[ ]:=

$$\frac{1}{2} \epsilon ca_{1,2} + \epsilon p_k x_k ca_{1,10} - \epsilon^2 p_k x_k ca_{1,2} ca_{1,10} + \frac{1}{2} \epsilon^3 p_k x_k ca_{1,2}^2 ca_{1,10} - \epsilon^2 ca_{2,1} +$$

$$2 \epsilon^3 p_k x_k ca_{1,10} ca_{2,1} - \epsilon^3 ca_{3,1} - \epsilon^2 p_k x_k cb_{2,10} + \epsilon^3 p_k x_k ca_{1,2} cb_{2,10} - \epsilon^3 p_k x_k cb_{3,10}$$

## Sw<sup>+</sup>

(Alt) In[ ]:=

```
lhs = CF[Module[{es = {i, j, i+, j+}},
  Times[
    Normal@Series[Exp[r_d[1, -1, -1, i, j] + \gamma_d[1, i+] + \gamma_d[1, j+]], {e, 0, d}],
    Exp[Sum[g_{\alpha,\beta} \pi_{\alpha} \xi_{\beta}, {\alpha, es}, {\beta, es}]]
  ] // Zip(p_{\#&/@es})U(x_{\#&/@es}) // Expand
] // gRules_{1,i,j}
]
```

(Alt) Out[ ]:=

$$1 - \frac{1}{2} \epsilon ca_{1,2} + \frac{1}{8} \epsilon^2 (ca_{1,2}^2 + 8 ca_{2,1}) + \frac{1}{48} \epsilon^3 (-ca_{1,2}^3 - 24 ca_{1,2} ca_{2,1} + 48 ca_{3,1}) + \epsilon ca_{1,2} g_{i^+,i^+} +$$

$$\epsilon^2 (-ca_{1,2}^2 - 2 ca_{2,1}) g_{i^+,i^+} + \frac{1}{24} \epsilon^3 (13 ca_{1,2}^3 + 96 ca_{1,2} ca_{2,1} - 48 ca_{3,1}) g_{i^+,i^+} + \epsilon^2 ca_{1,2}^2 g_{i^+,i^+} -$$

$$\frac{1}{2} \epsilon^3 ca_{1,2} (3 ca_{1,2}^2 + 8 ca_{2,1}) g_{i^+,i^+} + \epsilon^3 ca_{1,2}^3 g_{i^+,i^+} - \epsilon ca_{1,2} g_{j^+,i^+} + \epsilon^2 (ca_{1,2}^2 + 2 ca_{2,1}) g_{j^+,i^+} +$$

$$\frac{1}{24} \epsilon^3 (-13 ca_{1,2}^3 - 96 ca_{1,2} ca_{2,1} + 48 ca_{3,1}) g_{j^+,i^+} + \frac{(-1 + T) \epsilon ca_{1,10} g_{i^+,i^+} g_{j^+,i^+}}{T} -$$

$$\frac{\epsilon^2 (4 T ca_{1,2}^2 - 7 ca_{1,2} ca_{1,10} + 5 T ca_{1,2} ca_{1,10} + 2 ca_{1,10}^2 - 2 T ca_{1,10}^2 - 2 cb_{2,10} + 2 T cb_{2,10}) g_{i^+,i^+} g_{j^+,i^+}}{2 T} +$$

$$\frac{1}{8 T} \epsilon^3 (24 T ca_{1,2}^3 - 49 ca_{1,2}^2 ca_{1,10} + 25 T ca_{1,2} ca_{1,10} + 36 ca_{1,2} ca_{1,10}^2 - 28 T ca_{1,2} ca_{1,10}^2 -$$

$$8 ca_{1,10}^3 + 8 T ca_{1,10}^3 + 64 T ca_{1,2} ca_{2,1} - 56 ca_{1,10} ca_{2,1} + 40 T ca_{1,10} ca_{2,1} - 28 ca_{1,2} cb_{2,10} +$$

$$20 T ca_{1,2} cb_{2,10} + 16 ca_{1,10} cb_{2,10} - 16 T ca_{1,10} cb_{2,10} + 8 cb_{3,10} - 8 T cb_{3,10}) g_{i^+,i^+} g_{j^+,i^+} +$$

$$\frac{(-1 + T) \epsilon^2 (3 ca_{1,2} - ca_{1,10}) ca_{1,10} g_{i^+,i^+} g_{j^+,i^+}}{T} - \frac{1}{2 T} \epsilon^3 (6 T ca_{1,2}^3 - 24 ca_{1,2}^2 ca_{1,10} +$$

$$18 T ca_{1,2}^2 ca_{1,10} + 17 ca_{1,2} ca_{1,10}^2 - 15 T ca_{1,2} ca_{1,10}^2 - 4 ca_{1,10}^3 + 4 T ca_{1,10}^3 - 12 ca_{1,10} ca_{2,1} +$$

$$12 T ca_{1,10} ca_{2,1} - 6 ca_{1,2} cb_{2,10} + 6 T ca_{1,2} cb_{2,10} + 4 ca_{1,10} cb_{2,10} - 4 T ca_{1,10} cb_{2,10}) g_{i^+,i^+} g_{j^+,i^+} +$$

$$\frac{(-1 + T) \epsilon^3 ca_{1,10} (6 ca_{1,2}^2 - 4 ca_{1,2} ca_{1,10} + ca_{1,10}^2) g_{i^+,i^+} g_{j^+,i^+}}{T} + \epsilon ca_{1,10} g_{i^+,j^+} g_{j^+,i^+} +$$

$$\frac{1}{2} \epsilon^2 (-5 ca_{1,2} ca_{1,10} + 2 ca_{1,10}^2 - 2 cb_{2,10}) g_{i^+,j^+} g_{j^+,i^+} + \frac{1}{8} \epsilon^3$$

$$(25 ca_{1,2}^2 ca_{1,10} - 28 ca_{1,2} ca_{1,10}^2 + 8 ca_{1,10}^3 + 40 ca_{1,10} ca_{2,1} + 20 ca_{1,2} cb_{2,10} - 16 ca_{1,10} cb_{2,10} - 8 cb_{3,10})$$



$$\begin{aligned}
 & 3 T c a_{1,2} c b_{2,10} + 3 T^2 c a_{1,2} c b_{2,10} + 4 c a_{1,10} c b_{2,10} - 4 T c a_{1,10} c b_{2,10} \Big) g_{j^+,i^+}^3 - \\
 & \frac{6 (-1 + T)^2 \epsilon^2 c a_{1,10}^2 g_{i^+,i^+} g_{j^+,i^+}^3}{T^2} + \frac{1}{T^3} (-1 + T) \epsilon^3 c a_{1,10} (18 T^2 c a_{1,2}^2 - 55 T c a_{1,2} c a_{1,10} + \\
 & 23 T^2 c a_{1,2} c a_{1,10} + 6 c a_{1,10}^2 + 24 T c a_{1,10}^2 - 18 T^2 c a_{1,10}^2 - 12 T c b_{2,10} + 12 T^2 c b_{2,10}) g_{i^+,i^+} g_{j^+,i^+}^3 - \\
 & \frac{5 (-1 + T)^2 \epsilon^3 c a_{1,10}^2 (9 T c a_{1,2} - 4 c a_{1,10} - 4 T c a_{1,10}) g_{i^+,i^+}^2 g_{j^+,i^+}^3}{T^3} + \\
 & \frac{15 (-1 + T)^3 \epsilon^3 c a_{1,10}^3 g_{i^+,i^+}^3 g_{j^+,i^+}^3}{T^3} - \\
 & \frac{6 (-1 + T) \epsilon^2 c a_{1,10}^2 g_{i^+,j^+} g_{j^+,i^+}^3}{T} + \\
 & \frac{1}{T^2} 3 \epsilon^3 c a_{1,10} (3 T^2 c a_{1,2}^2 - 15 T c a_{1,2} c a_{1,10} + 8 T^2 c a_{1,2} c a_{1,10} + \\
 & 2 c a_{1,10}^2 + 8 T c a_{1,10}^2 - 6 T^2 c a_{1,10}^2 - 4 T c b_{2,10} + 4 T^2 c b_{2,10}) g_{i^+,j^+} g_{j^+,i^+}^3 - \\
 & \frac{8 (-1 + T) \epsilon^3 c a_{1,10}^2 (9 T c a_{1,2} - 5 c a_{1,10} - 5 T c a_{1,10}) g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^3}{T^2} + \\
 & \frac{45 (-1 + T)^2 \epsilon^3 c a_{1,10}^3 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^3}{T^2} - \\
 & \frac{3 \epsilon^3 c a_{1,10}^2 (6 T c a_{1,2} - 5 c a_{1,10} - 3 T c a_{1,10}) g_{i^+,j^+} g_{j^+,i^+}^3}{T} + \\
 & \frac{36 (-1 + T) \epsilon^3 c a_{1,10}^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^3}{T} + \\
 & 6 \epsilon^3 c a_{1,10}^3 g_{i^+,j^+} g_{j^+,i^+}^3 + \frac{3 (-1 + T)^2 \epsilon^2 c a_{1,10}^2 g_{j^+,i^+}^4}{T^2} - \\
 & \frac{1}{2 T^3} (-1 + T) \epsilon^3 c a_{1,10} (12 T^2 c a_{1,2}^2 - 55 T c a_{1,2} c a_{1,10} + 27 T^2 c a_{1,2} c a_{1,10} + \\
 & 12 c a_{1,10}^2 + 18 T c a_{1,10}^2 - 18 T^2 c a_{1,10}^2 - 12 T c b_{2,10} + 12 T^2 c b_{2,10}) g_{j^+,i^+}^4 + \\
 & \frac{5 (-1 + T)^2 \epsilon^3 c a_{1,10}^2 (9 T c a_{1,2} - 8 c a_{1,10} - 2 T c a_{1,10}) g_{i^+,i^+} g_{j^+,i^+}^4}{T^3} - \\
 & \frac{45 (-1 + T)^3 \epsilon^3 c a_{1,10}^3 g_{i^+,i^+}^2 g_{j^+,i^+}^4}{T^3} + \\
 & \frac{2 (-1 + T) \epsilon^3 c a_{1,10}^2 (18 T c a_{1,2} - 20 c a_{1,10} - 5 T c a_{1,10}) g_{i^+,j^+} g_{j^+,i^+}^4}{T^2} - \\
 & \frac{90 (-1 + T)^2 \epsilon^3 c a_{1,10}^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^4}{T^2} - \\
 & \frac{36 (-1 + T) \epsilon^3 c a_{1,10}^3 g_{i^+,j^+} g_{j^+,i^+}^4}{T} -
 \end{aligned}$$

$$\begin{aligned}
 & \frac{5(-1+T)^2 \epsilon^3 (3Tca_{1,2} - 4ca_{1,10}) ca_{1,10}^2 g_{j^+,i^+}^5}{T^3} + \\
 & \frac{45(-1+T)^3 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+}^5 g_{j^+,i^+}^5}{T^3} + \frac{45(-1+T)^2 \epsilon^3 ca_{1,10}^3 g_{i^+,j^+}^5 g_{j^+,i^+}^5}{T^2} - \\
 & \frac{15(-1+T)^3 \epsilon^3 ca_{1,10}^3 g_{j^+,i^+}^6}{T^3} + \epsilon ca_{1,10} g_{i^+,i^+} g_{j^+,j^+} + \\
 & \frac{1}{2} \epsilon^2 (-5ca_{1,2}ca_{1,10} + 2ca_{1,10}^2 - 2cb_{2,10}) g_{i^+,i^+} g_{j^+,j^+} + \frac{1}{8} \epsilon^3 \\
 & (25ca_{1,2}^2ca_{1,10} - 28ca_{1,2}ca_{1,10}^2 + 8ca_{1,10}^3 + 40ca_{1,10}ca_{2,1} + 20ca_{1,2}cb_{2,10} - 16ca_{1,10}cb_{2,10} - 8cb_{3,10}) \\
 & g_{i^+,i^+} g_{j^+,j^+} + \epsilon^2 (2ca_{1,2} - ca_{1,10}) ca_{1,10} g_{i^+,i^+}^2 g_{j^+,j^+} + \\
 & \frac{1}{2} \epsilon^3 (-12ca_{1,2}^2ca_{1,10} + 13ca_{1,2}ca_{1,10}^2 - 4ca_{1,10}^3 - 8ca_{1,10}ca_{2,1} - 4ca_{1,2}cb_{2,10} + 4ca_{1,10}cb_{2,10}) \\
 & g_{i^+,i^+}^2 g_{j^+,j^+} + \epsilon^3 ca_{1,10} (3ca_{1,2}^2 - 3ca_{1,2}ca_{1,10} + ca_{1,10}^2) g_{i^+,i^+}^3 g_{j^+,j^+} - \\
 & 2\epsilon ca_{1,10} g_{j^+,i^+} g_{j^+,j^+} + \epsilon^2 (5ca_{1,2}ca_{1,10} - 2ca_{1,10}^2 + 2cb_{2,10}) g_{j^+,i^+} g_{j^+,j^+} + \\
 & \frac{1}{4} \epsilon^3 (-25ca_{1,2}^2ca_{1,10} + 28ca_{1,2}ca_{1,10}^2 - 8ca_{1,10}^3 - 40ca_{1,10}ca_{2,1} - 20ca_{1,2}cb_{2,10} + 16ca_{1,10}cb_{2,10} + \\
 & 8cb_{3,10}) g_{j^+,i^+} g_{j^+,j^+} - \frac{2\epsilon^2 ca_{1,10} (4Tca_{1,2} - 2ca_{1,10} - Tca_{1,10}) g_{i^+,i^+} g_{j^+,i^+} g_{j^+,j^+}}{T} + \\
 & \frac{1}{T} \epsilon^3 (24Tca_{1,2}^2ca_{1,10} - 18ca_{1,2}ca_{1,10}^2 - 19Tca_{1,2}ca_{1,10}^2 + 10ca_{1,10}^3 + 4Tca_{1,10}^3 + \\
 & 16Tca_{1,10}ca_{2,1} + 8Tca_{1,2}cb_{2,10} - 8ca_{1,10}cb_{2,10} - 4Tca_{1,10}cb_{2,10}) g_{i^+,i^+} g_{j^+,i^+} g_{j^+,j^+} + \\
 & \frac{6(-1+T)\epsilon^2 ca_{1,10}^2 g_{i^+,i^+}^2 g_{j^+,i^+} g_{j^+,j^+}}{T} - \frac{1}{T} \epsilon^3 ca_{1,10} (18Tca_{1,2}^2 - 45ca_{1,2}ca_{1,10} + \\
 & 15Tca_{1,2}ca_{1,10} + 30ca_{1,10}^2 - 16Tca_{1,10}^2 - 12cb_{2,10} + 12Tcb_{2,10}) g_{i^+,i^+}^2 g_{j^+,i^+} g_{j^+,j^+} + \\
 & \frac{4(-1+T)\epsilon^3 (6ca_{1,2} - 5ca_{1,10}) ca_{1,10}^2 g_{i^+,i^+}^3 g_{j^+,i^+} g_{j^+,j^+}}{T} - 2\epsilon^2 ca_{1,10}^2 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} + \\
 & \epsilon^3 ca_{1,10} (7ca_{1,2}ca_{1,10} - 4ca_{1,10}^2 + 4cb_{2,10}) g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} + \\
 & 8\epsilon^2 ca_{1,10}^2 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} - \\
 & 4\epsilon^3 ca_{1,10} (11ca_{1,2}ca_{1,10} - 8ca_{1,10}^2 + 4cb_{2,10}) g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} + \\
 & 36\epsilon^3 (ca_{1,2} - ca_{1,10}) ca_{1,10}^2 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} + \\
 & \frac{6\epsilon^2 (Tca_{1,2} - ca_{1,10}) ca_{1,10} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} - \\
 & \frac{1}{T} 3\epsilon^3 (6Tca_{1,2}^2ca_{1,10} - 9ca_{1,2}ca_{1,10}^2 - 3Tca_{1,2}ca_{1,10}^2 + \\
 & 5ca_{1,10}^3 + 4Tca_{1,10}ca_{2,1} + 2Tca_{1,2}cb_{2,10} - 4ca_{1,10}cb_{2,10}) g_{j^+,i^+}^2 g_{j^+,j^+} - \\
 & \frac{18(-1+T)\epsilon^2 ca_{1,10}^2 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,i^+} g_{j^+,j^+}}{T} + \frac{1}{T^2} 9\epsilon^3 ca_{1,10} (3T^2ca_{1,2}^2 - 15Tca_{1,2}ca_{1,10} + \\
 & 8T^2ca_{1,2}ca_{1,10} + 2ca_{1,10}^2 + 8Tca_{1,10}^2 - 6T^2ca_{1,10}^2 - 4Tcb_{2,10} + 4T^2cb_{2,10}) g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+} -
 \end{aligned}$$

$$\begin{aligned}
& \frac{12 (-1 + T) \epsilon^3 ca_{1,10}^2 (9 T ca_{1,2} - 5 ca_{1,10} - 5 T ca_{1,10}) g_{i^+,i^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+}}{T^2} + \\
& \frac{45 (-1 + T)^2 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+}^3 g_{j^+,i^+}^2 g_{j^+,j^+}}{T^2} - 12 \epsilon^2 ca_{1,10}^2 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+} + \\
& \frac{3 \epsilon^3 ca_{1,10} (22 T ca_{1,2} ca_{1,10} - 4 ca_{1,10}^2 - 15 T ca_{1,10}^2 + 8 T cb_{2,10}) g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} - \\
& \frac{18 \epsilon^3 ca_{1,10}^2 (6 T ca_{1,2} - 5 ca_{1,10} - 3 T ca_{1,10}) g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} + \\
& \frac{108 (-1 + T) \epsilon^3 ca_{1,10}^3 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} - 18 \epsilon^3 ca_{1,10}^3 g_{i^+,j^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+} + \\
& 54 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+} + \frac{12 (-1 + T) \epsilon^2 ca_{1,10}^2 g_{j^+,i^+}^3 g_{j^+,j^+}}{T} - \\
& \frac{1}{T^2} 6 \epsilon^3 ca_{1,10} (2 T^2 ca_{1,2}^2 - 15 T ca_{1,2} ca_{1,10} + 9 T^2 ca_{1,2} ca_{1,10} + \\
& \quad 4 ca_{1,10}^2 + 6 T ca_{1,10}^2 - 6 T^2 ca_{1,10}^2 - 4 T cb_{2,10} + 4 T^2 cb_{2,10}) g_{j^+,i^+}^3 g_{j^+,j^+} + \\
& \frac{8 (-1 + T) \epsilon^3 ca_{1,10}^2 (18 T ca_{1,2} - 20 ca_{1,10} - 5 T ca_{1,10}) g_{i^+,i^+} g_{j^+,i^+}^3 g_{j^+,j^+}}{T^2} - \\
& \frac{180 (-1 + T)^2 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+}^2 g_{j^+,i^+}^3 g_{j^+,j^+}}{T^2} + \\
& \frac{24 \epsilon^3 (3 T ca_{1,2} - 5 ca_{1,10}) ca_{1,10}^2 g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}}{T} - \\
& \frac{288 (-1 + T) \epsilon^3 ca_{1,10}^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}}{T} - 72 \epsilon^3 ca_{1,10}^3 g_{i^+,j^+}^2 g_{j^+,i^+}^3 g_{j^+,j^+} - \\
& \frac{20 (-1 + T) \epsilon^3 (3 T ca_{1,2} - 5 ca_{1,10}) ca_{1,10}^2 g_{j^+,i^+}^4 g_{j^+,j^+}}{T^2} + \\
& \frac{225 (-1 + T)^2 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+} g_{j^+,i^+}^4 g_{j^+,j^+}}{T^2} + \frac{180 (-1 + T) \epsilon^3 ca_{1,10}^3 g_{i^+,j^+} g_{j^+,i^+}^4 g_{j^+,j^+}}{T} - \\
& \frac{90 (-1 + T)^2 \epsilon^3 ca_{1,10}^3 g_{j^+,i^+}^5 g_{j^+,j^+}}{T^2} - \epsilon^2 ca_{1,10}^2 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+} + \\
& \frac{1}{2} \epsilon^3 ca_{1,10} (7 ca_{1,2} ca_{1,10} - 4 ca_{1,10}^2 + 4 cb_{2,10}) g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+} + \\
& 2 \epsilon^2 ca_{1,10}^2 g_{i^+,i^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+} - \epsilon^3 ca_{1,10} (11 ca_{1,2} ca_{1,10} - 8 ca_{1,10}^2 + 4 cb_{2,10}) g_{i^+,i^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+} + \\
& 6 \epsilon^3 (ca_{1,2} - ca_{1,10}) ca_{1,10}^2 g_{i^+,i^+}^3 g_{j^+,i^+}^2 g_{j^+,j^+} + 3 \epsilon^2 ca_{1,10}^2 g_{j^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+} - \\
& \frac{3}{2} \epsilon^3 ca_{1,10} (7 ca_{1,2} ca_{1,10} - 4 ca_{1,10}^2 + 4 cb_{2,10}) g_{j^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+} - 12 \epsilon^2 ca_{1,10}^2 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+} + \\
& \frac{3 \epsilon^3 ca_{1,10} (22 T ca_{1,2} ca_{1,10} - 4 ca_{1,10}^2 - 15 T ca_{1,10}^2 + 8 T cb_{2,10}) g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} -
\end{aligned}$$

$$\begin{aligned}
 & \frac{9 \epsilon^3 ca_{1,10}^2 (6 T ca_{1,2} - 5 ca_{1,10} - 3 T ca_{1,10}) g_{i^+,i^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T} + \\
 & \frac{36 (-1 + T) \epsilon^3 ca_{1,10}^3 g_{i^+,i^+}^3 g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T} + \\
 & 3 \epsilon^3 ca_{1,10}^3 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^2 - 36 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^2 + \\
 & 54 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^2 + 12 \epsilon^2 ca_{1,10}^2 g_{i^+,i^+}^2 g_{j^+,j^+}^2 - \\
 & \frac{6 \epsilon^3 ca_{1,10} (11 T ca_{1,2} ca_{1,10} - 4 ca_{1,10}^2 - 7 T ca_{1,10}^2 + 4 T cb_{2,10}) g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T} + \\
 & \frac{36 \epsilon^3 (3 T ca_{1,2} - 5 ca_{1,10}) ca_{1,10}^2 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T} - \\
 & \frac{216 (-1 + T) \epsilon^3 ca_{1,10}^3 g_{i^+,i^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T} + \\
 & 72 \epsilon^3 ca_{1,10}^3 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^2 - 216 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^2 - \\
 & \frac{30 \epsilon^3 ca_{1,10}^2 (2 T ca_{1,2} - 5 ca_{1,10} + T ca_{1,10}) g_{j^+,i^+}^3 g_{j^+,j^+}^2}{T} + \\
 & \frac{360 (-1 + T) \epsilon^3 ca_{1,10}^3 g_{i^+,i^+} g_{j^+,i^+}^3 g_{j^+,j^+}^2}{T} + 180 \epsilon^3 ca_{1,10}^3 g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}^2 - \\
 & \frac{180 (-1 + T) \epsilon^3 ca_{1,10}^3 g_{j^+,i^+}^4 g_{j^+,j^+}^2}{T} + \epsilon^3 ca_{1,10}^3 g_{i^+,i^+} g_{j^+,j^+}^3 - \\
 & 6 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+}^2 g_{j^+,j^+}^3 + 6 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+} g_{j^+,j^+}^3 - 4 \epsilon^3 ca_{1,10}^3 g_{j^+,i^+} g_{j^+,j^+}^3 + \\
 & 48 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+} g_{j^+,i^+} g_{j^+,j^+}^3 - 72 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+}^2 g_{j^+,i^+} g_{j^+,j^+}^3 - \\
 & 60 \epsilon^3 ca_{1,10}^3 g_{j^+,i^+}^2 g_{j^+,j^+}^3 + 180 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}^3 - 120 \epsilon^3 ca_{1,10}^3 g_{j^+,i^+}^3 g_{j^+,j^+}^3.
 \end{aligned}$$

(Alt) In[ ]:=

```

rhs = CF[Module[{es = {i, j, i+, j+}},
  Times[
    Normal@Series[Exp[r_d[1, i, j]], {e, 0, d}],
    Exp[Sum[g_{a,b} pi_a xi_b, {a, es}, {b, es}]]
  ] // Zip(p_&/@es)U(x_&/@es) // Expand
] // gRules_{1,i,j}

```

(Alt) Out[ ]:=

$$\begin{aligned}
 & 1 - \frac{1}{2} \epsilon ca_{1,2} + \frac{1}{8} \epsilon^2 (ca_{1,2}^2 + 8 ca_{2,1}) + \frac{1}{48} \epsilon^3 (-ca_{1,2}^3 - 24 ca_{1,2} ca_{2,1} + 48 ca_{3,1}) + \epsilon ca_{1,2} g_{i^+,i^+} + \\
 & \epsilon^2 (-ca_{1,2}^2 - 2 ca_{2,1}) g_{i^+,i^+} + \frac{1}{24} \epsilon^3 (13 ca_{1,2}^3 + 96 ca_{1,2} ca_{2,1} - 48 ca_{3,1}) g_{i^+,i^+} + \epsilon^2 ca_{1,2}^2 g_{i^+,i^+}^2 - \\
 & \frac{1}{2} \epsilon^3 ca_{1,2} (3 ca_{1,2}^2 + 8 ca_{2,1}) g_{i^+,i^+}^2 + \epsilon^3 ca_{1,2}^2 g_{i^+,i^+}^3 - \epsilon ca_{1,2} g_{j^+,i^+} + \epsilon^2 (ca_{1,2}^2 + 2 ca_{2,1}) g_{j^+,i^+} + \\
 & \frac{1}{24} \epsilon^3 (-13 ca_{1,2}^3 - 96 ca_{1,2} ca_{2,1} + 48 ca_{3,1}) g_{j^+,i^+} + \frac{(-1 + T) \epsilon ca_{1,10} g_{i^+,i^+} g_{j^+,i^+}}{T} -
 \end{aligned}$$

$$\begin{aligned}
 & \frac{\epsilon^2 (4 T ca_{1,2}^2 - 7 ca_{1,2} ca_{1,10} + 5 T ca_{1,2} ca_{1,10} + 2 ca_{1,10}^2 - 2 T ca_{1,10}^2 - 2 cb_{2,10} + 2 T cb_{2,10}) g_{i^+,i^+} g_{j^+,i^+}}{2 T} + \\
 & \frac{1}{8 T} \epsilon^3 (24 T ca_{1,2}^3 - 49 ca_{1,2}^2 ca_{1,10} + 25 T ca_{1,2}^2 ca_{1,10} + 36 ca_{1,2} ca_{1,10}^2 - 28 T ca_{1,2} ca_{1,10}^2 - \\
 & \quad 8 ca_{1,10}^3 + 8 T ca_{1,10}^3 + 64 T ca_{1,2} ca_{2,1} - 56 ca_{1,10} ca_{2,1} + 40 T ca_{1,10} ca_{2,1} - 28 ca_{1,2} cb_{2,10} + \\
 & \quad 20 T ca_{1,2} cb_{2,10} + 16 ca_{1,10} cb_{2,10} - 16 T ca_{1,10} cb_{2,10} + 8 cb_{3,10} - 8 T cb_{3,10}) g_{i^+,i^+} g_{j^+,i^+} + \\
 & \frac{(-1 + T) \epsilon^2 (3 ca_{1,2} - ca_{1,10}) ca_{1,10} g_{i^+,i^+} g_{j^+,i^+}}{T} - \frac{1}{2 T} \epsilon^3 (6 T ca_{1,2}^3 - 24 ca_{1,2}^2 ca_{1,10} + \\
 & \quad 18 T ca_{1,2}^2 ca_{1,10} + 17 ca_{1,2} ca_{1,10}^2 - 15 T ca_{1,2} ca_{1,10}^2 - 4 ca_{1,10}^3 + 4 T ca_{1,10}^3 - 12 ca_{1,10} ca_{2,1} + \\
 & \quad 12 T ca_{1,10} ca_{2,1} - 6 ca_{1,2} cb_{2,10} + 6 T ca_{1,2} cb_{2,10} + 4 ca_{1,10} cb_{2,10} - 4 T ca_{1,10} cb_{2,10}) g_{i^+,i^+} g_{j^+,i^+} + \\
 & \frac{(-1 + T) \epsilon^3 ca_{1,10} (6 ca_{1,2}^2 - 4 ca_{1,2} ca_{1,10} + ca_{1,10}^2) g_{i^+,i^+} g_{j^+,i^+}}{T} + \epsilon ca_{1,10} g_{i^+,j^+} g_{j^+,i^+} + \\
 & \frac{1}{2} \epsilon^2 (-5 ca_{1,2} ca_{1,10} + 2 ca_{1,10}^2 - 2 cb_{2,10}) g_{i^+,j^+} g_{j^+,i^+} + \frac{1}{8} \epsilon^3 \\
 & \quad (25 ca_{1,2}^2 ca_{1,10} - 28 ca_{1,2} ca_{1,10}^2 + 8 ca_{1,10}^3 + 40 ca_{1,10} ca_{2,1} + 20 ca_{1,2} cb_{2,10} - 16 ca_{1,10} cb_{2,10} - 8 cb_{3,10}) \\
 & \quad g_{i^+,j^+} g_{j^+,i^+} + 2 \epsilon^2 (2 ca_{1,2} - ca_{1,10}) ca_{1,10} g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} + \\
 & \epsilon^3 (-12 ca_{1,2}^2 ca_{1,10} + 13 ca_{1,2} ca_{1,10}^2 - 4 ca_{1,10}^3 - 8 ca_{1,10} ca_{2,1} - 4 ca_{1,2} cb_{2,10} + 4 ca_{1,10} cb_{2,10}) g_{i^+,i^+} \\
 & \quad g_{i^+,j^+} g_{j^+,i^+} + 3 \epsilon^3 ca_{1,10} (3 ca_{1,2}^2 - 3 ca_{1,2} ca_{1,10} + ca_{1,10}^2) g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} - \frac{(-1 + T) \epsilon ca_{1,10} g_{j^+,i^+}^2}{T} + \\
 & \frac{\epsilon^2 (2 T ca_{1,2}^2 - 7 ca_{1,2} ca_{1,10} + 5 T ca_{1,2} ca_{1,10} + 2 ca_{1,10}^2 - 2 T ca_{1,10}^2 - 2 cb_{2,10} + 2 T cb_{2,10}) g_{j^+,i^+}^2}{2 T} - \\
 & \frac{1}{8 T} \epsilon^3 (12 T ca_{1,2}^3 - 49 ca_{1,2}^2 ca_{1,10} + 25 T ca_{1,2}^2 ca_{1,10} + 36 ca_{1,2} ca_{1,10}^2 - 28 T ca_{1,2} ca_{1,10}^2 - \\
 & \quad 8 ca_{1,10}^3 + 8 T ca_{1,10}^3 + 32 T ca_{1,2} ca_{2,1} - 56 ca_{1,10} ca_{2,1} + 40 T ca_{1,10} ca_{2,1} - 28 ca_{1,2} cb_{2,10} + \\
 & \quad 20 T ca_{1,2} cb_{2,10} + 16 ca_{1,10} cb_{2,10} - 16 T ca_{1,10} cb_{2,10} + 8 cb_{3,10} - 8 T cb_{3,10}) g_{j^+,i^+}^2 - \\
 & \frac{(-1 + T) \epsilon^2 ca_{1,10} (6 T ca_{1,2} - 2 ca_{1,10} - T ca_{1,10}) g_{i^+,i^+} g_{j^+,i^+}^2}{T^2} + \\
 & \frac{1}{2 T^2} \epsilon^3 (6 T^2 ca_{1,2}^3 - 48 T ca_{1,2}^2 ca_{1,10} + 36 T^2 ca_{1,2}^2 ca_{1,10} + 22 ca_{1,2} ca_{1,10}^2 + 7 T ca_{1,2} ca_{1,10}^2 - \\
 & \quad 23 T^2 ca_{1,2} ca_{1,10}^2 - 10 ca_{1,10}^3 + 6 T ca_{1,10}^3 + 4 T^2 ca_{1,10}^3 - 24 T ca_{1,10} ca_{2,1} + 24 T^2 ca_{1,10} ca_{2,1} - \\
 & \quad 12 T ca_{1,2} cb_{2,10} + 12 T^2 ca_{1,2} cb_{2,10} + 8 ca_{1,10} cb_{2,10} - 4 T ca_{1,10} cb_{2,10} - 4 T^2 ca_{1,10} cb_{2,10}) \\
 & \quad g_{i^+,i^+} g_{j^+,i^+}^2 + \frac{3 (-1 + T)^2 \epsilon^2 ca_{1,10}^2 g_{i^+,i^+} g_{j^+,i^+}^2}{T^2} - \frac{1}{2 T^2} (-1 + T) \epsilon^3 ca_{1,10} \\
 & \quad (36 T ca_{1,2}^2 - 55 ca_{1,2} ca_{1,10} + 11 T ca_{1,2} ca_{1,10} + 30 ca_{1,10}^2 - 16 T ca_{1,10}^2 - 12 cb_{2,10} + 12 T cb_{2,10}) \\
 & \quad g_{i^+,i^+}^2 g_{j^+,i^+}^2 + \frac{5 (-1 + T)^2 \epsilon^3 (3 ca_{1,2} - 2 ca_{1,10}) ca_{1,10}^2 g_{i^+,i^+}^3 g_{j^+,i^+}^2}{T^2} - \\
 & \frac{\epsilon^2 ca_{1,10} (4 T ca_{1,2} - 2 ca_{1,10} - T ca_{1,10}) g_{i^+,j^+} g_{j^+,i^+}^2}{T} +
 \end{aligned}$$

$$\begin{aligned}
& \frac{1}{2T} \epsilon^3 \left( 24 T ca_{1,2}^2 ca_{1,10} - 18 ca_{1,2} ca_{1,10}^2 - 19 T ca_{1,2} ca_{1,10}^2 + 10 ca_{1,10}^3 + 4 T ca_{1,10}^3 + \right. \\
& \quad \left. 16 T ca_{1,10} ca_{2,1} + 8 T ca_{1,2} cb_{2,10} - 8 ca_{1,10} cb_{2,10} - 4 T ca_{1,10} cb_{2,10} \right) g_{i^+,j^+}^2 g_{j^+,i^+}^2 + \\
& \frac{6(-1+T) \epsilon^2 ca_{1,10}^2 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^2}{T} - \frac{1}{T} \epsilon^3 ca_{1,10} \left( 18 T ca_{1,2}^2 - 45 ca_{1,2} ca_{1,10} + \right. \\
& \quad \left. 15 T ca_{1,2} ca_{1,10} + 30 ca_{1,10}^2 - 16 T ca_{1,10}^2 - 12 cb_{2,10} + 12 T cb_{2,10} \right) g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^2 + \\
& \frac{6(-1+T) \epsilon^3 (6 ca_{1,2} - 5 ca_{1,10}) ca_{1,10}^2 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^2}{T} + 2 \epsilon^2 ca_{1,10}^2 g_{i^+,j^+}^2 g_{j^+,i^+}^2 - \\
& \epsilon^3 ca_{1,10} \left( 11 ca_{1,2} ca_{1,10} - 8 ca_{1,10}^2 + 4 cb_{2,10} \right) g_{i^+,j^+}^2 g_{j^+,i^+}^2 + \\
& 18 \epsilon^3 (ca_{1,2} - ca_{1,10}) ca_{1,10}^2 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^2 + \\
& \frac{(-1+T) \epsilon^2 (3 T ca_{1,2} - 2 ca_{1,10}) ca_{1,10} g_{j^+,i^+}^3}{T^2} - \\
& \frac{1}{T^2} \epsilon^3 \left( T^2 ca_{1,2}^3 - 12 T ca_{1,2}^2 ca_{1,10} + 9 T^2 ca_{1,2}^2 ca_{1,10} + 11 ca_{1,2} ca_{1,10}^2 - 5 T ca_{1,2} ca_{1,10}^2 - \right. \\
& \quad \left. 4 T^2 ca_{1,2} ca_{1,10}^2 - 5 ca_{1,10}^3 + 5 T ca_{1,10}^3 - 6 T ca_{1,10} ca_{2,1} + 6 T^2 ca_{1,10} ca_{2,1} - \right. \\
& \quad \left. 3 T ca_{1,2} cb_{2,10} + 3 T^2 ca_{1,2} cb_{2,10} + 4 ca_{1,10} cb_{2,10} - 4 T ca_{1,10} cb_{2,10} \right) g_{j^+,i^+}^3 - \\
& \frac{6(-1+T)^2 \epsilon^2 ca_{1,10}^2 g_{i^+,i^+} g_{j^+,i^+}^3}{T^2} + \frac{1}{T^3} (-1+T) \epsilon^3 ca_{1,10} \left( 18 T^2 ca_{1,2}^2 - 55 T ca_{1,2} ca_{1,10} + \right. \\
& \quad \left. 23 T^2 ca_{1,2} ca_{1,10} + 6 ca_{1,10}^2 + 24 T ca_{1,10}^2 - 18 T^2 ca_{1,10}^2 - 12 T cb_{2,10} + 12 T^2 cb_{2,10} \right) g_{i^+,i^+} g_{j^+,i^+}^3 - \\
& \frac{5(-1+T)^2 \epsilon^3 ca_{1,10}^2 (9 T ca_{1,2} - 4 ca_{1,10} - 4 T ca_{1,10}) g_{i^+,i^+}^2 g_{j^+,i^+}^3}{T^3} + \\
& \frac{15(-1+T)^3 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+}^3 g_{j^+,i^+}^3}{T^3} - \\
& \frac{6(-1+T) \epsilon^2 ca_{1,10}^2 g_{i^+,j^+} g_{j^+,i^+}^3}{T} + \\
& \frac{1}{T^2} 3 \epsilon^3 ca_{1,10} \left( 3 T^2 ca_{1,2}^2 - 15 T ca_{1,2} ca_{1,10} + 8 T^2 ca_{1,2} ca_{1,10} + \right. \\
& \quad \left. 2 ca_{1,10}^2 + 8 T ca_{1,10}^2 - 6 T^2 ca_{1,10}^2 - 4 T cb_{2,10} + 4 T^2 cb_{2,10} \right) g_{i^+,j^+} g_{j^+,i^+}^3 - \\
& \frac{8(-1+T) \epsilon^3 ca_{1,10}^2 (9 T ca_{1,2} - 5 ca_{1,10} - 5 T ca_{1,10}) g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^3}{T^2} + \\
& \frac{45(-1+T)^2 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^3}{T^2} - \\
& \frac{3 \epsilon^3 ca_{1,10}^2 (6 T ca_{1,2} - 5 ca_{1,10} - 3 T ca_{1,10}) g_{i^+,j^+}^2 g_{j^+,i^+}^3}{T} + \\
& \frac{36(-1+T) \epsilon^3 ca_{1,10}^3 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^3}{T} +
\end{aligned}$$

$$\begin{aligned}
 & 6 \epsilon^3 ca_{1,10}^3 g_{i^+,j^+}^3 g_{j^+,i^+}^3 + \frac{3(-1+T)^2 \epsilon^2 ca_{1,10}^2 g_{j^+,i^+}^4}{T^2} - \\
 & \frac{1}{2T^3} (-1+T) \epsilon^3 ca_{1,10} (12T^2 ca_{1,2}^2 - 55T ca_{1,2} ca_{1,10} + 27T^2 ca_{1,2} ca_{1,10} + \\
 & \quad 12ca_{1,10}^2 + 18T ca_{1,10}^2 - 18T^2 ca_{1,10}^2 - 12T cb_{2,10} + 12T^2 cb_{2,10}) g_{j^+,i^+}^4 + \\
 & \frac{5(-1+T)^2 \epsilon^3 ca_{1,10}^2 (9T ca_{1,2} - 8ca_{1,10} - 2T ca_{1,10}) g_{i^+,i^+} g_{j^+,i^+}^4}{T^3} - \\
 & \frac{45(-1+T)^3 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+}^2 g_{j^+,i^+}^4}{T^3} + \\
 & \frac{2(-1+T) \epsilon^3 ca_{1,10}^2 (18T ca_{1,2} - 20ca_{1,10} - 5T ca_{1,10}) g_{i^+,j^+} g_{j^+,i^+}^4}{T^2} - \\
 & \frac{90(-1+T)^2 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^4}{T^2} - \\
 & \frac{36(-1+T) \epsilon^3 ca_{1,10}^3 g_{i^+,j^+}^2 g_{j^+,i^+}^4}{T} - \\
 & \frac{5(-1+T)^2 \epsilon^3 (3T ca_{1,2} - 4ca_{1,10}) ca_{1,10}^2 g_{j^+,i^+}^5}{T^3} + \\
 & \frac{45(-1+T)^3 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+} g_{j^+,i^+}^5}{T^3} + \frac{45(-1+T)^2 \epsilon^3 ca_{1,10}^3 g_{i^+,j^+} g_{j^+,i^+}^5}{T^2} - \\
 & \frac{15(-1+T)^3 \epsilon^3 ca_{1,10}^3 g_{j^+,i^+}^6}{T^3} + \epsilon ca_{1,10} g_{i^+,i^+} g_{j^+,j^+} + \\
 & \frac{1}{2} \epsilon^2 (-5ca_{1,2} ca_{1,10} + 2ca_{1,10}^2 - 2cb_{2,10}) g_{i^+,i^+} g_{j^+,j^+} + \frac{1}{8} \epsilon^3 \\
 & \quad (25ca_{1,2}^2 ca_{1,10} - 28ca_{1,2} ca_{1,10}^2 + 8ca_{1,10}^3 + 40ca_{1,10} ca_{2,1} + 20ca_{1,2} cb_{2,10} - 16ca_{1,10} cb_{2,10} - 8cb_{3,10}) \\
 & \quad g_{i^+,i^+} g_{j^+,j^+} + \epsilon^2 (2ca_{1,2} - ca_{1,10}) ca_{1,10} g_{i^+,i^+}^2 g_{j^+,j^+} + \\
 & \frac{1}{2} \epsilon^3 (-12ca_{1,2}^2 ca_{1,10} + 13ca_{1,2} ca_{1,10}^2 - 4ca_{1,10}^3 - 8ca_{1,10} ca_{2,1} - 4ca_{1,2} cb_{2,10} + 4ca_{1,10} cb_{2,10}) \\
 & \quad g_{i^+,i^+}^2 g_{j^+,j^+} + \epsilon^3 ca_{1,10} (3ca_{1,2}^2 - 3ca_{1,2} ca_{1,10} + ca_{1,10}^2) g_{i^+,i^+}^3 g_{j^+,j^+} - \\
 & 2 \epsilon ca_{1,10} g_{j^+,i^+} g_{j^+,j^+} + \epsilon^2 (5ca_{1,2} ca_{1,10} - 2ca_{1,10}^2 + 2cb_{2,10}) g_{j^+,i^+} g_{j^+,j^+} + \\
 & \frac{1}{4} \epsilon^3 (-25ca_{1,2}^2 ca_{1,10} + 28ca_{1,2} ca_{1,10}^2 - 8ca_{1,10}^3 - 40ca_{1,10} ca_{2,1} - 20ca_{1,2} cb_{2,10} + 16ca_{1,10} cb_{2,10} + \\
 & \quad 8cb_{3,10}) g_{j^+,i^+} g_{j^+,j^+} - \frac{2 \epsilon^2 ca_{1,10} (4T ca_{1,2} - 2ca_{1,10} - T ca_{1,10}) g_{i^+,i^+} g_{j^+,i^+} g_{j^+,j^+}}{T} + \\
 & \frac{1}{T} \epsilon^3 (24T ca_{1,2}^2 ca_{1,10} - 18ca_{1,2} ca_{1,10}^2 - 19T ca_{1,2} ca_{1,10}^2 + 10ca_{1,10}^3 + 4T ca_{1,10}^3 + \\
 & \quad 16T ca_{1,10} ca_{2,1} + 8T ca_{1,2} cb_{2,10} - 8ca_{1,10} cb_{2,10} - 4T ca_{1,10} cb_{2,10}) g_{i^+,i^+} g_{j^+,i^+} g_{j^+,j^+} + \\
 & \frac{6(-1+T) \epsilon^2 ca_{1,10}^2 g_{i^+,i^+}^2 g_{j^+,i^+} g_{j^+,j^+}}{T} - \frac{1}{T} \epsilon^3 ca_{1,10} (18T ca_{1,2}^2 - 45ca_{1,2} ca_{1,10} +
 \end{aligned}$$

