Pensieve header: Implementing ρ_1, and also ρ_d.

exec

```
nb2tex$TeXFileName = "Rho1.tex";
```

pdf

Preliminaries

pdf

This is Rho.nb of http://drorbn.net/oa22/ap.

In[•]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\Oaxaca-2210"];
```

pdf

In[•]:=
```
Once[<< KnotTheory`; << Rot.m];
```

pdf

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

pdf

```
Loading Rot.m from http://drorbn.net/la22/ap to compute rotation numbers.
```

pdf

The Program

pdf

In[•]:=
```
R1[s_, i_, j_] := s (gji (gj+,j + gj,j+ - gij) - gii (gj,j+ - 1) - 1 / 2);
Z[K_] := Module[{Cs, φ, n, A, s, i, j, k, Δ, G, ρ1},
  {Cs, φ} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ((-T^s  T^s - 1
                                                              0     -1  )))];
  Δ = T^((-Total[φ] - Total[Cs[[All, 1]]])/2) Det[A];
  G = Inverse[A];
  ρ1 = Sum_{k=1}^{n} R1 @@ Cs[[k]] - Sum_{k=1}^{2 n} φ[[k]] (gkk - 1 / 2);
  Factor@{Δ, Δ^2 ρ1 /. α_+ :> α + 1 /. gα_,β_ :> G[[α, β]]}];
```

pdf

The First Few Knots

pdf

In[•]:=
```
TableForm[Table[Join[{K[[1]]_K[[2]]}, Z[K]], {K, AllKnots[{3, 6}]}], TableAlignments → Center]
```

pdf

KnotTheory: Loading precomputed data in PD4Knots`.

Out[•]//TableForm=

pdf

3_1	$\frac{1-T+T^2}{T}$	$\frac{(-1+T)^2\,(1+T^2)}{T^2}$
4_1	$-\frac{1-3\,T+T^2}{T}$	0
5_1	$\frac{1-T+T^2-T^3+T^4}{T^2}$	$\frac{(-1+T)^2\,(1+T^2)\,(2+T^2+2\,T^4)}{T^4}$
5_2	$\frac{2-3\,T+2\,T^2}{T}$	$\frac{(-1+T)^2\,(5-4\,T+5\,T^2)}{T^2}$
6_1	$-\frac{(-2+T)\,(-1+2\,T)}{T}$	$\frac{(-1+T)^2\,(1-4\,T+T^2)}{T^2}$
6_2	$-\frac{1-3\,T+3\,T^2-3\,T^3+T^4}{T^2}$	$\frac{(-1+T)^2\,(1-4\,T+4\,T^2-4\,T^3+4\,T^4-4\,T^5+T^6)}{T^4}$
6_3	$\frac{1-3\,T+5\,T^2-3\,T^3+T^4}{T^2}$	0

tex

```
\def\nbpdfText#1{\vskip -3mm\[ \includegraphics[width=0.4\linewidth]{#1}\quad p=1-T^s \]}
```

pdf

tex