$$\begin{aligned}
 & \frac{15 T ca_{1,2} ca_{1,10} + 30 ca_{1,10}^2 - 16 T ca_{1,10}^2 - 12 cb_{2,10} + 12 T cb_{2,10}}{T} g_{i^+,i^+}^2 g_{j^+,i^+} g_{j^+,j^+} + \\
 & \frac{4 (-1 + T) \epsilon^3 (6 ca_{1,2} - 5 ca_{1,10}) ca_{1,10}^2 g_{i^+,i^+}^3 g_{j^+,i^+} g_{j^+,j^+}}{T} - 2 \epsilon^2 ca_{1,10}^2 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} + \\
 & \epsilon^3 ca_{1,10} (7 ca_{1,2} ca_{1,10} - 4 ca_{1,10}^2 + 4 cb_{2,10}) g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} + \\
 & 8 \epsilon^2 ca_{1,10}^2 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} - \\
 & \frac{4 \epsilon^3 ca_{1,10} (11 ca_{1,2} ca_{1,10} - 8 ca_{1,10}^2 + 4 cb_{2,10}) g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} +}{T} \\
 & \frac{36 \epsilon^3 (ca_{1,2} - ca_{1,10}) ca_{1,10}^2 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} +}{T} \\
 & \frac{6 \epsilon^2 (T ca_{1,2} - ca_{1,10}) ca_{1,10} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} - \\
 & \frac{1}{T} 3 \epsilon^3 (6 T ca_{1,2} ca_{1,10} - 9 ca_{1,2} ca_{1,10}^2 - 3 T ca_{1,2} ca_{1,10}^2 + \\
 & \quad 5 ca_{1,10}^3 + 4 T ca_{1,10} ca_{2,1} + 2 T ca_{1,2} cb_{2,10} - 4 ca_{1,10} cb_{2,10}) g_{j^+,i^+}^2 g_{j^+,j^+} - \\
 & \frac{18 (-1 + T) \epsilon^2 ca_{1,10}^2 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} + \frac{1}{T^2} 9 \epsilon^3 ca_{1,10} (3 T^2 ca_{1,2}^2 - 15 T ca_{1,2} ca_{1,10} + \\
 & \quad 8 T^2 ca_{1,2} ca_{1,10} + 2 ca_{1,10}^2 + 8 T ca_{1,10}^2 - 6 T^2 ca_{1,10}^2 - 4 T cb_{2,10} + 4 T^2 cb_{2,10}) g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+} - \\
 & \frac{12 (-1 + T) \epsilon^3 ca_{1,10}^2 (9 T ca_{1,2} - 5 ca_{1,10} - 5 T ca_{1,10}) g_{i^+,i^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+}}{T^2} + \\
 & \frac{45 (-1 + T)^2 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+}^3 g_{j^+,i^+}^2 g_{j^+,j^+}}{T^2} - 12 \epsilon^2 ca_{1,10}^2 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+} + \\
 & \frac{3 \epsilon^3 ca_{1,10} (22 T ca_{1,2} ca_{1,10} - 4 ca_{1,10}^2 - 15 T ca_{1,10}^2 + 8 T cb_{2,10}) g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} - \\
 & \frac{18 \epsilon^3 ca_{1,10}^2 (6 T ca_{1,2} - 5 ca_{1,10} - 3 T ca_{1,10}) g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} + \\
 & \frac{108 (-1 + T) \epsilon^3 ca_{1,10}^3 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} - 18 \epsilon^3 ca_{1,10}^3 g_{i^+,j^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+} + \\
 & 54 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+} + \frac{12 (-1 + T) \epsilon^2 ca_{1,10}^2 g_{j^+,i^+}^3 g_{j^+,j^+}}{T} - \\
 & \frac{1}{T^2} 6 \epsilon^3 ca_{1,10} (2 T^2 ca_{1,2}^2 - 15 T ca_{1,2} ca_{1,10} + 9 T^2 ca_{1,2} ca_{1,10} + \\
 & \quad 4 ca_{1,10}^2 + 6 T ca_{1,10}^2 - 6 T^2 ca_{1,10}^2 - 4 T cb_{2,10} + 4 T^2 cb_{2,10}) g_{j^+,i^+}^3 g_{j^+,j^+} + \\
 & \frac{8 (-1 + T) \epsilon^3 ca_{1,10}^2 (18 T ca_{1,2} - 20 ca_{1,10} - 5 T ca_{1,10}) g_{i^+,i^+} g_{j^+,i^+}^3 g_{j^+,j^+}}{T^2} - \\
 & \frac{180 (-1 + T)^2 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+}^2 g_{j^+,i^+}^3 g_{j^+,j^+}}{T^2} + \\
 & \frac{24 \epsilon^3 (3 T ca_{1,2} - 5 ca_{1,10}) ca_{1,10}^2 g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}}{T} - \\
 & \frac{288 (-1 + T) \epsilon^3 ca_{1,10}^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}}{T} - 72 \epsilon^3 ca_{1,10}^3 g_{i^+,j^+}^2 g_{j^+,i^+}^3 g_{j^+,j^+} -
 \end{aligned}$$

$$\begin{aligned}
& \frac{20 (-1 + T) \epsilon^3 (3 T ca_{1,2} - 5 ca_{1,10}) ca_{1,10}^2 g_{j^+,i^+}^4 g_{j^+,j^+}}{T^2} + \\
& \frac{225 (-1 + T)^2 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+}^4 g_{j^+,i^+}^4 g_{j^+,j^+}}{T^2} + \frac{180 (-1 + T) \epsilon^3 ca_{1,10}^3 g_{i^+,j^+}^4 g_{j^+,i^+}^4 g_{j^+,j^+}}{T} - \\
& \frac{90 (-1 + T)^2 \epsilon^3 ca_{1,10}^3 g_{j^+,i^+}^5 g_{j^+,j^+}}{T^2} - \epsilon^2 ca_{1,10}^2 g_{i^+,i^+}^2 g_{j^+,j^+}^2 + \\
& \frac{1}{2} \epsilon^3 ca_{1,10} (7 ca_{1,2} ca_{1,10} - 4 ca_{1,10}^2 + 4 cb_{2,10}) g_{i^+,i^+}^2 g_{j^+,j^+}^2 + \\
& 2 \epsilon^2 ca_{1,10}^2 g_{i^+,i^+}^2 g_{j^+,j^+}^2 - \epsilon^3 ca_{1,10} (11 ca_{1,2} ca_{1,10} - 8 ca_{1,10}^2 + 4 cb_{2,10}) g_{i^+,i^+}^2 g_{j^+,j^+}^2 + \\
& 6 \epsilon^3 (ca_{1,2} - ca_{1,10}) ca_{1,10}^2 g_{i^+,i^+}^3 g_{j^+,j^+}^2 + 3 \epsilon^2 ca_{1,10}^2 g_{j^+,i^+}^2 g_{j^+,j^+}^2 - \\
& \frac{3}{2} \epsilon^3 ca_{1,10} (7 ca_{1,2} ca_{1,10} - 4 ca_{1,10}^2 + 4 cb_{2,10}) g_{j^+,i^+}^2 g_{j^+,j^+}^2 - 12 \epsilon^2 ca_{1,10}^2 g_{i^+,i^+} g_{j^+,i^+} g_{j^+,j^+}^2 + \\
& \frac{3 \epsilon^3 ca_{1,10} (22 T ca_{1,2} ca_{1,10} - 4 ca_{1,10}^2 - 15 T ca_{1,10}^2 + 8 T cb_{2,10}) g_{i^+,i^+} g_{j^+,i^+} g_{j^+,j^+}^2}{T} - \\
& \frac{9 \epsilon^3 ca_{1,10}^2 (6 T ca_{1,2} - 5 ca_{1,10} - 3 T ca_{1,10}) g_{i^+,i^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T} + \\
& \frac{36 (-1 + T) \epsilon^3 ca_{1,10}^3 g_{i^+,i^+}^3 g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T} + \\
& 3 \epsilon^3 ca_{1,10}^3 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^2 - 36 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^2 + \\
& 54 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^2 + 12 \epsilon^2 ca_{1,10}^2 g_{j^+,i^+}^2 g_{j^+,j^+}^2 - \\
& \frac{6 \epsilon^3 ca_{1,10} (11 T ca_{1,2} ca_{1,10} - 4 ca_{1,10}^2 - 7 T ca_{1,10}^2 + 4 T cb_{2,10}) g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T} + \\
& \frac{36 \epsilon^3 (3 T ca_{1,2} - 5 ca_{1,10}) ca_{1,10}^2 g_{i^+,i^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T} - \\
& \frac{216 (-1 + T) \epsilon^3 ca_{1,10}^3 g_{i^+,i^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T} + \\
& 72 \epsilon^3 ca_{1,10}^3 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^2 - 216 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^2 - \\
& \frac{30 \epsilon^3 ca_{1,10}^2 (2 T ca_{1,2} - 5 ca_{1,10} + T ca_{1,10}) g_{j^+,i^+}^3 g_{j^+,j^+}^2}{T} + \\
& \frac{360 (-1 + T) \epsilon^3 ca_{1,10}^3 g_{i^+,i^+} g_{j^+,i^+}^3 g_{j^+,j^+}^2}{T} + 180 \epsilon^3 ca_{1,10}^3 g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}^2 - \\
& \frac{180 (-1 + T) \epsilon^3 ca_{1,10}^3 g_{j^+,i^+}^4 g_{j^+,j^+}^2}{T} + \epsilon^3 ca_{1,10}^3 g_{i^+,i^+}^3 g_{j^+,j^+}^3 - \\
& 6 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+}^2 g_{j^+,j^+}^3 + 6 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+}^3 g_{j^+,j^+}^3 - 4 \epsilon^3 ca_{1,10}^3 g_{j^+,i^+}^3 g_{j^+,j^+}^3 + \\
& 48 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+} g_{j^+,i^+}^3 g_{j^+,j^+}^3 - 72 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+}^2 g_{j^+,i^+}^3 g_{j^+,j^+}^3 - \\
& 60 \epsilon^3 ca_{1,10}^3 g_{j^+,i^+}^2 g_{j^+,j^+}^3 + 180 \epsilon^3 ca_{1,10}^3 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}^3 - 120 \epsilon^3 ca_{1,10}^3 g_{j^+,i^+}^3 g_{j^+,j^+}^3
\end{aligned}$$

(Alt) In[ ]:=

**me = Exponent [lhs - rhs, T, Min]**

(Alt) Out[ ]:=

$\infty$

(Alt) In[ ]:=

**covars = DeleteCases [Variables [lhs - rhs], T | (ca | cb | cc | cd) \_\_]**

(Alt) Out[ ]:=

{ }

(Alt) In[ ]:=

**eqnsSwp = { }**

(Alt) Out[ ]:=

{ }

## Solution

(Alt) In[ ]:=

**vars =**

**Cases [Variables [r<sub>d</sub>[1, i1, j1] + r<sub>d</sub>[-1, i2, j2] +  $\gamma_d$ [1, k1] +  $\gamma_d$ [-1, k2]], (ca | cb | cc | cd) \_\_]**

(Alt) Out[ ]:=

{ ca<sub>1,2</sub>, ca<sub>1,10</sub>, ca<sub>2,1</sub>, ca<sub>3,1</sub>, cb<sub>2,10</sub>, cb<sub>3,10</sub> }

(Alt) In[ ]:=

**{sol} = Solve [eqnsR3  $\cup$  eqnsR2b  $\cup$  eqnsR2c  $\cup$  eqnsR1l  $\cup$  eqnsR1r  $\cup$  eqnsSwp, vars]**

**Solve:** The solution set contains a full-dimensional component; use Reduce for complete solution information.

(Alt) Out[ ]:=

{ { } }

(Alt) In[ ]:=

**sol /. Rule  $\rightarrow$  Set**

(Alt) Out[ ]:=

{ }

(Alt) In[ ]:=

**r<sub>d</sub>[1, i, j] // CF**

**r<sub>d</sub>[-1, i, j] // CF**

**$\gamma_d$ [1, k] // CF**

**$\gamma_d$ [-1, k] // CF**

(Alt) Out[ ]:=

$$\begin{aligned}
 & -\frac{1}{2} \in ca_{1,2} + \in p_i x_i ca_{1,2} - \in p_j x_j ca_{1,2} - \frac{1}{2} (-1 + T) \in p_i p_j x_i^2 ca_{1,10} + \\
 & \frac{1}{2} (-1 + T) \in p_j^2 x_i^2 ca_{1,10} + \in p_i p_j x_i x_j ca_{1,10} - \in p_j^2 x_i x_j ca_{1,10} + \frac{1}{3} (-1 + T) \in^2 p_i^2 p_j x_i^3 ca_{1,10}^2 - \\
 & \frac{1}{6} (-1 + T) (5 + T) \in^2 p_i p_j^2 x_i^3 ca_{1,10}^2 + \frac{1}{6} (-1 + T) (3 + T) \in^2 p_j^3 x_i^3 ca_{1,10}^2 - \frac{1}{2} \in^2 p_i^2 p_j x_i^2 x_j ca_{1,10}^2 + \\
 & \frac{1}{2} (2 + T) \in^2 p_i p_j^2 x_i^2 x_j ca_{1,10}^2 - \frac{1}{2} (1 + T) \in^2 p_j^3 x_i^2 x_j ca_{1,10}^2 - \frac{1}{2} \in^2 p_i p_j^2 x_i x_j^2 ca_{1,10}^2 + \\
 & \frac{1}{2} \in^2 p_j^3 x_i x_j^2 ca_{1,10}^2 - \frac{1}{8} (-1 + T) \in^3 p_i^3 p_j x_i^4 ca_{1,10}^3 + \frac{1}{8} (-1 + T) (4 + 3 T) \in^3 p_i^2 p_j^2 x_i^4 ca_{1,10}^3 -
 \end{aligned}$$

$$\begin{aligned}
& \frac{1}{24} (-1 + T) (13 + 22 T + T^2) \epsilon^3 p_i p_j^3 x_i^4 ca_{1,10}^3 + \frac{1}{24} (-1 + T) (4 + 13 T + T^2) \epsilon^3 p_j^4 x_i^4 ca_{1,10}^3 + \\
& \frac{1}{6} \epsilon^3 p_i^3 p_j x_i^3 x_j ca_{1,10}^3 - \frac{7}{6} T \epsilon^3 p_i^2 p_j^2 x_i^3 x_j ca_{1,10}^3 + \frac{1}{6} (-6 + 17 T + T^2) \epsilon^3 p_i p_j^3 x_i^3 x_j ca_{1,10}^3 - \\
& \frac{1}{6} (-5 + 10 T + T^2) \epsilon^3 p_j^4 x_i^3 x_j ca_{1,10}^3 + \epsilon^3 p_i^2 p_j^2 x_i^2 x_j^2 ca_{1,10}^3 - \frac{1}{4} (10 + T) \epsilon^3 p_i p_j^3 x_i^2 x_j^2 ca_{1,10}^3 + \\
& \frac{1}{4} (6 + T) \epsilon^3 p_j^4 x_i^2 x_j^2 ca_{1,10}^3 + \frac{1}{6} \epsilon^3 p_i p_j^3 x_i x_j^3 ca_{1,10}^3 - \frac{1}{6} \epsilon^3 p_j^4 x_i x_j^3 ca_{1,10}^3 + \frac{1}{2} \epsilon^2 p_i x_i (-ca_{1,2}^2 - 4 ca_{2,1}) + \\
& \epsilon^2 ca_{2,1} + \frac{1}{2} \epsilon^2 p_j x_i (ca_{1,2}^2 + 4 ca_{2,1}) + \frac{1}{6} \epsilon^3 p_i x_i (ca_{1,2}^3 + 12 ca_{1,2} ca_{2,1} - 12 ca_{3,1}) + \epsilon^3 ca_{3,1} + \\
& \frac{1}{6} \epsilon^3 p_j x_i (-ca_{1,2}^3 - 12 ca_{1,2} ca_{2,1} + 12 ca_{3,1}) + \epsilon^2 p_i p_j x_i x_j (-2 ca_{1,2} ca_{1,10} + ca_{1,10}^2 - cb_{2,10}) + \\
& \epsilon^2 p_j^2 x_i x_j (2 ca_{1,2} ca_{1,10} - ca_{1,10}^2 + cb_{2,10}) + \epsilon^3 p_i^2 p_j x_i^2 x_j ca_{1,10} (2 ca_{1,2} ca_{1,10} - ca_{1,10}^2 + cb_{2,10}) + \\
& \frac{1}{2} \epsilon^3 p_i p_j^2 x_i x_j^2 ca_{1,10} (3 ca_{1,2} ca_{1,10} - 2 ca_{1,10}^2 + 2 cb_{2,10}) - \\
& \frac{1}{2} \epsilon^3 p_j^3 x_i x_j^2 ca_{1,10} (3 ca_{1,2} ca_{1,10} - 2 ca_{1,10}^2 + 2 cb_{2,10}) + \\
& \frac{1}{2} \epsilon^2 p_j^2 x_i^2 (ca_{1,2} ca_{1,10} - 2 T ca_{1,2} ca_{1,10} - ca_{1,10}^2 + T ca_{1,10}^2 + cb_{2,10} - T cb_{2,10}) + \\
& \frac{1}{2} \epsilon^2 p_i p_j x_i^2 (-ca_{1,2} ca_{1,10} + 2 T ca_{1,2} ca_{1,10} + ca_{1,10}^2 - T ca_{1,10}^2 - cb_{2,10} + T cb_{2,10}) + \\
& \frac{1}{2} \epsilon^3 p_j^3 x_i^2 x_j ca_{1,10} (6 ca_{1,2} ca_{1,10} + 3 T ca_{1,2} ca_{1,10} - 3 ca_{1,10}^2 - 2 T ca_{1,10}^2 + 2 cb_{2,10} + 2 T cb_{2,10}) - \\
& \frac{1}{2} \epsilon^3 p_i p_j^2 x_i^2 x_j ca_{1,10} (10 ca_{1,2} ca_{1,10} + 3 T ca_{1,2} ca_{1,10} - 5 ca_{1,10}^2 - 2 T ca_{1,10}^2 + 4 cb_{2,10} + 2 T cb_{2,10}) - \\
& \frac{1}{6} \epsilon^3 p_i^2 p_j x_i^3 ca_{1,10} (-7 ca_{1,2} ca_{1,10} + 8 T ca_{1,2} ca_{1,10} + 4 ca_{1,10}^2 - 4 T ca_{1,10}^2 - 4 cb_{2,10} + 4 T cb_{2,10}) - \\
& \frac{1}{6} \epsilon^3 p_j^3 x_i^3 ca_{1,10} (-12 ca_{1,2} ca_{1,10} + 11 T ca_{1,2} ca_{1,10} + 3 T^2 ca_{1,2} ca_{1,10} + \\
& 8 ca_{1,10}^2 - 6 T ca_{1,10}^2 - 2 T^2 ca_{1,10}^2 - 6 cb_{2,10} + 4 T cb_{2,10} + 2 T^2 cb_{2,10}) + \\
& \frac{1}{6} \epsilon^3 p_i p_j^2 x_i^3 ca_{1,10} (-19 ca_{1,2} ca_{1,10} + 19 T ca_{1,2} ca_{1,10} + 3 T^2 ca_{1,2} ca_{1,10} + 12 ca_{1,10}^2 - \\
& 10 T ca_{1,10}^2 - 2 T^2 ca_{1,10}^2 - 10 cb_{2,10} + 8 T cb_{2,10} + 2 T^2 cb_{2,10}) + \epsilon^3 p_i p_j x_i x_j \\
& (2 ca_{1,2}^2 ca_{1,10} - 3 ca_{1,2} ca_{1,10}^2 + ca_{1,10}^3 + 4 ca_{1,10} ca_{2,1} + 2 ca_{1,2} cb_{2,10} - 2 ca_{1,10} cb_{2,10} - cb_{3,10}) + \epsilon^3 p_j^2 \\
& x_i x_j (-2 ca_{1,2}^2 ca_{1,10} + 3 ca_{1,2} ca_{1,10}^2 - ca_{1,10}^3 - 4 ca_{1,10} ca_{2,1} - 2 ca_{1,2} cb_{2,10} + 2 ca_{1,10} cb_{2,10} + cb_{3,10}) + \\
& \frac{1}{4} \epsilon^3 p_j^2 x_i^2 (ca_{1,2}^2 ca_{1,10} + 4 T ca_{1,2}^2 ca_{1,10} + 4 ca_{1,2} ca_{1,10}^2 - 6 T ca_{1,2} ca_{1,10}^2 - 2 ca_{1,10}^3 + 2 T ca_{1,10}^3 - \\
& 4 ca_{1,10} ca_{2,1} + 8 T ca_{1,10} ca_{2,1} - 2 ca_{1,2} cb_{2,10} + 4 T ca_{1,2} cb_{2,10} + 4 ca_{1,10} cb_{2,10} - \\
& 4 T ca_{1,10} cb_{2,10} + 2 cb_{3,10} - 2 T cb_{3,10}) + \frac{1}{4} \epsilon^3 p_i p_j x_i^2 (-ca_{1,2}^2 ca_{1,10} - 4 T ca_{1,2}^2 ca_{1,10} - \\
& 4 ca_{1,2} ca_{1,10}^2 + 6 T ca_{1,2} ca_{1,10}^2 + 2 ca_{1,10}^3 - 2 T ca_{1,10}^3 + 4 ca_{1,10} ca_{2,1} - 8 T ca_{1,10} ca_{2,1} + \\
& 2 ca_{1,2} cb_{2,10} - 4 T ca_{1,2} cb_{2,10} - 4 ca_{1,10} cb_{2,10} + 4 T ca_{1,10} cb_{2,10} - 2 cb_{3,10} + 2 T cb_{3,10})
\end{aligned}$$

(Alt) Out[ ] =

$$\frac{1}{2} \epsilon ca_{1,2} - \epsilon p_i x_i ca_{1,2} + \epsilon p_j x_i ca_{1,2} - \frac{(-1 + T) \epsilon p_i p_j x_i^2 ca_{1,10}}{2 T} + \frac{(-1 + T) \epsilon p_j^2 x_i^2 ca_{1,10}}{2 T} -$$

$$\begin{aligned}
 & \in p_i p_j x_i x_j ca_{1,10} + \in p_j^2 x_i x_j ca_{1,10} - \frac{(-1+T) \in^2 p_i^2 p_j x_i^3 ca_{1,10}^2}{3T} + \frac{(-1+T)(1+5T) \in^2 p_i p_j^2 x_i^3 ca_{1,10}^2}{6T^2} - \\
 & \frac{(-1+T)(1+3T) \in^2 p_j^3 x_i^3 ca_{1,10}^2}{6T^2} - \frac{1}{2} \in^2 p_i^2 p_j x_i^2 x_j ca_{1,10}^2 + \frac{(1+2T) \in^2 p_i p_j^2 x_i^2 x_j ca_{1,10}^2}{2T} - \\
 & \frac{(1+T) \in^2 p_j^3 x_i^2 x_j ca_{1,10}^2}{2T} - \frac{1}{2} \in^2 p_i p_j^2 x_i x_j^2 ca_{1,10}^2 + \frac{1}{2} \in^2 p_j^3 x_i x_j^2 ca_{1,10}^2 - \frac{(-1+T) \in^3 p_i^3 p_j x_i^4 ca_{1,10}^3}{8T} + \\
 & \frac{(-1+T)(3+4T) \in^3 p_i^2 p_j^2 x_i^4 ca_{1,10}^3}{8T^2} - \frac{(-1+T)(1+22T+13T^2) \in^3 p_i p_j^3 x_i^4 ca_{1,10}^3}{24T^3} + \\
 & \frac{(-1+T)(1+13T+4T^2) \in^3 p_j^4 x_i^4 ca_{1,10}^3}{24T^3} - \frac{1}{6} \in^3 p_i^3 p_j x_i^3 x_j ca_{1,10}^3 + \frac{7 \in^3 p_i^2 p_j^2 x_i^3 x_j ca_{1,10}^3}{6T} + \\
 & \frac{(-1-17T+6T^2) \in^3 p_i p_j^3 x_i^3 x_j ca_{1,10}^3}{6T^2} - \frac{(-1-10T+5T^2) \in^3 p_j^4 x_i^3 x_j ca_{1,10}^3}{6T^2} - \in^3 p_i^2 p_j^2 x_i^2 x_j^2 ca_{1,10}^3 + \\
 & \frac{(1+10T) \in^3 p_i p_j^3 x_i^2 x_j^2 ca_{1,10}^3}{4T} - \frac{(1+6T) \in^3 p_j^4 x_i^2 x_j^2 ca_{1,10}^3}{4T} - \frac{1}{6} \in^3 p_i p_j^3 x_i x_j^3 ca_{1,10}^3 + \\
 & \frac{1}{6} \in^3 p_j^4 x_i x_j^3 ca_{1,10}^3 + \frac{1}{2} \in^2 p_j x_i (ca_{1,2}^2 - 4ca_{2,1}) - \in^2 ca_{2,1} + \frac{1}{2} \in^2 p_i x_i (-ca_{1,2}^2 + 4ca_{2,1}) + \\
 & \frac{1}{6} \in^3 p_j x_i (ca_{1,2}^3 - 12ca_{1,2}ca_{2,1} - 12ca_{3,1}) - \in^3 ca_{3,1} + \frac{1}{6} \in^3 p_i x_i (-ca_{1,2}^3 + 12ca_{1,2}ca_{2,1} + 12ca_{3,1}) + \\
 & \in^2 p_i p_j x_i x_j cb_{2,10} - \in^2 p_j^2 x_i x_j cb_{2,10} + \in^3 p_i^2 p_j x_i^2 x_j ca_{1,10} cb_{2,10} + \\
 & \frac{1}{2} \in^3 p_i p_j^2 x_i x_j^2 ca_{1,10} (ca_{1,2}ca_{1,10} + 2cb_{2,10}) - \frac{1}{2} \in^3 p_j^3 x_i x_j^2 ca_{1,10} (ca_{1,2}ca_{1,10} + 2cb_{2,10}) + \\
 & \frac{\in^3 p_i p_j^2 x_i^2 x_j ca_{1,10} (-ca_{1,2}ca_{1,10} + 2Tca_{1,2}ca_{1,10} - Tca_{1,10}^2 - 2cb_{2,10} - 4Tcb_{2,10})}{2T} - \\
 & \frac{\in^3 p_j^3 x_i^2 x_j ca_{1,10} (-ca_{1,2}ca_{1,10} + 2Tca_{1,2}ca_{1,10} - Tca_{1,10}^2 - 2cb_{2,10} - 2Tcb_{2,10})}{2T} + \\
 & \frac{\in^2 p_i p_j x_i^2 (Tca_{1,2}ca_{1,10} - cb_{2,10} + Tcb_{2,10})}{2T} - \frac{\in^2 p_j^2 x_i^2 (Tca_{1,2}ca_{1,10} - cb_{2,10} + Tcb_{2,10})}{2T} + \\
 & \frac{\in^3 p_i^2 p_j x_i^3 ca_{1,10} (Tca_{1,2}ca_{1,10} - 4cb_{2,10} + 4Tcb_{2,10})}{6T} + \\
 & \frac{\in^3 p_j^3 x_i^3 ca_{1,10} (-ca_{1,2}ca_{1,10} + 3Tca_{1,2}ca_{1,10} - 2Tca_{1,10}^2 + 2T^2ca_{1,10}^2 - 2cb_{2,10} - 4Tcb_{2,10} + 6T^2cb_{2,10})}{6T^2} \\
 & - \frac{1}{6T^2} \in^3 p_i p_j^2 x_i^3 ca_{1,10} (-ca_{1,2}ca_{1,10} + 3Tca_{1,2}ca_{1,10} + T^2ca_{1,2}ca_{1,10} - 2Tca_{1,10}^2 + \\
 & 2T^2ca_{1,10}^2 - 2cb_{2,10} - 8Tcb_{2,10} + 10T^2cb_{2,10}) + \in^3 p_i p_j x_i x_j cb_{3,10} - \in^3 p_j^2 x_i x_j cb_{3,10} + \\
 & \frac{\in^3 p_i p_j x_i^2 (Tca_{1,2}ca_{1,10} - 4Tca_{1,10}ca_{2,1} - 2Tca_{1,2}cb_{2,10} - 2cb_{3,10} + 2Tcb_{3,10})}{4T} - \\
 & \frac{\in^3 p_j^2 x_i^2 (Tca_{1,2}ca_{1,10} - 4Tca_{1,10}ca_{2,1} - 2Tca_{1,2}cb_{2,10} - 2cb_{3,10} + 2Tcb_{3,10})}{4T}
 \end{aligned}$$

(Alt) Out[\*]=

$$\frac{1}{2} \in \mathbf{ca}_{1,2} + \in \mathbf{pk} \mathbf{x}_k \mathbf{ca}_{1,10} - \in^2 \mathbf{ca}_{2,1} - \in^3 \mathbf{ca}_{3,1} + \in^2 \mathbf{pk} \mathbf{x}_k (-\mathbf{ca}_{1,2} \mathbf{ca}_{1,10} - \mathbf{cb}_{2,10}) +$$

$$\frac{1}{2} \in^3 \mathbf{pk} \mathbf{x}_k (\mathbf{ca}_{1,2}^2 \mathbf{ca}_{1,10} + 4 \mathbf{ca}_{1,10} \mathbf{ca}_{2,1} + 2 \mathbf{ca}_{1,2} \mathbf{cb}_{2,10} - 2 \mathbf{cb}_{3,10})$$

(Alt) Out[\*]=

$$-\frac{1}{2} \in \mathbf{ca}_{1,2} - \in \mathbf{pk} \mathbf{x}_k \mathbf{ca}_{1,10} + \in^2 \mathbf{ca}_{2,1} + \in^3 \mathbf{ca}_{3,1} + \in^2 \mathbf{pk} \mathbf{x}_k (\mathbf{ca}_{1,2} \mathbf{ca}_{1,10} - \mathbf{ca}_{1,10}^2 + \mathbf{cb}_{2,10}) + \frac{1}{2} \in^3 \mathbf{pk} \mathbf{x}_k$$

$$(-\mathbf{ca}_{1,2}^2 \mathbf{ca}_{1,10} + 4 \mathbf{ca}_{1,2} \mathbf{ca}_{1,10}^2 - 2 \mathbf{ca}_{1,10}^3 - 4 \mathbf{ca}_{1,10} \mathbf{ca}_{2,1} - 2 \mathbf{ca}_{1,2} \mathbf{cb}_{2,10} + 4 \mathbf{ca}_{1,10} \mathbf{cb}_{2,10} + 2 \mathbf{cb}_{3,10})$$

(Alt) In[\*]=

$$\{\mathbf{ca}_{1,2} = \mathbf{1}, \mathbf{ca}_{1,10} = -\mathbf{1}, \mathbf{ca}_{2,1} = \mathbf{0}, \mathbf{cb}_{2,10} = \mathbf{3} / \mathbf{2}\};$$

(Alt) In[ ]:=

**Column[Collect[#, ε, CF] & /@ {rd[1, i, j], rd[-1, i, j], γd[1, k], γd[-1, k]}]**

(Alt) Out[ ]:=

$$\begin{aligned}
& \in \left( -\frac{1}{2} + p_i x_i - p_j x_j + \frac{1}{2} (-1 + T) p_i p_j x_i^2 + \frac{1}{2} (1 - T) p_j^2 x_i^2 - p_i p_j x_i x_j + p_j^2 x_i x_j \right) + \\
& \in^2 \left( -\frac{1}{2} p_i x_i + \frac{p_j x_j}{2} + \frac{1}{4} (1 - 3T) p_i p_j x_i^2 + \frac{1}{4} (-1 + 3T) p_j^2 x_i^2 + \frac{1}{3} (-1 + T) p_i^2 p_j x_i^3 - \right. \\
& \quad \frac{1}{6} (-1 + T) (5 + T) p_i p_j^2 x_i^3 + \frac{1}{6} (-1 + T) (3 + T) p_j^3 x_i^3 + \frac{3}{2} p_i p_j x_i x_j - \frac{3}{2} p_j^2 x_i x_j - \\
& \quad \left. \frac{1}{2} p_i^2 p_j x_i^2 x_j + \frac{1}{2} (2 + T) p_i p_j^2 x_i^2 x_j + \frac{1}{2} (-1 - T) p_j^3 x_i^2 x_j - \frac{1}{2} p_i p_j^2 x_i x_j^2 + \frac{1}{2} p_j^3 x_i x_j^2 \right) + \\
& \in^3 \left( \frac{1}{6} (5 - 6T) p_i^2 p_j x_i^3 + \frac{1}{6} (-16 + 17T + 2T^2) p_i p_j^2 x_i^3 + \frac{1}{6} (11 - 11T - 2T^2) p_j^3 x_i^3 + \right. \\
& \quad \frac{1}{8} (-1 + T) p_i^3 p_j x_i^4 - \frac{1}{8} (-1 + T) (4 + 3T) p_i^2 p_j^2 x_i^4 + \frac{1}{24} (-1 + T) (13 + 22T + T^2) p_i p_j^3 x_i^4 - \\
& \quad \frac{1}{24} (-1 + T) (4 + 13T + T^2) p_j^4 x_i^4 + \frac{3}{2} p_i^2 p_j x_i^2 x_j + \frac{1}{2} (-9 - 2T) p_i p_j^2 x_i^2 x_j + (3 + T) p_j^3 x_i^2 x_j - \\
& \quad \frac{1}{6} p_i^3 p_j x_i^3 x_j + \frac{7}{6} T p_i^2 p_j^2 x_i^3 x_j + \frac{1}{6} (6 - 17T - T^2) p_i p_j^3 x_i^3 x_j + \frac{1}{6} (-5 + 10T + T^2) p_j^4 x_i^3 x_j + \\
& \quad p_i p_j^2 x_i x_j^2 - p_j^3 x_i x_j^2 - p_i^2 p_j^2 x_i^2 x_j^2 + \frac{1}{4} (10 + T) p_i p_j^3 x_i^2 x_j^2 + \frac{1}{4} (-6 - T) p_j^4 x_i^2 x_j^2 - \frac{1}{6} p_i p_j^3 x_i x_j^3 + \\
& \quad \frac{1}{6} p_j^4 x_i x_j^3 + \frac{1}{6} p_i x_i (1 - 12 ca_{3,1}) + ca_{3,1} + \frac{1}{6} p_j x_j (-1 + 12 ca_{3,1}) - p_i p_j x_i x_j cb_{3,10} + \\
& \quad \left. p_j^2 x_i x_j cb_{3,10} + \frac{1}{2} p_j^2 x_i^2 (-2 + cb_{3,10} - T cb_{3,10}) + \frac{1}{2} p_i p_j x_i^2 (2 - cb_{3,10} + T cb_{3,10}) \right) \\
& \in \left( \frac{1}{2} - p_i x_i + p_j x_j + \frac{(-1+T) p_i p_j x_i^2}{2T} - \frac{(-1+T) p_j^2 x_i^2}{2T} + p_i p_j x_i x_j - p_j^2 x_i x_j \right) + \\
& \in^2 \left( -\frac{1}{2} p_i x_i + \frac{p_j x_j}{2} + \frac{(-3+T) p_i p_j x_i^2}{4T} - \frac{(-3+T) p_j^2 x_i^2}{4T} - \frac{(-1+T) p_i^2 p_j x_i^3}{3T} + \frac{(-1+T) (1+5T) p_i p_j^2 x_i^3}{6T^2} - \frac{(-1+T) (1+3T) p_j^3 x_i^3}{6T^2} + \right. \\
& \quad \left. \frac{3}{2} p_i p_j x_i x_j - \frac{3}{2} p_j^2 x_i x_j - \frac{1}{2} p_i^2 p_j x_i^2 x_j + \frac{(1+2T) p_i p_j^2 x_i^2 x_j}{2T} - \frac{(1+T) p_j^3 x_i^2 x_j}{2T} - \frac{1}{2} p_i p_j^2 x_i x_j^2 + \frac{1}{2} p_j^3 x_i x_j^2 \right) + \\
& \in^3 \left( -\frac{(-6+5T) p_i^2 p_j x_i^3}{6T} + \frac{(-2-17T+16T^2) p_i p_j^2 x_i^3}{6T^2} - \frac{(-2-11T+11T^2) p_j^3 x_i^3}{6T^2} + \frac{(-1+T) p_i^3 p_j x_i^4}{8T} - \frac{(-1+T) (3+4T) p_i^2 p_j^2 x_i^4}{8T^2} + \right. \\
& \quad \frac{(-1+T) (1+22T+13T^2) p_i p_j^3 x_i^4}{24T^3} - \frac{(-1+T) (1+13T+4T^2) p_j^4 x_i^4}{24T^3} - \frac{3}{2} p_i^2 p_j x_i^2 x_j + \frac{(2+9T) p_i p_j^2 x_i^2 x_j}{2T} - \\
& \quad \frac{(1+3T) p_j^3 x_i^2 x_j}{T} + \frac{1}{6} p_i^3 p_j x_i^3 x_j - \frac{7 p_i^2 p_j^2 x_i^3 x_j}{6T} - \frac{(-1-17T+6T^2) p_i p_j^3 x_i^3 x_j}{6T^2} + \frac{(-1-10T+5T^2) p_j^4 x_i^3 x_j}{6T^2} - \\
& \quad p_i p_j^2 x_i x_j^2 + p_j^3 x_i x_j^2 + p_i^2 p_j^2 x_i^2 x_j^2 - \frac{(1+10T) p_i p_j^3 x_i^2 x_j^2}{4T} + \frac{(1+6T) p_j^4 x_i^2 x_j^2}{4T} + \frac{1}{6} p_i p_j^3 x_i x_j^3 - \\
& \quad \frac{1}{6} p_j^4 x_i x_j^3 + \frac{1}{6} p_j x_j (1 - 12 ca_{3,1}) - ca_{3,1} + \frac{1}{6} p_i x_i (-1 + 12 ca_{3,1}) + \\
& \quad \left. p_i p_j x_i x_j cb_{3,10} - p_j^2 x_i x_j cb_{3,10} + \frac{p_i p_j x_i^2 (-2T - cb_{3,10} + T cb_{3,10})}{2T} - \frac{p_j^2 x_i^2 (-2T - cb_{3,10} + T cb_{3,10})}{2T} \right) \\
& - \frac{1}{2} \in^2 p_k x_k + \in \left( \frac{1}{2} - p_k x_k \right) + \in^3 (-ca_{3,1} + p_k x_k (1 - cb_{3,10})) \\
& - \frac{1}{2} \in^2 p_k x_k + \in \left( -\frac{1}{2} + p_k x_k \right) + \in^3 (ca_{3,1} + p_k x_k (-1 + cb_{3,10}))
\end{aligned}$$

## Non-Universally Solving at d=4

(Alt) In[ ]:=

**{ca<sub>1,2</sub> = 1, ca<sub>1,10</sub> = -1, ca<sub>2,1</sub> = 0, cb<sub>2,10</sub> = 3 / 2, cb<sub>3,10</sub> = (7 - 12 ca<sub>3,1</sub>) / 6, ca<sub>3,1</sub> = 0};**

(Alt) In[ ]:=

**d = 4;****vars =****Cases [Variables[r<sub>d</sub>[1, i1, j1] + r<sub>d</sub>[-1, i2, j2] + γ<sub>d</sub>[1, k1] + γ<sub>d</sub>[-1, k2]], (ca | cb | cc | cd) \_\_]**

(Alt) Out[ ]:=

```
{ca4,1, ca4,2, ca4,3, ca4,4, ca4,5, ca4,6, ca4,7, ca4,8, ca4,9, ca4,10, ca4,11, ca4,12, ca4,13,
ca4,14, ca4,15, ca4,16, ca4,17, ca4,18, ca4,19, ca4,20, ca4,21, ca4,22, ca4,23, ca4,24, ca4,25,
ca4,26, ca4,27, ca4,28, ca4,29, ca4,30, ca4,31, ca4,32, ca4,33, ca4,34, ca4,35, ca4,36, ca4,37,
ca4,38, ca4,39, ca4,40, ca4,41, ca4,42, ca4,43, ca4,44, ca4,45, ca4,46, ca4,47, ca4,48, ca4,49,
ca4,50, ca4,51, ca4,52, ca4,53, ca4,54, ca4,55, ca4,56, ca4,57, ca4,58, ca4,59, ca4,60, ca4,61,
ca4,62, ca4,63, ca4,64, ca4,65, ca4,66, ca4,67, ca4,68, ca4,69, ca4,70, ca4,71, ca4,72, ca4,73,
ca4,74, ca4,75, ca4,76, ca4,77, ca4,78, ca4,79, ca4,80, ca4,81, ca4,82, ca4,83, ca4,84, ca4,85,
ca4,86, ca4,87, ca4,88, ca4,89, ca4,90, ca4,91, cb4,1, cb4,2, cb4,3, cb4,4, cb4,5, cb4,6,
cb4,7, cb4,8, cb4,9, cb4,10, cb4,11, cb4,12, cb4,13, cb4,14, cb4,15, cb4,16, cb4,17, cb4,18,
cb4,19, cb4,20, cb4,21, cb4,22, cb4,23, cb4,24, cb4,25, cb4,26, cb4,27, cb4,28, cb4,29, cb4,30,
cb4,31, cb4,32, cb4,33, cb4,34, cb4,35, cb4,36, cb4,37, cb4,38, cb4,39, cb4,40, cb4,41, cb4,42,
cb4,43, cb4,44, cb4,45, cb4,46, cb4,47, cb4,48, cb4,49, cb4,50, cb4,51, cb4,52, cb4,53, cb4,54,
cb4,55, cb4,56, cb4,57, cb4,58, cb4,59, cb4,60, cb4,61, cb4,62, cb4,63, cb4,64, cb4,65, cb4,66,
cb4,67, cb4,68, cb4,69, cb4,70, cb4,71, cb4,72, cb4,73, cb4,74, cb4,75, cb4,76, cb4,77, cb4,78,
cb4,79, cb4,80, cb4,81, cb4,82, cb4,83, cb4,84, cb4,85, cb4,86, cb4,87, cb4,88, cb4,89, cb4,90,
cb4,91, cc4,1, cc4,2, cc4,3, cc4,4, cc4,5, cc4,6, cd4,1, cd4,2, cd4,3, cd4,4, cd4,5, cd4,6}
```

**c̄**

(Alt) In[ ]:=

**lhs = Module[{x1, p1},****{x1\*, p1\*} = {p1, x1};****Normal[****Log[0[ε]<sup>d+1</sup> + Zip<sub>{x1}</sub>[Exp[0[ε]<sup>d+1</sup> + (γ<sub>d</sub>[1, i] /. x<sub>i</sub> → x<sub>i</sub> + x1) + (γ<sub>d</sub>[-1, i] /. p<sub>i</sub> → p<sub>i</sub> - p1)]]]]]****rhs = 0**

(Alt) Out[ ]:=

$$\epsilon^4 \left( \frac{p_i x_i}{12} + cc_{4,1} + p_i x_i cc_{4,2} + p_i^2 x_i^2 cc_{4,3} + p_i^3 x_i^3 cc_{4,4} + p_i^4 x_i^4 cc_{4,5} + \right. \\ \left. p_i^5 x_i^5 cc_{4,6} + cd_{4,1} + p_i x_i cd_{4,2} + p_i^2 x_i^2 cd_{4,3} + p_i^3 x_i^3 cd_{4,4} + p_i^4 x_i^4 cd_{4,5} + p_i^5 x_i^5 cd_{4,6} \right)$$

(Alt) Out[ ]:=

**0**

(Alt) In[ ]:=

**covars = DeleteCases [Variables [lhs - rhs], T | (ca | cb | cc | cd) \_\_]**

(Alt) Out[ ]:=

**{ε, p<sub>i</sub>, x<sub>i</sub>}**

(Alt) In[ ]:=

```
eqnsCCbar = (# == 0) & /@ Union[Last /@ CoefficientRules[Expand[lhs - rhs], covars]]
```

(Alt) Out[ ]:=

$$\left\{ \begin{aligned} &cc_{4,1} + cd_{4,1} == 0, \frac{1}{12} + cc_{4,2} + cd_{4,2} == 0, \\ &cc_{4,3} + cd_{4,3} == 0, cc_{4,4} + cd_{4,4} == 0, cc_{4,5} + cd_{4,5} == 0, cc_{4,6} + cd_{4,6} == 0 \end{aligned} \right\}$$

(Alt) In[ ]:=

```
vars =
Cases[Variables[r_d[1, i1, j1] + r_d[-1, i2, j2] + \gamma_d[1, k1] + \gamma_d[-1, k2]], (ca | cb | cc | cd) __]
{sol} = Solve[eqnsCCbar, vars]
sol /. Rule -> Set;
\gamma_d[1, k]
\gamma_d[-1, k]
```

(Alt) Out[ ]:=

- {ca<sub>4,1</sub>, ca<sub>4,2</sub>, ca<sub>4,3</sub>, ca<sub>4,4</sub>, ca<sub>4,5</sub>, ca<sub>4,6</sub>, ca<sub>4,7</sub>, ca<sub>4,8</sub>, ca<sub>4,9</sub>, ca<sub>4,10</sub>, ca<sub>4,11</sub>, ca<sub>4,12</sub>, ca<sub>4,13</sub>,
- ca<sub>4,14</sub>, ca<sub>4,15</sub>, ca<sub>4,16</sub>, ca<sub>4,17</sub>, ca<sub>4,18</sub>, ca<sub>4,19</sub>, ca<sub>4,20</sub>, ca<sub>4,21</sub>, ca<sub>4,22</sub>, ca<sub>4,23</sub>, ca<sub>4,24</sub>, ca<sub>4,25</sub>,
- ca<sub>4,26</sub>, ca<sub>4,27</sub>, ca<sub>4,28</sub>, ca<sub>4,29</sub>, ca<sub>4,30</sub>, ca<sub>4,31</sub>, ca<sub>4,32</sub>, ca<sub>4,33</sub>, ca<sub>4,34</sub>, ca<sub>4,35</sub>, ca<sub>4,36</sub>, ca<sub>4,37</sub>,
- ca<sub>4,38</sub>, ca<sub>4,39</sub>, ca<sub>4,40</sub>, ca<sub>4,41</sub>, ca<sub>4,42</sub>, ca<sub>4,43</sub>, ca<sub>4,44</sub>, ca<sub>4,45</sub>, ca<sub>4,46</sub>, ca<sub>4,47</sub>, ca<sub>4,48</sub>, ca<sub>4,49</sub>,
- ca<sub>4,50</sub>, ca<sub>4,51</sub>, ca<sub>4,52</sub>, ca<sub>4,53</sub>, ca<sub>4,54</sub>, ca<sub>4,55</sub>, ca<sub>4,56</sub>, ca<sub>4,57</sub>, ca<sub>4,58</sub>, ca<sub>4,59</sub>, ca<sub>4,60</sub>, ca<sub>4,61</sub>,
- ca<sub>4,62</sub>, ca<sub>4,63</sub>, ca<sub>4,64</sub>, ca<sub>4,65</sub>, ca<sub>4,66</sub>, ca<sub>4,67</sub>, ca<sub>4,68</sub>, ca<sub>4,69</sub>, ca<sub>4,70</sub>, ca<sub>4,71</sub>, ca<sub>4,72</sub>, ca<sub>4,73</sub>,
- ca<sub>4,74</sub>, ca<sub>4,75</sub>, ca<sub>4,76</sub>, ca<sub>4,77</sub>, ca<sub>4,78</sub>, ca<sub>4,79</sub>, ca<sub>4,80</sub>, ca<sub>4,81</sub>, ca<sub>4,82</sub>, ca<sub>4,83</sub>, ca<sub>4,84</sub>, ca<sub>4,85</sub>,
- ca<sub>4,86</sub>, ca<sub>4,87</sub>, ca<sub>4,88</sub>, ca<sub>4,89</sub>, ca<sub>4,90</sub>, ca<sub>4,91</sub>, cb<sub>4,1</sub>, cb<sub>4,2</sub>, cb<sub>4,3</sub>, cb<sub>4,4</sub>, cb<sub>4,5</sub>, cb<sub>4,6</sub>,
- cb<sub>4,7</sub>, cb<sub>4,8</sub>, cb<sub>4,9</sub>, cb<sub>4,10</sub>, cb<sub>4,11</sub>, cb<sub>4,12</sub>, cb<sub>4,13</sub>, cb<sub>4,14</sub>, cb<sub>4,15</sub>, cb<sub>4,16</sub>, cb<sub>4,17</sub>, cb<sub>4,18</sub>,
- cb<sub>4,19</sub>, cb<sub>4,20</sub>, cb<sub>4,21</sub>, cb<sub>4,22</sub>, cb<sub>4,23</sub>, cb<sub>4,24</sub>, cb<sub>4,25</sub>, cb<sub>4,26</sub>, cb<sub>4,27</sub>, cb<sub>4,28</sub>, cb<sub>4,29</sub>, cb<sub>4,30</sub>,
- cb<sub>4,31</sub>, cb<sub>4,32</sub>, cb<sub>4,33</sub>, cb<sub>4,34</sub>, cb<sub>4,35</sub>, cb<sub>4,36</sub>, cb<sub>4,37</sub>, cb<sub>4,38</sub>, cb<sub>4,39</sub>, cb<sub>4,40</sub>, cb<sub>4,41</sub>, cb<sub>4,42</sub>,
- cb<sub>4,43</sub>, cb<sub>4,44</sub>, cb<sub>4,45</sub>, cb<sub>4,46</sub>, cb<sub>4,47</sub>, cb<sub>4,48</sub>, cb<sub>4,49</sub>, cb<sub>4,50</sub>, cb<sub>4,51</sub>, cb<sub>4,52</sub>, cb<sub>4,53</sub>, cb<sub>4,54</sub>,
- cb<sub>4,55</sub>, cb<sub>4,56</sub>, cb<sub>4,57</sub>, cb<sub>4,58</sub>, cb<sub>4,59</sub>, cb<sub>4,60</sub>, cb<sub>4,61</sub>, cb<sub>4,62</sub>, cb<sub>4,63</sub>, cb<sub>4,64</sub>, cb<sub>4,65</sub>, cb<sub>4,66</sub>,
- cb<sub>4,67</sub>, cb<sub>4,68</sub>, cb<sub>4,69</sub>, cb<sub>4,70</sub>, cb<sub>4,71</sub>, cb<sub>4,72</sub>, cb<sub>4,73</sub>, cb<sub>4,74</sub>, cb<sub>4,75</sub>, cb<sub>4,76</sub>, cb<sub>4,77</sub>, cb<sub>4,78</sub>,
- cb<sub>4,79</sub>, cb<sub>4,80</sub>, cb<sub>4,81</sub>, cb<sub>4,82</sub>, cb<sub>4,83</sub>, cb<sub>4,84</sub>, cb<sub>4,85</sub>, cb<sub>4,86</sub>, cb<sub>4,87</sub>, cb<sub>4,88</sub>, cb<sub>4,89</sub>, cb<sub>4,90</sub>,
- cb<sub>4,91</sub>, cc<sub>4,1</sub>, cc<sub>4,2</sub>, cc<sub>4,3</sub>, cc<sub>4,4</sub>, cc<sub>4,5</sub>, cc<sub>4,6</sub>, cd<sub>4,1</sub>, cd<sub>4,2</sub>, cd<sub>4,3</sub>, cd<sub>4,4</sub>, cd<sub>4,5</sub>, cd<sub>4,6</sub>}

Solve: Equations may not give solutions for all "solve" variables.

(Alt) Out[ ]:=

$$\left\{ \left\{ \begin{aligned} &cd_{4,1} \rightarrow -cc_{4,1}, cd_{4,2} \rightarrow -\frac{1}{12} - cc_{4,2}, cd_{4,3} \rightarrow -cc_{4,3}, cd_{4,4} \rightarrow -cc_{4,4}, cd_{4,5} \rightarrow -cc_{4,5}, cd_{4,6} \rightarrow -cc_{4,6} \end{aligned} \right\} \right\}$$

(Alt) Out[ ]:=

$$\begin{aligned} &\frac{\epsilon}{2} - \epsilon p_k x_k - \frac{1}{2} \epsilon^2 p_k x_k - \frac{1}{6} \epsilon^3 p_k x_k + \epsilon^4 cc_{4,1} + \\ &\epsilon^4 p_k x_k cc_{4,2} + \epsilon^4 p_k^2 x_k^2 cc_{4,3} + \epsilon^4 p_k^3 x_k^3 cc_{4,4} + \epsilon^4 p_k^4 x_k^4 cc_{4,5} + \epsilon^4 p_k^5 x_k^5 cc_{4,6} \end{aligned}$$

(Alt) Out[ ]:=

$$\begin{aligned} &-\frac{\epsilon}{2} + \epsilon p_k x_k - \frac{1}{2} \epsilon^2 p_k x_k + \frac{1}{6} \epsilon^3 p_k x_k - \frac{1}{12} \epsilon^4 p_k x_k - \epsilon^4 cc_{4,1} - \\ &\epsilon^4 p_k x_k cc_{4,2} - \epsilon^4 p_k^2 x_k^2 cc_{4,3} - \epsilon^4 p_k^3 x_k^3 cc_{4,4} - \epsilon^4 p_k^4 x_k^4 cc_{4,5} - \epsilon^4 p_k^5 x_k^5 cc_{4,6} \end{aligned}$$

### R3 @ d = 4

(Alt) In[ ]:=

```
Short[lhs = CF[Module[{es = {i, j, k, i+, j+, k+}},
  Times[
    Normal@Series[Exp[r_d[1, j, k] + r_d[1, i, k+] + r_d[1, i+, j+]], {e, 0, d}],
    Exp[Sum[g_{a,b} pi_a xi_b, {a, es}, {b, es}]]
  ] // Zip_{(p_x & /@ es) U (x_x & /@ es)} // Expand
] // . gRules_{1,j,k} U gRules_{1,i,k+} U gRules_{1,i+,j+}, 5]
```

(Alt) Out[ ]:=

$$1 - \frac{3\epsilon}{2} + \ll 17759 \gg + 6720 \ll 3 \gg \ll 1 \gg + 1680 \epsilon^4 g_{\ll 1 \gg, \ll 1 \gg}^4 g_{k^{++}, \ll 1 \gg}^4 -$$

$$240 \epsilon^4 \left( -ca_{4,61} + 5 T ca_{4,61} - 10 T^2 ca_{4,61} + 10 T^3 ca_{4,61} - 5 T^4 ca_{4,61} + T^5 ca_{4,61} - \right.$$

$$ca_{4,67} + 4 T ca_{4,67} - 6 T^2 ca_{4,67} + 4 T^3 ca_{4,67} - T^4 ca_{4,67} - ca_{4,73} + 3 T ca_{4,73} -$$

$$\left. 3 T^2 ca_{4,73} + T^3 ca_{4,73} - ca_{4,79} + 2 T ca_{4,79} - T^2 ca_{4,79} - ca_{4,85} + T ca_{4,85} - ca_{4,91} \right) g_{k^{++}, k^{++}}^5$$

(Alt) In[ ]:=

```
Short[rhs = CF[Module[{es = {i, j, k, i+, j+, k+}},
  Times[
    Normal@Series[Exp[r_d[1, i, j] + r_d[1, i+, k] + r_d[1, j+, k+]], {e, 0, d}],
    Exp[Sum[g_{a,b} pi_a xi_b, {a, es}, {b, es}]]
  ] // Zip_{(p_x & /@ es) U (x_x & /@ es)} // Expand
] // . gRules_{1,i,j} U gRules_{1,i+,k} U gRules_{1,j+,k+}, 5]
```

(Alt) Out[ ]:=

$$1 - \frac{3\epsilon}{2} + \ll 17770 \gg + 6720 \ll 3 \gg \ll 1 \gg + 1680 \epsilon^4 g_{\ll 1 \gg, \ll 1 \gg}^4 g_{k^{++}, \ll 1 \gg}^4 -$$

$$240 \epsilon^4 \left( -ca_{4,61} + 5 T ca_{4,61} - 10 T^2 ca_{4,61} + 10 T^3 ca_{4,61} - 5 T^4 ca_{4,61} + T^5 ca_{4,61} - \right.$$

$$ca_{4,67} + 4 T ca_{4,67} - 6 T^2 ca_{4,67} + 4 T^3 ca_{4,67} - T^4 ca_{4,67} - ca_{4,73} + 3 T ca_{4,73} -$$

$$\left. 3 T^2 ca_{4,73} + T^3 ca_{4,73} - ca_{4,79} + 2 T ca_{4,79} - T^2 ca_{4,79} - ca_{4,85} + T ca_{4,85} - ca_{4,91} \right) g_{k^{++}, k^{++}}^5$$

(Alt) In[ ]:=

```
me = Exponent[lhs - rhs, T, Min]
```

(Alt) Out[ ]:=

$$-10$$

(Alt) In[ ]:=

```
covars = DeleteCases[Variables[lhs - rhs], T | (ca | cb | cc | cd) _]
```

(Alt) Out[ ]:=

$$\{\epsilon, g_{i^{++}, i^{++}}, g_{i^{++}, j^{++}}, g_{i^{++}, k^{++}}, g_{j^{++}, i^{++}}, g_{j^{++}, j^{++}}, g_{j^{++}, k^{++}}, g_{k^{++}, i^{++}}, g_{k^{++}, j^{++}}, g_{k^{++}, k^{++}}\}$$

(Alt) In[ ]:=

```
Short[
  eqnsR3 = (# == 0) & /@ Union[Last /@ CoefficientRules[Expand[T^{-me} (lhs - rhs)], covars]]]
]
```

(Alt) Out[ ]:=

$$\{-T^9 ca_{4,3} + T^{10} ca_{4,3} == 0, \ll 1 \gg, \ll 868 \gg,$$

$$-126 T^4 + \ll 367 \gg + \ll 1 \gg == 0, \ll 310 \gg + 7200 T^{10} ca_{4,91} == 0\}$$

## R2b @ d = 4

(Alt) In[ ]:=

```
Short[lhs = CF[Module[{es = {i, j, i+, j+}},
  Times[
    Normal@Series[Exp[r_d[1, i, j] + r_d[-1, i+, j+]], {e, 0, d}],
    Exp[Sum[g_{a,b} pi_{a,b}, {a, es}, {b, es}]]
  ] // Zip((p_&/@es) U (x_&/@es) // Expand
] // . gRules_{1,i,j} U gRules_{-1,i+,j+}]
```

(Alt) Out[ ]//Short=

$$1 + e^4 (ca_{4,1} + cb_{4,1}) + \frac{\langle\langle 1 \rangle\rangle}{12 T} + \langle\langle 121 \rangle\rangle + \frac{\langle\langle 1 \rangle\rangle}{\langle\langle 1 \rangle\rangle} + \frac{10 \langle\langle 3 \rangle\rangle \langle\langle 1 \rangle\rangle}{T^5} + \frac{120 e^4 (\langle\langle 1 \rangle\rangle) g^5_{\langle\langle 1 \rangle\rangle, \langle\langle 1 \rangle\rangle}}{T^5}$$

(Alt) In[ ]:=

```
rhs = 1
```

(Alt) Out[ ]:=

```
1
```

(Alt) In[ ]:=

```
me = Exponent[lhs - rhs, T, Min]
```

(Alt) Out[ ]:=

```
-5
```

(Alt) In[ ]:=

```
covars = DeleteCases[Variables[lhs - rhs], T | (ca | cb | cc | cd) _]
```

(Alt) Out[ ]:=

```
{e, g_{i+,i+}, g_{i+,j+}, g_{j+,i+}, g_{j+,j+}}
```

(Alt) In[ ]:=

```
Short[eqnsR2b =
  (Factor[#] == 0) & /@ Union[Last /@ CoefficientRules[Expand[T^-me (lhs - rhs)], covars]]]
```

(Alt) Out[ ]//Short=

$$\{T^5 (ca_{4,1} + cb_{4,1}) == 0, T^4 (T ca_{4,3} + cb_{4,3}) == 0, \langle\langle 116 \rangle\rangle, 5 (\langle\langle 1 \rangle\rangle) == 0, 5 (-11 T^5 + \langle\langle 330 \rangle\rangle) == 0\}$$

(Alt) In[ ]:=

```
vars = Cases[Variables[r_d[1, i1, j1] + r_d[-1, i2, j2] + \gamma_d[1, k]], (ca | cb | cc | cd) _]
{sol} = Solve[eqnsR3 U eqnsR2b U eqnsR2c U eqnsR1l U eqnsR1r U eqnsSwp, vars]
sol /. Rule -> Set;
r_d[1, i, j]
\gamma_d[1, k]
```

(Alt) Out[\*]=

{ca<sub>4,1</sub>, ca<sub>4,2</sub>, ca<sub>4,3</sub>, ca<sub>4,4</sub>, ca<sub>4,5</sub>, ca<sub>4,6</sub>, ca<sub>4,7</sub>, ca<sub>4,8</sub>, ca<sub>4,9</sub>, ca<sub>4,10</sub>, ca<sub>4,11</sub>, ca<sub>4,12</sub>, ca<sub>4,13</sub>, ca<sub>4,14</sub>, ca<sub>4,15</sub>, ca<sub>4,16</sub>, ca<sub>4,17</sub>, ca<sub>4,18</sub>, ca<sub>4,19</sub>, ca<sub>4,20</sub>, ca<sub>4,21</sub>, ca<sub>4,22</sub>, ca<sub>4,23</sub>, ca<sub>4,24</sub>, ca<sub>4,25</sub>, ca<sub>4,26</sub>, ca<sub>4,27</sub>, ca<sub>4,28</sub>, ca<sub>4,29</sub>, ca<sub>4,30</sub>, ca<sub>4,31</sub>, ca<sub>4,32</sub>, ca<sub>4,33</sub>, ca<sub>4,34</sub>, ca<sub>4,35</sub>, ca<sub>4,36</sub>, ca<sub>4,37</sub>, ca<sub>4,38</sub>, ca<sub>4,39</sub>, ca<sub>4,40</sub>, ca<sub>4,41</sub>, ca<sub>4,42</sub>, ca<sub>4,43</sub>, ca<sub>4,44</sub>, ca<sub>4,45</sub>, ca<sub>4,46</sub>, ca<sub>4,47</sub>, ca<sub>4,48</sub>, ca<sub>4,49</sub>, ca<sub>4,50</sub>, ca<sub>4,51</sub>, ca<sub>4,52</sub>, ca<sub>4,53</sub>, ca<sub>4,54</sub>, ca<sub>4,55</sub>, ca<sub>4,56</sub>, ca<sub>4,57</sub>, ca<sub>4,58</sub>, ca<sub>4,59</sub>, ca<sub>4,60</sub>, ca<sub>4,61</sub>, ca<sub>4,62</sub>, ca<sub>4,63</sub>, ca<sub>4,64</sub>, ca<sub>4,65</sub>, ca<sub>4,66</sub>, ca<sub>4,67</sub>, ca<sub>4,68</sub>, ca<sub>4,69</sub>, ca<sub>4,70</sub>, ca<sub>4,71</sub>, ca<sub>4,72</sub>, ca<sub>4,73</sub>, ca<sub>4,74</sub>, ca<sub>4,75</sub>, ca<sub>4,76</sub>, ca<sub>4,77</sub>, ca<sub>4,78</sub>, ca<sub>4,79</sub>, ca<sub>4,80</sub>, ca<sub>4,81</sub>, ca<sub>4,82</sub>, ca<sub>4,83</sub>, ca<sub>4,84</sub>, ca<sub>4,85</sub>, ca<sub>4,86</sub>, ca<sub>4,87</sub>, ca<sub>4,88</sub>, ca<sub>4,89</sub>, ca<sub>4,90</sub>, ca<sub>4,91</sub>, cb<sub>4,1</sub>, cb<sub>4,2</sub>, cb<sub>4,3</sub>, cb<sub>4,4</sub>, cb<sub>4,5</sub>, cb<sub>4,6</sub>, cb<sub>4,7</sub>, cb<sub>4,8</sub>, cb<sub>4,9</sub>, cb<sub>4,10</sub>, cb<sub>4,11</sub>, cb<sub>4,12</sub>, cb<sub>4,13</sub>, cb<sub>4,14</sub>, cb<sub>4,15</sub>, cb<sub>4,16</sub>, cb<sub>4,17</sub>, cb<sub>4,18</sub>, cb<sub>4,19</sub>, cb<sub>4,20</sub>, cb<sub>4,21</sub>, cb<sub>4,22</sub>, cb<sub>4,23</sub>, cb<sub>4,24</sub>, cb<sub>4,25</sub>, cb<sub>4,26</sub>, cb<sub>4,27</sub>, cb<sub>4,28</sub>, cb<sub>4,29</sub>, cb<sub>4,30</sub>, cb<sub>4,31</sub>, cb<sub>4,32</sub>, cb<sub>4,33</sub>, cb<sub>4,34</sub>, cb<sub>4,35</sub>, cb<sub>4,36</sub>, cb<sub>4,37</sub>, cb<sub>4,38</sub>, cb<sub>4,39</sub>, cb<sub>4,40</sub>, cb<sub>4,41</sub>, cb<sub>4,42</sub>, cb<sub>4,43</sub>, cb<sub>4,44</sub>, cb<sub>4,45</sub>, cb<sub>4,46</sub>, cb<sub>4,47</sub>, cb<sub>4,48</sub>, cb<sub>4,49</sub>, cb<sub>4,50</sub>, cb<sub>4,51</sub>, cb<sub>4,52</sub>, cb<sub>4,53</sub>, cb<sub>4,54</sub>, cb<sub>4,55</sub>, cb<sub>4,56</sub>, cb<sub>4,57</sub>, cb<sub>4,58</sub>, cb<sub>4,59</sub>, cb<sub>4,60</sub>, cb<sub>4,61</sub>, cb<sub>4,62</sub>, cb<sub>4,63</sub>, cb<sub>4,64</sub>, cb<sub>4,65</sub>, cb<sub>4,66</sub>, cb<sub>4,67</sub>, cb<sub>4,68</sub>, cb<sub>4,69</sub>, cb<sub>4,70</sub>, cb<sub>4,71</sub>, cb<sub>4,72</sub>, cb<sub>4,73</sub>, cb<sub>4,74</sub>, cb<sub>4,75</sub>, cb<sub>4,76</sub>, cb<sub>4,77</sub>, cb<sub>4,78</sub>, cb<sub>4,79</sub>, cb<sub>4,80</sub>, cb<sub>4,81</sub>, cb<sub>4,82</sub>, cb<sub>4,83</sub>, cb<sub>4,84</sub>, cb<sub>4,85</sub>, cb<sub>4,86</sub>, cb<sub>4,87</sub>, cb<sub>4,88</sub>, cb<sub>4,89</sub>, cb<sub>4,90</sub>, cb<sub>4,91</sub>, cc<sub>4,1</sub>, cc<sub>4,2</sub>, cc<sub>4,3</sub>, cc<sub>4,4</sub>, cc<sub>4,5</sub>, cc<sub>4,6</sub>}

**Solve:** Equations may not give solutions for all "solve" variables.

(Alt) Out[\*]=

$$\left\{ \begin{array}{l} ca_{4,2} \rightarrow -\frac{1}{12} - cb_{4,2}, ca_{4,3} \rightarrow 0, ca_{4,4} \rightarrow \frac{1}{12} + cb_{4,2} + T cb_{4,5}, ca_{4,5} \rightarrow -cb_{4,5}, ca_{4,6} \rightarrow 0, ca_{4,7} \rightarrow 0, \\ ca_{4,8} \rightarrow 0, ca_{4,10} \rightarrow \frac{5}{4} - cb_{4,10}, ca_{4,11} \rightarrow 0, ca_{4,12} \rightarrow \frac{5}{24} (2T + 3T^2) - (1-T) ca_{4,9} - \frac{1}{2} (-T + T^2) cb_{4,10}, \\ ca_{4,13} \rightarrow -\frac{5}{24} (8 + 3T) - ca_{4,9} - \frac{1}{2} (-1 - T) cb_{4,10}, ca_{4,14} \rightarrow 0, ca_{4,15} \rightarrow 0, ca_{4,16} \rightarrow 0, ca_{4,17} \rightarrow 0, \\ ca_{4,18} \rightarrow 0, ca_{4,20} \rightarrow -\frac{55}{24}, ca_{4,21} \rightarrow 0, ca_{4,22} \rightarrow 0, ca_{4,23} \rightarrow \frac{1}{24} (67 - 72T - 45T^2) - (1-T) ca_{4,19}, \\ ca_{4,24} \rightarrow \frac{1}{24} (212 + 25T), ca_{4,25} \rightarrow -\frac{25}{24}, ca_{4,26} \rightarrow 0, ca_{4,27} \rightarrow \frac{1}{72} (-201 + 284T + 25T^2), \\ ca_{4,28} \rightarrow \frac{1}{72} (-539 + 103T - 110T^2) - (1-T) ca_{4,19}, ca_{4,29} \rightarrow \frac{1}{72} (7 + 110T) - ca_{4,19}, ca_{4,30} \rightarrow 0, \\ ca_{4,31} \rightarrow 0, ca_{4,32} \rightarrow 0, ca_{4,33} \rightarrow 0, ca_{4,34} \rightarrow 0, ca_{4,35} \rightarrow 0, ca_{4,37} \rightarrow \frac{3}{4}, ca_{4,38} \rightarrow 0, ca_{4,39} \rightarrow 0, \\ ca_{4,40} \rightarrow 0, ca_{4,41} \rightarrow \frac{1}{96} (-237 + 46T + 219T^2) + \frac{3}{2} (-1 + T) ca_{4,36}, ca_{4,42} \rightarrow \frac{1}{12} (-30 - 53T), \\ ca_{4,43} \rightarrow \frac{15}{4}, ca_{4,44} \rightarrow 0, ca_{4,45} \rightarrow 0, ca_{4,46} \rightarrow \frac{1}{24} (110 - T - 132T^2 + 11T^3) - (-1 + 2T - T^2) ca_{4,36}, \\ ca_{4,47} \rightarrow \frac{1}{12} (12 + 147T + 5T^2), ca_{4,48} \rightarrow \frac{1}{8} (-86 - 5T), ca_{4,49} \rightarrow \frac{5}{12}, ca_{4,50} \rightarrow 0, \\ ca_{4,51} \rightarrow \frac{1}{96} (-203 - 92T + 388T^2 - 96T^3 + 27T^4) - \frac{1}{2} (1 - 3T + 3T^2 - T^3) ca_{4,36}, \\ ca_{4,52} \rightarrow \frac{1}{48} (61 - 453T + 59T^2 - 27T^3) - (1 - 2T + T^2) ca_{4,36}, \\ ca_{4,53} \rightarrow \frac{1}{32} (249 - 32T + 27T^2) + \frac{3}{2} (-1 + T) ca_{4,36}, ca_{4,54} \rightarrow \frac{1}{48} (5 - 27T) - ca_{4,36}, \\ ca_{4,55} \rightarrow 0, ca_{4,56} \rightarrow 0, ca_{4,57} \rightarrow 0, ca_{4,58} \rightarrow 0, ca_{4,59} \rightarrow 0, ca_{4,60} \rightarrow 0, \end{array} \right.$$

$$\begin{aligned}
ca_{4,61} &\rightarrow 0, ca_{4,63} \rightarrow -\frac{1}{24}, ca_{4,64} \rightarrow 0, ca_{4,65} \rightarrow 0, ca_{4,66} \rightarrow 0, ca_{4,67} \rightarrow 0, \\
ca_{4,68} &\rightarrow \frac{1}{2} (T - T^2) + 2(-1 + T) ca_{4,62}, ca_{4,69} \rightarrow \frac{1}{24} (-16 + 31T), ca_{4,70} \rightarrow -1, ca_{4,71} \rightarrow 0, \\
ca_{4,72} &\rightarrow 0, ca_{4,73} \rightarrow 0, ca_{4,74} \rightarrow \frac{1}{12} (5 - 28T + 21T^2 + 2T^3) + 2(1 - 2T + T^2) ca_{4,62}, \\
ca_{4,75} &\rightarrow -\frac{5}{24} (-18 + 23T + 5T^2), ca_{4,76} \rightarrow \frac{5}{12} (9 + 4T), ca_{4,77} \rightarrow -1, ca_{4,78} \rightarrow 0, \\
ca_{4,79} &\rightarrow 0, ca_{4,80} \rightarrow \frac{1}{24} (-21 + 78T - 46T^2 - 10T^3 - T^4) - (1 - 3T + 3T^2 - T^3) ca_{4,62}, \\
ca_{4,81} &\rightarrow \frac{1}{24} (-132 + 131T + 60T^2 + T^3), ca_{4,82} \rightarrow \frac{1}{12} (-51 - 49T - T^2), ca_{4,83} \rightarrow \frac{30 + T}{12}, \\
ca_{4,84} &\rightarrow -\frac{1}{24}, ca_{4,85} \rightarrow 0, ca_{4,86} \rightarrow \frac{1}{120} (55 - 166T + 72T^2 + 38T^3 + T^4), \\
ca_{4,87} &\rightarrow \frac{1}{120} (291 - 219T - 199T^2 + 11T^3 - 4T^4) - (1 - 3T + 3T^2 - T^3) ca_{4,62}, \\
ca_{4,88} &\rightarrow \frac{1}{60} (86 + 157T - 7T^2 + 4T^3) - 2(1 - 2T + T^2) ca_{4,62}, \\
ca_{4,89} &\rightarrow \frac{1}{60} (-94 + 3T - 4T^2) + 2(-1 + T) ca_{4,62}, ca_{4,90} \rightarrow \frac{1}{120} (1 + 4T) - ca_{4,62}, \\
ca_{4,91} &\rightarrow 0, cb_{4,1} \rightarrow -ca_{4,1}, cb_{4,3} \rightarrow 0, cb_{4,4} \rightarrow -cb_{4,2} - \frac{cb_{4,5}}{T}, cb_{4,6} \rightarrow 0, \\
cb_{4,7} &\rightarrow 0, cb_{4,8} \rightarrow 0, cb_{4,9} \rightarrow -\frac{5(1 + 4T)}{12T} - \frac{ca_{4,9}}{T} - \frac{(1 - T)cb_{4,10}}{T}, \\
cb_{4,11} &\rightarrow 0, cb_{4,12} \rightarrow \frac{5(-2 - T + 8T^2)}{24T^2} - \frac{(1 - T)ca_{4,9}}{T^2} - \frac{(1 - 3T + 2T^2)cb_{4,10}}{2T^2}, \\
cb_{4,13} &\rightarrow \frac{5(2 + 3T)}{24T} + \frac{ca_{4,9}}{T} - \frac{(-1 + 3T)cb_{4,10}}{2T}, cb_{4,14} \rightarrow 0, cb_{4,15} \rightarrow 0, cb_{4,16} \rightarrow 0, \\
cb_{4,17} &\rightarrow 0, cb_{4,18} \rightarrow 0, cb_{4,19} \rightarrow \frac{7(1 + T)}{12T} - \frac{ca_{4,19}}{T}, cb_{4,20} \rightarrow -\frac{55}{24}, cb_{4,21} \rightarrow 0, cb_{4,22} \rightarrow 0, \\
cb_{4,23} &\rightarrow -\frac{31 + 72T - 53T^2}{24T^2} - \frac{(1 - T)ca_{4,19}}{T^2}, cb_{4,24} \rightarrow -\frac{-25 - 212T}{24T}, cb_{4,25} \rightarrow -\frac{25}{24}, \\
cb_{4,26} &\rightarrow 0, cb_{4,27} \rightarrow -\frac{-25 - 284T + 201T^2}{72T^2}, cb_{4,28} \rightarrow -\frac{68 - 103T + 581T^2}{72T^2} - \frac{(1 - T)ca_{4,19}}{T^2}, \\
cb_{4,29} &\rightarrow -\frac{-68 + 35T}{72T} + \frac{ca_{4,19}}{T}, cb_{4,30} \rightarrow 0, cb_{4,31} \rightarrow 0, cb_{4,32} \rightarrow 0, cb_{4,33} \rightarrow 0, cb_{4,34} \rightarrow 0, \\
cb_{4,35} &\rightarrow 0, cb_{4,36} \rightarrow -\frac{1 + T}{24T} - \frac{ca_{4,36}}{T}, cb_{4,37} \rightarrow \frac{3}{4}, cb_{4,38} \rightarrow 0, cb_{4,39} \rightarrow 0, cb_{4,40} \rightarrow 0, \\
cb_{4,41} &\rightarrow -\frac{-213 - 46T + 231T^2}{96T^2} + \frac{3(-1 + T)ca_{4,36}}{2T^2}, cb_{4,42} \rightarrow -\frac{53 + 30T}{12T}, cb_{4,43} \rightarrow \frac{15}{4}, \\
cb_{4,44} &\rightarrow 0, cb_{4,45} \rightarrow 0, cb_{4,46} \rightarrow -\frac{-10 + 131T - 109T^3}{24T^3} - \frac{(1 - 2T + T^2)ca_{4,36}}{T^3},
\end{aligned}$$

$$\begin{aligned}
 &cb_{4,47} \rightarrow -\frac{-5 - 147 T - 12 T^2}{12 T^2}, \quad cb_{4,48} \rightarrow -\frac{5 + 86 T}{8 T}, \quad cb_{4,49} \rightarrow \frac{5}{12}, \quad cb_{4,50} \rightarrow 0, \\
 &cb_{4,51} \rightarrow -\frac{-25 + 92 T - 388 T^2 + 96 T^3 + 201 T^4}{96 T^4} - \frac{(1 - 3 T + 3 T^2 - T^3) ca_{4,36}}{2 T^4}, \\
 &cb_{4,52} \rightarrow -\frac{25 - 57 T + 455 T^2 - 63 T^3}{48 T^3} - \frac{(-1 + 2 T - T^2) ca_{4,36}}{T^3}, \\
 &cb_{4,53} \rightarrow -\frac{-25 + 32 T - 251 T^2}{32 T^2} + \frac{3(-1 + T) ca_{4,36}}{2 T^2}, \quad cb_{4,54} \rightarrow -\frac{25 - 7 T}{48 T} + \frac{ca_{4,36}}{T}, \\
 &cb_{4,55} \rightarrow 0, \quad cb_{4,56} \rightarrow 0, \quad cb_{4,57} \rightarrow 0, \quad cb_{4,58} \rightarrow 0, \quad cb_{4,59} \rightarrow 0, \quad cb_{4,60} \rightarrow 0, \quad cb_{4,61} \rightarrow 0, \\
 &cb_{4,62} \rightarrow -\frac{ca_{4,62}}{T}, \quad cb_{4,63} \rightarrow -\frac{1}{24}, \quad cb_{4,64} \rightarrow 0, \quad cb_{4,65} \rightarrow 0, \quad cb_{4,66} \rightarrow 0, \quad cb_{4,67} \rightarrow 0, \\
 &cb_{4,68} \rightarrow -\frac{1 - T}{2 T^2} + \frac{2(-1 + T) ca_{4,62}}{T^2}, \quad cb_{4,69} \rightarrow -\frac{-31 + 16 T}{24 T}, \quad cb_{4,70} \rightarrow -1, \quad cb_{4,71} \rightarrow 0, \\
 &cb_{4,72} \rightarrow 0, \quad cb_{4,73} \rightarrow 0, \quad cb_{4,74} \rightarrow -\frac{-2 - 21 T + 28 T^2 - 5 T^3}{12 T^3} - \frac{2(1 - 2 T + T^2) ca_{4,62}}{T^3}, \\
 &cb_{4,75} \rightarrow \frac{5(-5 - 23 T + 18 T^2)}{24 T^2}, \quad cb_{4,76} \rightarrow \frac{5(4 + 9 T)}{12 T}, \quad cb_{4,77} \rightarrow -1, \quad cb_{4,78} \rightarrow 0, \\
 &cb_{4,79} \rightarrow 0, \quad cb_{4,80} \rightarrow -\frac{1 + 10 T + 46 T^2 - 78 T^3 + 21 T^4}{24 T^4} - \frac{(1 - 3 T + 3 T^2 - T^3) ca_{4,62}}{T^4}, \\
 &cb_{4,81} \rightarrow -\frac{-1 - 60 T - 131 T^2 + 132 T^3}{24 T^3}, \quad cb_{4,82} \rightarrow -\frac{1 + 49 T + 51 T^2}{12 T^2}, \quad cb_{4,83} \rightarrow -\frac{-1 - 30 T}{12 T}, \\
 &cb_{4,84} \rightarrow -\frac{1}{24}, \quad cb_{4,85} \rightarrow 0, \quad cb_{4,86} \rightarrow -\frac{-1 - 38 T - 72 T^2 + 166 T^3 - 55 T^4}{120 T^4}, \\
 &cb_{4,87} \rightarrow -\frac{4 - 11 T + 199 T^2 + 219 T^3 - 291 T^4}{120 T^4} - \frac{(1 - 3 T + 3 T^2 - T^3) ca_{4,62}}{T^4}, \\
 &cb_{4,88} \rightarrow -\frac{-4 + 7 T - 157 T^2 - 86 T^3}{60 T^3} + \frac{2(1 - 2 T + T^2) ca_{4,62}}{T^3}, \\
 &cb_{4,89} \rightarrow -\frac{4 - 3 T + 94 T^2}{60 T^2} + \frac{2(-1 + T) ca_{4,62}}{T^2}, \quad cb_{4,90} \rightarrow -\frac{-4 - T}{120 T} + \frac{ca_{4,62}}{T}, \quad cb_{4,91} \rightarrow 0 \}}
 \end{aligned}$$