```
\def\nbpdfText#1{\vskip 1mm\par\noindent\includegraphics{#1}}
```

tex

```
\needspace{2in}
```

pdf

Fast!

tex

```
\[ \resizebox{\linewidth}{!}{\import{../Waco-2203/}{GST48-Marked.pdf_t}} \]
```

pdf

In[•]:=
```
Timing@
 Z[GST48 = EPD[X14,1, X2,29, X3,40, X43,4, X26,5, X6,95, X96,7, X13,8, X9,28, X10,41, X42,11, X27,12, X30,15,
    X16,61, X17,72, X18,83, X19,34, X89,20, X21,92, X79,22, X68,23, X57,24, X25,56, X62,31, X73,32,
    X84,33, X50,35, X36,81, X37,70, X38,59, X39,54, X44,55, X58,45, X69,46, X80,47, X48,91, X90,49,
    X51,82, X52,71, X53,60, X63,74, X64,85, X76,65, X87,66, X67,94, X75,86, X88,77, X78,93]]
```

Out[•]=

pdf

$$\left\{170.313,\ \left\{-\frac{\left(-1+2\,T-T^2-T^3+2\,T^4-T^5+T^8\right)\,\left(-1+T^3-2\,T^4+T^5+T^6-2\,T^7+T^8\right)}{T^8},\right.\right.$$
$$\frac{1}{T^{16}}\,(-1+T)^2\,\left(5-18\,T+33\,T^2-32\,T^3+2\,T^4+42\,T^5-62\,T^6-8\,T^7+166\,T^8-242\,T^9+108\,T^{10}+\right.$$
$$\left.\left.\left.132\,T^{11}-226\,T^{12}+148\,T^{13}-11\,T^{14}-36\,T^{15}-11\,T^{16}+148\,T^{17}-226\,T^{18}+132\,T^{19}+108\,T^{20}-\right.\right.\right.$$
$$\left.\left.\left.242\,T^{21}+166\,T^{22}-8\,T^{23}-62\,T^{24}+42\,T^{25}+2\,T^{26}-32\,T^{27}+33\,T^{28}-18\,T^{29}+5\,T^{30}\right)\right\}\right\}$$

pdf

Strong!

pdf

```
{NumberOfKnots[{3, 12}],
 Length@Union@Table[Z[K], {K, AllKnots[{3, 12}]}],
 Length@Union@Table[{HOMFLYPT[K], Kh[K]}, {K, AllKnots[{3, 12}]}]}
```

Out[•]=

pdf

{2977, 2882, 2785}

In[•]:=
```
2977 - {2882, 2785}
```

Out[•]=

{95, 192}

tex

So the pair (Δ,ρ_1) attains 2,882 distinct values on the 2,977 prime knots with up to 12 crossings (a deficit of 95), whereas the pair (HOMFLYPT, Khovanov Homology) attains only 2,785 distinct values on the same knots (a deficit of 192).

tex

\def\nbpdfText#1{\vskip 1mm\par\noindent\includegraphics[width=\linewidth]{#1}}

pdf

tex

\def\nbpdfText#1{\vskip 1mm\par\noindent\includegraphics{#1}}

Invariance under R3

exec

```
nb2tex$TeXFileName = "Invariance.tex";
```

pdf

In[•]:=
```
δ_,_ := If[i === j, 1, 0];
gRules_,_,_ := {g_iβ_ :> δ_iβ + T^s g_i⁺,β + (1 - T^s) g_j⁺,β, g_jβ_ :> δ_jβ + g_j⁺,β,
  g_α_,i :> T^-s (g_α,i⁺ - δ_α,i⁺), g_α_,j :> g_α,j⁺ - (1 - T^s) g_αi - δ_α,j⁺}
```

Proof of Reidemeister 3:

pdf

In[•]:=
```
lhs = R_1[1, j, k] + R_1[1, i, k⁺] + R_1[1, i⁺, j⁺] //. gRules_1,j,k ⋃ gRules_1,i,k⁺ ⋃ gRules_1,i⁺,j⁺;
rhs = R_1[1, i, j] + R_1[1, i⁺, k] + R_1[1, j⁺, k⁺] //. gRules_1,i,j ⋃ gRules_1,i⁺,k ⋃ gRules_1,j⁺,k⁺;
Simplify[lhs == rhs]
```

Out[•]=
pdf

```
True
```

tex

Next comes Reid1, where we use results from an earlier example:

In[•]:=
```
{{1, T^-1, 1}, {0, T^-1, 1}, {0, 0, 1}} // Inverse // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & T & -T \\ 0 & 0 & 1 \end{pmatrix}$$

pdf

In[•]:=
```
R_1[1, 2, 1] - 1 (g_22 - 1 / 2) /. g_α_,β_ :> {{1, T^-1, 1}, {0, T^-1, 1}, {0, 0, 1}}[[α, β]]
```

Out[•]=
pdf

$$\frac{1}{T^2} - \frac{1}{T} - \frac{-1 + \frac{1}{T}}{T}$$

tex

Invariance under the other moves is proven similarly.

exec

```
nb2tex$TeXFileName = "Rhod.tex";
nb2tex$PDFWidth = 4.2 / 0.7;
```

On to ρ_d!

tex

{\bf\red Implementation.} Data, then program (with output using the \text{Conway} variable $z=\sqrt{T}-1/\sqrt{T}$), and then a demo. See {\tt Rho.nb} of \web{ap}.