(Alt) Out[ ] =

$$\begin{aligned}
 &-\frac{\epsilon}{2} + \epsilon p_i x_i - \frac{1}{2} \epsilon^2 p_i x_i + \frac{1}{6} \epsilon^3 p_i x_i - \frac{1}{12} \epsilon^4 p_i x_i - \epsilon p_j x_i + \frac{1}{2} \epsilon^2 p_j x_i - \frac{1}{6} \epsilon^3 p_j x_i + \frac{1}{12} \epsilon^4 p_j x_i - \\
 &\frac{1}{2} \epsilon p_i p_j x_i^2 + \frac{1}{2} T \epsilon p_i p_j x_i^2 + \frac{1}{4} \epsilon^2 p_i p_j x_i^2 - \frac{3}{4} T \epsilon^2 p_i p_j x_i^2 + \frac{5}{12} \epsilon^3 p_i p_j x_i^2 + \frac{7}{12} T \epsilon^3 p_i p_j x_i^2 + \\
 &\frac{1}{2} \epsilon p_j^2 x_i^2 - \frac{1}{2} T \epsilon p_j^2 x_i^2 - \frac{1}{4} \epsilon^2 p_j^2 x_i^2 + \frac{3}{4} T \epsilon^2 p_j^2 x_i^2 - \frac{5}{12} \epsilon^3 p_j^2 x_i^2 - \frac{7}{12} T \epsilon^3 p_j^2 x_i^2 + \frac{5}{12} T \epsilon^4 p_j^2 x_i^2 + \\
 &\frac{5}{8} T^2 \epsilon^4 p_j^2 x_i^2 - \frac{1}{3} \epsilon^2 p_i^2 p_j x_i^3 + \frac{1}{3} T \epsilon^2 p_i^2 p_j x_i^3 + \frac{5}{6} \epsilon^3 p_i^2 p_j x_i^3 - T \epsilon^3 p_i^2 p_j x_i^3 + \frac{5}{6} \epsilon^2 p_i p_j^2 x_i^3 - \\
 &\frac{2}{3} T \epsilon^2 p_i p_j^2 x_i^3 - \frac{1}{6} T^2 \epsilon^2 p_i p_j^2 x_i^3 - \frac{8}{3} \epsilon^3 p_i p_j^2 x_i^3 + \frac{17}{6} T \epsilon^3 p_i p_j^2 x_i^3 + \frac{1}{3} T^2 \epsilon^3 p_i p_j^2 x_i^3 + \frac{67}{24} \epsilon^4 p_i p_j^2 x_i^3 -
 \end{aligned}$$

$$\begin{aligned}
 & 3 T \epsilon^4 p_i p_j^2 x_i^3 - \frac{15}{8} T^2 \epsilon^4 p_i p_j^2 x_i^3 - \frac{1}{2} \epsilon^2 p_j^3 x_i^3 + \frac{1}{3} T \epsilon^2 p_j^3 x_i^3 + \frac{1}{6} T^2 \epsilon^2 p_j^3 x_i^3 + \frac{11}{6} \epsilon^3 p_j^3 x_i^3 - \frac{11}{6} T \epsilon^3 p_j^3 x_i^3 - \\
 & \frac{1}{3} T^2 \epsilon^3 p_j^3 x_i^3 - \frac{67}{24} \epsilon^4 p_j^3 x_i^3 + \frac{71}{18} T \epsilon^4 p_j^3 x_i^3 + \frac{25}{72} T^2 \epsilon^4 p_j^3 x_i^3 - \frac{1}{8} \epsilon^3 p_i p_j x_i^4 + \frac{1}{8} T \epsilon^3 p_i p_j x_i^4 + \\
 & \frac{1}{2} \epsilon^3 p_i^2 p_j^2 x_i^4 - \frac{1}{8} T \epsilon^3 p_i^2 p_j^2 x_i^4 - \frac{3}{8} T^2 \epsilon^3 p_i^2 p_j^2 x_i^4 - \frac{79}{32} \epsilon^4 p_i^2 p_j^2 x_i^4 + \frac{23}{48} T \epsilon^4 p_i^2 p_j^2 x_i^4 + \frac{73}{32} T^2 \epsilon^4 p_i^2 p_j^2 x_i^4 - \\
 & \frac{13}{24} \epsilon^3 p_i p_j^3 x_i^4 - \frac{3}{8} T \epsilon^3 p_i p_j^3 x_i^4 + \frac{7}{8} T^2 \epsilon^3 p_i p_j^3 x_i^4 + \frac{1}{24} T^3 \epsilon^3 p_i p_j^3 x_i^4 + \frac{55}{12} \epsilon^4 p_i p_j^3 x_i^4 - \frac{1}{24} T \epsilon^4 p_i p_j^3 x_i^4 - \\
 & \frac{11}{2} T^2 \epsilon^4 p_i p_j^3 x_i^4 + \frac{11}{24} T^3 \epsilon^4 p_i p_j^3 x_i^4 + \frac{1}{6} \epsilon^3 p_j^4 x_i^4 + \frac{3}{8} T \epsilon^3 p_j^4 x_i^4 - \frac{1}{2} T^2 \epsilon^3 p_j^4 x_i^4 - \frac{1}{24} T^3 \epsilon^3 p_j^4 x_i^4 - \\
 & \frac{203}{96} \epsilon^4 p_j^4 x_i^4 - \frac{23}{24} T \epsilon^4 p_j^4 x_i^4 + \frac{97}{24} T^2 \epsilon^4 p_j^4 x_i^4 - T^3 \epsilon^4 p_j^4 x_i^4 + \frac{9}{32} T^4 \epsilon^4 p_j^4 x_i^4 + \frac{1}{2} T \epsilon^4 p_i^3 p_j^2 x_i^5 - \\
 & \frac{1}{2} T^2 \epsilon^4 p_i^3 p_j^2 x_i^5 + \frac{5}{12} \epsilon^4 p_i^2 p_j^3 x_i^5 - \frac{7}{3} T \epsilon^4 p_i^2 p_j^3 x_i^5 + \frac{7}{4} T^2 \epsilon^4 p_i^2 p_j^3 x_i^5 + \frac{1}{6} T^3 \epsilon^4 p_i^2 p_j^3 x_i^5 - \frac{7}{8} \epsilon^4 p_i p_j^4 x_i^5 + \\
 & \frac{13}{4} T \epsilon^4 p_i p_j^4 x_i^5 - \frac{23}{12} T^2 \epsilon^4 p_i p_j^4 x_i^5 - \frac{5}{12} T^3 \epsilon^4 p_i p_j^4 x_i^5 - \frac{1}{24} T^4 \epsilon^4 p_i p_j^4 x_i^5 + \frac{11}{24} \epsilon^4 p_j^5 x_i^5 - \frac{83}{60} T \epsilon^4 p_j^5 x_i^5 + \\
 & \frac{3}{5} T^2 \epsilon^4 p_j^5 x_i^5 + \frac{19}{60} T^3 \epsilon^4 p_j^5 x_i^5 + \frac{1}{120} T^4 \epsilon^4 p_j^5 x_i^5 - \epsilon p_i p_j x_i x_j + \frac{3}{2} \epsilon^2 p_i p_j x_i x_j - \frac{7}{6} \epsilon^3 p_i p_j x_i x_j + \\
 & \frac{5}{4} \epsilon^4 p_i p_j x_i x_j + \epsilon p_j^2 x_i x_j - \frac{3}{2} \epsilon^2 p_j^2 x_i x_j + \frac{7}{6} \epsilon^3 p_j^2 x_i x_j - \frac{5}{3} \epsilon^4 p_j^2 x_i x_j - \frac{5}{8} T \epsilon^4 p_j^2 x_i x_j - \\
 & \frac{1}{2} \epsilon^2 p_i^2 p_j x_i^2 x_j + \frac{3}{2} \epsilon^3 p_i^2 p_j x_i^2 x_j - \frac{55}{24} \epsilon^4 p_i^2 p_j x_i^2 x_j + \epsilon^2 p_i p_j^2 x_i^2 x_j + \frac{1}{2} T \epsilon^2 p_i p_j^2 x_i^2 x_j - \\
 & \frac{9}{2} \epsilon^3 p_i p_j^2 x_i^2 x_j - T \epsilon^3 p_i p_j^2 x_i^2 x_j + \frac{53}{6} \epsilon^4 p_i p_j^2 x_i^2 x_j + \frac{25}{24} T \epsilon^4 p_i p_j^2 x_i^2 x_j - \frac{1}{2} \epsilon^2 p_j^3 x_i^2 x_j - \\
 & \frac{1}{2} T \epsilon^2 p_j^3 x_i^2 x_j + 3 \epsilon^3 p_j^3 x_i^2 x_j + T \epsilon^3 p_j^3 x_i^2 x_j - \frac{539}{72} \epsilon^4 p_j^3 x_i^2 x_j + \frac{103}{72} T \epsilon^4 p_j^3 x_i^2 x_j - \frac{55}{36} T^2 \epsilon^4 p_j^3 x_i^2 x_j - \\
 & \frac{1}{6} \epsilon^3 p_i^3 p_j x_i^3 x_j + \frac{3}{4} \epsilon^4 p_i^3 p_j x_i^3 x_j + \frac{7}{6} T \epsilon^3 p_i^2 p_j^2 x_i^3 x_j - \frac{5}{2} \epsilon^4 p_i^2 p_j^2 x_i^3 x_j - \frac{53}{12} T \epsilon^4 p_i^2 p_j^2 x_i^3 x_j + \\
 & \epsilon^3 p_i p_j^3 x_i^3 x_j - \frac{17}{6} T \epsilon^3 p_i p_j^3 x_i^3 x_j - \frac{1}{6} T^2 \epsilon^3 p_i p_j^3 x_i^3 x_j + \epsilon^4 p_i p_j^3 x_i^3 x_j + \frac{49}{4} T \epsilon^4 p_i p_j^3 x_i^3 x_j + \\
 & \frac{5}{12} T^2 \epsilon^4 p_i p_j^3 x_i^3 x_j - \frac{5}{6} \epsilon^3 p_j^4 x_i^3 x_j + \frac{5}{3} T \epsilon^3 p_j^4 x_i^3 x_j + \frac{1}{6} T^2 \epsilon^3 p_j^4 x_i^3 x_j + \frac{61}{48} \epsilon^4 p_j^4 x_i^3 x_j - \\
 & \frac{151}{16} T \epsilon^4 p_j^4 x_i^3 x_j + \frac{59}{48} T^2 \epsilon^4 p_j^4 x_i^3 x_j - \frac{9}{16} T^3 \epsilon^4 p_j^4 x_i^3 x_j - \frac{1}{24} \epsilon^4 p_i^4 p_j x_i^4 x_j - \frac{2}{3} \epsilon^4 p_i^3 p_j^2 x_i^4 x_j + \\
 & \frac{31}{24} T \epsilon^4 p_i^3 p_j^2 x_i^4 x_j + \frac{15}{4} \epsilon^4 p_i^2 p_j^3 x_i^4 x_j - \frac{115}{24} T \epsilon^4 p_i^2 p_j^3 x_i^4 x_j - \frac{25}{24} T^2 \epsilon^4 p_i^2 p_j^3 x_i^4 x_j - \frac{11}{2} \epsilon^4 p_i p_j^4 x_i^4 x_j + \\
 & \frac{131}{24} T \epsilon^4 p_i p_j^4 x_i^4 x_j + \frac{5}{2} T^2 \epsilon^4 p_i p_j^4 x_i^4 x_j + \frac{1}{24} T^3 \epsilon^4 p_i p_j^4 x_i^4 x_j + \frac{97}{40} \epsilon^4 p_j^5 x_i^4 x_j - \frac{73}{40} T \epsilon^4 p_j^5 x_i^4 x_j - \\
 & \frac{199}{120} T^2 \epsilon^4 p_j^5 x_i^4 x_j + \frac{11}{120} T^3 \epsilon^4 p_j^5 x_i^4 x_j - \frac{1}{30} T^4 \epsilon^4 p_j^5 x_i^4 x_j - \frac{1}{2} \epsilon^2 p_i p_j^2 x_i x_j^2 + \epsilon^3 p_i p_j^2 x_i x_j^2 - \\
 & \frac{25}{24} \epsilon^4 p_i p_j^2 x_i x_j^2 + \frac{1}{2} \epsilon^2 p_j^3 x_i x_j^2 - \epsilon^3 p_j^3 x_i x_j^2 + \frac{7}{72} \epsilon^4 p_j^3 x_i x_j^2 + \frac{55}{36} T \epsilon^4 p_j^3 x_i x_j^2 - \epsilon^3 p_i^2 p_j^2 x_i^2 x_j^2 + \\
 & \frac{15}{4} \epsilon^4 p_i^2 p_j^2 x_i^2 x_j^2 + \frac{5}{2} \epsilon^3 p_i p_j^3 x_i^2 x_j^2 + \frac{1}{4} T \epsilon^3 p_i p_j^3 x_i^2 x_j^2 - \frac{43}{4} \epsilon^4 p_i p_j^3 x_i^2 x_j^2 - \frac{5}{8} T \epsilon^4 p_i p_j^3 x_i^2 x_j^2 -
 \end{aligned}$$

$$\begin{aligned}
 & \frac{3}{2} \epsilon^3 p_j^4 x_i^2 x_j^2 - \frac{1}{4} T \epsilon^3 p_j^4 x_i^2 x_j^2 + \frac{249}{32} \epsilon^4 p_j^4 x_i^2 x_j^2 - T \epsilon^4 p_j^4 x_i^2 x_j^2 + \frac{27}{32} T^2 \epsilon^4 p_j^4 x_i^2 x_j^2 - \epsilon^4 p_i^3 p_j^2 x_i^3 x_j^2 + \\
 & \frac{15}{4} \epsilon^4 p_i^2 p_j^3 x_i^3 x_j^2 + \frac{5}{3} T \epsilon^4 p_i^2 p_j^3 x_i^3 x_j^2 - \frac{17}{4} \epsilon^4 p_i p_j^4 x_i^3 x_j^2 - \frac{49}{12} T \epsilon^4 p_i p_j^4 x_i^3 x_j^2 - \frac{1}{12} T^2 \epsilon^4 p_i p_j^4 x_i^3 x_j^2 + \\
 & \frac{43}{30} \epsilon^4 p_j^5 x_i^3 x_j^2 + \frac{157}{60} T \epsilon^4 p_j^5 x_i^3 x_j^2 - \frac{7}{60} T^2 \epsilon^4 p_j^5 x_i^3 x_j^2 + \frac{1}{15} T^3 \epsilon^4 p_j^5 x_i^3 x_j^2 - \frac{1}{6} \epsilon^3 p_i p_j^3 x_i x_j^3 + \\
 & \frac{5}{12} \epsilon^4 p_i p_j^3 x_i x_j^3 + \frac{1}{6} \epsilon^3 p_j^4 x_i x_j^3 + \frac{5}{48} \epsilon^4 p_j^4 x_i x_j^3 - \frac{9}{16} T \epsilon^4 p_j^4 x_i x_j^3 - \epsilon^4 p_i^2 p_j^3 x_i^2 x_j^3 + \frac{5}{2} \epsilon^4 p_i p_j^4 x_i^2 x_j^3 + \\
 & \frac{1}{12} T \epsilon^4 p_i p_j^4 x_i^2 x_j^3 - \frac{47}{30} \epsilon^4 p_j^5 x_i^2 x_j^3 + \frac{1}{20} T \epsilon^4 p_j^5 x_i^2 x_j^3 - \frac{1}{15} T^2 \epsilon^4 p_j^5 x_i^2 x_j^3 - \frac{1}{24} \epsilon^4 p_i p_j^4 x_i x_j^4 + \\
 & \frac{1}{120} \epsilon^4 p_j^5 x_i x_j^4 + \frac{1}{30} T \epsilon^4 p_j^5 x_i x_j^4 + \epsilon^4 ca_{4,1} + \epsilon^4 p_i p_j x_i^2 ca_{4,9} - \epsilon^4 p_j^2 x_i^2 ca_{4,9} + T \epsilon^4 p_j^2 x_i^2 ca_{4,9} - \\
 & \epsilon^4 p_j^2 x_i x_j ca_{4,9} + \epsilon^4 p_i^2 p_j x_i^3 ca_{4,19} - \epsilon^4 p_i p_j^2 x_i^3 ca_{4,19} + T \epsilon^4 p_i p_j^2 x_i^3 ca_{4,19} - \epsilon^4 p_j^3 x_i^2 x_j ca_{4,19} + \\
 & T \epsilon^4 p_j^3 x_i^2 x_j ca_{4,19} - \epsilon^4 p_j^3 x_i x_j^2 ca_{4,19} + \epsilon^4 p_i^3 p_j x_i^4 ca_{4,36} - \frac{3}{2} \epsilon^4 p_i^2 p_j^2 x_i^4 ca_{4,36} + \frac{3}{2} T \epsilon^4 p_i^2 p_j^2 x_i^4 ca_{4,36} + \\
 & \epsilon^4 p_i p_j^3 x_i^4 ca_{4,36} - 2 T \epsilon^4 p_i p_j^3 x_i^4 ca_{4,36} + T^2 \epsilon^4 p_i p_j^3 x_i^4 ca_{4,36} - \frac{1}{2} \epsilon^4 p_j^4 x_i^4 ca_{4,36} + \frac{3}{2} T \epsilon^4 p_j^4 x_i^4 ca_{4,36} - \\
 & \frac{3}{2} T^2 \epsilon^4 p_j^4 x_i^4 ca_{4,36} + \frac{1}{2} T^3 \epsilon^4 p_j^4 x_i^4 ca_{4,36} - \epsilon^4 p_j^4 x_i^3 x_j ca_{4,36} + 2 T \epsilon^4 p_j^4 x_i^3 x_j ca_{4,36} - T^2 \epsilon^4 p_j^4 x_i^3 x_j ca_{4,36} - \\
 & \frac{3}{2} \epsilon^4 p_j^4 x_i^2 x_j^2 ca_{4,36} + \frac{3}{2} T \epsilon^4 p_j^4 x_i^2 x_j^2 ca_{4,36} - \epsilon^4 p_j^4 x_i x_j^3 ca_{4,36} + \epsilon^4 p_i^4 p_j x_i^5 ca_{4,62} - 2 \epsilon^4 p_i^3 p_j^2 x_i^5 ca_{4,62} + \\
 & 2 T \epsilon^4 p_i^3 p_j^2 x_i^5 ca_{4,62} + 2 \epsilon^4 p_i^2 p_j^3 x_i^5 ca_{4,62} - 4 T \epsilon^4 p_i^2 p_j^3 x_i^5 ca_{4,62} + 2 T^2 \epsilon^4 p_i^2 p_j^3 x_i^5 ca_{4,62} - \\
 & \epsilon^4 p_i p_j^4 x_i^5 ca_{4,62} + 3 T \epsilon^4 p_i p_j^4 x_i^5 ca_{4,62} - 3 T^2 \epsilon^4 p_i p_j^4 x_i^5 ca_{4,62} + T^3 \epsilon^4 p_i p_j^4 x_i^5 ca_{4,62} - \epsilon^4 p_j^5 x_i^4 x_j ca_{4,62} + \\
 & 3 T \epsilon^4 p_j^5 x_i^4 x_j ca_{4,62} - 3 T^2 \epsilon^4 p_j^5 x_i^4 x_j ca_{4,62} + T^3 \epsilon^4 p_j^5 x_i^4 x_j ca_{4,62} - 2 \epsilon^4 p_j^5 x_i^3 x_j^2 ca_{4,62} + \\
 & 4 T \epsilon^4 p_j^5 x_i^3 x_j^2 ca_{4,62} - 2 T^2 \epsilon^4 p_j^5 x_i^3 x_j^2 ca_{4,62} - 2 \epsilon^4 p_j^5 x_i^2 x_j^3 ca_{4,62} + 2 T \epsilon^4 p_j^5 x_i^2 x_j^3 ca_{4,62} - \\
 & \epsilon^4 p_j^5 x_i x_j^4 ca_{4,62} - \epsilon^4 p_i x_i cb_{4,2} + \epsilon^4 p_j x_i cb_{4,2} + T \epsilon^4 p_j x_i cb_{4,5} - \epsilon^4 p_j x_j cb_{4,5} + \frac{1}{2} T \epsilon^4 p_j^2 x_i^2 cb_{4,10} - \\
 & \frac{1}{2} T^2 \epsilon^4 p_j^2 x_i^2 cb_{4,10} - \epsilon^4 p_i p_j x_i x_j cb_{4,10} + \frac{1}{2} \epsilon^4 p_j^2 x_i x_j cb_{4,10} + \frac{1}{2} T \epsilon^4 p_j^2 x_i x_j cb_{4,10}
 \end{aligned}$$

(Alt) Out[\*]=

$$\begin{aligned}
 & \frac{\epsilon}{2} - \epsilon p_k x_k - \frac{1}{2} \epsilon^2 p_k x_k - \frac{1}{6} \epsilon^3 p_k x_k + \epsilon^4 cc_{4,1} + \\
 & \epsilon^4 p_k x_k cc_{4,2} + \epsilon^4 p_k^2 x_k^2 cc_{4,3} + \epsilon^4 p_k^3 x_k^3 cc_{4,4} + \epsilon^4 p_k^4 x_k^4 cc_{4,5} + \epsilon^4 p_k^5 x_k^5 cc_{4,6}
 \end{aligned}$$

## R2c @ d = 4

(Alt) In[ ]:=

```

lhs = CF[Module[{es = {i, j, i+, j+}},
  Times[
    Normal@Series[Exp[r_d[-1, 0, 1, i, j+] + r_d[1, i+, j]], {e, 0, d}],
    Exp[Sum[g_{\alpha,\beta} \pi_{\alpha} \xi_{\beta}, {\alpha, es}, {\beta, es}]]
  ] // Zip_{(p_{\alpha}&/@es)\cup(x_{\alpha}&/@es)} // Expand
] // . gRules_{-1,i,j+} \cup gRules_{1,i+,j}

```

(Alt) Out[ ]:=

$$\begin{aligned}
 & 1 + \frac{e}{2} + \frac{e^2}{8} + \frac{e^3}{48} + \frac{1}{384} e^4 (1 + 384 cc_{4,1}) - \frac{(-1+T) e^4 (-7 + 12 cb_{4,10} + 12 cc_{4,2}) g_{j^{++}, i^{++}}}{12 T} + \\
 & \frac{2 (-1+T)^2 e^4 cc_{4,3} g_{j^{++}, i^{++}}^2}{T^2} - \frac{6 (-1+T)^3 e^4 cc_{4,4} g_{j^{++}, i^{++}}^3}{T^3} + \frac{24 (-1+T)^4 e^4 cc_{4,5} g_{j^{++}, i^{++}}^4}{T^4} - \\
 & \frac{120 (-1+T)^5 e^4 cc_{4,6} g_{j^{++}, i^{++}}^5}{T^5} - e g_{j^{++}, j^{++}} - e^2 g_{j^{++}, j^{++}}^2 - \frac{13}{24} e^3 g_{j^{++}, j^{++}}^3 + \frac{1}{6} e^4 (-1 + 6 cc_{4,2}) g_{j^{++}, j^{++}} - \\
 & \frac{4 (-1+T) e^4 cc_{4,3} g_{j^{++}, i^{++}} g_{j^{++}, j^{++}}}{T} + \frac{18 (-1+T)^2 e^4 cc_{4,4} g_{j^{++}, i^{++}}^2 g_{j^{++}, j^{++}}}{T^2} - \\
 & \frac{96 (-1+T)^3 e^4 cc_{4,5} g_{j^{++}, i^{++}}^3 g_{j^{++}, j^{++}}}{T^3} + \frac{600 (-1+T)^4 e^4 cc_{4,6} g_{j^{++}, i^{++}}^4 g_{j^{++}, j^{++}}}{T^4} + e^2 g_{j^{++}, j^{++}}^2 + \\
 & \frac{3}{2} e^3 g_{j^{++}, j^{++}}^2 + \frac{1}{24} e^4 (29 + 48 cc_{4,3}) g_{j^{++}, j^{++}}^2 - \frac{18 (-1+T) e^4 cc_{4,4} g_{j^{++}, i^{++}} g_{j^{++}, j^{++}}^2}{T} + \\
 & \frac{144 (-1+T)^2 e^4 cc_{4,5} g_{j^{++}, i^{++}}^2 g_{j^{++}, j^{++}}^2}{T^2} - \frac{1200 (-1+T)^3 e^4 cc_{4,6} g_{j^{++}, i^{++}}^3 g_{j^{++}, j^{++}}^2}{T^3} - e^3 g_{j^{++}, j^{++}}^3 + \\
 & 2 e^4 (-1 + 3 cc_{4,4}) g_{j^{++}, j^{++}}^3 - \frac{96 (-1+T) e^4 cc_{4,5} g_{j^{++}, i^{++}} g_{j^{++}, j^{++}}^3}{T} + \frac{1200 (-1+T)^2 e^4 cc_{4,6} g_{j^{++}, i^{++}}^2 g_{j^{++}, j^{++}}^3}{T^2} + \\
 & e^4 (1 + 24 cc_{4,5}) g_{j^{++}, j^{++}}^4 - \frac{600 (-1+T) e^4 cc_{4,6} g_{j^{++}, i^{++}} g_{j^{++}, j^{++}}^4}{T} + 120 e^4 cc_{4,6} g_{j^{++}, j^{++}}^5
 \end{aligned}$$

(Alt) In[ ]:=

```

rhs = CF[Module[{es = {(j+)+}},
  Times[
    Normal@Series[Exp[γd[1, (j+)+]], {ε, 0, d}],
    Exp[Sum[gα,β πα ξβ, {α, es}, {β, es}]]
  ] // Zip (p#&/@es) ∪ (x#&/@es) // Expand
]]

```

(Alt) Out[ ]:=

$$\begin{aligned}
& 1 + \frac{\epsilon}{2} + \frac{\epsilon^2}{8} + \frac{\epsilon^3}{48} + \frac{1}{384} \epsilon^4 (1 + 384 \text{cc}_{4,1}) - \epsilon \mathbf{g}_{j^{++}, j^{++}} - \epsilon^2 \mathbf{g}_{j^{++}, j^{++}}^2 - \frac{13}{24} \epsilon^3 \mathbf{g}_{j^{++}, j^{++}}^3 + \\
& \frac{1}{6} \epsilon^4 (-1 + 6 \text{cc}_{4,2}) \mathbf{g}_{j^{++}, j^{++}} + \epsilon^2 \mathbf{g}_{j^{++}, j^{++}}^2 + \frac{3}{2} \epsilon^3 \mathbf{g}_{j^{++}, j^{++}}^2 + \frac{1}{24} \epsilon^4 (29 + 48 \text{cc}_{4,3}) \mathbf{g}_{j^{++}, j^{++}}^2 - \\
& \epsilon^3 \mathbf{g}_{j^{++}, j^{++}}^3 + 2 \epsilon^4 (-1 + 3 \text{cc}_{4,4}) \mathbf{g}_{j^{++}, j^{++}}^3 + \epsilon^4 (1 + 24 \text{cc}_{4,5}) \mathbf{g}_{j^{++}, j^{++}}^4 + 120 \epsilon^4 \text{cc}_{4,6} \mathbf{g}_{j^{++}, j^{++}}^5
\end{aligned}$$

(Alt) In[ ]:=

```
me = Exponent[lhs - rhs, T, Min]
```

(Alt) Out[ ]:=

-5

(Alt) In[ ]:=

```
covars = DeleteCases[Variables[lhs - rhs], T | (ca | cb | cc | cd) _]
```

(Alt) Out[ ]:=

```
{ε, gj++, i++, gj++, j++}
```

(Alt) In[ ]:=

```
CoefficientRules[Expand[T-me (lhs - rhs)], covars] // Column
```

(Alt) Out[ ]:=

$$\begin{aligned}
\{4, 5, 0\} &\rightarrow 120 \text{cc}_{4,6} - 600 T \text{cc}_{4,6} + 1200 T^2 \text{cc}_{4,6} - 1200 T^3 \text{cc}_{4,6} + 600 T^4 \text{cc}_{4,6} - 120 T^5 \text{cc}_{4,6} \\
\{4, 4, 1\} &\rightarrow 600 T \text{cc}_{4,6} - 2400 T^2 \text{cc}_{4,6} + 3600 T^3 \text{cc}_{4,6} - 2400 T^4 \text{cc}_{4,6} + 600 T^5 \text{cc}_{4,6} \\
\{4, 4, 0\} &\rightarrow 24 T \text{cc}_{4,5} - 96 T^2 \text{cc}_{4,5} + 144 T^3 \text{cc}_{4,5} - 96 T^4 \text{cc}_{4,5} + 24 T^5 \text{cc}_{4,5} \\
\{4, 3, 2\} &\rightarrow 1200 T^2 \text{cc}_{4,6} - 3600 T^3 \text{cc}_{4,6} + 3600 T^4 \text{cc}_{4,6} - 1200 T^5 \text{cc}_{4,6} \\
\{4, 3, 1\} &\rightarrow 96 T^2 \text{cc}_{4,5} - 288 T^3 \text{cc}_{4,5} + 288 T^4 \text{cc}_{4,5} - 96 T^5 \text{cc}_{4,5} \\
\{4, 3, 0\} &\rightarrow 6 T^2 \text{cc}_{4,4} - 18 T^3 \text{cc}_{4,4} + 18 T^4 \text{cc}_{4,4} - 6 T^5 \text{cc}_{4,4} \\
\{4, 2, 3\} &\rightarrow 1200 T^3 \text{cc}_{4,6} - 2400 T^4 \text{cc}_{4,6} + 1200 T^5 \text{cc}_{4,6} \\
\{4, 2, 2\} &\rightarrow 144 T^3 \text{cc}_{4,5} - 288 T^4 \text{cc}_{4,5} + 144 T^5 \text{cc}_{4,5} \\
\{4, 2, 1\} &\rightarrow 18 T^3 \text{cc}_{4,4} - 36 T^4 \text{cc}_{4,4} + 18 T^5 \text{cc}_{4,4} \\
\{4, 2, 0\} &\rightarrow 2 T^3 \text{cc}_{4,3} - 4 T^4 \text{cc}_{4,3} + 2 T^5 \text{cc}_{4,3} \\
\{4, 1, 4\} &\rightarrow 600 T^4 \text{cc}_{4,6} - 600 T^5 \text{cc}_{4,6} \\
\{4, 1, 3\} &\rightarrow 96 T^4 \text{cc}_{4,5} - 96 T^5 \text{cc}_{4,5} \\
\{4, 1, 2\} &\rightarrow 18 T^4 \text{cc}_{4,4} - 18 T^5 \text{cc}_{4,4} \\
\{4, 1, 1\} &\rightarrow 4 T^4 \text{cc}_{4,3} - 4 T^5 \text{cc}_{4,3} \\
\{4, 1, 0\} &\rightarrow -\frac{7T^4}{12} + \frac{7T^5}{12} + T^4 \text{cb}_{4,10} - T^5 \text{cb}_{4,10} + T^4 \text{cc}_{4,2} - T^5 \text{cc}_{4,2}
\end{aligned}$$

(Alt) In[ ]:=

```
eqnsR2c =
  (Factor[#] == 0) & /@ Union[Last /@ CoefficientRules[Expand[T^me (lhs - rhs)], covars]]
```

(Alt) Out[ ]:=

$$\left\{ -\frac{1}{12} (-1 + T) T^4 (-7 + 12 cb_{4,10} + 12 cc_{4,2}) = 0, -4 (-1 + T) T^4 cc_{4,3} = 0, 2 (-1 + T)^2 T^3 cc_{4,3} = 0, \right.$$

$$-18 (-1 + T) T^4 cc_{4,4} = 0, -6 (-1 + T)^3 T^2 cc_{4,4} = 0, 18 (-1 + T)^2 T^3 cc_{4,4} = 0,$$

$$-96 (-1 + T) T^4 cc_{4,5} = 0, -96 (-1 + T)^3 T^2 cc_{4,5} = 0, 24 (-1 + T)^4 T cc_{4,5} = 0,$$

$$144 (-1 + T)^2 T^3 cc_{4,5} = 0, -1200 (-1 + T)^3 T^2 cc_{4,6} = 0, -600 (-1 + T) T^4 cc_{4,6} = 0,$$

$$\left. -120 (-1 + T)^5 cc_{4,6} = 0, 600 (-1 + T)^4 T cc_{4,6} = 0, 1200 (-1 + T)^2 T^3 cc_{4,6} = 0 \right\}$$

(Alt) In[ ]:=

```
vars = Cases[Variables[r_d[1, i1, j1] + r_d[-1, i2, j2] + \gamma_d[1, k]], (ca | cb | cc | cd) __]
{sol} = Solve[eqnsR3 \cup eqnsR2b \cup eqnsR2c \cup eqnsR11 \cup eqnsR1r \cup eqnsSwp, vars]
sol /. Rule -> Set;
r_d[1, i, j]
\gamma_d[1, k]
```

(Alt) Out[ ]:=

$$\{ca_{4,9}, ca_{4,19}, ca_{4,36}, ca_{4,62}, cb_{4,2}, cb_{4,5}, cb_{4,10}, cc_{4,1}, cc_{4,2}, cc_{4,3}, cc_{4,4}, cc_{4,5}, cc_{4,6}\}$$

Solve: Equations may not give solutions for all "solve" variables.

(Alt) Out[ ]:=

$$\left\{ \left\{ cc_{4,2} \rightarrow \frac{7}{12} - cb_{4,10}, cc_{4,3} \rightarrow 0, cc_{4,4} \rightarrow 0, cc_{4,5} \rightarrow 0, cc_{4,6} \rightarrow 0 \right\} \right\}$$

(Alt) Out[ ]:=

$$-\frac{\epsilon}{2} + \epsilon p_i x_i - \frac{1}{2} \epsilon^2 p_i x_i + \frac{1}{6} \epsilon^3 p_i x_i - \frac{1}{12} \epsilon^4 p_i x_i - \epsilon p_j x_i + \frac{1}{2} \epsilon^2 p_j x_i - \frac{1}{6} \epsilon^3 p_j x_i + \frac{1}{12} \epsilon^4 p_j x_i -$$

$$\frac{1}{2} \epsilon p_i p_j x_i^2 + \frac{1}{2} T \epsilon p_i p_j x_i^2 + \frac{1}{4} \epsilon^2 p_i p_j x_i^2 - \frac{3}{4} T \epsilon^2 p_i p_j x_i^2 + \frac{5}{12} \epsilon^3 p_i p_j x_i^2 + \frac{7}{12} T \epsilon^3 p_i p_j x_i^2 +$$

$$\frac{1}{2} \epsilon p_j^2 x_i^2 - \frac{1}{2} T \epsilon p_j^2 x_i^2 - \frac{1}{4} \epsilon^2 p_j^2 x_i^2 + \frac{3}{4} T \epsilon^2 p_j^2 x_i^2 - \frac{5}{12} \epsilon^3 p_j^2 x_i^2 - \frac{7}{12} T \epsilon^3 p_j^2 x_i^2 + \frac{5}{12} T \epsilon^4 p_j^2 x_i^2 +$$

$$\frac{5}{8} T^2 \epsilon^4 p_j^2 x_i^2 - \frac{1}{3} \epsilon^2 p_i^2 p_j x_i^3 + \frac{1}{3} T \epsilon^2 p_i^2 p_j x_i^3 + \frac{5}{6} \epsilon^3 p_i^2 p_j x_i^3 - T \epsilon^3 p_i^2 p_j x_i^3 + \frac{5}{6} \epsilon^2 p_i p_j^2 x_i^3 -$$

$$\frac{2}{3} T \epsilon^2 p_i p_j^2 x_i^3 - \frac{1}{6} T^2 \epsilon^2 p_i p_j^2 x_i^3 - \frac{8}{3} \epsilon^3 p_i p_j^2 x_i^3 + \frac{17}{6} T \epsilon^3 p_i p_j^2 x_i^3 + \frac{1}{3} T^2 \epsilon^3 p_i p_j^2 x_i^3 + \frac{67}{24} \epsilon^4 p_i p_j^2 x_i^3 -$$

$$3 T \epsilon^4 p_i p_j^2 x_i^3 - \frac{15}{8} T^2 \epsilon^4 p_i p_j^2 x_i^3 - \frac{1}{2} \epsilon^2 p_j^3 x_i^3 + \frac{1}{3} T \epsilon^2 p_j^3 x_i^3 + \frac{1}{6} T^2 \epsilon^2 p_j^3 x_i^3 + \frac{11}{6} \epsilon^3 p_j^3 x_i^3 - \frac{11}{6} T \epsilon^3 p_j^3 x_i^3 -$$

$$\frac{1}{3} T^2 \epsilon^3 p_j^3 x_i^3 - \frac{67}{24} \epsilon^4 p_j^3 x_i^3 + \frac{71}{18} T \epsilon^4 p_j^3 x_i^3 + \frac{25}{72} T^2 \epsilon^4 p_j^3 x_i^3 - \frac{1}{8} \epsilon^3 p_i^3 p_j x_i^4 + \frac{1}{8} T \epsilon^3 p_i^3 p_j x_i^4 +$$

$$\frac{1}{2} \epsilon^3 p_i^2 p_j^2 x_i^4 - \frac{1}{8} T \epsilon^3 p_i^2 p_j^2 x_i^4 - \frac{3}{8} T^2 \epsilon^3 p_i^2 p_j^2 x_i^4 - \frac{79}{32} \epsilon^4 p_i^2 p_j^2 x_i^4 + \frac{23}{48} T \epsilon^4 p_i^2 p_j^2 x_i^4 + \frac{73}{32} T^2 \epsilon^4 p_i^2 p_j^2 x_i^4 -$$

$$\frac{13}{24} \epsilon^3 p_i p_j^3 x_i^4 - \frac{3}{8} T \epsilon^3 p_i p_j^3 x_i^4 + \frac{7}{8} T^2 \epsilon^3 p_i p_j^3 x_i^4 + \frac{1}{24} T^3 \epsilon^3 p_i p_j^3 x_i^4 + \frac{55}{12} \epsilon^4 p_i p_j^3 x_i^4 - \frac{1}{24} T \epsilon^4 p_i p_j^3 x_i^4 -$$

$$\frac{11}{2} T^2 \epsilon^4 p_i p_j^3 x_i^4 + \frac{11}{24} T^3 \epsilon^4 p_i p_j^3 x_i^4 + \frac{1}{6} \epsilon^3 p_j^4 x_i^4 + \frac{3}{8} T \epsilon^3 p_j^4 x_i^4 - \frac{1}{2} T^2 \epsilon^3 p_j^4 x_i^4 - \frac{1}{24} T^3 \epsilon^3 p_j^4 x_i^4 -$$

$$\begin{aligned}
 & \frac{203}{96} \epsilon^4 p_j^4 x_i^4 - \frac{23}{24} T \epsilon^4 p_j^4 x_i^4 + \frac{97}{24} T^2 \epsilon^4 p_j^4 x_i^4 - T^3 \epsilon^4 p_j^4 x_i^4 + \frac{9}{32} T^4 \epsilon^4 p_j^4 x_i^4 + \frac{1}{2} T \epsilon^4 p_i^3 p_j^2 x_i^5 - \\
 & \frac{1}{2} T^2 \epsilon^4 p_i^3 p_j^2 x_i^5 + \frac{5}{12} \epsilon^4 p_i^2 p_j^3 x_i^5 - \frac{7}{3} T \epsilon^4 p_i^2 p_j^3 x_i^5 + \frac{7}{4} T^2 \epsilon^4 p_i^2 p_j^3 x_i^5 + \frac{1}{6} T^3 \epsilon^4 p_i^2 p_j^3 x_i^5 - \frac{7}{8} \epsilon^4 p_i p_j^4 x_i^5 + \\
 & \frac{13}{4} T \epsilon^4 p_i p_j^4 x_i^5 - \frac{23}{12} T^2 \epsilon^4 p_i p_j^4 x_i^5 - \frac{5}{12} T^3 \epsilon^4 p_i p_j^4 x_i^5 - \frac{1}{24} T^4 \epsilon^4 p_i p_j^4 x_i^5 + \frac{11}{24} \epsilon^4 p_j^5 x_i^5 - \frac{83}{60} T \epsilon^4 p_j^5 x_i^5 + \\
 & \frac{3}{5} T^2 \epsilon^4 p_j^5 x_i^5 + \frac{19}{60} T^3 \epsilon^4 p_j^5 x_i^5 + \frac{1}{120} T^4 \epsilon^4 p_j^5 x_i^5 - \epsilon p_i p_j x_i x_j + \frac{3}{2} \epsilon^2 p_i p_j x_i x_j - \frac{7}{6} \epsilon^3 p_i p_j x_i x_j + \\
 & \frac{5}{4} \epsilon^4 p_i p_j x_i x_j + \epsilon p_j^2 x_i x_j - \frac{3}{2} \epsilon^2 p_j^2 x_i x_j + \frac{7}{6} \epsilon^3 p_j^2 x_i x_j - \frac{5}{3} \epsilon^4 p_j^2 x_i x_j - \frac{5}{8} T \epsilon^4 p_j^2 x_i x_j - \\
 & \frac{1}{2} \epsilon^2 p_i^2 p_j x_i^2 x_j + \frac{3}{2} \epsilon^3 p_i^2 p_j x_i^2 x_j - \frac{55}{24} \epsilon^4 p_i^2 p_j x_i^2 x_j + \epsilon^2 p_i p_j^2 x_i^2 x_j + \frac{1}{2} T \epsilon^2 p_i p_j^2 x_i^2 x_j - \\
 & \frac{9}{2} \epsilon^3 p_i p_j^2 x_i^2 x_j - T \epsilon^3 p_i p_j^2 x_i^2 x_j + \frac{53}{6} \epsilon^4 p_i p_j^2 x_i^2 x_j + \frac{25}{24} T \epsilon^4 p_i p_j^2 x_i^2 x_j - \frac{1}{2} \epsilon^2 p_j^3 x_i^2 x_j - \\
 & \frac{1}{2} T \epsilon^2 p_j^3 x_i^2 x_j + 3 \epsilon^3 p_j^3 x_i^2 x_j + T \epsilon^3 p_j^3 x_i^2 x_j - \frac{539}{72} \epsilon^4 p_j^3 x_i^2 x_j + \frac{103}{72} T \epsilon^4 p_j^3 x_i^2 x_j - \frac{55}{36} T^2 \epsilon^4 p_j^3 x_i^2 x_j - \\
 & \frac{1}{6} \epsilon^3 p_i p_j^3 x_i^3 x_j + \frac{3}{4} \epsilon^4 p_i p_j^3 x_i^3 x_j + \frac{7}{6} T \epsilon^3 p_i^2 p_j^2 x_i^3 x_j - \frac{5}{2} \epsilon^4 p_i^2 p_j^2 x_i^3 x_j - \frac{53}{12} T \epsilon^4 p_i^2 p_j^2 x_i^3 x_j + \\
 & \epsilon^3 p_i p_j^3 x_i^3 x_j - \frac{17}{6} T \epsilon^3 p_i p_j^3 x_i^3 x_j - \frac{1}{6} T^2 \epsilon^3 p_i p_j^3 x_i^3 x_j + \epsilon^4 p_i p_j^3 x_i^3 x_j + \frac{49}{4} T \epsilon^4 p_i p_j^3 x_i^3 x_j + \\
 & \frac{5}{12} T^2 \epsilon^4 p_i p_j^3 x_i^3 x_j - \frac{5}{6} \epsilon^3 p_j^4 x_i^3 x_j + \frac{5}{3} T \epsilon^3 p_j^4 x_i^3 x_j + \frac{1}{6} T^2 \epsilon^3 p_j^4 x_i^3 x_j + \frac{61}{48} \epsilon^4 p_j^4 x_i^3 x_j - \\
 & \frac{151}{16} T \epsilon^4 p_j^4 x_i^3 x_j + \frac{59}{48} T^2 \epsilon^4 p_j^4 x_i^3 x_j - \frac{9}{16} T^3 \epsilon^4 p_j^4 x_i^3 x_j - \frac{1}{24} \epsilon^4 p_i^4 p_j x_i^4 x_j - \frac{2}{3} \epsilon^4 p_i^3 p_j^2 x_i^4 x_j + \\
 & \frac{31}{24} T \epsilon^4 p_i^3 p_j^2 x_i^4 x_j + \frac{15}{4} \epsilon^4 p_i^2 p_j^3 x_i^4 x_j - \frac{115}{24} T \epsilon^4 p_i^2 p_j^3 x_i^4 x_j - \frac{25}{24} T^2 \epsilon^4 p_i^2 p_j^3 x_i^4 x_j - \frac{11}{2} \epsilon^4 p_i p_j^4 x_i^4 x_j + \\
 & \frac{131}{24} T \epsilon^4 p_i p_j^4 x_i^4 x_j + \frac{5}{2} T^2 \epsilon^4 p_i p_j^4 x_i^4 x_j + \frac{1}{24} T^3 \epsilon^4 p_i p_j^4 x_i^4 x_j + \frac{97}{40} \epsilon^4 p_j^5 x_i^4 x_j - \frac{73}{40} T \epsilon^4 p_j^5 x_i^4 x_j - \\
 & \frac{199}{120} T^2 \epsilon^4 p_j^5 x_i^4 x_j + \frac{11}{120} T^3 \epsilon^4 p_j^5 x_i^4 x_j - \frac{1}{30} T^4 \epsilon^4 p_j^5 x_i^4 x_j - \frac{1}{2} \epsilon^2 p_i p_j^2 x_i x_j^2 + \epsilon^3 p_i p_j^2 x_i x_j^2 - \\
 & \frac{25}{24} \epsilon^4 p_i p_j^2 x_i x_j^2 + \frac{1}{2} \epsilon^2 p_j^3 x_i x_j^2 - \epsilon^3 p_j^3 x_i x_j^2 + \frac{7}{72} \epsilon^4 p_j^3 x_i x_j^2 + \frac{55}{36} T \epsilon^4 p_j^3 x_i x_j^2 - \epsilon^3 p_i^2 p_j^2 x_i^2 x_j^2 + \\
 & \frac{15}{4} \epsilon^4 p_i^2 p_j^2 x_i^2 x_j^2 + \frac{5}{2} \epsilon^3 p_i p_j^3 x_i^2 x_j^2 + \frac{1}{4} T \epsilon^3 p_i p_j^3 x_i^2 x_j^2 - \frac{43}{4} \epsilon^4 p_i p_j^3 x_i^2 x_j^2 - \frac{5}{8} T \epsilon^4 p_i p_j^3 x_i^2 x_j^2 - \\
 & \frac{3}{2} \epsilon^3 p_j^4 x_i^2 x_j^2 - \frac{1}{4} T \epsilon^3 p_j^4 x_i^2 x_j^2 + \frac{249}{32} \epsilon^4 p_j^4 x_i^2 x_j^2 - T \epsilon^4 p_j^4 x_i^2 x_j^2 + \frac{27}{32} T^2 \epsilon^4 p_j^4 x_i^2 x_j^2 - \epsilon^4 p_i^3 p_j^2 x_i^3 x_j^2 + \\
 & \frac{15}{4} \epsilon^4 p_i^2 p_j^3 x_i^3 x_j^2 + \frac{5}{3} T \epsilon^4 p_i^2 p_j^3 x_i^3 x_j^2 - \frac{17}{4} \epsilon^4 p_i p_j^4 x_i^3 x_j^2 - \frac{49}{12} T \epsilon^4 p_i p_j^4 x_i^3 x_j^2 - \frac{1}{12} T^2 \epsilon^4 p_i p_j^4 x_i^3 x_j^2 + \\
 & \frac{43}{30} \epsilon^4 p_j^5 x_i^3 x_j^2 + \frac{157}{60} T \epsilon^4 p_j^5 x_i^3 x_j^2 - \frac{7}{60} T^2 \epsilon^4 p_j^5 x_i^3 x_j^2 + \frac{1}{15} T^3 \epsilon^4 p_j^5 x_i^3 x_j^2 - \frac{1}{6} \epsilon^3 p_i p_j^3 x_i x_j^3 + \\
 & \frac{5}{12} \epsilon^4 p_i p_j^3 x_i x_j^3 + \frac{1}{6} \epsilon^3 p_j^4 x_i x_j^3 + \frac{5}{48} \epsilon^4 p_j^4 x_i x_j^3 - \frac{9}{16} T \epsilon^4 p_j^4 x_i x_j^3 - \epsilon^4 p_i^2 p_j^2 x_i^2 x_j^3 + \frac{5}{2} \epsilon^4 p_i p_j^4 x_i^2 x_j^3 + \\
 & \frac{1}{12} T \epsilon^4 p_i p_j^4 x_i^2 x_j^3 - \frac{47}{30} \epsilon^4 p_j^5 x_i^2 x_j^3 + \frac{1}{20} T \epsilon^4 p_j^5 x_i^2 x_j^3 - \frac{1}{15} T^2 \epsilon^4 p_j^5 x_i^2 x_j^3 - \frac{1}{24} \epsilon^4 p_i p_j^4 x_i x_j^4 +
 \end{aligned}$$

$$\begin{aligned} & \frac{1}{120} \epsilon^4 p_j^5 x_i x_j^4 + \frac{1}{30} T \epsilon^4 p_j^5 x_i x_j^4 + \epsilon^4 ca_{4,1} + \epsilon^4 p_i p_j x_i^2 ca_{4,9} - \epsilon^4 p_j^2 x_i^2 ca_{4,9} + T \epsilon^4 p_j^2 x_i^2 ca_{4,9} - \\ & \epsilon^4 p_j^2 x_i x_j ca_{4,9} + \epsilon^4 p_i^2 p_j x_i^3 ca_{4,19} - \epsilon^4 p_i p_j^2 x_i^3 ca_{4,19} + T \epsilon^4 p_i p_j^2 x_i^3 ca_{4,19} - \epsilon^4 p_j^3 x_i^2 x_j ca_{4,19} + \\ & T \epsilon^4 p_j^3 x_i^2 x_j ca_{4,19} - \epsilon^4 p_j^3 x_i x_j^2 ca_{4,19} + \epsilon^4 p_i^3 p_j x_i^4 ca_{4,36} - \frac{3}{2} \epsilon^4 p_i^2 p_j^2 x_i^4 ca_{4,36} + \frac{3}{2} T \epsilon^4 p_i^2 p_j^2 x_i^4 ca_{4,36} + \\ & \epsilon^4 p_i p_j^3 x_i^4 ca_{4,36} - 2 T \epsilon^4 p_i p_j^3 x_i^4 ca_{4,36} + T^2 \epsilon^4 p_i p_j^3 x_i^4 ca_{4,36} - \frac{1}{2} \epsilon^4 p_j^4 x_i^4 ca_{4,36} + \frac{3}{2} T \epsilon^4 p_j^4 x_i^4 ca_{4,36} - \\ & \frac{3}{2} T^2 \epsilon^4 p_j^4 x_i^4 ca_{4,36} + \frac{1}{2} T^3 \epsilon^4 p_j^4 x_i^4 ca_{4,36} - \epsilon^4 p_j^4 x_i^3 x_j ca_{4,36} + 2 T \epsilon^4 p_j^4 x_i^3 x_j ca_{4,36} - T^2 \epsilon^4 p_j^4 x_i^3 x_j ca_{4,36} - \\ & \frac{3}{2} \epsilon^4 p_j^4 x_i^2 x_j^2 ca_{4,36} + \frac{3}{2} T \epsilon^4 p_j^4 x_i^2 x_j^2 ca_{4,36} - \epsilon^4 p_j^4 x_i x_j^3 ca_{4,36} + \epsilon^4 p_i^4 p_j x_i^5 ca_{4,62} - 2 \epsilon^4 p_i^3 p_j^2 x_i^5 ca_{4,62} + \\ & 2 T \epsilon^4 p_i^3 p_j^2 x_i^5 ca_{4,62} + 2 \epsilon^4 p_i^2 p_j^3 x_i^5 ca_{4,62} - 4 T \epsilon^4 p_i^2 p_j^3 x_i^5 ca_{4,62} + 2 T^2 \epsilon^4 p_i^2 p_j^3 x_i^5 ca_{4,62} - \\ & \epsilon^4 p_i p_j^4 x_i^5 ca_{4,62} + 3 T \epsilon^4 p_i p_j^4 x_i^5 ca_{4,62} - 3 T^2 \epsilon^4 p_i p_j^4 x_i^5 ca_{4,62} + T^3 \epsilon^4 p_i p_j^4 x_i^5 ca_{4,62} - \epsilon^4 p_i^5 x_i^4 x_j ca_{4,62} + \\ & 3 T \epsilon^4 p_j^5 x_i^4 x_j ca_{4,62} - 3 T^2 \epsilon^4 p_j^5 x_i^4 x_j ca_{4,62} + T^3 \epsilon^4 p_j^5 x_i^4 x_j ca_{4,62} - 2 \epsilon^4 p_j^5 x_i^3 x_j^2 ca_{4,62} + \\ & 4 T \epsilon^4 p_j^5 x_i^3 x_j^2 ca_{4,62} - 2 T^2 \epsilon^4 p_j^5 x_i^3 x_j^2 ca_{4,62} - 2 \epsilon^4 p_j^5 x_i^2 x_j^3 ca_{4,62} + 2 T \epsilon^4 p_j^5 x_i^2 x_j^3 ca_{4,62} - \\ & \epsilon^4 p_j^5 x_i x_j^4 ca_{4,62} - \epsilon^4 p_i x_i cb_{4,2} + \epsilon^4 p_j x_i cb_{4,2} + T \epsilon^4 p_j x_i cb_{4,5} - \epsilon^4 p_j x_j cb_{4,5} + \frac{1}{2} T \epsilon^4 p_j^2 x_i^2 cb_{4,10} - \\ & \frac{1}{2} T^2 \epsilon^4 p_j^2 x_i^2 cb_{4,10} - \epsilon^4 p_i p_j x_i x_j cb_{4,10} + \frac{1}{2} \epsilon^4 p_j^2 x_i x_j cb_{4,10} + \frac{1}{2} T \epsilon^4 p_j^2 x_i x_j cb_{4,10} \end{aligned}$$

(Alt) Out[ ]:=

$$\frac{\epsilon}{2} - \epsilon p_k x_k - \frac{1}{2} \epsilon^2 p_k x_k - \frac{1}{6} \epsilon^3 p_k x_k + \frac{7}{12} \epsilon^4 p_k x_k - \epsilon^4 p_k x_k cb_{4,10} + \epsilon^4 cc_{4,1}$$

## R1l

(Alt) In[ ]:=

```
lhs = CF[Module[{es = {i, i^+}},
  Times[
    Normal@Series[Exp[r_d[1, 1, 0, i^+, i]], {e, 0, d}],
    Exp[Sum[g_{\alpha,\beta} \pi_{\alpha} \xi_{\beta}, {\alpha, es}, {\beta, es}]]
  ] // Zip((p_{\alpha}&/@es) \cup (x_{\alpha}&/@es) // Expand
] // . {g_{i^+,\beta} \to T^{-1} \delta_{i^+,\beta} + g_{i^{++},\beta}, g_{i,\beta} \to \delta_{i,\beta} + g_{i^+,\beta}}
```

(Alt) Out[ ]:=

$$\begin{aligned} & 1 + \epsilon^4 (ca_{4,1} + cc_{4,1}) - \\ & \frac{1}{12 T^4} \epsilon^4 (48 - 150 T + 80 T^2 - 20 T^3 + 19 T^4 + 24 T^3 ca_{4,9} + 72 T^2 ca_{4,19} + 72 T^3 ca_{4,19} + 288 T ca_{4,36} + \\ & 288 T^2 ca_{4,36} + 288 T^3 ca_{4,36} + 1440 ca_{4,62} + 1440 T ca_{4,62} + 1440 T^2 ca_{4,62} + \\ & 1440 T^3 ca_{4,62} + 12 T^4 cb_{4,5} + 12 T^3 cb_{4,10} - 12 T^4 cb_{4,10}) g_{i^{++},i} - \\ & \frac{1}{12 T^3} \epsilon^4 (96 - 225 T + 86 T^2 + 37 T^3 + 72 T^2 ca_{4,19} + 432 T ca_{4,36} + 432 T^2 ca_{4,36} + \\ & 2880 ca_{4,62} + 2880 T ca_{4,62} + 2880 T^2 ca_{4,62}) g_{i^{++},i} - \\ & \frac{\epsilon^4 (16 - 25 T + 11 T^2 + 48 T ca_{4,36} + 480 ca_{4,62} + 480 T ca_{4,62}) g_{i^{++},i}}{2 T^2} + \end{aligned}$$

$$\begin{aligned}
 & \frac{4 \epsilon^4 (-1 + T - 30 ca_{4,62}) g_{i^{++},i}^4}{T} + \\
 & \frac{1}{12 T^3} \epsilon^4 (48 - 150 T + 80 T^2 - 20 T^3 + 19 T^4 + 24 T^3 ca_{4,9} + 72 T^2 ca_{4,19} + \\
 & \quad 72 T^3 ca_{4,19} + 288 T ca_{4,36} + 288 T^2 ca_{4,36} + 288 T^3 ca_{4,36} + 1440 ca_{4,62} + 1440 T ca_{4,62} + \\
 & \quad 1440 T^2 ca_{4,62} + 1440 T^3 ca_{4,62} + 12 T^4 cb_{4,5} + 12 T^3 cb_{4,10} - 12 T^4 cb_{4,10}) g_{i^{++},i^+} + \\
 & \frac{1}{12 T^3} \epsilon^4 (-192 + 642 T - 622 T^2 + 210 T^3 - 15 T^4 - 24 T^3 ca_{4,9} - 144 T^2 ca_{4,19} - \\
 & \quad 864 T ca_{4,36} - 5760 ca_{4,62} - 12 T^3 cb_{4,10} + 12 T^4 cb_{4,10}) g_{i^{++},i} g_{i^{++},i^+} + \\
 & \frac{\epsilon^4 (-144 + 369 T - 277 T^2 + 55 T^3 - 36 T^2 ca_{4,19} - 432 T ca_{4,36} - 4320 ca_{4,62}) g_{i^{++},i}^2 g_{i^{++},i^+}}{6 T^2} - \\
 & \frac{\epsilon^4 (32 - 57 T + 27 T^2 + 48 T ca_{4,36} + 960 ca_{4,62}) g_{i^{++},i}^3 g_{i^{++},i^+}}{2 T} + \\
 & 4 \epsilon^4 (-1 + T - 30 ca_{4,62}) g_{i^{++},i}^4 g_{i^{++},i^+} - \\
 & \frac{1}{12 T^2} \epsilon^4 (-192 + 546 T - 397 T^2 + 124 T^3 - 52 T^4 - 24 T^3 ca_{4,9} - 144 T^2 ca_{4,19} - \\
 & \quad 72 T^3 ca_{4,19} - 864 T ca_{4,36} - 432 T^2 ca_{4,36} - 432 T^3 ca_{4,36} - 5760 ca_{4,62} - \\
 & \quad 2880 T ca_{4,62} - 2880 T^2 ca_{4,62} - 2880 T^3 ca_{4,62} - 12 T^3 cb_{4,10} + 12 T^4 cb_{4,10}) g_{i^{++},i^+}^2 - \\
 & \frac{(-1 + T) \epsilon^4 (-144 + 369 T - 277 T^2 + 55 T^3 - 36 T^2 ca_{4,19} - 432 T ca_{4,36} - 4320 ca_{4,62}) g_{i^{++},i} g_{i^{++},i^+}^2}{6 T^2} + \\
 & \frac{3 (-1 + T) \epsilon^4 (32 - 57 T + 27 T^2 + 48 T ca_{4,36} + 960 ca_{4,62}) g_{i^{++},i}^2 g_{i^{++},i^+}^2}{4 T} - \\
 & 8 (-1 + T) \epsilon^4 (-1 + T - 30 ca_{4,62}) g_{i^{++},i}^3 g_{i^{++},i^+}^2 + \\
 & \frac{1}{6 T} \epsilon^4 (144 - 369 T + 325 T^2 - 130 T^3 + 33 T^4 + 36 T^2 ca_{4,19} + 432 T ca_{4,36} + \\
 & \quad 144 T^3 ca_{4,36} + 4320 ca_{4,62} + 1440 T^2 ca_{4,62} + 1440 T^3 ca_{4,62}) g_{i^{++},i^+}^3 - \\
 & \frac{(-1 + T)^2 \epsilon^4 (32 - 57 T + 27 T^2 + 48 T ca_{4,36} + 960 ca_{4,62}) g_{i^{++},i} g_{i^{++},i^+}^3}{2 T} + \\
 & 8 (-1 + T)^2 \epsilon^4 (-1 + T - 30 ca_{4,62}) g_{i^{++},i}^2 g_{i^{++},i^+}^3 + \\
 & \frac{1}{4} \epsilon^4 (64 - 146 T + 143 T^2 - 68 T^3 + 11 T^4 + 96 T ca_{4,36} - 48 T^2 ca_{4,36} + \\
 & \quad 48 T^3 ca_{4,36} + 1920 ca_{4,62} - 960 T ca_{4,62} + 960 T^2 ca_{4,62} + 480 T^3 ca_{4,62}) g_{i^{++},i^+}^4 - \\
 & 4 (-1 + T)^3 \epsilon^4 (-1 + T - 30 ca_{4,62}) g_{i^{++},i} g_{i^{++},i^+}^4 - 4 T (1 - T + T^2) \epsilon^4 (-1 + T - 30 ca_{4,62}) g_{i^{++},i^+}^5
 \end{aligned}$$

(Alt) In[ ]:=

**rhs = 1**

(Alt) Out[ ]:=

**1**

(Alt) In[\*]:=

```
me = Exponent [lhs - rhs, T, Min]
```

(Alt) Out[\*]=

```
- 4
```

(Alt) In[\*]:=

```
covars = DeleteCases [Variables [lhs - rhs], T | (ca | cb | cc | cd) __]
```

(Alt) Out[\*]=

```
{ $\epsilon$ ,  $\mathfrak{g}_{i^{++},i}$ ,  $\mathfrak{g}_{i^{++},i^+}$ }
```

(Alt) In[ ]:=

eqnsR11 =

(Factor[#] == 0) & /@Union[Last /@ CoefficientRules[Expand[T^me (lhs - rhs)], covars]]

(Alt) Out[ ]:=

$$\left\{ \begin{aligned} &-\frac{1}{6} T^2 (-144 + 369 T - 277 T^2 + 55 T^3 - 36 T^2 ca_{4,19} - 432 T ca_{4,36} - 4320 ca_{4,62}) = 0, \\ &-\frac{1}{2} T^3 (32 - 57 T + 27 T^2 + 48 T ca_{4,36} + 960 ca_{4,62}) = 0, \\ &-\frac{1}{2} T^2 (16 - 25 T + 11 T^2 + 48 T ca_{4,36} + 480 ca_{4,62} + 480 T ca_{4,62}) = 0, \\ &-\frac{1}{12} T (96 - 225 T + 86 T^2 + 37 T^3 + 72 T^2 ca_{4,19} + 432 T ca_{4,36} + 432 T^2 ca_{4,36} + \\ &\quad 2880 ca_{4,62} + 2880 T ca_{4,62} + 2880 T^2 ca_{4,62}) = 0, 4 T^3 (-1 + T - 30 ca_{4,62}) = 0, \\ &-\frac{1}{6} (-1 + T) T^2 (-144 + 369 T - 277 T^2 + 55 T^3 - 36 T^2 ca_{4,19} - 432 T ca_{4,36} - 4320 ca_{4,62}) = 0, \\ &4 T^4 (-1 + T - 30 ca_{4,62}) = 0, \frac{3}{4} (-1 + T) T^3 (32 - 57 T + 27 T^2 + 48 T ca_{4,36} + 960 ca_{4,62}) = 0, \\ &-\frac{1}{2} (-1 + T)^2 T^3 (32 - 57 T + 27 T^2 + 48 T ca_{4,36} + 960 ca_{4,62}) = 0, \\ &-8 (-1 + T) T^4 (-1 + T - 30 ca_{4,62}) = 0, 8 (-1 + T)^2 T^4 (-1 + T - 30 ca_{4,62}) = 0, \\ &\frac{1}{6} T^3 (144 - 369 T + 325 T^2 - 130 T^3 + 33 T^4 + 36 T^2 ca_{4,19} + \\ &\quad 432 T ca_{4,36} + 144 T^3 ca_{4,36} + 4320 ca_{4,62} + 1440 T^2 ca_{4,62} + 1440 T^3 ca_{4,62}) = 0, \\ &-4 (-1 + T)^3 T^4 (-1 + T - 30 ca_{4,62}) = 0, -4 T^5 (1 - T + T^2) (-1 + T - 30 ca_{4,62}) = 0, \\ &\frac{1}{4} T^4 (64 - 146 T + 143 T^2 - 68 T^3 + 11 T^4 + 96 T ca_{4,36} - 48 T^2 ca_{4,36} + \\ &\quad 48 T^3 ca_{4,36} + 1920 ca_{4,62} - 960 T ca_{4,62} + 960 T^2 ca_{4,62} + 480 T^3 ca_{4,62}) = 0, \\ &\frac{1}{12} (-48 + 150 T - 80 T^2 + 20 T^3 - 19 T^4 - 24 T^3 ca_{4,9} - 72 T^2 ca_{4,19} - 72 T^3 ca_{4,19} - \\ &\quad 288 T ca_{4,36} - 288 T^2 ca_{4,36} - 288 T^3 ca_{4,36} - 1440 ca_{4,62} - 1440 T ca_{4,62} - \\ &\quad 1440 T^2 ca_{4,62} - 1440 T^3 ca_{4,62} - 12 T^4 cb_{4,5} - 12 T^3 cb_{4,10} + 12 T^4 cb_{4,10}) = 0, \\ &\frac{1}{12} T (48 - 150 T + 80 T^2 - 20 T^3 + 19 T^4 + 24 T^3 ca_{4,9} + 72 T^2 ca_{4,19} + 72 T^3 ca_{4,19} + \\ &\quad 288 T ca_{4,36} + 288 T^2 ca_{4,36} + 288 T^3 ca_{4,36} + 1440 ca_{4,62} + 1440 T ca_{4,62} + \\ &\quad 1440 T^2 ca_{4,62} + 1440 T^3 ca_{4,62} + 12 T^4 cb_{4,5} + 12 T^3 cb_{4,10} - 12 T^4 cb_{4,10}) = 0, \\ &\frac{1}{12} T (-192 + 642 T - 622 T^2 + 210 T^3 - 15 T^4 - 24 T^3 ca_{4,9} - 144 T^2 ca_{4,19} - \\ &\quad 864 T ca_{4,36} - 5760 ca_{4,62} - 12 T^3 cb_{4,10} + 12 T^4 cb_{4,10}) = 0, \\ &-\frac{1}{12} T^2 (-192 + 546 T - 397 T^2 + 124 T^3 - 52 T^4 - 24 T^3 ca_{4,9} - 144 T^2 ca_{4,19} - 72 T^3 ca_{4,19} - \\ &\quad 864 T ca_{4,36} - 432 T^2 ca_{4,36} - 432 T^3 ca_{4,36} - 5760 ca_{4,62} - 2880 T ca_{4,62} - 2880 T^2 ca_{4,62} - \\ &\quad 2880 T^3 ca_{4,62} - 12 T^3 cb_{4,10} + 12 T^4 cb_{4,10}) = 0, T^4 (ca_{4,1} + cc_{4,1}) = 0 \end{aligned} \right\}$$

(Alt) In[ ]:=

```
vars = Cases[Variables[r_d[1, i1, j1] + r_d[-1, i2, j2] + \gamma_d[1, k]], (ca | cb | cc | cd) __]
{sol} = Solve[eqnsR3 \cup eqnsR2b \cup eqnsR2c \cup eqnsR11 \cup eqnsR1r \cup eqnsSwp, vars]
sol /. Rule -> Set;
r_d[1, i, j]
\gamma_d[1, k]
```

(Alt) Out[ ]:=

{ca<sub>4,9</sub>, ca<sub>4,19</sub>, ca<sub>4,36</sub>, ca<sub>4,62</sub>, cb<sub>4,2</sub>, cb<sub>4,5</sub>, cb<sub>4,10</sub>, cc<sub>4,1</sub>}

Solve: Equations may not give solutions for all "solve" variables.

(Alt) Out[ ]:=

$$\left\{ \left\{ \begin{aligned} ca_{4,9} &\rightarrow -\frac{5}{24} (2 + 3 T) - \frac{1}{2} (1 - T) cb_{4,10}, ca_{4,19} \rightarrow \frac{1}{36} (-34 + 55 T), \\ ca_{4,36} &\rightarrow \frac{1}{48} (25 - 27 T), ca_{4,62} \rightarrow \frac{1}{30} (-1 + T), cb_{4,5} \rightarrow 0, cc_{4,1} \rightarrow -ca_{4,1} \end{aligned} \right\} \right\}$$

(Alt) Out[ ]:=

$$\begin{aligned} &-\frac{\epsilon}{2} + \epsilon p_i x_i - \frac{1}{2} \epsilon^2 p_i x_i + \frac{1}{6} \epsilon^3 p_i x_i - \frac{1}{12} \epsilon^4 p_i x_i - \epsilon p_j x_i + \frac{1}{2} \epsilon^2 p_j x_i - \frac{1}{6} \epsilon^3 p_j x_i + \frac{1}{12} \epsilon^4 p_j x_i - \\ &\frac{1}{2} \epsilon p_i p_j x_i^2 + \frac{1}{2} T \epsilon p_i p_j x_i^2 + \frac{1}{4} \epsilon^2 p_i p_j x_i^2 - \frac{3}{4} T \epsilon^2 p_i p_j x_i^2 + \frac{5}{12} \epsilon^3 p_i p_j x_i^2 + \frac{7}{12} T \epsilon^3 p_i p_j x_i^2 - \\ &\frac{5}{12} \epsilon^4 p_i p_j x_i^2 - \frac{5}{8} T \epsilon^4 p_i p_j x_i^2 + \frac{1}{2} \epsilon p_j^2 x_i^2 - \frac{1}{2} T \epsilon p_j^2 x_i^2 - \frac{1}{4} \epsilon^2 p_j^2 x_i^2 + \frac{3}{4} T \epsilon^2 p_j^2 x_i^2 - \frac{5}{12} \epsilon^3 p_j^2 x_i^2 - \\ &\frac{7}{12} T \epsilon^3 p_j^2 x_i^2 + \frac{5}{12} \epsilon^4 p_j^2 x_i^2 + \frac{5}{8} T \epsilon^4 p_j^2 x_i^2 - \frac{1}{3} \epsilon^2 p_i^2 p_j x_i^3 + \frac{1}{3} T \epsilon^2 p_i^2 p_j x_i^3 + \frac{5}{6} \epsilon^3 p_i^2 p_j x_i^3 - T \epsilon^3 p_i^2 p_j x_i^3 - \\ &\frac{17}{18} \epsilon^4 p_i^2 p_j x_i^3 + \frac{55}{36} T \epsilon^4 p_i^2 p_j x_i^3 + \frac{5}{6} \epsilon^2 p_i p_j^2 x_i^3 - \frac{2}{3} T \epsilon^2 p_i p_j^2 x_i^3 - \frac{1}{6} T^2 \epsilon^2 p_i p_j^2 x_i^3 - \frac{8}{3} \epsilon^3 p_i p_j^2 x_i^3 + \\ &\frac{17}{6} T \epsilon^3 p_i p_j^2 x_i^3 + \frac{1}{3} T^2 \epsilon^3 p_i p_j^2 x_i^3 + \frac{269}{72} \epsilon^4 p_i p_j^2 x_i^3 - \frac{197}{36} T \epsilon^4 p_i p_j^2 x_i^3 - \frac{25}{72} T^2 \epsilon^4 p_i p_j^2 x_i^3 - \\ &\frac{1}{2} \epsilon^2 p_j^3 x_i^3 + \frac{1}{3} T \epsilon^2 p_j^3 x_i^3 + \frac{1}{6} T^2 \epsilon^2 p_j^3 x_i^3 + \frac{11}{6} \epsilon^3 p_j^3 x_i^3 - \frac{11}{6} T \epsilon^3 p_j^3 x_i^3 - \frac{1}{3} T^2 \epsilon^3 p_j^3 x_i^3 - \frac{67}{24} \epsilon^4 p_j^3 x_i^3 + \\ &\frac{71}{18} T \epsilon^4 p_j^3 x_i^3 + \frac{25}{72} T^2 \epsilon^4 p_j^3 x_i^3 - \frac{1}{8} \epsilon^3 p_i^3 p_j x_i^4 + \frac{1}{8} T \epsilon^3 p_i^3 p_j x_i^4 + \frac{25}{48} \epsilon^4 p_i^3 p_j x_i^4 - \frac{9}{16} T \epsilon^4 p_i^3 p_j x_i^4 + \\ &\frac{1}{2} \epsilon^3 p_i^2 p_j^2 x_i^4 - \frac{1}{8} T \epsilon^3 p_i^2 p_j^2 x_i^4 - \frac{3}{8} T^2 \epsilon^3 p_i^2 p_j^2 x_i^4 - \frac{13}{4} \epsilon^4 p_i^2 p_j^2 x_i^4 + \frac{101}{48} T \epsilon^4 p_i^2 p_j^2 x_i^4 + \frac{23}{16} T^2 \epsilon^4 p_i^2 p_j^2 x_i^4 - \\ &\frac{13}{24} \epsilon^3 p_i p_j^3 x_i^4 - \frac{3}{8} T \epsilon^3 p_i p_j^3 x_i^4 + \frac{7}{8} T^2 \epsilon^3 p_i p_j^3 x_i^4 + \frac{1}{24} T^3 \epsilon^3 p_i p_j^3 x_i^4 + \frac{245}{48} \epsilon^4 p_i p_j^3 x_i^4 - \frac{79}{48} T \epsilon^4 p_i p_j^3 x_i^4 - \\ &\frac{185}{48} T^2 \epsilon^4 p_i p_j^3 x_i^4 - \frac{5}{48} T^3 \epsilon^4 p_i p_j^3 x_i^4 + \frac{1}{6} \epsilon^3 p_j^4 x_i^4 + \frac{3}{8} T \epsilon^3 p_j^4 x_i^4 - \frac{1}{2} T^2 \epsilon^3 p_j^4 x_i^4 - \frac{1}{24} T^3 \epsilon^3 p_j^4 x_i^4 - \\ &\frac{19}{8} \epsilon^4 p_i^4 x_i^4 + \frac{5}{48} T \epsilon^4 p_i^4 x_i^4 + \frac{29}{12} T^2 \epsilon^4 p_i^4 x_i^4 + \frac{5}{48} T^3 \epsilon^4 p_i^4 x_i^4 - \frac{1}{30} \epsilon^4 p_i^4 p_j x_i^5 + \frac{1}{30} T \epsilon^4 p_i^4 p_j x_i^5 + \\ &\frac{1}{15} \epsilon^4 p_i^3 p_j^2 x_i^5 + \frac{11}{30} T \epsilon^4 p_i^3 p_j^2 x_i^5 - \frac{13}{30} T^2 \epsilon^4 p_i^3 p_j^2 x_i^5 + \frac{7}{20} \epsilon^4 p_i^2 p_j^3 x_i^5 - \frac{32}{15} T \epsilon^4 p_i^2 p_j^3 x_i^5 + \\ &\frac{31}{20} T^2 \epsilon^4 p_i^2 p_j^3 x_i^5 + \frac{7}{30} T^3 \epsilon^4 p_i^2 p_j^3 x_i^5 - \frac{101}{120} \epsilon^4 p_i p_j^4 x_i^5 + \frac{187}{60} T \epsilon^4 p_i p_j^4 x_i^5 - \frac{103}{60} T^2 \epsilon^4 p_i p_j^4 x_i^5 - \\ &\frac{11}{20} T^3 \epsilon^4 p_i p_j^4 x_i^5 - \frac{1}{120} T^4 \epsilon^4 p_i p_j^4 x_i^5 + \frac{11}{24} \epsilon^4 p_j^5 x_i^5 - \frac{83}{60} T \epsilon^4 p_j^5 x_i^5 + \frac{3}{5} T^2 \epsilon^4 p_j^5 x_i^5 + \frac{19}{60} T^3 \epsilon^4 p_j^5 x_i^5 + \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{120} T^4 \in^4 p_j^5 x_i^5 - \in p_i p_j x_i x_j + \frac{3}{2} \in^2 p_i p_j x_i x_j - \frac{7}{6} \in^3 p_i p_j x_i x_j + \frac{5}{4} \in^4 p_i p_j x_i x_j + \in p_j^2 x_i x_j - \\
 & \frac{3}{2} \in^2 p_j^2 x_i x_j + \frac{7}{6} \in^3 p_j^2 x_i x_j - \frac{5}{4} \in^4 p_j^2 x_i x_j - \frac{1}{2} \in^2 p_i^2 p_j x_i^2 x_j + \frac{3}{2} \in^3 p_i^2 p_j x_i^2 x_j - \frac{55}{24} \in^4 p_i^2 p_j x_i^2 x_j + \\
 & \in^2 p_i p_j^2 x_i^2 x_j + \frac{1}{2} T \in^2 p_i p_j^2 x_i^2 x_j - \frac{9}{2} \in^3 p_i p_j^2 x_i^2 x_j - T \in^3 p_i p_j^2 x_i^2 x_j + \frac{53}{6} \in^4 p_i p_j^2 x_i^2 x_j + \\
 & \frac{25}{24} T \in^4 p_i p_j^2 x_i^2 x_j - \frac{1}{2} \in^2 p_j^3 x_i^2 x_j - \frac{1}{2} T \in^2 p_j^3 x_i^2 x_j + 3 \in^3 p_j^3 x_i^2 x_j + T \in^3 p_j^3 x_i^2 x_j - \frac{157}{24} \in^4 p_j^3 x_i^2 x_j - \\
 & \frac{25}{24} T \in^4 p_j^3 x_i^2 x_j - \frac{1}{6} \in^3 p_i^3 p_j x_i^3 x_j + \frac{3}{4} \in^4 p_i^3 p_j x_i^3 x_j + \frac{7}{6} T \in^3 p_i^2 p_j^2 x_i^3 x_j - \frac{5}{2} \in^4 p_i^2 p_j^2 x_i^3 x_j - \\
 & \frac{53}{12} T \in^4 p_i^2 p_j^2 x_i^3 x_j + \in^3 p_i p_j^3 x_i^3 x_j - \frac{17}{6} T \in^3 p_i p_j^3 x_i^3 x_j - \frac{1}{6} T^2 \in^3 p_i p_j^3 x_i^3 x_j + \in^4 p_i p_j^3 x_i^3 x_j + \\
 & \frac{49}{4} T \in^4 p_i p_j^3 x_i^3 x_j + \frac{5}{12} T^2 \in^4 p_i p_j^3 x_i^3 x_j - \frac{5}{6} \in^3 p_j^4 x_i^3 x_j + \frac{5}{3} T \in^3 p_j^4 x_i^3 x_j + \frac{1}{6} T^2 \in^3 p_j^4 x_i^3 x_j + \frac{3}{4} \in^4 p_j^4 x_i^3 x_j - \\
 & \frac{47}{6} T \in^4 p_j^4 x_i^3 x_j - \frac{5}{12} T^2 \in^4 p_j^4 x_i^3 x_j - \frac{1}{24} \in^4 p_i^4 p_j x_i^4 x_j - \frac{2}{3} \in^4 p_i^3 p_j^2 x_i^4 x_j + \frac{31}{24} T \in^4 p_i^3 p_j^2 x_i^4 x_j + \\
 & \frac{15}{4} \in^4 p_i^2 p_j^3 x_i^4 x_j - \frac{115}{24} T \in^4 p_i^2 p_j^3 x_i^4 x_j - \frac{25}{24} T^2 \in^4 p_i^2 p_j^3 x_i^4 x_j - \frac{11}{2} \in^4 p_i p_j^4 x_i^4 x_j + \frac{131}{24} T \in^4 p_i p_j^4 x_i^4 x_j + \\
 & \frac{5}{2} T^2 \in^4 p_i p_j^4 x_i^4 x_j + \frac{1}{24} T^3 \in^4 p_i p_j^4 x_i^4 x_j + \frac{59}{24} \in^4 p_j^5 x_i^4 x_j - \frac{47}{24} T \in^4 p_j^5 x_i^4 x_j - \frac{35}{24} T^2 \in^4 p_j^5 x_i^4 x_j - \\
 & \frac{1}{24} T^3 \in^4 p_j^5 x_i^4 x_j - \frac{1}{2} \in^2 p_i p_j^2 x_i x_j^2 + \in^3 p_i p_j^2 x_i x_j^2 - \frac{25}{24} \in^4 p_i p_j^2 x_i x_j^2 + \frac{1}{2} \in^2 p_j^3 x_i x_j^2 - \in^3 p_j^3 x_i x_j^2 + \\
 & \frac{25}{24} \in^4 p_j^3 x_i x_j^2 - \in^3 p_i^2 p_j^2 x_i^2 x_j^2 + \frac{15}{4} \in^4 p_i^2 p_j^2 x_i^2 x_j^2 + \frac{5}{2} \in^3 p_i p_j^3 x_i^2 x_j^2 + \frac{1}{4} T \in^3 p_i p_j^3 x_i^2 x_j^2 - \frac{43}{4} \in^4 p_i p_j^3 x_i^2 x_j^2 - \\
 & \frac{5}{8} T \in^4 p_i p_j^3 x_i^2 x_j^2 - \frac{3}{2} \in^3 p_j^4 x_i^2 x_j^2 - \frac{1}{4} T \in^3 p_j^4 x_i^2 x_j^2 + 7 \in^4 p_j^4 x_i^2 x_j^2 + \frac{5}{8} T \in^4 p_j^4 x_i^2 x_j^2 - \in^4 p_i^3 p_j^2 x_i^3 x_j^2 + \\
 & \frac{15}{4} \in^4 p_i^2 p_j^3 x_i^3 x_j^2 + \frac{5}{3} T \in^4 p_i^2 p_j^3 x_i^3 x_j^2 - \frac{17}{4} \in^4 p_i p_j^4 x_i^3 x_j^2 - \frac{49}{12} T \in^4 p_i p_j^4 x_i^3 x_j^2 - \frac{1}{12} T^2 \in^4 p_i p_j^4 x_i^3 x_j^2 + \\
 & \frac{3}{2} \in^4 p_j^5 x_i^3 x_j^2 + \frac{29}{12} T \in^4 p_j^5 x_i^3 x_j^2 + \frac{1}{12} T^2 \in^4 p_j^5 x_i^3 x_j^2 - \frac{1}{6} \in^3 p_i p_j^3 x_i x_j^3 + \frac{5}{12} \in^4 p_i p_j^3 x_i x_j^3 + \frac{1}{6} \in^3 p_j^4 x_i x_j^3 - \\
 & \frac{5}{12} \in^4 p_j^4 x_i x_j^3 - \in^4 p_i^2 p_j^3 x_i^2 x_j^3 + \frac{5}{2} \in^4 p_i p_j^4 x_i^2 x_j^3 + \frac{1}{12} T \in^4 p_i p_j^4 x_i^2 x_j^3 - \frac{3}{2} \in^4 p_j^5 x_i^2 x_j^3 - \frac{1}{12} T \in^4 p_j^5 x_i^2 x_j^3 - \\
 & \frac{1}{24} \in^4 p_i p_j^4 x_i x_j^4 + \frac{1}{24} \in^4 p_j^5 x_i x_j^4 + \in^4 ca_{4,1} - \in^4 p_i x_i cb_{4,2} + \in^4 p_j x_i cb_{4,2} - \frac{1}{2} \in^4 p_i p_j x_i^2 cb_{4,10} + \\
 & \frac{1}{2} T \in^4 p_i p_j x_i^2 cb_{4,10} + \frac{1}{2} \in^4 p_j^2 x_i^2 cb_{4,10} - \frac{1}{2} T \in^4 p_j^2 x_i^2 cb_{4,10} - \in^4 p_i p_j x_i x_j cb_{4,10} + \in^4 p_j^2 x_i x_j cb_{4,10}
 \end{aligned}$$

(Alt) Out[\*]=

$$\frac{\in}{2} - \in p_k x_k - \frac{1}{2} \in^2 p_k x_k - \frac{1}{6} \in^3 p_k x_k + \frac{7}{12} \in^4 p_k x_k - \in^4 ca_{4,1} - \in^4 p_k x_k cb_{4,10}$$

## R1r

```
(Alt) In[ ]:=
lhs = CF[Module[{es = {i, i^+}},
  Times[
    Normal@Series[Exp[r_d[1, 0, -1, i, i^+]], {e, 0, d}],
    Exp[Sum[g_{\alpha,\beta} \pi_{\alpha} \xi_{\beta}, {\alpha, es}, {\beta, es}]]
  ] // Zip((p_#&/@es) \cup (x_#&/@es) // Expand
] // . {
  g_{i,\beta} \to \delta_{i,\beta} + T g_{i^+,\beta} + (1 - T) g_{i^+,\beta}, g_{i^+,\beta} \to \delta_{i^+,\beta} + g_{i^+,\beta},
  g_{\alpha,i} \to T^{-1} (g_{\alpha,i^+} - \delta_{\alpha,i^+}), g_{\alpha,i^+} \to T g_{\alpha,i^+} - (1 - T) \delta_{\alpha,i^+} - T \delta_{\alpha,i^+} }
]

(Alt) Out[ ]:=
1 + \frac{1}{24} \epsilon^4 (-1 + 48 ca_{4,1} - 24 cb_{4,2})

(Alt) In[ ]:=
rhs = 1

(Alt) Out[ ]:=
1

(Alt) In[ ]:=
me = Exponent[lhs - rhs, T, Min]

(Alt) Out[ ]:=
0

(Alt) In[ ]:=
covars = DeleteCases[Variables[lhs - rhs], T | (ca | cb | cc | cd) __]

(Alt) Out[ ]:=
{ \epsilon }

(Alt) In[ ]:=
eqnsR1r =
(Factor[#] == 0) & /@ Union[Last /@ CoefficientRules[Expand[T^{-me} (lhs - rhs)], covars]]

(Alt) Out[ ]:=
\left\{ \frac{1}{24} (-1 + 48 ca_{4,1} - 24 cb_{4,2}) == 0 \right\}

(Alt) In[ ]:=
vars = Cases[Variables[r_d[1, i1, j1] + r_d[-1, i2, j2] + \gamma_d[1, k]], (ca | cb | cc | cd) __]
{sol} = Solve[eqnsR3 \cup eqnsR2b \cup eqnsR2c \cup eqnsR1l \cup eqnsR1r \cup eqnsSwp, vars]
sol /. Rule -> Set;
r_d[1, i, j]
\gamma_d[1, k]

(Alt) Out[ ]:=
{ca_{4,1}, cb_{4,2}, cb_{4,10}}
```

**Solve:** Equations may not give solutions for all "solve" variables.

(Alt) Out[\*]=

$$\left\{ \left\{ \text{cb}_{4,2} \rightarrow -\frac{1}{24} + 2 \text{ca}_{4,1} \right\} \right\}$$