\def\nbpdfInput#1{\vskip 1mm\par\noindent\includegraphics[scale=0.7]{#1}}
\def\nbpdfOutput#1{\vskip 1mm\par\noindent\includegraphics[scale=0.7]{#1}}

pdf

In[•]:=
```
V@γ1,φ_[k_] = φ (1 / 2 - p̄k x̄k); V@γ2,φ_[k_] = -φ^2 p̄k x̄k / 2; V@γ3,φ_[k_] := -φ^3 p̄k x̄k / 6
```

pdf

In[•]:=
```
V@r1,s_[i_, j_] :=
 s (-1 + 2 pi xi - 2 pj xi + (-1 + T^s) pi pj xi^2 + (1 - T^s) pj^2 xi^2 - 2 pi pj xi xj + 2 pj^2 xi xj) / 2
```

pdf

In[•]:=
```
V@r2,1[i_, j_] := (-6 pi xi + 6 pj xi - 3 (-1 + 3 T) pi pj xi^2 + 3 (-1 + 3 T) pj^2 xi^2 + 4 (-1 + T) pi^2 pj xi^3 -
    2 (-1 + T) (5 + T) pi pj^2 xi^3 + 2 (-1 + T) (3 + T) pj^3 xi^3 + 18 pi pj xi xj - 18 pj^2 xi xj -
    6 pi^2 pj xi^2 xj + 6 (2 + T) pi pj^2 xi^2 xj - 6 (1 + T) pj^3 xi^2 xj - 6 pi pj^2 xi xj^2 + 6 pj^3 xi xj^2) / 12
```

pdf

In[•]:=
```
V@r2,-1[i_, j_] :=
 (-6 T^2 pi xi + 6 T^2 pj xi + 3 (-3 + T) T pi pj xi^2 - 3 (-3 + T) T pj^2 xi^2 - 4 (-1 + T) T pi^2 pj xi^3 + 2 (-1 + T)
     (1 + 5 T) pi pj^2 xi^3 - 2 (-1 + T) (1 + 3 T) pj^3 xi^3 + 18 T^2 pi pj xi xj - 18 T^2 pj^2 xi xj - 6 T^2 pi^2 pj xi^2 xj +
   6 T (1 + 2 T) pi pj^2 xi^2 xj - 6 T (1 + T) pj^3 xi^2 xj - 6 T^2 pi pj^2 xi xj^2 + 6 T^2 pj^3 xi xj^2) / (12 T^2)
```

pdf

In[•]:=
```
V@r3,1[i_, j_] :=
 (4 pi xi - 4 pj xi + 2 (5 + 7 T) pi pj xi^2 - 2 (5 + 7 T) pj^2 xi^2 - 4 (-5 + 6 T) pi^2 pj xi^3 + 4 (-16 + 17 T + 2 T^2)
     pi pj^2 xi^3 - 4 (-11 + 11 T + 2 T^2) pj^3 xi^3 + 3 (-1 + T) pi^3 pj xi^4 - 3 (-1 + T) (4 + 3 T) pi^2 pj^2 xi^4 +
   (-1 + T) (13 + 22 T + T^2) pi pj^3 xi^4 - (-1 + T) (4 + 13 T + T^2) pj^4 xi^4 - 28 pi pj xi xj + 28 pj^2 xi xj +
   36 pi^2 pj xi^2 xj - 12 (9 + 2 T) pi pj^2 xi^2 xj + 24 (3 + T) pj^3 xi^2 xj - 4 pi^3 pj xi^3 xj + 28 T pi^2 pj^2 xi^3 xj -
   4 (-6 + 17 T + T^2) pi pj^3 xi^3 xj + 4 (-5 + 10 T + T^2) pj^4 xi^3 xj + 24 pi pj^2 xi xj^2 - 24 pj^3 xi xj^2 -
   24 pi^2 pj^2 xi^2 xj^2 + 6 (10 + T) pi pj^3 xi^2 xj^2 - 6 (6 + T) pj^4 xi^2 xj^2 - 4 pi pj^3 xi xj^3 + 4 pj^4 xi xj^3) / 24
```