(Alt) Out[\*]=

$$\begin{aligned} & -\frac{\epsilon}{2} + \epsilon p_i x_i - \frac{1}{2} \epsilon^2 p_i x_i + \frac{1}{6} \epsilon^3 p_i x_i - \frac{1}{24} \epsilon^4 p_i x_i - \epsilon p_j x_i + \frac{1}{2} \epsilon^2 p_j x_i - \frac{1}{6} \epsilon^3 p_j x_i + \frac{1}{24} \epsilon^4 p_j x_i - \\ & \frac{1}{2} \epsilon p_i p_j x_i^2 + \frac{1}{2} T \epsilon p_i p_j x_i^2 + \frac{1}{4} \epsilon^2 p_i p_j x_i^2 - \frac{3}{4} T \epsilon^2 p_i p_j x_i^2 + \frac{5}{12} \epsilon^3 p_i p_j x_i^2 + \frac{7}{12} T \epsilon^3 p_i p_j x_i^2 - \\ & \frac{5}{12} \epsilon^4 p_i p_j x_i^2 - \frac{5}{8} T \epsilon^4 p_i p_j x_i^2 + \frac{1}{2} \epsilon p_j^2 x_i^2 - \frac{1}{2} T \epsilon p_j^2 x_i^2 - \frac{1}{4} \epsilon^2 p_j^2 x_i^2 + \frac{3}{4} T \epsilon^2 p_j^2 x_i^2 - \frac{5}{12} \epsilon^3 p_j^2 x_i^2 - \\ & \frac{7}{12} T \epsilon^3 p_j^2 x_i^2 + \frac{5}{12} \epsilon^4 p_j^2 x_i^2 + \frac{5}{8} T \epsilon^4 p_j^2 x_i^2 - \frac{1}{3} \epsilon^2 p_i^2 p_j x_i^3 + \frac{1}{3} T \epsilon^2 p_i^2 p_j x_i^3 + \frac{5}{6} \epsilon^3 p_i^2 p_j x_i^3 - T \epsilon^3 p_i^2 p_j x_i^3 - \\ & \frac{17}{18} \epsilon^4 p_i^2 p_j x_i^3 + \frac{55}{36} T \epsilon^4 p_i^2 p_j x_i^3 + \frac{5}{6} \epsilon^2 p_i p_j^2 x_i^3 - \frac{2}{3} T \epsilon^2 p_i p_j^2 x_i^3 - \frac{1}{6} T^2 \epsilon^2 p_i p_j^2 x_i^3 - \frac{8}{3} \epsilon^3 p_i p_j^2 x_i^3 + \\ & \frac{17}{6} T \epsilon^3 p_i p_j^2 x_i^3 + \frac{1}{3} T^2 \epsilon^3 p_i p_j^2 x_i^3 + \frac{269}{72} \epsilon^4 p_i p_j^2 x_i^3 - \frac{197}{36} T \epsilon^4 p_i p_j^2 x_i^3 - \frac{25}{72} T^2 \epsilon^4 p_i p_j^2 x_i^3 - \\ & \frac{1}{2} \epsilon^2 p_j^3 x_i^3 + \frac{1}{3} T \epsilon^2 p_j^3 x_i^3 + \frac{1}{6} T^2 \epsilon^2 p_j^3 x_i^3 + \frac{11}{6} \epsilon^3 p_j^3 x_i^3 - \frac{11}{6} T \epsilon^3 p_j^3 x_i^3 - \frac{1}{3} T^2 \epsilon^3 p_j^3 x_i^3 - \frac{67}{24} \epsilon^4 p_j^3 x_i^3 + \\ & \frac{71}{18} T \epsilon^4 p_j^3 x_i^3 + \frac{25}{72} T^2 \epsilon^4 p_j^3 x_i^3 - \frac{1}{8} \epsilon^3 p_i^3 p_j x_i^4 + \frac{1}{8} T \epsilon^3 p_i^3 p_j x_i^4 + \frac{25}{48} \epsilon^4 p_i^3 p_j x_i^4 - \frac{9}{16} T \epsilon^4 p_i^3 p_j x_i^4 + \\ & \frac{1}{2} \epsilon^3 p_i^2 p_j^2 x_i^4 - \frac{1}{8} T \epsilon^3 p_i^2 p_j^2 x_i^4 - \frac{3}{8} T^2 \epsilon^3 p_i^2 p_j^2 x_i^4 - \frac{13}{4} \epsilon^4 p_i^2 p_j^2 x_i^4 + \frac{101}{48} T \epsilon^4 p_i^2 p_j^2 x_i^4 + \frac{23}{16} T^2 \epsilon^4 p_i^2 p_j^2 x_i^4 - \\ & \frac{13}{24} \epsilon^3 p_i p_j^3 x_i^4 - \frac{3}{8} T \epsilon^3 p_i p_j^3 x_i^4 + \frac{7}{8} T^2 \epsilon^3 p_i p_j^3 x_i^4 + \frac{1}{24} T^3 \epsilon^3 p_i p_j^3 x_i^4 + \frac{245}{48} \epsilon^4 p_i p_j^3 x_i^4 - \frac{79}{48} T \epsilon^4 p_i p_j^3 x_i^4 - \\ & \frac{185}{48} T^2 \epsilon^4 p_i p_j^3 x_i^4 - \frac{5}{48} T^3 \epsilon^4 p_i p_j^3 x_i^4 + \frac{1}{6} \epsilon^3 p_j^4 x_i^4 + \frac{3}{8} T \epsilon^3 p_j^4 x_i^4 - \frac{1}{2} T^2 \epsilon^3 p_j^4 x_i^4 - \frac{1}{24} T^3 \epsilon^3 p_j^4 x_i^4 - \\ & \frac{19}{8} \epsilon^4 p_j^4 x_i^4 + \frac{5}{48} T \epsilon^4 p_j^4 x_i^4 + \frac{29}{12} T^2 \epsilon^4 p_j^4 x_i^4 + \frac{5}{48} T^3 \epsilon^4 p_j^4 x_i^4 - \frac{1}{30} \epsilon^4 p_i^4 p_j x_i^5 + \frac{1}{30} T \epsilon^4 p_i^4 p_j x_i^5 + \\ & \frac{1}{15} \epsilon^4 p_i^3 p_j^2 x_i^5 + \frac{11}{30} T \epsilon^4 p_i^3 p_j^2 x_i^5 - \frac{13}{30} T^2 \epsilon^4 p_i^3 p_j^2 x_i^5 + \frac{7}{20} \epsilon^4 p_i^2 p_j^3 x_i^5 - \frac{32}{15} T \epsilon^4 p_i^2 p_j^3 x_i^5 + \\ & \frac{31}{20} T^2 \epsilon^4 p_i^2 p_j^3 x_i^5 + \frac{7}{30} T^3 \epsilon^4 p_i^2 p_j^3 x_i^5 - \frac{101}{120} \epsilon^4 p_i p_j^4 x_i^5 + \frac{187}{60} T \epsilon^4 p_i p_j^4 x_i^5 - \frac{103}{60} T^2 \epsilon^4 p_i p_j^4 x_i^5 - \\ & \frac{11}{20} T^3 \epsilon^4 p_i p_j^4 x_i^5 - \frac{1}{120} T^4 \epsilon^4 p_i p_j^4 x_i^5 + \frac{11}{24} \epsilon^4 p_j^5 x_i^5 - \frac{83}{60} T \epsilon^4 p_j^5 x_i^5 + \frac{3}{5} T^2 \epsilon^4 p_j^5 x_i^5 + \frac{19}{60} T^3 \epsilon^4 p_j^5 x_i^5 + \\ & \frac{1}{120} T^4 \epsilon^4 p_j^5 x_i^5 - \epsilon p_i p_j x_i x_j + \frac{3}{2} \epsilon^2 p_i p_j x_i x_j - \frac{7}{6} \epsilon^3 p_i p_j x_i x_j + \frac{5}{4} \epsilon^4 p_i p_j x_i x_j + \epsilon p_j^2 x_i x_j - \\ & \frac{3}{2} \epsilon^2 p_j^2 x_i x_j + \frac{7}{6} \epsilon^3 p_j^2 x_i x_j - \frac{5}{4} \epsilon^4 p_j^2 x_i x_j - \frac{1}{2} \epsilon^2 p_i^2 p_j x_i^2 x_j + \frac{3}{2} \epsilon^3 p_i^2 p_j x_i^2 x_j - \frac{55}{24} \epsilon^4 p_i^2 p_j x_i^2 x_j + \\ & \epsilon^2 p_i p_j^2 x_i^2 x_j + \frac{1}{2} T \epsilon^2 p_i p_j^2 x_i^2 x_j - \frac{9}{2} \epsilon^3 p_i p_j^2 x_i^2 x_j - T \epsilon^3 p_i p_j^2 x_i^2 x_j + \frac{53}{6} \epsilon^4 p_i p_j^2 x_i^2 x_j + \\ & \frac{25}{24} T \epsilon^4 p_i p_j^2 x_i^2 x_j - \frac{1}{2} \epsilon^2 p_j^3 x_i^2 x_j - \frac{1}{2} T \epsilon^2 p_j^3 x_i^2 x_j + 3 \epsilon^3 p_j^3 x_i^2 x_j + T \epsilon^3 p_j^3 x_i^2 x_j - \frac{157}{24} \epsilon^4 p_j^3 x_i^2 x_j - \\ & \frac{25}{24} T \epsilon^4 p_j^3 x_i^2 x_j - \frac{1}{6} \epsilon^3 p_i^3 p_j x_i^3 x_j + \frac{3}{4} \epsilon^4 p_i^3 p_j x_i^3 x_j + \frac{7}{6} T \epsilon^3 p_i^2 p_j^2 x_i^3 x_j - \frac{5}{2} \epsilon^4 p_i^2 p_j^2 x_i^3 x_j - \end{aligned}$$

$$\begin{aligned}
 & \frac{53}{12} T \epsilon^4 p_i^2 p_j^2 x_i^3 x_j + \epsilon^3 p_i p_j^3 x_i^3 x_j - \frac{17}{6} T \epsilon^3 p_i p_j^3 x_i^3 x_j - \frac{1}{6} T^2 \epsilon^3 p_i p_j^3 x_i^3 x_j + \epsilon^4 p_i p_j^3 x_i^3 x_j + \\
 & \frac{49}{4} T \epsilon^4 p_i p_j^3 x_i^3 x_j + \frac{5}{12} T^2 \epsilon^4 p_i p_j^3 x_i^3 x_j - \frac{5}{6} \epsilon^3 p_j^4 x_i^3 x_j + \frac{5}{3} T \epsilon^3 p_j^4 x_i^3 x_j + \frac{1}{6} T^2 \epsilon^3 p_j^4 x_i^3 x_j + \frac{3}{4} \epsilon^4 p_j^4 x_i^3 x_j - \\
 & \frac{47}{6} T \epsilon^4 p_j^4 x_i^3 x_j - \frac{5}{12} T^2 \epsilon^4 p_j^4 x_i^3 x_j - \frac{1}{24} \epsilon^4 p_i^4 p_j x_i^4 x_j - \frac{2}{3} \epsilon^4 p_i^3 p_j^2 x_i^4 x_j + \frac{31}{24} T \epsilon^4 p_i^3 p_j^2 x_i^4 x_j + \\
 & \frac{15}{4} \epsilon^4 p_i^2 p_j^3 x_i^4 x_j - \frac{115}{24} T \epsilon^4 p_i^2 p_j^3 x_i^4 x_j - \frac{25}{24} T^2 \epsilon^4 p_i^2 p_j^3 x_i^4 x_j - \frac{11}{2} \epsilon^4 p_i p_j^4 x_i^4 x_j + \frac{131}{24} T \epsilon^4 p_i p_j^4 x_i^4 x_j + \\
 & \frac{5}{2} T^2 \epsilon^4 p_i p_j^4 x_i^4 x_j + \frac{1}{24} T^3 \epsilon^4 p_i p_j^4 x_i^4 x_j + \frac{59}{24} \epsilon^4 p_j^5 x_i^4 x_j - \frac{47}{24} T \epsilon^4 p_j^5 x_i^4 x_j - \frac{35}{24} T^2 \epsilon^4 p_j^5 x_i^4 x_j - \\
 & \frac{1}{24} T^3 \epsilon^4 p_j^5 x_i^4 x_j - \frac{1}{2} \epsilon^2 p_i p_j^2 x_i x_j^2 + \epsilon^3 p_i p_j^2 x_i x_j^2 - \frac{25}{24} \epsilon^4 p_i p_j^2 x_i x_j^2 + \frac{1}{2} \epsilon^2 p_j^3 x_i x_j^2 - \epsilon^3 p_j^3 x_i x_j^2 + \\
 & \frac{25}{24} \epsilon^4 p_j^3 x_i x_j^2 - \epsilon^3 p_i^2 p_j^2 x_i^2 x_j^2 + \frac{15}{4} \epsilon^4 p_i^2 p_j^2 x_i^2 x_j^2 + \frac{5}{2} \epsilon^3 p_i p_j^3 x_i^2 x_j^2 + \frac{1}{4} T \epsilon^3 p_i p_j^3 x_i^2 x_j^2 - \frac{43}{4} \epsilon^4 p_i p_j^3 x_i^2 x_j^2 - \\
 & \frac{5}{8} T \epsilon^4 p_i p_j^3 x_i^2 x_j^2 - \frac{3}{2} \epsilon^3 p_j^4 x_i^2 x_j^2 - \frac{1}{4} T \epsilon^3 p_j^4 x_i^2 x_j^2 + 7 \epsilon^4 p_j^4 x_i^2 x_j^2 + \frac{5}{8} T \epsilon^4 p_j^4 x_i^2 x_j^2 - \epsilon^4 p_i^3 p_j^2 x_i^3 x_j^2 + \\
 & \frac{15}{4} \epsilon^4 p_i^2 p_j^3 x_i^3 x_j^2 + \frac{5}{3} T \epsilon^4 p_i^2 p_j^3 x_i^3 x_j^2 - \frac{17}{4} \epsilon^4 p_i p_j^4 x_i^3 x_j^2 - \frac{49}{12} T \epsilon^4 p_i p_j^4 x_i^3 x_j^2 - \frac{1}{12} T^2 \epsilon^4 p_i p_j^4 x_i^3 x_j^2 + \\
 & \frac{3}{2} \epsilon^4 p_j^5 x_i^3 x_j^2 + \frac{29}{12} T \epsilon^4 p_j^5 x_i^3 x_j^2 + \frac{1}{12} T^2 \epsilon^4 p_j^5 x_i^3 x_j^2 - \frac{1}{6} \epsilon^3 p_i p_j^3 x_i x_j^3 + \frac{5}{12} \epsilon^4 p_i p_j^3 x_i x_j^3 + \frac{1}{6} \epsilon^3 p_j^4 x_i x_j^3 - \\
 & \frac{5}{12} \epsilon^4 p_j^4 x_i x_j^3 - \epsilon^4 p_i^2 p_j^3 x_i^2 x_j^3 + \frac{5}{2} \epsilon^4 p_i p_j^4 x_i^2 x_j^3 + \frac{1}{12} T \epsilon^4 p_i p_j^4 x_i^2 x_j^3 - \frac{3}{2} \epsilon^4 p_j^5 x_i^2 x_j^3 - \frac{1}{12} T \epsilon^4 p_j^5 x_i^2 x_j^3 - \\
 & \frac{1}{24} \epsilon^4 p_i p_j^4 x_i x_j^4 + \frac{1}{24} \epsilon^4 p_j^5 x_i x_j^4 + \epsilon^4 ca_{4,1} - 2 \epsilon^4 p_i x_i ca_{4,1} + 2 \epsilon^4 p_j x_i ca_{4,1} - \frac{1}{2} \epsilon^4 p_i p_j x_i^2 cb_{4,10} + \\
 & \frac{1}{2} T \epsilon^4 p_i p_j x_i^2 cb_{4,10} + \frac{1}{2} \epsilon^4 p_j^2 x_i^2 cb_{4,10} - \frac{1}{2} T \epsilon^4 p_j^2 x_i^2 cb_{4,10} - \epsilon^4 p_i p_j x_i x_j cb_{4,10} + \epsilon^4 p_j^2 x_i x_j cb_{4,10}
 \end{aligned}$$

(Alt) Out[ ]:=

$$\frac{\epsilon}{2} - \epsilon p_k x_k - \frac{1}{2} \epsilon^2 p_k x_k - \frac{1}{6} \epsilon^3 p_k x_k + \frac{7}{12} \epsilon^4 p_k x_k - \epsilon^4 ca_{4,1} - \epsilon^4 p_k x_k cb_{4,10}$$

### Sw<sup>+</sup>

(Alt) In[ ]:=

```

lhs = CF[Module[{es = {i, j, i+, j+}},
  Times[
    Normal@Series[Exp[rd[1, -1, -1, i, j] + γd[1, i+] + γd[1, j+]], {ε, 0, d}],
    Exp[Sum[gα,β πα ξβ, {α, es}, {β, es}]]
  ] // Zip(pα&/@es) ∪ (xα&/@es) // Expand
] // . gRules1,i,j

```

(Alt) Out[ ]:=

$$\begin{aligned}
 & 1 - \frac{\epsilon}{2} + \frac{\epsilon^2}{8} - \frac{\epsilon^3}{48} + \frac{1}{384} \epsilon^4 (1 + 384 ca_{4,1}) + \epsilon g_{i^+,i^+} - \epsilon^2 g_{i^+,i^+} + \frac{13}{24} \epsilon^3 g_{i^+,i^+} + \\
 & \frac{1}{24} \epsilon^4 (-5 - 48 ca_{4,1}) g_{i^+,i^+} + \epsilon^2 g_{i^+,i^+}^2 - \frac{3}{2} \epsilon^3 g_{i^+,i^+}^2 + \frac{29}{24} \epsilon^4 g_{i^+,i^+}^2 + \epsilon^3 g_{i^+,i^+}^3 - 2 \epsilon^4 g_{i^+,i^+}^3 +
 \end{aligned}$$

$$\begin{aligned}
 & \epsilon^4 g_{i^+,i^+}^4 - \epsilon g_{j^+,i^+} + \epsilon^2 g_{j^+,i^+} - \frac{13}{24} \epsilon^3 g_{j^+,i^+} + \frac{1}{24} \epsilon^4 (5 + 48 ca_{4,1}) g_{j^+,i^+} - \frac{(-1 + T) \epsilon g_{i^+,i^+} g_{j^+,i^+}}{T} - \\
 & \frac{3 \epsilon^2 g_{i^+,i^+} g_{j^+,i^+}}{T} + \frac{(109 + 23 T) \epsilon^3 g_{i^+,i^+} g_{j^+,i^+}}{24 T} - \frac{\epsilon^4 (42 + 3 T - 8 cb_{4,10} + 8 T cb_{4,10}) g_{i^+,i^+} g_{j^+,i^+}}{8 T} - \\
 & \frac{4 (-1 + T) \epsilon^2 g_{i^+,i^+}^2 g_{j^+,i^+}}{T} + \frac{(-15 + 8 T) \epsilon^3 g_{i^+,i^+}^2 g_{j^+,i^+}}{T} - \frac{(-173 + 59 T) \epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+}}{6 T} - \\
 & \frac{11 (-1 + T) \epsilon^3 g_{i^+,i^+}^3 g_{j^+,i^+}}{T} + \frac{5 (-10 + 7 T) \epsilon^4 g_{i^+,i^+}^3 g_{j^+,i^+}}{T} - \frac{26 (-1 + T) \epsilon^4 g_{i^+,i^+}^4 g_{j^+,i^+}}{T} - \epsilon g_{i^+,j^+} g_{j^+,i^+} + \\
 & 2 \epsilon^2 g_{i^+,j^+} g_{j^+,i^+} - \frac{49}{24} \epsilon^3 g_{i^+,j^+} g_{j^+,i^+} + \frac{1}{24} \epsilon^4 (49 - 24 cb_{4,10}) g_{i^+,j^+} g_{j^+,i^+} - 6 \epsilon^2 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} + \\
 & 17 \epsilon^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} - \frac{101}{4} \epsilon^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} - 21 \epsilon^3 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+} + 78 \epsilon^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+} - \\
 & 60 \epsilon^4 g_{i^+,i^+}^3 g_{i^+,j^+} g_{j^+,i^+} + \frac{(-1 + T) \epsilon g_{j^+,i^+}^2}{T} - \frac{(-3 + T) \epsilon^2 g_{j^+,i^+}^2}{T} + \frac{(-109 + 13 T) \epsilon^3 g_{j^+,i^+}^2}{24 T} + \\
 & \frac{\epsilon^4 (63 - 10 T - 12 cb_{4,10} + 12 T cb_{4,10}) g_{j^+,i^+}^2}{12 T} + \frac{(-1 + T) (2 + 7 T) \epsilon^2 g_{i^+,i^+} g_{j^+,i^+}^2}{T^2} - \\
 & \frac{(-20 - 37 T + 33 T^2) \epsilon^3 g_{i^+,i^+} g_{j^+,i^+}^2}{2 T^2} + \frac{(-610 - 841 T + 539 T^2) \epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^2}{24 T^2} + \\
 & \frac{3 (-1 + T)^2 \epsilon^2 g_{i^+,i^+}^2 g_{j^+,i^+}^2}{T^2} + \frac{(-1 + T) (67 + 27 T) \epsilon^3 g_{i^+,i^+}^2 g_{j^+,i^+}^2}{2 T^2} - \\
 & \frac{(-1197 + 322 T + 475 T^2) \epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+}^2}{8 T^2} + \frac{25 (-1 + T)^2 \epsilon^3 g_{i^+,i^+}^3 g_{j^+,i^+}^2}{T^2} - \\
 & \frac{(-1 + T) (-253 + 58 T) \epsilon^4 g_{i^+,i^+}^3 g_{j^+,i^+}^2}{T^2} + \frac{130 (-1 + T)^2 \epsilon^4 g_{i^+,i^+}^4 g_{j^+,i^+}^2}{T^2} + \frac{(2 + 5 T) \epsilon^2 g_{i^+,j^+} g_{j^+,i^+}^2}{T} - \\
 & \frac{(16 + 29 T) \epsilon^3 g_{i^+,j^+} g_{j^+,i^+}^2}{2 T} + \frac{(394 + 529 T) \epsilon^4 g_{i^+,j^+} g_{j^+,i^+}^2}{24 T} + \frac{6 (-1 + T) \epsilon^2 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^2}{T} + \\
 & \frac{(57 + 5 T) \epsilon^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^2}{T} - \frac{(869 + 235 T) \epsilon^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^2}{4 T} + \frac{66 (-1 + T) \epsilon^3 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^2}{T} - \\
 & \frac{3 (-196 + 81 T) \epsilon^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^2}{T} + \frac{420 (-1 + T) \epsilon^4 g_{i^+,i^+}^3 g_{i^+,j^+} g_{j^+,i^+}^2}{T} + \\
 & 2 \epsilon^2 g_{i^+,j^+}^2 g_{j^+,i^+}^2 - 13 \epsilon^3 g_{i^+,j^+}^2 g_{j^+,i^+}^2 + \frac{425}{12} \epsilon^4 g_{i^+,j^+}^2 g_{j^+,i^+}^2 + 36 \epsilon^3 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^2 - \\
 & 243 \epsilon^4 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^2 + 300 \epsilon^4 g_{i^+,i^+}^2 g_{i^+,j^+}^2 g_{j^+,i^+}^2 - \frac{(-1 + T) (2 + 3 T) \epsilon^2 g_{j^+,i^+}^3}{T^2} + \\
 & \frac{(-20 - 7 T + 15 T^2) \epsilon^3 g_{j^+,i^+}^3}{2 T^2} - \frac{(-610 - 149 T + 255 T^2) \epsilon^4 g_{j^+,i^+}^3}{24 T^2} - \frac{6 (-1 + T)^2 \epsilon^2 g_{i^+,i^+} g_{j^+,i^+}^3}{T^2} + \\
 & \frac{(-1 + T) (-6 - 61 T + 5 T^2) \epsilon^3 g_{i^+,i^+} g_{j^+,i^+}^3}{T^3} + \frac{(-180 - 901 T + 790 T^2 + 51 T^3) \epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^3}{4 T^3} -
 \end{aligned}$$

$$\begin{aligned}
 & \frac{5(-1+T)^2(4+13T)\epsilon^3 g_{i^+,i^+}^2 g_{j^+,i^+}^3}{T^3} + \frac{(-1+T)(-252-481T+268T^2)\epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+}^3}{T^3} - \\
 & \frac{15(-1+T)^3\epsilon^3 g_{i^+,i^+}^3 g_{j^+,i^+}^3}{T^3} - \frac{5(-1+T)^2(83+58T)\epsilon^4 g_{i^+,i^+}^3 g_{j^+,i^+}^3}{T^3} - \frac{210(-1+T)^3\epsilon^4 g_{i^+,i^+}^4 g_{j^+,i^+}^3}{T^3} - \\
 & \frac{6(-1+T)\epsilon^2 g_{i^+,j^+} g_{j^+,i^+}^3}{T} + \frac{3(-2-17T+5T^2)\epsilon^3 g_{i^+,j^+} g_{j^+,i^+}^3}{T^2} - \frac{(-156-753T+65T^2)\epsilon^4 g_{i^+,j^+} g_{j^+,i^+}^3}{4T^2} - \\
 & \frac{8(-1+T)(5+14T)\epsilon^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^3}{T^2} + \frac{4(-113-163T+136T^2)\epsilon^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^3}{T^2} - \\
 & \frac{45(-1+T)^2\epsilon^3 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^3}{T^2} - \frac{60(-1+T)(19+10T)\epsilon^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^3}{T^2} - \\
 & \frac{780(-1+T)^2\epsilon^4 g_{i^+,i^+}^3 g_{i^+,j^+} g_{j^+,i^+}^3}{T^2} - \frac{3(5+9T)\epsilon^3 g_{i^+,j^+}^2 g_{j^+,i^+}^3}{T} + \frac{2(69+94T)\epsilon^4 g_{i^+,j^+}^2 g_{j^+,i^+}^3}{T} - \\
 & \frac{36(-1+T)\epsilon^3 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^3}{T} - \frac{24(32+3T)\epsilon^4 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^3}{T} - \\
 & \frac{840(-1+T)\epsilon^4 g_{i^+,i^+}^2 g_{i^+,j^+}^2 g_{j^+,i^+}^3}{T} - 6\epsilon^3 g_{i^+,j^+}^3 g_{j^+,i^+}^3 + 93\epsilon^4 g_{i^+,j^+}^3 g_{j^+,i^+}^3 - 240\epsilon^4 g_{i^+,i^+} g_{i^+,j^+}^3 g_{j^+,i^+}^3 + \\
 & \frac{3(-1+T)^2\epsilon^2 g_{j^+,i^+}^4}{T^2} - \frac{(-1+T)(-12-55T+15T^2)\epsilon^3 g_{j^+,i^+}^4}{2T^3} + \frac{(360+605T-858T^2+85T^3)\epsilon^4 g_{j^+,i^+}^4}{8T^3} + \\
 & \frac{5(-1+T)^2(8+11T)\epsilon^3 g_{i^+,i^+} g_{j^+,i^+}^4}{T^3} - \frac{(-1+T)(-24-468T-217T^2+259T^3)\epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^4}{T^4} + \\
 & \frac{45(-1+T)^3\epsilon^3 g_{i^+,i^+}^2 g_{j^+,i^+}^4}{T^3} + \frac{5(-1+T)^2(26+223T+23T^2)\epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+}^4}{T^4} + \\
 & \frac{105(-1+T)^3(2+7T)\epsilon^4 g_{i^+,i^+}^3 g_{j^+,i^+}^4}{T^4} + \frac{105(-1+T)^4\epsilon^4 g_{i^+,i^+}^4 g_{j^+,i^+}^4}{T^4} + \\
 & \frac{2(-1+T)(20+23T)\epsilon^3 g_{i^+,j^+} g_{j^+,i^+}^4}{T^2} - \frac{2(-12-208T-39T^2+121T^3)\epsilon^4 g_{i^+,j^+} g_{j^+,i^+}^4}{T^3} + \\
 & \frac{90(-1+T)^2\epsilon^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^4}{T^2} - \frac{10(-1+T)(-26-202T+T^2)\epsilon^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^4}{T^3} + \\
 & \frac{45(-1+T)^2(14+45T)\epsilon^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^4}{T^3} + \frac{420(-1+T)^3\epsilon^4 g_{i^+,i^+}^3 g_{i^+,j^+} g_{j^+,i^+}^4}{T^3} + \\
 & \frac{36(-1+T)\epsilon^3 g_{i^+,j^+}^2 g_{j^+,i^+}^4}{T} - \frac{2(-52-327T+102T^2)\epsilon^4 g_{i^+,j^+}^2 g_{j^+,i^+}^4}{T^2} + \\
 & \frac{15(-1+T)(35+93T)\epsilon^4 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^4}{T^2} + \frac{540(-1+T)^2\epsilon^4 g_{i^+,i^+}^2 g_{i^+,j^+}^2 g_{j^+,i^+}^4}{T^2} + \\
 & \frac{12(9+14T)\epsilon^4 g_{i^+,j^+}^3 g_{j^+,i^+}^4}{T} + \frac{240(-1+T)\epsilon^4 g_{i^+,i^+} g_{i^+,j^+}^3 g_{j^+,i^+}^4}{T} + 24\epsilon^4 g_{i^+,j^+}^4 g_{j^+,i^+}^4 -
 \end{aligned}$$

$$\begin{aligned}
 & \frac{5(-1+T)^2(4+3T)\epsilon^3 g_{j^+,i^+}^5}{T^3} + \frac{(-1+T)(-24-216T+11T^2+75T^3)\epsilon^4 g_{j^+,i^+}^5}{T^4} - \\
 & \frac{45(-1+T)^3\epsilon^3 g_{i^+,i^+} g_{j^+,i^+}^5}{T^3} + \frac{5(-1+T)^2(-52-197T+24T^2)\epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^5}{T^4} - \\
 & \frac{315(-1+T)^3(2+3T)\epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+}^5}{T^4} - \frac{420(-1+T)^4\epsilon^4 g_{i^+,i^+}^3 g_{j^+,i^+}^5}{T^4} - \frac{45(-1+T)^2\epsilon^3 g_{i^+,j^+} g_{j^+,i^+}^5}{T^2} + \\
 & \frac{10(-1+T)(-26-88T+19T^2)\epsilon^4 g_{i^+,j^+} g_{j^+,i^+}^5}{T^3} - \frac{90(-1+T)^2(14+19T)\epsilon^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^5}{T^3} - \\
 & \frac{1260(-1+T)^3\epsilon^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^5}{T^3} - \frac{15(-1+T)(35+37T)\epsilon^4 g_{i^+,j^+}^2 g_{j^+,i^+}^5}{T^2} - \\
 & \frac{1080(-1+T)^2\epsilon^4 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^5}{T^2} - \frac{240(-1+T)\epsilon^4 g_{i^+,j^+}^3 g_{j^+,i^+}^5}{T} + \frac{15(-1+T)^3\epsilon^3 g_{j^+,i^+}^6}{T^3} - \\
 & \frac{5(-1+T)^2(-26-57T+15T^2)\epsilon^4 g_{j^+,i^+}^6}{T^4} + \frac{105(-1+T)^3(6+5T)\epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^6}{T^4} + \\
 & \frac{630(-1+T)^4\epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+}^6}{T^4} + \frac{15(-1+T)^2(42+31T)\epsilon^4 g_{i^+,j^+} g_{j^+,i^+}^6}{T^3} + \\
 & \frac{1260(-1+T)^3\epsilon^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^6}{T^3} + \frac{540(-1+T)^2\epsilon^4 g_{i^+,j^+}^2 g_{j^+,i^+}^6}{T^2} - \\
 & \frac{105(-1+T)^3(2+T)\epsilon^4 g_{j^+,i^+}^7}{T^4} - \frac{420(-1+T)^4\epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^7}{T^4} - \frac{420(-1+T)^3\epsilon^4 g_{i^+,j^+} g_{j^+,i^+}^7}{T^3} + \\
 & \frac{105(-1+T)^4\epsilon^4 g_{j^+,i^+}^8}{T^4} - \epsilon g_{i^+,i^+} g_{j^+,j^+} + 2\epsilon^2 g_{i^+,i^+} g_{j^+,j^+} - \frac{49}{24}\epsilon^3 g_{i^+,i^+} g_{j^+,j^+} + \\
 & \frac{1}{24}\epsilon^4(49-24cb_{4,10})g_{i^+,i^+} g_{j^+,j^+} - 3\epsilon^2 g_{i^+,i^+}^2 g_{j^+,j^+} + \frac{17}{2}\epsilon^3 g_{i^+,i^+}^2 g_{j^+,j^+} - \frac{101}{8}\epsilon^4 g_{i^+,i^+}^2 g_{j^+,j^+} - \\
 & 7\epsilon^3 g_{i^+,i^+}^3 g_{j^+,j^+} + 26\epsilon^4 g_{i^+,i^+}^3 g_{j^+,j^+} - 15\epsilon^4 g_{i^+,i^+}^4 g_{j^+,j^+} + 2\epsilon g_{j^+,i^+} g_{j^+,j^+} - 4\epsilon^2 g_{j^+,i^+} g_{j^+,j^+} + \\
 & \frac{49}{12}\epsilon^3 g_{j^+,i^+} g_{j^+,j^+} + \frac{1}{12}\epsilon^4(-49+24cb_{4,10})g_{j^+,i^+} g_{j^+,j^+} + \frac{2(2+5T)\epsilon^2 g_{i^+,i^+} g_{j^+,i^+} g_{j^+,j^+}}{T} - \\
 & \frac{(16+29T)\epsilon^3 g_{i^+,i^+} g_{j^+,i^+} g_{j^+,j^+}}{T} + \frac{(394+529T)\epsilon^4 g_{i^+,i^+} g_{j^+,i^+} g_{j^+,j^+}}{12T} + \frac{6(-1+T)\epsilon^2 g_{i^+,i^+}^2 g_{j^+,i^+} g_{j^+,j^+}}{T} + \\
 & \frac{(57+5T)\epsilon^3 g_{i^+,i^+}^2 g_{j^+,i^+} g_{j^+,j^+}}{T} - \frac{(869+235T)\epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+} g_{j^+,j^+}}{4T} + \frac{44(-1+T)\epsilon^3 g_{i^+,i^+}^3 g_{j^+,i^+} g_{j^+,j^+}}{T} - \\
 & \frac{2(-196+81T)\epsilon^4 g_{i^+,i^+}^3 g_{j^+,i^+} g_{j^+,j^+}}{T} + \frac{210(-1+T)\epsilon^4 g_{i^+,i^+}^4 g_{j^+,i^+} g_{j^+,j^+}}{T} - 2\epsilon^2 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} + \\
 & 5\epsilon^3 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} - \frac{77}{12}\epsilon^4 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} + 8\epsilon^2 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} - \\
 & 52\epsilon^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} + \frac{425}{3}\epsilon^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} + 72\epsilon^3 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} -
 \end{aligned}$$

$$\begin{aligned}
 & 486 \in^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} + 400 \in^4 g_{i^+,i^+}^3 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} - \frac{6(1+T) \in^2 g_{j^+,i^+}^2 g_{j^+,j^+}}{T} + \\
 & \frac{6(4+3T) \in^3 g_{j^+,i^+}^2 g_{j^+,j^+}}{T} - \frac{(197+113T) \in^4 g_{j^+,i^+}^2 g_{j^+,j^+}}{4T} - \frac{18(-1+T) \in^2 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} + \\
 & \frac{9(-2-17T+5T^2) \in^3 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T^2} - \frac{3(-156-753T+65T^2) \in^4 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{4T^2} - \\
 & \frac{12(-1+T)(5+14T) \in^3 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T^2} + \frac{6(-113-163T+136T^2) \in^4 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T^2} - \\
 & \frac{45(-1+T)^2 \in^3 g_{i^+,i^+}^3 g_{j^+,i^+}^2 g_{j^+,j^+}}{T^2} - \frac{60(-1+T)(19+10T) \in^4 g_{i^+,i^+}^3 g_{j^+,i^+}^2 g_{j^+,j^+}}{T^2} - \\
 & \frac{585(-1+T)^2 \in^4 g_{i^+,i^+}^4 g_{j^+,i^+}^2 g_{j^+,j^+}}{T^2} - 12 \in^2 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+} + \frac{3(4+25T) \in^3 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} - \\
 & \frac{(120+407T) \in^4 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{2T} - \frac{18(5+9T) \in^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} + \\
 & \frac{12(69+94T) \in^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} - \frac{108(-1+T) \in^3 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} - \\
 & \frac{72(32+3T) \in^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} - \frac{1680(-1+T) \in^4 g_{i^+,i^+}^3 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} + \\
 & 18 \in^3 g_{i^+,j^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+} - 114 \in^4 g_{i^+,j^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+} - 54 \in^3 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+} + \\
 & 837 \in^4 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+} - 1080 \in^4 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+} + \frac{12(-1+T) \in^2 g_{j^+,i^+}^3 g_{j^+,j^+}}{T} - \\
 & \frac{6(-4-15T+7T^2) \in^3 g_{j^+,i^+}^3 g_{j^+,j^+}}{T^2} + \frac{(-312-637T+157T^2) \in^4 g_{j^+,i^+}^3 g_{j^+,j^+}}{2T^2} + \\
 & \frac{8(-1+T)(20+23T) \in^3 g_{i^+,i^+} g_{j^+,i^+}^3 g_{j^+,j^+}}{T^2} - \frac{8(-12-208T-39T^2+121T^3) \in^4 g_{i^+,i^+} g_{j^+,i^+}^3 g_{j^+,j^+}}{T^3} + \\
 & \frac{180(-1+T)^2 \in^3 g_{i^+,i^+} g_{j^+,i^+}^3 g_{j^+,j^+}}{T^2} - \frac{20(-1+T)(-26-202T+T^2) \in^4 g_{i^+,i^+} g_{j^+,i^+}^3 g_{j^+,j^+}}{T^3} + \\
 & \frac{60(-1+T)^2(14+45T) \in^4 g_{i^+,i^+}^3 g_{j^+,i^+}^3 g_{j^+,j^+}}{T^3} + \frac{420(-1+T)^3 \in^4 g_{i^+,i^+}^4 g_{j^+,i^+}^3 g_{j^+,j^+}}{T^3} + \\
 & \frac{24(5+3T) \in^3 g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}}{T} - \frac{8(9+132T+67T^2) \in^4 g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}}{T^2} + \\
 & \frac{288(-1+T) \in^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}}{T} - \frac{16(-52-327T+102T^2) \in^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}}{T^2} + \\
 & \frac{60(-1+T)(35+93T) \in^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}}{T^2} + \frac{1440(-1+T)^2 \in^4 g_{i^+,i^+}^3 g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}}{T^2} +
 \end{aligned}$$

$$\begin{aligned}
 & 72 \epsilon^3 g_{i^+,j^+}^2 g_{j^+,i^+}^3 g_{j^+,j^+} - \frac{12 (19 + 86 T) \epsilon^4 g_{i^+,j^+}^2 g_{j^+,i^+}^3 g_{j^+,j^+}}{T} + \frac{144 (9 + 14 T) \epsilon^4 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^3 g_{j^+,j^+}}{T} + \\
 & \frac{1440 (-1 + T) \epsilon^4 g_{i^+,i^+}^2 g_{i^+,j^+}^2 g_{j^+,i^+}^3 g_{j^+,j^+}}{T} - 144 \epsilon^4 g_{i^+,j^+}^3 g_{j^+,i^+}^3 g_{j^+,j^+} + 384 \epsilon^4 g_{i^+,i^+} g_{i^+,j^+}^3 g_{j^+,i^+}^3 g_{j^+,j^+} - \\
 & \frac{20 (-1 + T) (5 + 3 T) \epsilon^3 g_{j^+,i^+}^4 g_{j^+,j^+}}{T^2} + \frac{10 (-12 - 95 T + 26 T^2 + 33 T^3) \epsilon^4 g_{j^+,i^+}^4 g_{j^+,j^+}}{T^3} - \\
 & \frac{225 (-1 + T)^2 \epsilon^3 g_{i^+,i^+} g_{j^+,i^+}^4 g_{j^+,j^+}}{T^2} + \frac{50 (-1 + T) (-26 - 88 T + 19 T^2) \epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^4 g_{j^+,j^+}}{T^3} - \\
 & \frac{225 (-1 + T)^2 (14 + 19 T) \epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+}^4 g_{j^+,j^+}}{T^3} - \frac{2100 (-1 + T)^3 \epsilon^4 g_{i^+,i^+}^3 g_{j^+,i^+}^4 g_{j^+,j^+}}{T^3} - \\
 & \frac{180 (-1 + T) \epsilon^3 g_{i^+,j^+} g_{j^+,i^+}^4 g_{j^+,j^+}}{T} + \frac{20 (-52 - 135 T + 70 T^2) \epsilon^4 g_{i^+,j^+} g_{j^+,i^+}^4 g_{j^+,j^+}}{T^2} - \\
 & \frac{150 (-1 + T) (35 + 37 T) \epsilon^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^4 g_{j^+,j^+}}{T^2} - \frac{5400 (-1 + T)^2 \epsilon^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^4 g_{j^+,j^+}}{T^2} - \\
 & \frac{180 (9 + 4 T) \epsilon^4 g_{i^+,j^+}^2 g_{j^+,i^+}^4 g_{j^+,j^+}}{T} - \frac{3600 (-1 + T) \epsilon^4 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^4 g_{j^+,j^+}}{T} - \\
 & 480 \epsilon^4 g_{i^+,j^+}^3 g_{j^+,i^+}^4 g_{j^+,j^+} + \frac{90 (-1 + T)^2 \epsilon^3 g_{j^+,i^+}^5 g_{j^+,j^+}}{T^2} - \frac{60 (-1 + T) (-13 - 25 T + 9 T^2) \epsilon^4 g_{j^+,i^+}^5 g_{j^+,j^+}}{T^3} + \\
 & \frac{90 (-1 + T)^2 (42 + 31 T) \epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^5 g_{j^+,j^+}}{T^3} + \frac{3780 (-1 + T)^3 \epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+}^5 g_{j^+,j^+}}{T^3} + \\
 & \frac{150 (-1 + T) (21 + 11 T) \epsilon^4 g_{i^+,j^+} g_{j^+,i^+}^5 g_{j^+,j^+}}{T^2} + \frac{6480 (-1 + T)^2 \epsilon^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^5 g_{j^+,j^+}}{T^2} + \\
 & \frac{2160 (-1 + T) \epsilon^4 g_{i^+,j^+}^2 g_{j^+,i^+}^5 g_{j^+,j^+}}{T} - \frac{210 (-1 + T)^2 (7 + 3 T) \epsilon^4 g_{j^+,i^+}^6 g_{j^+,j^+}}{T^3} - \\
 & \frac{2940 (-1 + T)^3 \epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^6 g_{j^+,j^+}}{T^3} - \frac{2520 (-1 + T)^2 \epsilon^4 g_{i^+,j^+} g_{j^+,i^+}^6 g_{j^+,j^+}}{T^2} + \\
 & \frac{840 (-1 + T)^3 \epsilon^4 g_{j^+,i^+}^7 g_{j^+,j^+}}{T^3} - \epsilon^2 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+} + \frac{5}{2} \epsilon^3 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+} - \frac{77}{24} \epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+} + \\
 & 2 \epsilon^2 g_{i^+,i^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+} - 13 \epsilon^3 g_{i^+,i^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+} + \frac{425}{12} \epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+} + 12 \epsilon^3 g_{i^+,i^+}^3 g_{j^+,i^+}^2 g_{j^+,j^+} - \\
 & 81 \epsilon^4 g_{i^+,i^+}^3 g_{j^+,i^+}^2 g_{j^+,j^+} + 50 \epsilon^4 g_{i^+,i^+}^4 g_{j^+,i^+}^2 g_{j^+,j^+} + 3 \epsilon^2 g_{j^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+} - \frac{15}{2} \epsilon^3 g_{j^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+} + \frac{77}{8} \epsilon^4 g_{j^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+} - \\
 & 12 \epsilon^2 g_{i^+,i^+} g_{j^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+} + \frac{3 (4 + 25 T) \epsilon^3 g_{i^+,i^+} g_{j^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} - \frac{(120 + 407 T) \epsilon^4 g_{i^+,i^+} g_{j^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{2 T} - \\
 & \frac{9 (5 + 9 T) \epsilon^3 g_{i^+,i^+}^2 g_{j^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} + \frac{6 (69 + 94 T) \epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} - \frac{36 (-1 + T) \epsilon^3 g_{i^+,i^+}^3 g_{j^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} -
 \end{aligned}$$

$$\begin{aligned}
 & \frac{24 (32 + 3 T) \in^4 g_{i^+,i^+}^3 g_{j^+,i^+} g_{j^+,j^+}^2}{T} - \frac{420 (-1 + T) \in^4 g_{i^+,i^+}^4 g_{j^+,i^+} g_{j^+,j^+}^2}{T} - 3 \in^3 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^2 + \\
 & 9 \in^4 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^2 + 36 \in^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^2 - 228 \in^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^2 - \\
 & 54 \in^3 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^2 + 837 \in^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^2 - 720 \in^4 g_{i^+,i^+}^3 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^2 + \\
 & 12 \in^2 g_{j^+,i^+}^2 g_{j^+,j^+}^2 - \frac{24 (1 + 3 T) \in^3 g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T} + \frac{(240 + 389 T) \in^4 g_{j^+,i^+}^2 g_{j^+,j^+}^2}{2 T} + \\
 & \frac{36 (5 + 3 T) \in^3 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T} - \frac{12 (9 + 132 T + 67 T^2) \in^4 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T^2} + \\
 & \frac{216 (-1 + T) \in^3 g_{i^+,i^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T} - \frac{12 (-52 - 327 T + 102 T^2) \in^4 g_{i^+,i^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T^2} + \\
 & \frac{30 (-1 + T) (35 + 93 T) \in^4 g_{i^+,i^+}^3 g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T^2} + \frac{540 (-1 + T)^2 \in^4 g_{i^+,i^+}^4 g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T^2} - \\
 & 72 \in^3 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}^2 + \frac{6 (8 + 75 T) \in^4 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T} + 216 \in^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}^2 - \\
 & \frac{36 (19 + 86 T) \in^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T} + \frac{216 (9 + 14 T) \in^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T} + \\
 & \frac{1440 (-1 + T) \in^4 g_{i^+,i^+}^3 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T} + 84 \in^4 g_{i^+,j^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+}^2 - 648 \in^4 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+}^2 + \\
 & 864 \in^4 g_{i^+,i^+}^2 g_{i^+,j^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+}^2 - \frac{30 (5 + T) \in^3 g_{j^+,i^+}^3 g_{j^+,j^+}^2}{T} + \frac{90 (2 + 14 T + 3 T^2) \in^4 g_{j^+,i^+}^3 g_{j^+,j^+}^2}{T^2} - \\
 & \frac{360 (-1 + T) \in^3 g_{i^+,i^+} g_{j^+,i^+}^3 g_{j^+,j^+}^2}{T} + \frac{40 (-52 - 135 T + 70 T^2) \in^4 g_{i^+,i^+} g_{j^+,i^+}^3 g_{j^+,j^+}^2}{T^2} - \\
 & \frac{150 (-1 + T) (35 + 37 T) \in^4 g_{i^+,i^+}^2 g_{j^+,i^+}^3 g_{j^+,j^+}^2}{T^2} - \frac{3600 (-1 + T)^2 \in^4 g_{i^+,i^+}^3 g_{j^+,i^+}^3 g_{j^+,j^+}^2}{T^2} - \\
 & 180 \in^3 g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}^2 + \frac{30 (38 + 79 T) \in^4 g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}^2}{T} - \frac{720 (9 + 4 T) \in^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}^2}{T} - \\
 & \frac{7200 (-1 + T) \in^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}^2}{T} + 1080 \in^4 g_{i^+,j^+}^2 g_{j^+,i^+}^3 g_{j^+,j^+}^2 - 2880 \in^4 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^3 g_{j^+,j^+}^2 + \\
 & \frac{180 (-1 + T) \in^3 g_{j^+,i^+}^4 g_{j^+,j^+}^2}{T} - \frac{30 (-52 - 71 T + 51 T^2) \in^4 g_{j^+,i^+}^4 g_{j^+,j^+}^2}{T^2} + \\
 & \frac{375 (-1 + T) (21 + 11 T) \in^4 g_{i^+,i^+} g_{j^+,i^+}^4 g_{j^+,j^+}^2}{T^2} + \frac{8100 (-1 + T)^2 \in^4 g_{i^+,i^+}^2 g_{j^+,i^+}^4 g_{j^+,j^+}^2}{T^2} + \\
 & \frac{180 (27 + 2 T) \in^4 g_{i^+,j^+} g_{j^+,i^+}^4 g_{j^+,j^+}^2}{T} + \frac{10800 (-1 + T) \in^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^4 g_{j^+,j^+}^2}{T} + \\
 & 2160 \in^4 g_{i^+,j^+}^2 g_{j^+,i^+}^4 g_{j^+,j^+}^2 - \frac{105 (-1 + T) (35 + 9 T) \in^4 g_{j^+,i^+}^5 g_{j^+,j^+}^2}{T^2} - \frac{7560 (-1 + T)^2 \in^4 g_{i^+,i^+} g_{j^+,i^+}^5 g_{j^+,j^+}^2}{T^2} -
 \end{aligned}$$

$$\begin{aligned}
 & \frac{5040 (-1 + T) \epsilon^4 g_{i^+,j^+} g_{j^+,i^+}^5 g_{j^+,j^+}^2}{T} + \frac{2520 (-1 + T)^2 \epsilon^4 g_{j^+,i^+}^6 g_{j^+,j^+}^2}{T^2} - \epsilon^3 g_{i^+,i^+} g_{j^+,j^+}^3 + 3 \epsilon^4 g_{i^+,i^+} g_{j^+,j^+}^3 + \\
 & 6 \epsilon^3 g_{i^+,i^+}^2 g_{j^+,j^+}^3 - 38 \epsilon^4 g_{i^+,i^+}^2 g_{j^+,j^+}^3 - 6 \epsilon^3 g_{i^+,i^+}^3 g_{j^+,j^+}^3 + 93 \epsilon^4 g_{i^+,i^+}^3 g_{j^+,j^+}^3 - 60 \epsilon^4 g_{i^+,i^+}^4 g_{j^+,j^+}^3 + \\
 & 4 \epsilon^3 g_{j^+,i^+} g_{j^+,j^+}^3 - 12 \epsilon^4 g_{j^+,i^+} g_{j^+,j^+}^3 - 48 \epsilon^3 g_{i^+,i^+} g_{j^+,i^+} g_{j^+,j^+}^3 + \frac{4 (8 + 75 T) \epsilon^4 g_{i^+,i^+} g_{j^+,i^+} g_{j^+,j^+}^3}{T} + \\
 & 72 \epsilon^3 g_{i^+,i^+}^2 g_{j^+,i^+} g_{j^+,j^+}^3 - \frac{12 (19 + 86 T) \epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+} g_{j^+,j^+}^3}{T} + \frac{48 (9 + 14 T) \epsilon^4 g_{i^+,i^+}^3 g_{j^+,i^+} g_{j^+,j^+}^3}{T} + \\
 & \frac{240 (-1 + T) \epsilon^4 g_{i^+,i^+}^4 g_{j^+,i^+} g_{j^+,j^+}^3}{T} - 4 \epsilon^4 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^3 + 112 \epsilon^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^3 - \\
 & 432 \epsilon^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^3 + 384 \epsilon^4 g_{i^+,i^+}^3 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^3 + 60 \epsilon^3 g_{j^+,i^+}^2 g_{j^+,j^+}^3 - \\
 & \frac{10 (8 + 37 T) \epsilon^4 g_{j^+,i^+}^2 g_{j^+,j^+}^3}{T} - 180 \epsilon^3 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}^3 + \frac{30 (38 + 79 T) \epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}^3}{T} - \\
 & \frac{360 (9 + 4 T) \epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+}^3}{T} - \frac{2400 (-1 + T) \epsilon^4 g_{i^+,i^+}^3 g_{j^+,i^+}^2 g_{j^+,j^+}^3}{T} - \\
 & 280 \epsilon^4 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}^3 + 2160 \epsilon^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}^3 - 2880 \epsilon^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}^3 + \\
 & 120 \epsilon^3 g_{j^+,i^+}^3 g_{j^+,j^+}^3 - \frac{60 (19 + 24 T) \epsilon^4 g_{j^+,i^+}^3 g_{j^+,j^+}^3}{T} + \frac{240 (27 + 2 T) \epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^3 g_{j^+,j^+}^3}{T} + \\
 & \frac{7200 (-1 + T) \epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+}^3 g_{j^+,j^+}^3}{T} - 2160 \epsilon^4 g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}^3 + 5760 \epsilon^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}^3 + \\
 & \frac{420 (-9 + T) \epsilon^4 g_{j^+,i^+}^4 g_{j^+,j^+}^3}{T} - \frac{8400 (-1 + T) \epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^4 g_{j^+,j^+}^3}{T} - 3360 \epsilon^4 g_{i^+,j^+} g_{j^+,i^+}^4 g_{j^+,j^+}^3 + \\
 & \frac{3360 (-1 + T) \epsilon^4 g_{j^+,i^+}^5 g_{j^+,j^+}^3}{T} - \epsilon^4 g_{i^+,i^+} g_{j^+,j^+}^4 + 14 \epsilon^4 g_{i^+,i^+}^2 g_{j^+,j^+}^4 - 36 \epsilon^4 g_{i^+,i^+}^3 g_{j^+,j^+}^4 + \\
 & 24 \epsilon^4 g_{i^+,i^+}^4 g_{j^+,j^+}^4 + 5 \epsilon^4 g_{j^+,i^+} g_{j^+,j^+}^4 - 140 \epsilon^4 g_{i^+,i^+} g_{j^+,i^+} g_{j^+,j^+}^4 + 540 \epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+} g_{j^+,j^+}^4 - \\
 & 480 \epsilon^4 g_{i^+,i^+}^3 g_{j^+,i^+} g_{j^+,j^+}^4 + 210 \epsilon^4 g_{j^+,i^+}^2 g_{j^+,j^+}^4 - 1620 \epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}^4 + \\
 & 2160 \epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+}^4 + 1260 \epsilon^4 g_{j^+,i^+}^3 g_{j^+,j^+}^4 - 3360 \epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^3 g_{j^+,j^+}^4 + 1680 \epsilon^4 g_{j^+,i^+}^4 g_{j^+,j^+}^4
 \end{aligned}$$

(Alt) In[ ]:=

```

rhs = CF[Module[{es = {i, j, i^+, j^+}},
  Times[
    Normal@Series[Exp[r_d[1, i, j]], {e, 0, d}],
    Exp[Sum[g_{\alpha,\beta} \pi_{\alpha} \xi_{\beta}, {\alpha, es}, {\beta, es}]]
  ] // Zip(p_{\alpha}&/@es)U(x_{\alpha}&/@es) // Expand
] // . gRules_{i,j}
]

```

(Alt) Out[ ]:=

$$\begin{aligned}
 & 1 - \frac{\epsilon}{2} + \frac{\epsilon^2}{8} - \frac{\epsilon^3}{48} + \frac{1}{384} \epsilon^4 (1 + 384 ca_{4,1}) + \epsilon g_{i^+,i^+} - \epsilon^2 g_{i^+,i^+}^2 + \frac{13}{24} \epsilon^3 g_{i^+,i^+}^3 + \\
 & \frac{1}{24} \epsilon^4 (-5 - 48 ca_{4,1}) g_{i^+,i^+} + \epsilon^2 g_{i^+,i^+}^2 - \frac{3}{2} \epsilon^3 g_{i^+,i^+}^3 + \frac{29}{24} \epsilon^4 g_{i^+,i^+}^4 + \epsilon^3 g_{i^+,i^+}^3 - 2 \epsilon^4 g_{i^+,i^+}^4 +
 \end{aligned}$$

$$\begin{aligned}
 & \epsilon^4 g_{i^+,i^+}^4 - \epsilon g_{j^+,i^+} + \epsilon^2 g_{j^+,i^+} - \frac{13}{24} \epsilon^3 g_{j^+,i^+} + \frac{1}{24} \epsilon^4 (5 + 48 ca_{4,1}) g_{j^+,i^+} - \frac{(-1 + T) \epsilon g_{i^+,i^+} g_{j^+,i^+}}{T} - \\
 & \frac{3 \epsilon^2 g_{i^+,i^+} g_{j^+,i^+}}{T} + \frac{(109 + 23 T) \epsilon^3 g_{i^+,i^+} g_{j^+,i^+}}{24 T} - \frac{\epsilon^4 (42 + 3 T - 8 cb_{4,10} + 8 T cb_{4,10}) g_{i^+,i^+} g_{j^+,i^+}}{8 T} - \\
 & \frac{4 (-1 + T) \epsilon^2 g_{i^+,i^+}^2 g_{j^+,i^+}}{T} + \frac{(-15 + 8 T) \epsilon^3 g_{i^+,i^+}^2 g_{j^+,i^+}}{T} - \frac{(-173 + 59 T) \epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+}}{6 T} - \\
 & \frac{11 (-1 + T) \epsilon^3 g_{i^+,i^+}^3 g_{j^+,i^+}}{T} + \frac{5 (-10 + 7 T) \epsilon^4 g_{i^+,i^+}^3 g_{j^+,i^+}}{T} - \frac{26 (-1 + T) \epsilon^4 g_{i^+,i^+}^4 g_{j^+,i^+}}{T} - \epsilon g_{i^+,j^+} g_{j^+,i^+} + \\
 & 2 \epsilon^2 g_{i^+,j^+} g_{j^+,i^+} - \frac{49}{24} \epsilon^3 g_{i^+,j^+} g_{j^+,i^+} + \frac{1}{24} \epsilon^4 (49 - 24 cb_{4,10}) g_{i^+,j^+} g_{j^+,i^+} - 6 \epsilon^2 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} + \\
 & 17 \epsilon^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} - \frac{101}{4} \epsilon^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} - 21 \epsilon^3 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+} + 78 \epsilon^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+} - \\
 & 60 \epsilon^4 g_{i^+,i^+}^3 g_{i^+,j^+} g_{j^+,i^+} + \frac{(-1 + T) \epsilon g_{j^+,i^+}^2}{T} - \frac{(-3 + T) \epsilon^2 g_{j^+,i^+}^2}{T} + \frac{(-109 + 13 T) \epsilon^3 g_{j^+,i^+}^2}{24 T} + \\
 & \frac{\epsilon^4 (63 - 10 T - 12 cb_{4,10} + 12 T cb_{4,10}) g_{j^+,i^+}^2}{12 T} + \frac{(-1 + T) (2 + 7 T) \epsilon^2 g_{i^+,i^+} g_{j^+,i^+}^2}{T^2} - \\
 & \frac{(-20 - 37 T + 33 T^2) \epsilon^3 g_{i^+,i^+} g_{j^+,i^+}^2}{2 T^2} + \frac{(-610 - 841 T + 539 T^2) \epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^2}{24 T^2} + \\
 & \frac{3 (-1 + T)^2 \epsilon^2 g_{i^+,i^+}^2 g_{j^+,i^+}^2}{T^2} + \frac{(-1 + T) (67 + 27 T) \epsilon^3 g_{i^+,i^+}^2 g_{j^+,i^+}^2}{2 T^2} - \\
 & \frac{(-1197 + 322 T + 475 T^2) \epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+}^2}{8 T^2} + \frac{25 (-1 + T)^2 \epsilon^3 g_{i^+,i^+}^3 g_{j^+,i^+}^2}{T^2} - \\
 & \frac{(-1 + T) (-253 + 58 T) \epsilon^4 g_{i^+,i^+}^3 g_{j^+,i^+}^2}{T^2} + \frac{130 (-1 + T)^2 \epsilon^4 g_{i^+,i^+}^4 g_{j^+,i^+}^2}{T^2} + \frac{(2 + 5 T) \epsilon^2 g_{i^+,j^+} g_{j^+,i^+}^2}{T} - \\
 & \frac{(16 + 29 T) \epsilon^3 g_{i^+,j^+} g_{j^+,i^+}^2}{2 T} + \frac{(394 + 529 T) \epsilon^4 g_{i^+,j^+} g_{j^+,i^+}^2}{24 T} + \frac{6 (-1 + T) \epsilon^2 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^2}{T} + \\
 & \frac{(57 + 5 T) \epsilon^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^2}{T} - \frac{(869 + 235 T) \epsilon^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^2}{4 T} + \frac{66 (-1 + T) \epsilon^3 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^2}{T} - \\
 & \frac{3 (-196 + 81 T) \epsilon^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^2}{T} + \frac{420 (-1 + T) \epsilon^4 g_{i^+,i^+}^3 g_{i^+,j^+} g_{j^+,i^+}^2}{T} + \\
 & 2 \epsilon^2 g_{i^+,j^+}^2 g_{j^+,i^+}^2 - 13 \epsilon^3 g_{i^+,j^+}^2 g_{j^+,i^+}^2 + \frac{425}{12} \epsilon^4 g_{i^+,j^+}^2 g_{j^+,i^+}^2 + 36 \epsilon^3 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^2 - \\
 & 243 \epsilon^4 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^2 + 300 \epsilon^4 g_{i^+,i^+}^2 g_{i^+,j^+}^2 g_{j^+,i^+}^2 - \frac{(-1 + T) (2 + 3 T) \epsilon^2 g_{j^+,i^+}^3}{T^2} + \\
 & \frac{(-20 - 7 T + 15 T^2) \epsilon^3 g_{j^+,i^+}^3}{2 T^2} - \frac{(-610 - 149 T + 255 T^2) \epsilon^4 g_{j^+,i^+}^3}{24 T^2} - \frac{6 (-1 + T)^2 \epsilon^2 g_{i^+,i^+} g_{j^+,i^+}^3}{T^2} + \\
 & \frac{(-1 + T) (-6 - 61 T + 5 T^2) \epsilon^3 g_{i^+,i^+} g_{j^+,i^+}^3}{T^3} + \frac{(-180 - 901 T + 790 T^2 + 51 T^3) \epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^3}{4 T^3} -
 \end{aligned}$$

$$\begin{aligned}
 & \frac{5(-1+T)^2(4+13T)\epsilon^3 g_{i^+,i^+}^2 g_{j^+,i^+}^3}{T^3} + \frac{(-1+T)(-252-481T+268T^2)\epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+}^3}{T^3} - \\
 & \frac{15(-1+T)^3\epsilon^3 g_{i^+,i^+}^3 g_{j^+,i^+}^3}{T^3} - \frac{5(-1+T)^2(83+58T)\epsilon^4 g_{i^+,i^+}^3 g_{j^+,i^+}^3}{T^3} - \frac{210(-1+T)^3\epsilon^4 g_{i^+,i^+}^4 g_{j^+,i^+}^3}{T^3} - \\
 & \frac{6(-1+T)\epsilon^2 g_{i^+,j^+} g_{j^+,i^+}^3}{T} + \frac{3(-2-17T+5T^2)\epsilon^3 g_{i^+,j^+} g_{j^+,i^+}^3}{T^2} - \frac{(-156-753T+65T^2)\epsilon^4 g_{i^+,j^+} g_{j^+,i^+}^3}{4T^2} - \\
 & \frac{8(-1+T)(5+14T)\epsilon^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^3}{T^2} + \frac{4(-113-163T+136T^2)\epsilon^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^3}{T^2} - \\
 & \frac{45(-1+T)^2\epsilon^3 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^3}{T^2} - \frac{60(-1+T)(19+10T)\epsilon^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^3}{T^2} - \\
 & \frac{780(-1+T)^2\epsilon^4 g_{i^+,i^+}^3 g_{i^+,j^+} g_{j^+,i^+}^3}{T^2} - \frac{3(5+9T)\epsilon^3 g_{i^+,j^+}^2 g_{j^+,i^+}^3}{T} + \frac{2(69+94T)\epsilon^4 g_{i^+,j^+}^2 g_{j^+,i^+}^3}{T} - \\
 & \frac{36(-1+T)\epsilon^3 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^3}{T} - \frac{24(32+3T)\epsilon^4 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^3}{T} - \\
 & \frac{840(-1+T)\epsilon^4 g_{i^+,i^+}^2 g_{i^+,j^+}^2 g_{j^+,i^+}^3}{T} - 6\epsilon^3 g_{i^+,j^+}^3 g_{j^+,i^+}^3 + 93\epsilon^4 g_{i^+,j^+}^3 g_{j^+,i^+}^3 - 240\epsilon^4 g_{i^+,i^+} g_{i^+,j^+}^3 g_{j^+,i^+}^3 + \\
 & \frac{3(-1+T)^2\epsilon^2 g_{j^+,i^+}^4}{T^2} - \frac{(-1+T)(-12-55T+15T^2)\epsilon^3 g_{j^+,i^+}^4}{2T^3} + \frac{(360+605T-858T^2+85T^3)\epsilon^4 g_{j^+,i^+}^4}{8T^3} + \\
 & \frac{5(-1+T)^2(8+11T)\epsilon^3 g_{i^+,i^+} g_{j^+,i^+}^4}{T^3} - \frac{(-1+T)(-24-468T-217T^2+259T^3)\epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^4}{T^4} + \\
 & \frac{45(-1+T)^3\epsilon^3 g_{i^+,i^+}^2 g_{j^+,i^+}^4}{T^3} + \frac{5(-1+T)^2(26+223T+23T^2)\epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+}^4}{T^4} + \\
 & \frac{105(-1+T)^3(2+7T)\epsilon^4 g_{i^+,i^+}^3 g_{j^+,i^+}^4}{T^4} + \frac{105(-1+T)^4\epsilon^4 g_{i^+,i^+}^4 g_{j^+,i^+}^4}{T^4} + \\
 & \frac{2(-1+T)(20+23T)\epsilon^3 g_{i^+,j^+} g_{j^+,i^+}^4}{T^2} - \frac{2(-12-208T-39T^2+121T^3)\epsilon^4 g_{i^+,j^+} g_{j^+,i^+}^4}{T^3} + \\
 & \frac{90(-1+T)^2\epsilon^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^4}{T^2} - \frac{10(-1+T)(-26-202T+T^2)\epsilon^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^4}{T^3} + \\
 & \frac{45(-1+T)^2(14+45T)\epsilon^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^4}{T^3} + \frac{420(-1+T)^3\epsilon^4 g_{i^+,i^+}^3 g_{i^+,j^+} g_{j^+,i^+}^4}{T^3} + \\
 & \frac{36(-1+T)\epsilon^3 g_{i^+,j^+}^2 g_{j^+,i^+}^4}{T} - \frac{2(-52-327T+102T^2)\epsilon^4 g_{i^+,j^+}^2 g_{j^+,i^+}^4}{T^2} + \\
 & \frac{15(-1+T)(35+93T)\epsilon^4 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^4}{T^2} + \frac{540(-1+T)^2\epsilon^4 g_{i^+,i^+}^2 g_{i^+,j^+}^2 g_{j^+,i^+}^4}{T^2} + \\
 & \frac{12(9+14T)\epsilon^4 g_{i^+,j^+}^3 g_{j^+,i^+}^4}{T} + \frac{240(-1+T)\epsilon^4 g_{i^+,i^+} g_{i^+,j^+}^3 g_{j^+,i^+}^4}{T} + 24\epsilon^4 g_{i^+,j^+}^4 g_{j^+,i^+}^4 -
 \end{aligned}$$

$$\begin{aligned}
 & \frac{5(-1+T)^2(4+3T)\epsilon^3 g_{j^+,i^+}^5}{T^3} + \frac{(-1+T)(-24-216T+11T^2+75T^3)\epsilon^4 g_{j^+,i^+}^5}{T^4} - \\
 & \frac{45(-1+T)^3\epsilon^3 g_{i^+,i^+} g_{j^+,i^+}^5}{T^3} + \frac{5(-1+T)^2(-52-197T+24T^2)\epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^5}{T^4} - \\
 & \frac{315(-1+T)^3(2+3T)\epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+}^5}{T^4} - \frac{420(-1+T)^4\epsilon^4 g_{i^+,i^+}^3 g_{j^+,i^+}^5}{T^4} - \frac{45(-1+T)^2\epsilon^3 g_{i^+,j^+} g_{j^+,i^+}^5}{T^2} + \\
 & \frac{10(-1+T)(-26-88T+19T^2)\epsilon^4 g_{i^+,j^+} g_{j^+,i^+}^5}{T^3} - \frac{90(-1+T)^2(14+19T)\epsilon^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^5}{T^3} - \\
 & \frac{1260(-1+T)^3\epsilon^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^5}{T^3} - \frac{15(-1+T)(35+37T)\epsilon^4 g_{i^+,j^+}^2 g_{j^+,i^+}^5}{T^2} - \\
 & \frac{1080(-1+T)^2\epsilon^4 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^5}{T^2} - \frac{240(-1+T)\epsilon^4 g_{i^+,j^+}^3 g_{j^+,i^+}^5}{T} + \frac{15(-1+T)^3\epsilon^3 g_{j^+,i^+}^6}{T^3} - \\
 & \frac{5(-1+T)^2(-26-57T+15T^2)\epsilon^4 g_{j^+,i^+}^6}{T^4} + \frac{105(-1+T)^3(6+5T)\epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^6}{T^4} + \\
 & \frac{630(-1+T)^4\epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+}^6}{T^4} + \frac{15(-1+T)^2(42+31T)\epsilon^4 g_{i^+,j^+} g_{j^+,i^+}^6}{T^3} + \\
 & \frac{1260(-1+T)^3\epsilon^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^6}{T^3} + \frac{540(-1+T)^2\epsilon^4 g_{i^+,j^+}^2 g_{j^+,i^+}^6}{T^2} - \\
 & \frac{105(-1+T)^3(2+T)\epsilon^4 g_{j^+,i^+}^7}{T^4} - \frac{420(-1+T)^4\epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^7}{T^4} - \frac{420(-1+T)^3\epsilon^4 g_{i^+,j^+} g_{j^+,i^+}^7}{T^3} + \\
 & \frac{105(-1+T)^4\epsilon^4 g_{j^+,i^+}^8}{T^4} - \epsilon g_{i^+,i^+} g_{j^+,j^+} + 2\epsilon^2 g_{i^+,i^+} g_{j^+,j^+} - \frac{49}{24}\epsilon^3 g_{i^+,i^+} g_{j^+,j^+} + \\
 & \frac{1}{24}\epsilon^4(49-24cb_{4,10})g_{i^+,i^+} g_{j^+,j^+} - 3\epsilon^2 g_{i^+,i^+}^2 g_{j^+,j^+} + \frac{17}{2}\epsilon^3 g_{i^+,i^+}^2 g_{j^+,j^+} - \frac{101}{8}\epsilon^4 g_{i^+,i^+}^2 g_{j^+,j^+} - \\
 & 7\epsilon^3 g_{i^+,i^+}^3 g_{j^+,j^+} + 26\epsilon^4 g_{i^+,i^+}^3 g_{j^+,j^+} - 15\epsilon^4 g_{i^+,i^+}^4 g_{j^+,j^+} + 2\epsilon g_{j^+,i^+} g_{j^+,j^+} - 4\epsilon^2 g_{j^+,i^+} g_{j^+,j^+} + \\
 & \frac{49}{12}\epsilon^3 g_{j^+,i^+} g_{j^+,j^+} + \frac{1}{12}\epsilon^4(-49+24cb_{4,10})g_{j^+,i^+} g_{j^+,j^+} + \frac{2(2+5T)\epsilon^2 g_{i^+,i^+} g_{j^+,i^+} g_{j^+,j^+}}{T} - \\
 & \frac{(16+29T)\epsilon^3 g_{i^+,i^+} g_{j^+,i^+} g_{j^+,j^+}}{T} + \frac{(394+529T)\epsilon^4 g_{i^+,i^+} g_{j^+,i^+} g_{j^+,j^+}}{12T} + \frac{6(-1+T)\epsilon^2 g_{i^+,i^+}^2 g_{j^+,i^+} g_{j^+,j^+}}{T} + \\
 & \frac{(57+5T)\epsilon^3 g_{i^+,i^+}^2 g_{j^+,i^+} g_{j^+,j^+}}{T} - \frac{(869+235T)\epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+} g_{j^+,j^+}}{4T} + \frac{44(-1+T)\epsilon^3 g_{i^+,i^+}^3 g_{j^+,i^+} g_{j^+,j^+}}{T} - \\
 & \frac{2(-196+81T)\epsilon^4 g_{i^+,i^+}^3 g_{j^+,i^+} g_{j^+,j^+}}{T} + \frac{210(-1+T)\epsilon^4 g_{i^+,i^+}^4 g_{j^+,i^+} g_{j^+,j^+}}{T} - 2\epsilon^2 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} + \\
 & 5\epsilon^3 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} - \frac{77}{12}\epsilon^4 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} + 8\epsilon^2 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} - \\
 & 52\epsilon^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} + \frac{425}{3}\epsilon^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} + 72\epsilon^3 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} -
 \end{aligned}$$

$$\begin{aligned}
 & 486 \in^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} + 400 \in^4 g_{i^+,i^+}^3 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+} - \frac{6(1+T) \in^2 g_{j^+,i^+}^2 g_{j^+,j^+}}{T} + \\
 & \frac{6(4+3T) \in^3 g_{j^+,i^+}^2 g_{j^+,j^+}}{T} - \frac{(197+113T) \in^4 g_{j^+,i^+}^2 g_{j^+,j^+}}{4T} - \frac{18(-1+T) \in^2 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} + \\
 & \frac{9(-2-17T+5T^2) \in^3 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T^2} - \frac{3(-156-753T+65T^2) \in^4 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{4T^2} - \\
 & \frac{12(-1+T)(5+14T) \in^3 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T^2} + \frac{6(-113-163T+136T^2) \in^4 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T^2} - \\
 & \frac{45(-1+T)^2 \in^3 g_{i^+,i^+}^3 g_{j^+,i^+}^2 g_{j^+,j^+}}{T^2} - \frac{60(-1+T)(19+10T) \in^4 g_{i^+,i^+}^3 g_{j^+,i^+}^2 g_{j^+,j^+}}{T^2} - \\
 & \frac{585(-1+T)^2 \in^4 g_{i^+,i^+}^4 g_{j^+,i^+}^2 g_{j^+,j^+}}{T^2} - 12 \in^2 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+} + \frac{3(4+25T) \in^3 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} - \\
 & \frac{(120+407T) \in^4 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{2T} - \frac{18(5+9T) \in^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} + \\
 & \frac{12(69+94T) \in^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} - \frac{108(-1+T) \in^3 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} - \\
 & \frac{72(32+3T) \in^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} - \frac{1680(-1+T) \in^4 g_{i^+,i^+}^3 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} + \\
 & 18 \in^3 g_{i^+,j^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+} - 114 \in^4 g_{i^+,j^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+} - 54 \in^3 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+} + \\
 & 837 \in^4 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+} - 1080 \in^4 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+} + \frac{12(-1+T) \in^2 g_{j^+,i^+}^3 g_{j^+,j^+}}{T} - \\
 & \frac{6(-4-15T+7T^2) \in^3 g_{j^+,i^+}^3 g_{j^+,j^+}}{T^2} + \frac{(-312-637T+157T^2) \in^4 g_{j^+,i^+}^3 g_{j^+,j^+}}{2T^2} + \\
 & \frac{8(-1+T)(20+23T) \in^3 g_{i^+,i^+} g_{j^+,i^+}^3 g_{j^+,j^+}}{T^2} - \frac{8(-12-208T-39T^2+121T^3) \in^4 g_{i^+,i^+} g_{j^+,i^+}^3 g_{j^+,j^+}}{T^3} + \\
 & \frac{180(-1+T)^2 \in^3 g_{i^+,i^+} g_{j^+,i^+}^3 g_{j^+,j^+}}{T^2} - \frac{20(-1+T)(-26-202T+T^2) \in^4 g_{i^+,i^+} g_{j^+,i^+}^3 g_{j^+,j^+}}{T^3} + \\
 & \frac{60(-1+T)^2(14+45T) \in^4 g_{i^+,i^+}^3 g_{j^+,i^+}^3 g_{j^+,j^+}}{T^3} + \frac{420(-1+T)^3 \in^4 g_{i^+,i^+}^4 g_{j^+,i^+}^3 g_{j^+,j^+}}{T^3} + \\
 & \frac{24(5+3T) \in^3 g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}}{T} - \frac{8(9+132T+67T^2) \in^4 g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}}{T^2} + \\
 & \frac{288(-1+T) \in^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}}{T} - \frac{16(-52-327T+102T^2) \in^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}}{T^2} + \\
 & \frac{60(-1+T)(35+93T) \in^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}}{T^2} + \frac{1440(-1+T)^2 \in^4 g_{i^+,i^+}^3 g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}}{T^2} +
 \end{aligned}$$

$$\begin{aligned}
 & 72 \epsilon^3 g_{i^+,j^+}^2 g_{j^+,i^+}^3 g_{j^+,j^+} - \frac{12 (19 + 86 T) \epsilon^4 g_{i^+,j^+}^2 g_{j^+,i^+}^3 g_{j^+,j^+}}{T} + \frac{144 (9 + 14 T) \epsilon^4 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^3 g_{j^+,j^+}}{T} + \\
 & \frac{1440 (-1 + T) \epsilon^4 g_{i^+,i^+}^2 g_{i^+,j^+}^2 g_{j^+,i^+}^3 g_{j^+,j^+}}{T} - 144 \epsilon^4 g_{i^+,j^+}^3 g_{j^+,i^+}^3 g_{j^+,j^+} + 384 \epsilon^4 g_{i^+,i^+} g_{i^+,j^+}^3 g_{j^+,i^+}^3 g_{j^+,j^+} - \\
 & \frac{20 (-1 + T) (5 + 3 T) \epsilon^3 g_{j^+,i^+}^4 g_{j^+,j^+}}{T^2} + \frac{10 (-12 - 95 T + 26 T^2 + 33 T^3) \epsilon^4 g_{j^+,i^+}^4 g_{j^+,j^+}}{T^3} - \\
 & \frac{225 (-1 + T)^2 \epsilon^3 g_{i^+,i^+} g_{j^+,i^+}^4 g_{j^+,j^+}}{T^2} + \frac{50 (-1 + T) (-26 - 88 T + 19 T^2) \epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^4 g_{j^+,j^+}}{T^3} - \\
 & \frac{225 (-1 + T)^2 (14 + 19 T) \epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+}^4 g_{j^+,j^+}}{T^3} - \frac{2100 (-1 + T)^3 \epsilon^4 g_{i^+,i^+}^3 g_{j^+,i^+}^4 g_{j^+,j^+}}{T^3} - \\
 & \frac{180 (-1 + T) \epsilon^3 g_{i^+,j^+} g_{j^+,i^+}^4 g_{j^+,j^+}}{T} + \frac{20 (-52 - 135 T + 70 T^2) \epsilon^4 g_{i^+,j^+} g_{j^+,i^+}^4 g_{j^+,j^+}}{T^2} - \\
 & \frac{150 (-1 + T) (35 + 37 T) \epsilon^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^4 g_{j^+,j^+}}{T^2} - \frac{5400 (-1 + T)^2 \epsilon^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^4 g_{j^+,j^+}}{T^2} - \\
 & \frac{180 (9 + 4 T) \epsilon^4 g_{i^+,j^+}^2 g_{j^+,i^+}^4 g_{j^+,j^+}}{T} - \frac{3600 (-1 + T) \epsilon^4 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^4 g_{j^+,j^+}}{T} - \\
 & 480 \epsilon^4 g_{i^+,j^+}^3 g_{j^+,i^+}^4 g_{j^+,j^+} + \frac{90 (-1 + T)^2 \epsilon^3 g_{j^+,i^+}^5 g_{j^+,j^+}}{T^2} - \frac{60 (-1 + T) (-13 - 25 T + 9 T^2) \epsilon^4 g_{j^+,i^+}^5 g_{j^+,j^+}}{T^3} + \\
 & \frac{90 (-1 + T)^2 (42 + 31 T) \epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^5 g_{j^+,j^+}}{T^3} + \frac{3780 (-1 + T)^3 \epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+}^5 g_{j^+,j^+}}{T^3} + \\
 & \frac{150 (-1 + T) (21 + 11 T) \epsilon^4 g_{i^+,j^+} g_{j^+,i^+}^5 g_{j^+,j^+}}{T^2} + \frac{6480 (-1 + T)^2 \epsilon^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^5 g_{j^+,j^+}}{T^2} + \\
 & \frac{2160 (-1 + T) \epsilon^4 g_{i^+,j^+}^2 g_{j^+,i^+}^5 g_{j^+,j^+}}{T} - \frac{210 (-1 + T)^2 (7 + 3 T) \epsilon^4 g_{j^+,i^+}^6 g_{j^+,j^+}}{T^3} - \\
 & \frac{2940 (-1 + T)^3 \epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^6 g_{j^+,j^+}}{T^3} - \frac{2520 (-1 + T)^2 \epsilon^4 g_{i^+,j^+} g_{j^+,i^+}^6 g_{j^+,j^+}}{T^2} + \\
 & \frac{840 (-1 + T)^3 \epsilon^4 g_{j^+,i^+}^7 g_{j^+,j^+}}{T^3} - \epsilon^2 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+} + \frac{5}{2} \epsilon^3 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+} - \frac{77}{24} \epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+} + \\
 & 2 \epsilon^2 g_{i^+,i^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+} - 13 \epsilon^3 g_{i^+,i^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+} + \frac{425}{12} \epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+} + 12 \epsilon^3 g_{i^+,i^+}^3 g_{j^+,i^+}^2 g_{j^+,j^+} - \\
 & 81 \epsilon^4 g_{i^+,i^+}^3 g_{j^+,i^+}^2 g_{j^+,j^+} + 50 \epsilon^4 g_{i^+,i^+}^4 g_{j^+,i^+}^2 g_{j^+,j^+} + 3 \epsilon^2 g_{j^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+} - \frac{15}{2} \epsilon^3 g_{j^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+} + \frac{77}{8} \epsilon^4 g_{j^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+} - \\
 & 12 \epsilon^2 g_{i^+,i^+} g_{j^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+} + \frac{3 (4 + 25 T) \epsilon^3 g_{i^+,i^+} g_{j^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} - \frac{(120 + 407 T) \epsilon^4 g_{i^+,i^+} g_{j^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{2 T} - \\
 & \frac{9 (5 + 9 T) \epsilon^3 g_{i^+,i^+}^2 g_{j^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} + \frac{6 (69 + 94 T) \epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} - \frac{36 (-1 + T) \epsilon^3 g_{i^+,i^+}^3 g_{j^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}}{T} -
 \end{aligned}$$

$$\begin{aligned}
 & \frac{24 (32 + 3 T) \in^4 g_{i^+,i^+}^3 g_{j^+,i^+} g_{j^+,j^+}^2}{T} - \frac{420 (-1 + T) \in^4 g_{i^+,i^+}^4 g_{j^+,i^+} g_{j^+,j^+}^2}{T} - 3 \in^3 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^2 + \\
 & 9 \in^4 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^2 + 36 \in^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^2 - 228 \in^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^2 - \\
 & 54 \in^3 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^2 + 837 \in^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^2 - 720 \in^4 g_{i^+,i^+}^3 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^2 + \\
 & 12 \in^2 g_{j^+,i^+}^2 g_{j^+,j^+}^2 - \frac{24 (1 + 3 T) \in^3 g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T} + \frac{(240 + 389 T) \in^4 g_{j^+,i^+}^2 g_{j^+,j^+}^2}{2 T} + \\
 & \frac{36 (5 + 3 T) \in^3 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T} - \frac{12 (9 + 132 T + 67 T^2) \in^4 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T^2} + \\
 & \frac{216 (-1 + T) \in^3 g_{i^+,i^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T} - \frac{12 (-52 - 327 T + 102 T^2) \in^4 g_{i^+,i^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T^2} + \\
 & \frac{30 (-1 + T) (35 + 93 T) \in^4 g_{i^+,i^+}^3 g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T^2} + \frac{540 (-1 + T)^2 \in^4 g_{i^+,i^+}^4 g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T^2} - \\
 & 72 \in^3 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}^2 + \frac{6 (8 + 75 T) \in^4 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T} + 216 \in^3 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}^2 - \\
 & \frac{36 (19 + 86 T) \in^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T} + \frac{216 (9 + 14 T) \in^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T} + \\
 & \frac{1440 (-1 + T) \in^4 g_{i^+,i^+}^3 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}^2}{T} + 84 \in^4 g_{i^+,j^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+}^2 - 648 \in^4 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+}^2 + \\
 & 864 \in^4 g_{i^+,i^+}^2 g_{i^+,j^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+}^2 - \frac{30 (5 + T) \in^3 g_{j^+,i^+}^3 g_{j^+,j^+}^2}{T} + \frac{90 (2 + 14 T + 3 T^2) \in^4 g_{j^+,i^+}^3 g_{j^+,j^+}^2}{T^2} - \\
 & \frac{360 (-1 + T) \in^3 g_{i^+,i^+} g_{j^+,i^+}^3 g_{j^+,j^+}^2}{T} + \frac{40 (-52 - 135 T + 70 T^2) \in^4 g_{i^+,i^+} g_{j^+,i^+}^3 g_{j^+,j^+}^2}{T^2} - \\
 & \frac{150 (-1 + T) (35 + 37 T) \in^4 g_{i^+,i^+}^2 g_{j^+,i^+}^3 g_{j^+,j^+}^2}{T^2} - \frac{3600 (-1 + T)^2 \in^4 g_{i^+,i^+}^3 g_{j^+,i^+}^3 g_{j^+,j^+}^2}{T^2} - \\
 & 180 \in^3 g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}^2 + \frac{30 (38 + 79 T) \in^4 g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}^2}{T} - \frac{720 (9 + 4 T) \in^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}^2}{T} - \\
 & \frac{7200 (-1 + T) \in^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}^2}{T} + 1080 \in^4 g_{i^+,j^+}^2 g_{j^+,i^+}^3 g_{j^+,j^+}^2 - 2880 \in^4 g_{i^+,i^+} g_{i^+,j^+}^2 g_{j^+,i^+}^3 g_{j^+,j^+}^2 + \\
 & \frac{180 (-1 + T) \in^3 g_{j^+,i^+}^4 g_{j^+,j^+}^2}{T} - \frac{30 (-52 - 71 T + 51 T^2) \in^4 g_{j^+,i^+}^4 g_{j^+,j^+}^2}{T^2} + \\
 & \frac{375 (-1 + T) (21 + 11 T) \in^4 g_{i^+,i^+} g_{j^+,i^+}^4 g_{j^+,j^+}^2}{T^2} + \frac{8100 (-1 + T)^2 \in^4 g_{i^+,i^+}^2 g_{j^+,i^+}^4 g_{j^+,j^+}^2}{T^2} + \\
 & \frac{180 (27 + 2 T) \in^4 g_{i^+,j^+} g_{j^+,i^+}^4 g_{j^+,j^+}^2}{T} + \frac{10800 (-1 + T) \in^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^4 g_{j^+,j^+}^2}{T} + \\
 & 2160 \in^4 g_{i^+,j^+}^2 g_{j^+,i^+}^4 g_{j^+,j^+}^2 - \frac{105 (-1 + T) (35 + 9 T) \in^4 g_{j^+,i^+}^5 g_{j^+,j^+}^2}{T^2} - \frac{7560 (-1 + T)^2 \in^4 g_{i^+,i^+} g_{j^+,i^+}^5 g_{j^+,j^+}^2}{T^2} -
 \end{aligned}$$

$$\begin{aligned}
 & \frac{5040 (-1 + T) \epsilon^4 g_{i^+,j^+} g_{j^+,i^+}^5 g_{j^+,j^+}^2}{T} + \frac{2520 (-1 + T)^2 \epsilon^4 g_{j^+,i^+}^6 g_{j^+,j^+}^2}{T^2} - \epsilon^3 g_{i^+,i^+} g_{j^+,j^+}^3 + 3 \epsilon^4 g_{i^+,i^+} g_{j^+,j^+}^3 + \\
 & 6 \epsilon^3 g_{i^+,i^+}^2 g_{j^+,j^+}^3 - 38 \epsilon^4 g_{i^+,i^+}^2 g_{j^+,j^+}^3 - 6 \epsilon^3 g_{i^+,i^+}^3 g_{j^+,j^+}^3 + 93 \epsilon^4 g_{i^+,i^+}^3 g_{j^+,j^+}^3 - 60 \epsilon^4 g_{i^+,i^+}^4 g_{j^+,j^+}^3 + \\
 & 4 \epsilon^3 g_{j^+,i^+} g_{j^+,j^+}^3 - 12 \epsilon^4 g_{j^+,i^+} g_{j^+,j^+}^3 - 48 \epsilon^3 g_{i^+,i^+} g_{j^+,i^+} g_{j^+,j^+}^3 + \frac{4 (8 + 75 T) \epsilon^4 g_{i^+,i^+} g_{j^+,i^+} g_{j^+,j^+}^3}{T} + \\
 & 72 \epsilon^3 g_{i^+,i^+}^2 g_{j^+,i^+} g_{j^+,j^+}^3 - \frac{12 (19 + 86 T) \epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+} g_{j^+,j^+}^3}{T} + \frac{48 (9 + 14 T) \epsilon^4 g_{i^+,i^+}^3 g_{j^+,i^+} g_{j^+,j^+}^3}{T} + \\
 & \frac{240 (-1 + T) \epsilon^4 g_{i^+,i^+}^4 g_{j^+,i^+} g_{j^+,j^+}^3}{T} - 4 \epsilon^4 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^3 + 112 \epsilon^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^3 - \\
 & 432 \epsilon^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^3 + 384 \epsilon^4 g_{i^+,i^+}^3 g_{i^+,j^+} g_{j^+,i^+} g_{j^+,j^+}^3 + 60 \epsilon^3 g_{j^+,i^+}^2 g_{j^+,j^+}^3 - \\
 & \frac{10 (8 + 37 T) \epsilon^4 g_{j^+,i^+}^2 g_{j^+,j^+}^3}{T} - 180 \epsilon^3 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}^3 + \frac{30 (38 + 79 T) \epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}^3}{T} - \\
 & \frac{360 (9 + 4 T) \epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+}^3}{T} - \frac{2400 (-1 + T) \epsilon^4 g_{i^+,i^+}^3 g_{j^+,i^+}^2 g_{j^+,j^+}^3}{T} - \\
 & 280 \epsilon^4 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}^3 + 2160 \epsilon^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}^3 - 2880 \epsilon^4 g_{i^+,i^+}^2 g_{i^+,j^+} g_{j^+,i^+}^2 g_{j^+,j^+}^3 + \\
 & 120 \epsilon^3 g_{j^+,i^+}^3 g_{j^+,j^+}^3 - \frac{60 (19 + 24 T) \epsilon^4 g_{j^+,i^+}^3 g_{j^+,j^+}^3}{T} + \frac{240 (27 + 2 T) \epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^3 g_{j^+,j^+}^3}{T} + \\
 & \frac{7200 (-1 + T) \epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+}^3 g_{j^+,j^+}^3}{T} - 2160 \epsilon^4 g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}^3 + 5760 \epsilon^4 g_{i^+,i^+} g_{i^+,j^+} g_{j^+,i^+}^3 g_{j^+,j^+}^3 + \\
 & \frac{420 (-9 + T) \epsilon^4 g_{j^+,i^+}^4 g_{j^+,j^+}^3}{T} - \frac{8400 (-1 + T) \epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^4 g_{j^+,j^+}^3}{T} - 3360 \epsilon^4 g_{i^+,j^+} g_{j^+,i^+}^4 g_{j^+,j^+}^3 + \\
 & \frac{3360 (-1 + T) \epsilon^4 g_{j^+,i^+}^5 g_{j^+,j^+}^3}{T} - \epsilon^4 g_{i^+,i^+} g_{j^+,j^+}^4 + 14 \epsilon^4 g_{i^+,i^+}^2 g_{j^+,j^+}^4 - 36 \epsilon^4 g_{i^+,i^+}^3 g_{j^+,j^+}^4 + \\
 & 24 \epsilon^4 g_{i^+,i^+}^4 g_{j^+,j^+}^4 + 5 \epsilon^4 g_{j^+,i^+} g_{j^+,j^+}^4 - 140 \epsilon^4 g_{i^+,i^+} g_{j^+,i^+} g_{j^+,j^+}^4 + 540 \epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+} g_{j^+,j^+}^4 - \\
 & 480 \epsilon^4 g_{i^+,i^+}^3 g_{j^+,i^+} g_{j^+,j^+}^4 + 210 \epsilon^4 g_{j^+,i^+}^2 g_{j^+,j^+}^4 - 1620 \epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^2 g_{j^+,j^+}^4 + \\
 & 2160 \epsilon^4 g_{i^+,i^+}^2 g_{j^+,i^+}^2 g_{j^+,j^+}^4 + 1260 \epsilon^4 g_{j^+,i^+}^3 g_{j^+,j^+}^4 - 3360 \epsilon^4 g_{i^+,i^+} g_{j^+,i^+}^3 g_{j^+,j^+}^4 + 1680 \epsilon^4 g_{j^+,i^+}^4 g_{j^+,j^+}^4
 \end{aligned}$$

(Alt) In[\*]:=

**me = Exponent[lhs - rhs, T, Min]**

(Alt) Out[\*]=

∞

(Alt) In[\*]:=

**covars = DeleteCases[Variables[lhs - rhs], T | (ca | cb | cc | cd) \_]**

(Alt) Out[\*]=

{}

(Alt) In[\*]:=

**eqnsSwp = {}**

(Alt) Out[\*]=

{}

## Solution

(Alt) In[ ]:=

**vars =**

**Cases [Variables [r<sub>d</sub>[1, i1, j1] + r<sub>d</sub>[-1, i2, j2] + γ<sub>d</sub>[1, k1] + γ<sub>d</sub>[-1, k2]], (ca | cb | cc | cd) \_]**

(Alt) Out[ ]:=

{ca<sub>4,1</sub>, cb<sub>4,10</sub>}

(Alt) In[ ]:=

**{sol} = Solve [eqnsR3 ∪ eqnsR2b ∪ eqnsR2c ∪ eqnsR11 ∪ eqnsR1r ∪ eqnsSwp, vars]**

**Solve:** The solution set contains a full-dimensional component; use Reduce for complete solution information.

(Alt) Out[ ]:=

{ {} }

(Alt) In[ ]:=

**sol /. Rule → Set**

(Alt) Out[ ]:=

{ }

(Alt) In[ ]:=

**r<sub>d</sub>[1, i, j] // CF**

**r<sub>d</sub>[-1, i, j] // CF**

**γ<sub>d</sub>[1, k] // CF**

**γ<sub>d</sub>[-1, k] // CF**

(Alt) Out[ ]:=

$$\begin{aligned}
 & -\frac{\epsilon}{2} + \epsilon p_i x_i - \frac{1}{2} \epsilon^2 p_i x_i + \frac{1}{6} \epsilon^3 p_i x_i - \epsilon p_j x_i + \frac{1}{2} \epsilon^2 p_j x_i - \frac{1}{6} \epsilon^3 p_j x_i + \frac{1}{2} (-1 + T) \epsilon p_i p_j x_i^2 + \\
 & \frac{1}{4} (1 - 3T) \epsilon^2 p_i p_j x_i^2 + \frac{1}{12} (5 + 7T) \epsilon^3 p_i p_j x_i^2 + \frac{1}{2} (1 - T) \epsilon p_j^2 x_i^2 + \frac{1}{4} (-1 + 3T) \epsilon^2 p_j^2 x_i^2 + \\
 & \frac{1}{12} (-5 - 7T) \epsilon^3 p_j^2 x_i^2 + \frac{1}{3} (-1 + T) \epsilon^2 p_i^2 p_j x_i^3 + \frac{1}{6} (5 - 6T) \epsilon^3 p_i^2 p_j x_i^3 + \frac{1}{36} (-34 + 55T) \epsilon^4 p_i^2 p_j x_i^3 - \\
 & \frac{1}{6} (-1 + T) (5 + T) \epsilon^2 p_i p_j^2 x_i^3 + \frac{1}{6} (-16 + 17T + 2T^2) \epsilon^3 p_i p_j^2 x_i^3 + \frac{1}{72} (269 - 394T - 25T^2) \epsilon^4 p_i p_j^2 x_i^3 + \\
 & \frac{1}{6} (-1 + T) (3 + T) \epsilon^2 p_j^3 x_i^3 + \frac{1}{6} (11 - 11T - 2T^2) \epsilon^3 p_j^3 x_i^3 + \frac{1}{72} (-201 + 284T + 25T^2) \epsilon^4 p_j^3 x_i^3 + \\
 & \frac{1}{8} (-1 + T) \epsilon^3 p_i^3 p_j x_i^4 + \frac{1}{48} (25 - 27T) \epsilon^4 p_i^3 p_j x_i^4 - \frac{1}{8} (-1 + T) (4 + 3T) \epsilon^3 p_i^2 p_j^2 x_i^4 + \\
 & \frac{1}{48} (-156 + 101T + 69T^2) \epsilon^4 p_i^2 p_j^2 x_i^4 + \frac{1}{24} (-1 + T) (13 + 22T + T^2) \epsilon^3 p_i p_j^3 x_i^4 + \\
 & \frac{1}{48} (245 - 79T - 185T^2 - 5T^3) \epsilon^4 p_i p_j^3 x_i^4 - \frac{1}{24} (-1 + T) (4 + 13T + T^2) \epsilon^3 p_j^4 x_i^4 + \\
 & \frac{1}{48} (-114 + 5T + 116T^2 + 5T^3) \epsilon^4 p_j^4 x_i^4 + \frac{1}{30} (-1 + T) \epsilon^4 p_i^4 p_j x_i^5 - \frac{1}{30} (-1 + T) (2 + 13T) \epsilon^4 p_i^3 p_j^2 x_i^5 + \\
 & \frac{1}{60} (-1 + T) (-21 + 107T + 14T^2) \epsilon^4 p_i^2 p_j^3 x_i^5 - \frac{1}{120} (-1 + T) (-101 + 273T + 67T^2 + T^3) \epsilon^4 p_i p_j^4 x_i^5 + \\
 & \frac{1}{120} (-1 + T) (-55 + 111T + 39T^2 + T^3) \epsilon^4 p_j^5 x_i^5 - \epsilon p_i p_j x_i x_j + \frac{3}{2} \epsilon^2 p_i p_j x_i x_j - \frac{7}{6} \epsilon^3 p_i p_j x_i x_j +
 \end{aligned}$$

$$\begin{aligned}
 & \in p_j^2 x_i x_j - \frac{3}{2} \in^2 p_j^2 x_i x_j + \frac{7}{6} \in^3 p_j^2 x_i x_j - \frac{1}{2} \in^2 p_i^2 p_j x_i^2 x_j + \frac{3}{2} \in^3 p_i^2 p_j x_i^2 x_j - \frac{55}{24} \in^4 p_i^2 p_j x_i^2 x_j + \\
 & \frac{1}{2} (2 + T) \in^2 p_i p_j^2 x_i^2 x_j + \frac{1}{2} (-9 - 2 T) \in^3 p_i p_j^2 x_i^2 x_j + \frac{1}{24} (212 + 25 T) \in^4 p_i p_j^2 x_i^2 x_j + \\
 & \frac{1}{2} (-1 - T) \in^2 p_j^3 x_i^2 x_j + (3 + T) \in^3 p_j^3 x_i^2 x_j + \frac{1}{24} (-157 - 25 T) \in^4 p_j^3 x_i^2 x_j - \frac{1}{6} \in^3 p_i^3 p_j x_i^3 x_j + \\
 & \frac{3}{4} \in^4 p_i^3 p_j x_i^3 x_j + \frac{7}{6} T \in^3 p_i^2 p_j^2 x_i^3 x_j + \frac{1}{12} (-30 - 53 T) \in^4 p_i^2 p_j^2 x_i^3 x_j + \frac{1}{6} (6 - 17 T - T^2) \in^3 p_i p_j^3 x_i^3 x_j + \\
 & \frac{1}{12} (12 + 147 T + 5 T^2) \in^4 p_i p_j^3 x_i^3 x_j + \frac{1}{6} (-5 + 10 T + T^2) \in^3 p_j^4 x_i^3 x_j + \frac{1}{12} (9 - 94 T - 5 T^2) \in^4 p_j^4 x_i^3 x_j - \\
 & \frac{1}{24} \in^4 p_i^4 p_j x_i^4 x_j + \frac{1}{24} (-16 + 31 T) \in^4 p_i^3 p_j^2 x_i^4 x_j - \frac{5}{24} (-18 + 23 T + 5 T^2) \in^4 p_i^2 p_j^3 x_i^4 x_j + \\
 & \frac{1}{24} (-132 + 131 T + 60 T^2 + T^3) \in^4 p_i p_j^4 x_i^4 x_j + \frac{1}{24} (59 - 47 T - 35 T^2 - T^3) \in^4 p_j^5 x_i^4 x_j - \\
 & \frac{1}{2} \in^2 p_i p_j^2 x_i x_j^2 + \in^3 p_i p_j^2 x_i x_j^2 - \frac{25}{24} \in^4 p_i p_j^2 x_i x_j^2 + \frac{1}{2} \in^2 p_j^3 x_i x_j^2 - \in^3 p_j^3 x_i x_j^2 + \frac{25}{24} \in^4 p_j^3 x_i x_j^2 - \\
 & \in^3 p_i^2 p_j^2 x_i^2 x_j^2 + \frac{15}{4} \in^4 p_i^2 p_j^2 x_i^2 x_j^2 + \frac{1}{4} (10 + T) \in^3 p_i p_j^3 x_i^2 x_j^2 + \frac{1}{8} (-86 - 5 T) \in^4 p_i p_j^3 x_i^2 x_j^2 + \\
 & \frac{1}{4} (-6 - T) \in^3 p_j^4 x_i^2 x_j^2 + \frac{1}{8} (56 + 5 T) \in^4 p_j^4 x_i^2 x_j^2 - \in^4 p_i^3 p_j^2 x_i^3 x_j^2 + \frac{5}{12} (9 + 4 T) \in^4 p_i^2 p_j^3 x_i^3 x_j^2 + \\
 & \frac{1}{12} (-51 - 49 T - T^2) \in^4 p_i p_j^4 x_i^3 x_j^2 + \frac{1}{12} (18 + 29 T + T^2) \in^4 p_j^5 x_i^3 x_j^2 - \frac{1}{6} \in^3 p_i p_j^3 x_i x_j^3 + \\
 & \frac{5}{12} \in^4 p_i p_j^3 x_i x_j^3 + \frac{1}{6} \in^3 p_j^4 x_i x_j^3 - \frac{5}{12} \in^4 p_j^4 x_i x_j^3 - \in^4 p_i^2 p_j^3 x_i^2 x_j^3 + \frac{1}{12} (30 + T) \in^4 p_i p_j^4 x_i^2 x_j^3 + \\
 & \frac{1}{12} (-18 - T) \in^4 p_j^5 x_i^2 x_j^3 - \frac{1}{24} \in^4 p_i p_j^4 x_i x_j^4 + \frac{1}{24} \in^4 p_j^5 x_i x_j^4 + \frac{1}{24} \in^4 p_i x_i (-1 - 48 ca_{4,1}) + \\
 & \in^4 ca_{4,1} + \frac{1}{24} \in^4 p_j x_i (1 + 48 ca_{4,1}) + \frac{1}{4} \in^4 p_i p_j x_i x_j (5 - 4 cb_{4,10}) + \frac{1}{4} \in^4 p_j^2 x_i x_j (-5 + 4 cb_{4,10}) + \\
 & \frac{1}{24} \in^4 p_j^2 x_i^2 (10 + 15 T + 12 cb_{4,10} - 12 T cb_{4,10}) + \frac{1}{24} \in^4 p_i p_j x_i^2 (-10 - 15 T - 12 cb_{4,10} + 12 T cb_{4,10})
 \end{aligned}$$

(Alt) Out[\*]=

$$\begin{aligned}
 & \frac{\in}{2} - \in p_i x_i - \frac{1}{2} \in^2 p_i x_i - \frac{1}{6} \in^3 p_i x_i + \in p_j x_i + \frac{1}{2} \in^2 p_j x_i + \frac{1}{6} \in^3 p_j x_i + \frac{(-1 + T) \in p_i p_j x_i^2}{2 T} + \\
 & \frac{(-3 + T) \in^2 p_i p_j x_i^2}{4 T} - \frac{(7 + 5 T) \in^3 p_i p_j x_i^2}{12 T} - \frac{(-1 + T) \in p_j^2 x_i^2}{2 T} - \frac{(-3 + T) \in^2 p_j^2 x_i^2}{4 T} + \\
 & \frac{(7 + 5 T) \in^3 p_j^2 x_i^2}{12 T} - \frac{(-1 + T) \in^2 p_i^2 p_j x_i^3}{3 T} - \frac{(-6 + 5 T) \in^3 p_i^2 p_j x_i^3}{6 T} - \frac{(-55 + 34 T) \in^4 p_i^2 p_j x_i^3}{36 T} + \\
 & \frac{(-1 + T) (1 + 5 T) \in^2 p_i p_j^2 x_i^3}{6 T^2} + \frac{(-2 - 17 T + 16 T^2) \in^3 p_i p_j^2 x_i^3}{6 T^2} + \frac{(-25 - 394 T + 269 T^2) \in^4 p_i p_j^2 x_i^3}{72 T^2} - \\
 & \frac{(-1 + T) (1 + 3 T) \in^2 p_j^3 x_i^3}{6 T^2} - \frac{(-2 - 11 T + 11 T^2) \in^3 p_j^3 x_i^3}{6 T^2} - \frac{(-25 - 284 T + 201 T^2) \in^4 p_j^3 x_i^3}{72 T^2} +
 \end{aligned}$$

$$\begin{aligned}
 & \frac{(-1+T) \epsilon^3 p_i^3 p_j x_i^4}{8 T} + \frac{(-27+25 T) \epsilon^4 p_i^3 p_j x_i^4}{48 T} - \frac{(-1+T) (3+4 T) \epsilon^3 p_i^2 p_j^2 x_i^4}{8 T^2} - \\
 & \frac{(-69-101 T+156 T^2) \epsilon^4 p_i^2 p_j^2 x_i^4}{48 T^2} + \frac{(-1+T) (1+22 T+13 T^2) \epsilon^3 p_i p_j^3 x_i^4}{24 T^3} + \\
 & \frac{(-5-185 T-79 T^2+245 T^3) \epsilon^4 p_i p_j^3 x_i^4}{48 T^3} - \frac{(-1+T) (1+13 T+4 T^2) \epsilon^3 p_j^4 x_i^4}{24 T^3} - \\
 & \frac{(-5-116 T-5 T^2+114 T^3) \epsilon^4 p_j^4 x_i^4}{48 T^3} - \frac{(-1+T) \epsilon^4 p_i^4 p_j x_i^5}{30 T} + \frac{(-1+T) (13+2 T) \epsilon^4 p_i^3 p_j^2 x_i^5}{30 T^2} + \\
 & \frac{(-1+T) (-14-107 T+21 T^2) \epsilon^4 p_i^2 p_j^3 x_i^5}{60 T^3} - \frac{(-1+T) (-1-67 T-273 T^2+101 T^3) \epsilon^4 p_i p_j^4 x_i^5}{120 T^4} + \\
 & \frac{(-1+T) (-1-39 T-111 T^2+55 T^3) \epsilon^4 p_j^5 x_i^5}{120 T^4} + \epsilon p_i p_j x_i x_j + \frac{3}{2} \epsilon^2 p_i p_j x_i x_j + \frac{7}{6} \epsilon^3 p_i p_j x_i x_j - \\
 & \epsilon p_j^2 x_i x_j - \frac{3}{2} \epsilon^2 p_j^2 x_i x_j - \frac{7}{6} \epsilon^3 p_j^2 x_i x_j - \frac{1}{2} \epsilon^2 p_i^2 p_j x_i^2 x_j - \frac{3}{2} \epsilon^3 p_i^2 p_j x_i^2 x_j - \frac{55}{24} \epsilon^4 p_i^2 p_j x_i^2 x_j + \\
 & \frac{(1+2 T) \epsilon^2 p_i p_j^2 x_i^2 x_j}{2 T} + \frac{(2+9 T) \epsilon^3 p_i p_j^2 x_i^2 x_j}{2 T} + \frac{(25+212 T) \epsilon^4 p_i p_j^2 x_i^2 x_j}{24 T} - \\
 & \frac{(1+T) \epsilon^2 p_j^3 x_i^2 x_j}{2 T} - \frac{(1+3 T) \epsilon^3 p_j^3 x_i^2 x_j}{T} - \frac{(25+157 T) \epsilon^4 p_j^3 x_i^2 x_j}{24 T} + \frac{1}{6} \epsilon^3 p_i^3 p_j x_i^3 x_j + \\
 & \frac{3}{4} \epsilon^4 p_i^3 p_j x_i^3 x_j - \frac{7 \epsilon^3 p_i^2 p_j^2 x_i^3 x_j}{6 T} - \frac{(53+30 T) \epsilon^4 p_i^2 p_j^2 x_i^3 x_j}{12 T} - \frac{(-1-17 T+6 T^2) \epsilon^3 p_i p_j^3 x_i^3 x_j}{6 T^2} + \\
 & \frac{(5+147 T+12 T^2) \epsilon^4 p_i p_j^3 x_i^3 x_j}{12 T^2} + \frac{(-1-10 T+5 T^2) \epsilon^3 p_j^4 x_i^3 x_j}{6 T^2} + \frac{(-5-94 T+9 T^2) \epsilon^4 p_j^4 x_i^3 x_j}{12 T^2} - \\
 & \frac{1}{24} \epsilon^4 p_i^4 p_j x_i^4 x_j - \frac{(-31+16 T) \epsilon^4 p_i^3 p_j^2 x_i^4 x_j}{24 T} + \frac{5 (-5-23 T+18 T^2) \epsilon^4 p_i^2 p_j^3 x_i^4 x_j}{24 T^2} - \\
 & \frac{(-1-60 T-131 T^2+132 T^3) \epsilon^4 p_i p_j^4 x_i^4 x_j}{24 T^3} + \frac{(-1-35 T-47 T^2+59 T^3) \epsilon^4 p_j^5 x_i^4 x_j}{24 T^3} - \frac{1}{2} \epsilon^2 p_i p_j^2 x_i x_j^2 - \\
 & \epsilon^3 p_i p_j^2 x_i x_j^2 - \frac{25}{24} \epsilon^4 p_i p_j^2 x_i x_j^2 + \frac{1}{2} \epsilon^2 p_j^3 x_i x_j^2 + \epsilon^3 p_j^3 x_i x_j^2 + \frac{25}{24} \epsilon^4 p_j^3 x_i x_j^2 + \epsilon^3 p_i^2 p_j^2 x_i^2 x_j^2 + \\
 & \frac{15}{4} \epsilon^4 p_i^2 p_j^2 x_i^2 x_j^2 - \frac{(1+10 T) \epsilon^3 p_i p_j^3 x_i^2 x_j^2}{4 T} - \frac{(5+86 T) \epsilon^4 p_i p_j^3 x_i^2 x_j^2}{8 T} + \frac{(1+6 T) \epsilon^3 p_j^4 x_i^2 x_j^2}{4 T} + \\
 & \frac{(5+56 T) \epsilon^4 p_j^4 x_i^2 x_j^2}{8 T} - \epsilon^4 p_i^3 p_j^2 x_i^3 x_j^2 + \frac{5 (4+9 T) \epsilon^4 p_i^2 p_j^3 x_i^3 x_j^2}{12 T} - \frac{(1+49 T+51 T^2) \epsilon^4 p_i p_j^4 x_i^3 x_j^2}{12 T^2} + \\
 & \frac{(1+29 T+18 T^2) \epsilon^4 p_j^5 x_i^3 x_j^2}{12 T^2} + \frac{1}{6} \epsilon^3 p_i p_j^3 x_i x_j^3 + \frac{5}{12} \epsilon^4 p_i p_j^3 x_i x_j^3 - \frac{1}{6} \epsilon^3 p_j^4 x_i x_j^3 - \frac{5}{12} \epsilon^4 p_j^4 x_i x_j^3 - \\
 & \epsilon^4 p_i^2 p_j^3 x_i^2 x_j^3 + \frac{(1+30 T) \epsilon^4 p_i p_j^4 x_i^2 x_j^3}{12 T} - \frac{(1+18 T) \epsilon^4 p_j^5 x_i^2 x_j^3}{12 T} - \frac{1}{24} \epsilon^4 p_i p_j^4 x_i x_j^4 + \frac{1}{24} \epsilon^4 p_j^5 x_i x_j^4 +
 \end{aligned}$$

$$\frac{1}{24} \epsilon^4 p_j x_i (1 - 48 ca_{4,1}) - \epsilon^4 ca_{4,1} + \frac{1}{24} \epsilon^4 p_i x_i (-1 + 48 ca_{4,1}) + \epsilon^4 p_i p_j x_i x_j cb_{4,10} -$$

$$\epsilon^4 p_j^2 x_i x_j cb_{4,10} + \frac{\epsilon^4 p_i p_j x_i^2 (-25 T - 12 cb_{4,10} + 12 T cb_{4,10})}{24 T} - \frac{\epsilon^4 p_j^2 x_i^2 (-25 T - 12 cb_{4,10} + 12 T cb_{4,10})}{24 T}$$

(Alt) Out[ ]:=

$$\frac{\epsilon}{2} - \epsilon p_k x_k - \frac{1}{2} \epsilon^2 p_k x_k - \frac{1}{6} \epsilon^3 p_k x_k - \epsilon^4 ca_{4,1} + \frac{1}{12} \epsilon^4 p_k x_k (7 - 12 cb_{4,10})$$

(Alt) Out[ ]:=

$$-\frac{\epsilon}{2} + \epsilon p_k x_k - \frac{1}{2} \epsilon^2 p_k x_k + \frac{1}{6} \epsilon^3 p_k x_k + \epsilon^4 ca_{4,1} + \frac{1}{3} \epsilon^4 p_k x_k (-2 + 3 cb_{4,10})$$

(Alt) In[ ]:=

{ca<sub>1,2</sub> = 1, ca<sub>1,10</sub> = -1, ca<sub>2,1</sub> = 0, cb<sub>2,10</sub> = 3 / 2};

(Alt) In[ ]:=

Column[Collect[#, ε, CF] & /@ {r<sub>d</sub>[1, i, j], r<sub>d</sub>[-1, i, j], γ<sub>d</sub>[1, k], γ<sub>d</sub>[-1, k]}]

(Alt) Out[ ]:=

$$\epsilon \left( -\frac{1}{2} + p_i x_i - p_j x_i + \frac{1}{2} (-1 + T) p_i p_j x_i^2 + \frac{1}{2} (1 - T) p_j^2 x_i^2 - p_i p_j x_i x_j + p_j^2 x_i x_j \right) +$$

$$\epsilon^2 \left( -\frac{1}{2} p_i x_i + \frac{p_j x_i}{2} + \frac{1}{4} (1 - 3 T) p_i p_j x_i^2 + \frac{1}{4} (-1 + 3 T) p_j^2 x_i^2 + \frac{1}{3} (-1 + T) p_i^2 p_j x_i^3 - \right.$$

$$\frac{1}{6} (-1 + T) (5 + T) p_i p_j^2 x_i^3 + \frac{1}{6} (-1 + T) (3 + T) p_j^3 x_i^3 + \frac{3}{2} p_i p_j x_i x_j - \frac{3}{2} p_j^2 x_i x_j -$$

$$\left. \frac{1}{2} p_i^2 p_j x_i^2 x_j + \frac{1}{2} (2 + T) p_i p_j^2 x_i^2 x_j + \frac{1}{2} (-1 - T) p_j^3 x_i^2 x_j - \frac{1}{2} p_i p_j^2 x_i x_j^2 + \frac{1}{2} p_j^3 x_i x_j^2 \right) +$$

$$\epsilon^3 \left( \frac{p_i x_i}{6} - \frac{p_j x_i}{6} + \frac{1}{12} (5 + 7 T) p_i p_j x_i^2 + \frac{1}{12} (-5 - 7 T) p_j^2 x_i^2 + \frac{1}{6} (5 - 6 T) p_i^2 p_j x_i^3 + \frac{1}{6} (-16 + 17 T + 2 T^2) \right.$$

$$p_i p_j^2 x_i^3 + \frac{1}{6} (11 - 11 T - 2 T^2) p_j^3 x_i^3 + \frac{1}{8} (-1 + T) p_i^3 p_j x_i^4 - \frac{1}{8} (-1 + T) (4 + 3 T) p_i^2 p_j^2 x_i^4 +$$

$$\frac{1}{24} (-1 + T) (13 + 22 T + T^2) p_i p_j^3 x_i^4 - \frac{1}{24} (-1 + T) (4 + 13 T + T^2) p_j^4 x_i^4 - \frac{7}{6} p_i p_j x_i x_j +$$

$$\frac{7}{6} p_j^2 x_i x_j + \frac{3}{2} p_i^2 p_j x_i^2 x_j + \frac{1}{2} (-9 - 2 T) p_i p_j^2 x_i^2 x_j + (3 + T) p_j^3 x_i^2 x_j - \frac{1}{6} p_i^3 p_j x_i^3 x_j +$$

$$\frac{7}{6} T p_i^2 p_j^2 x_i^3 x_j + \frac{1}{6} (6 - 17 T - T^2) p_i p_j^3 x_i^3 x_j + \frac{1}{6} (-5 + 10 T + T^2) p_j^4 x_i^3 x_j + p_i p_j^2 x_i x_j^2 -$$

$$\left. p_j^3 x_i x_j^2 - p_i^2 p_j^2 x_i^2 x_j^2 + \frac{1}{4} (10 + T) p_i p_j^3 x_i^2 x_j^2 + \frac{1}{4} (-6 - T) p_j^4 x_i^2 x_j^2 - \frac{1}{6} p_i p_j^3 x_i x_j^3 + \frac{1}{6} p_j^4 x_i x_j^3 \right) +$$

$$\epsilon^4 \left( \frac{1}{36} (-34 + 55 T) p_i^2 p_j x_i^3 + \frac{1}{72} (269 - 394 T - 25 T^2) p_i p_j^2 x_i^3 + \frac{1}{72} (-201 + 284 T + 25 T^2) p_j^3 x_i^3 + \right.$$

$$\frac{1}{48} (25 - 27 T) p_i^3 p_j x_i^4 + \frac{1}{48} (-156 + 101 T + 69 T^2) p_i^2 p_j^2 x_i^4 +$$

$$\frac{1}{48} (245 - 79 T - 185 T^2 - 5 T^3) p_i p_j^3 x_i^4 + \frac{1}{48} (-114 + 5 T + 116 T^2 + 5 T^3) p_j^4 x_i^4 +$$

$$\frac{1}{30} (-1 + T) p_i^4 p_j x_i^5 - \frac{1}{30} (-1 + T) (2 + 13 T) p_i^3 p_j^2 x_i^5 + \frac{1}{60} (-1 + T) (-21 + 107 T + 14 T^2) p_i^2 p_j^3 x_i^5 -$$

$$\frac{1}{120} (-1 + T) (-101 + 273 T + 67 T^2 + T^3) p_i p_j^4 x_i^5 + \frac{1}{120} (-1 + T) (-55 + 111 T + 39 T^2 + T^3) p_j^5 x_i^5 -$$

$$\frac{55}{24} p_i^2 p_j x_i^2 x_j + \frac{1}{24} (212 + 25 T) p_i p_j^2 x_i^2 x_j + \frac{1}{24} (-157 - 25 T) p_j^3 x_i^2 x_j + \frac{3}{4} p_i^3 p_j x_i^3 x_j +$$

$$\frac{1}{12} (-30 - 53 T) p_i^2 p_j^2 x_i^3 x_j + \frac{1}{12} (12 + 147 T + 5 T^2) p_i p_j^3 x_i^3 x_j + \frac{1}{12} (9 - 94 T - 5 T^2) p_j^4 x_i^3 x_j -$$

$$\frac{1}{24} p_i^4 p_j x_i^4 x_j + \frac{1}{24} (-16 + 31 T) p_i^3 p_j^2 x_i^4 x_j - \frac{5}{24} (-18 + 23 T + 5 T^2) p_i^2 p_j^3 x_i^4 x_j +$$

$$\frac{1}{24} (-132 + 131 T + 60 T^2 + T^3) p_i p_j^4 x_i^4 x_j + \frac{1}{24} (59 - 47 T - 35 T^2 - T^3) p_j^5 x_i^4 x_j -$$

$$\frac{25}{24} p_i p_j^2 x_i x_j^2 + \frac{25}{24} p_j^3 x_i x_j^2 + \frac{15}{4} p_i^2 p_j^2 x_i^2 x_j^2 + \frac{1}{8} (-86 - 5 T) p_i p_j^3 x_i^2 x_j^2 + \frac{1}{8} (56 + 5 T) p_j^4 x_i^2 x_j^2 -$$

$$\left. p_i^3 p_j^2 x_i^3 x_j^2 + \frac{5}{12} (9 + 4 T) p_i^2 p_j^3 x_i^3 x_j^2 + \frac{1}{12} (-51 - 49 T - T^2) p_i p_j^4 x_i^3 x_j^2 + \right.$$

$$\frac{1}{12} (18 + 29 T + T^2) p_j^5 x_i^3 x_j^2 + \frac{5}{12} p_i p_j^3 x_i x_j^3 - \frac{5}{12} p_j^4 x_i x_j^3 - p_i^2 p_j^3 x_i^2 x_j^3 + \frac{1}{12} (30 + T) p_i p_j^4 x_i^2 x_j^3 +$$

$$\left. \frac{1}{12} (-18 - T) p_i^5 x_i^2 x_j^3 - \frac{1}{12} p_i p_j^4 x_i x_j^4 + \frac{1}{12} p_i^5 x_i x_j^4 + \frac{1}{12} p_i x_i (-1 - 48 ca_{4,1}) + ca_{4,1} + \right.$$

$$\begin{aligned}
 & \frac{1}{24} p_j x_i (1 + 48 ca_{4,1}) + \frac{1}{4} p_i p_j x_i x_j (5 - 4 cb_{4,10}) + \frac{1}{4} p_j^2 x_i x_j (-5 + 4 cb_{4,10}) + \\
 & \frac{1}{24} p_j^2 x_i^2 (10 + 15 T + 12 cb_{4,10} - 12 T cb_{4,10}) + \frac{1}{24} p_i p_j x_i^2 (-10 - 15 T - 12 cb_{4,10} + 12 T cb_{4,10}) \\
 \in & \left( \frac{1}{2} - p_i x_i + p_j x_i + \frac{(-1+T) p_i p_j x_i^2}{2T} - \frac{(-1+T) p_j^2 x_i^2}{2T} + p_i p_j x_i x_j - p_j^2 x_i x_j \right) + \\
 \in &^2 \left( -\frac{1}{2} p_i x_i + \frac{p_j x_i}{2} + \frac{(-3+T) p_i p_j x_i^2}{4T} - \frac{(-3+T) p_j^2 x_i^2}{4T} - \frac{(-1+T) p_i^2 p_j x_i^3}{3T} + \frac{(-1+T) (1+5T) p_i p_j^2 x_i^3}{6T^2} - \frac{(-1+T) (1+3T) p_j^3 x_i^3}{6T^2} + \right. \\
 & \left. \frac{3}{2} p_i p_j x_i x_j - \frac{3}{2} p_j^2 x_i x_j - \frac{1}{2} p_i^2 p_j x_i^2 x_j + \frac{(1+2T) p_i p_j^2 x_i^2 x_j}{2T} - \frac{(1+T) p_j^3 x_i^2 x_j}{2T} - \frac{1}{2} p_i p_j^2 x_i x_j^2 + \frac{1}{2} p_j^3 x_i x_j^2 \right) + \\
 \in &^3 \left( -\frac{1}{6} p_i x_i + \frac{p_j x_i}{6} - \frac{(7+5T) p_i p_j x_i^2}{12T} + \frac{(7+5T) p_j^2 x_i^2}{12T} - \frac{(-6+5T) p_i^2 p_j x_i^3}{6T} + \frac{(-2-17T+16T^2) p_i p_j^2 x_i^3}{6T^2} - \right. \\
 & \frac{(-2-11T+11T^2) p_j^3 x_i^3}{6T^2} + \frac{(-1+T) p_i^3 p_j x_i^4}{8T} - \frac{(-1+T) (3+4T) p_i^2 p_j^2 x_i^4}{8T^2} + \frac{(-1+T) (1+22T+13T^2) p_i p_j^3 x_i^4}{24T^3} - \\
 & \frac{(-1+T) (1+13T+4T^2) p_j^4 x_i^4}{24T^3} + \frac{7}{6} p_i p_j x_i x_j - \frac{7}{6} p_j^2 x_i x_j - \frac{3}{2} p_i^2 p_j x_i^2 x_j + \frac{(2+9T) p_i p_j^2 x_i^2 x_j}{2T} - \\
 & \frac{(1+3T) p_j^3 x_i^2 x_j}{T} + \frac{1}{6} p_i^3 p_j x_i^3 x_j - \frac{7 p_i^2 p_j^2 x_i^3 x_j}{6T} - \frac{(-1-17T+6T^2) p_i p_j^3 x_i^3 x_j}{6T^2} + \frac{(-1-10T+5T^2) p_j^4 x_i^3 x_j}{6T^2} - \\
 & \left. p_i p_j^2 x_i x_j^2 + p_j^3 x_i x_j^2 + p_i^2 p_j^2 x_i^2 x_j^2 - \frac{(1+10T) p_i p_j^3 x_i^2 x_j^2}{4T} + \frac{(1+6T) p_j^4 x_i^2 x_j^2}{4T} + \frac{1}{6} p_i p_j^3 x_i x_j^3 - \frac{1}{6} p_j^4 x_i x_j^3 \right) + \\
 \in &^4 \left( -\frac{(-55+34T) p_i^2 p_j x_i^3}{36T} + \frac{(-25-394T+269T^2) p_i p_j^2 x_i^3}{72T^2} - \frac{(-25-284T+201T^2) p_j^3 x_i^3}{72T^2} + \frac{(-27+25T) p_i^3 p_j x_i^4}{48T} - \right. \\
 & \frac{(-69-101T+156T^2) p_i^2 p_j^2 x_i^4}{48T^2} + \frac{(-5-185T-79T^2+245T^3) p_i p_j^3 x_i^4}{48T^3} - \frac{(-5-116T-5T^2+114T^3) p_j^4 x_i^4}{48T^3} - \frac{(-1+T) p_i^4 p_j x_i^5}{30T} + \\
 & \frac{(-1+T) (13+2T) p_i^3 p_j^2 x_i^5}{30T^2} + \frac{(-1+T) (-14-107T+21T^2) p_i^2 p_j^3 x_i^5}{60T^3} - \frac{(-1+T) (-1-67T-273T^2+101T^3) p_i p_j^4 x_i^5}{120T^4} + \\
 & \frac{(-1+T) (-1-39T-111T^2+55T^3) p_j^5 x_i^5}{120T^4} - \frac{55}{24} p_i^2 p_j x_i^2 x_j + \frac{(25+212T) p_i p_j^2 x_i^2 x_j}{24T} - \frac{(25+157T) p_j^3 x_i^2 x_j}{24T} + \frac{3}{4} p_i^3 p_j x_i^3 x_j - \\
 & \frac{(53+30T) p_i^2 p_j^2 x_i^3 x_j}{12T} + \frac{(5+147T+12T^2) p_i p_j^3 x_i^3 x_j}{12T^2} + \frac{(-5-94T+9T^2) p_j^4 x_i^3 x_j}{12T^2} - \frac{1}{24} p_i^4 p_j x_i^4 x_j - \frac{(-31+16T) p_i^3 p_j^2 x_i^4 x_j}{24T} + \\
 & \frac{5(-5-23T+18T^2) p_i^2 p_j^3 x_i^4 x_j}{24T^2} - \frac{(-1-60T-131T^2+132T^3) p_i p_j^4 x_i^4 x_j}{24T^3} + \frac{(-1-35T-47T^2+59T^3) p_j^5 x_i^4 x_j}{24T^3} - \frac{25}{24} p_i p_j^2 x_i x_j^2 + \\
 & \frac{25}{24} p_j^3 x_i x_j^2 + \frac{15}{4} p_i^2 p_j^2 x_i^2 x_j^2 - \frac{(5+86T) p_i p_j^3 x_i^2 x_j^2}{8T} + \frac{(5+56T) p_j^4 x_i^2 x_j^2}{8T} - p_i^3 p_j^2 x_i^3 x_j^2 + \frac{5(4+9T) p_i^2 p_j^3 x_i^3 x_j^2}{12T} - \\
 & \frac{(1+49T+51T^2) p_i p_j^4 x_i^3 x_j^2}{12T^2} + \frac{(1+29T+18T^2) p_j^5 x_i^3 x_j^2}{12T^2} + \frac{5}{12} p_i p_j^3 x_i x_j^3 - \frac{5}{12} p_j^4 x_i x_j^3 - p_i^2 p_j^3 x_i^2 x_j^3 + \frac{(1+30T) p_i p_j^4 x_i^2 x_j^3}{12T} - \\
 & \frac{(1+18T) p_j^5 x_i^2 x_j^3}{12T} - \frac{1}{24} p_i p_j^4 x_i x_j^4 + \frac{1}{24} p_j^5 x_i x_j^4 + \frac{1}{24} p_j x_i (1 - 48 ca_{4,1}) - ca_{4,1} + \frac{1}{24} p_i x_i (-1 + 48 ca_{4,1}) + \\
 & \left. p_i p_j x_i x_j cb_{4,10} - p_j^2 x_i x_j cb_{4,10} + \frac{p_i p_j x_i^2 (-25T-12cb_{4,10}+12Tcb_{4,10})}{24T} - \frac{p_j^2 x_i^2 (-25T-12cb_{4,10}+12Tcb_{4,10})}{24T} \right) \\
 - & \frac{1}{2} \in^2 p_k x_k - \frac{1}{6} \in^3 p_k x_k + \in \left( \frac{1}{2} - p_k x_k \right) + \in^4 \left( -ca_{4,1} + \frac{1}{12} p_k x_k (7 - 12 cb_{4,10}) \right) \\
 - & \frac{1}{2} \in^2 p_k x_k + \frac{1}{6} \in^3 p_k x_k + \in \left( -\frac{1}{2} + p_k x_k \right) + \in^4 \left( ca_{4,1} + \frac{1}{3} p_k x_k (-2 + 3 cb_{4,10}) \right)
 \end{aligned}$$