pdf

In[•]:=
```
V@r3,-1[i_, j_] :=
 (-4 T^3 pi xi + 4 T^3 pj xi - 2 T^2 (7 + 5 T) pi pj xi^2 + 2 T^2 (7 + 5 T) pj^2 xi^2 - 4 T^2 (-6 + 5 T) pi^2 pj xi^3 +
   4 T (-2 - 17 T + 16 T^2) pi pj^2 xi^3 - 4 T (-2 - 11 T + 11 T^2) pj^3 xi^3 + 3 (-1 + T) T^2 pi^3 pj xi^4 -
   3 (-1 + T) T (3 + 4 T) pi^2 pj^2 xi^4 + (-1 + T) (1 + 22 T + 13 T^2) pi pj^3 xi^4 -
   (-1 + T) (1 + 13 T + 4 T^2) pj^4 xi^4 + 28 T^3 pi pj xi xj - 28 T^3 pj^2 xi xj - 36 T^3 pi^2 pj xi^2 xj +
   12 T^2 (2 + 9 T) pi pj^2 xi^2 xj - 24 T^2 (1 + 3 T) pj^3 xi^2 xj + 4 T^3 pi^3 pj xi^3 xj -
   28 T^2 pi^2 pj^2 xi^3 xj - 4 T (-1 - 17 T + 6 T^2) pi pj^3 xi^3 xj + 4 T (-1 - 10 T + 5 T^2) pj^4 xi^3 xj -
   24 T^3 pi pj^2 xi xj^2 + 24 T^3 pj^3 xi xj^2 + 24 T^3 pi^2 pj^2 xi^2 xj^2 - 6 T^2 (1 + 10 T) pi pj^3 xi^2 xj^2 +
   6 T^2 (1 + 6 T) pj^4 xi^2 xj^2 + 4 T^3 pi pj^3 xi xj^3 - 4 T^3 pj^4 xi xj^3) / (24 T^3)
```

pdf

```
In[•]:= {p*, x*, p̄*, x̄*} = {π, ξ, π̄, ξ̄};  (z_i__)* := (z*)_i;
        Zip_{}[ℰ_] := ℰ;
        Zip_{z_,zs___}[ℰ_] := (Collect[ℰ // Zip_{zs}, z] /. f_. z^d_. :> (D[f, {z*, d}])) /. z* → 0
```

pdf

```
In[•]:= gPair[fs_, w_] := gPair[fs, w] = Collect[Zip_{Join@@Table[{p_α,p̄_α,x_α,x̄_α},{α,w}]}[(Times @@ (V /@ fs))
          Exp[Sum[g_{α,β} (π_α + π̄_α) (ξ_β + ξ̄_β), {α, w}, {β, w}] - Sum[ξ̄_α π_α, {α, w}]]], g__, Factor]
```

pdf

```
In[•]:= T2z[p_] := Module[{q = Expand[p], n, c},
          If[q === 0, 0, c = Coefficient[q, T, n = Exponent[q, T]];
           c z^(2 n) + T2z[q - c (T^(1/2) - T^(-1/2))^(2 n)]]];
```

pdf

```
In[•]:= Z_d_[K_] := Module[{Cs, φ, n, A, s, i, j, k, Δ, G, d1, Z1, Z2, Z3},
          {Cs, φ} = Rot[K]; n = Length[Cs]; A = IdentityMatrix[2 n + 1];
          Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ((-T^s  T^s - 1
                                                                      0     -1))))];
          {Δ, G} = Factor@{T^((-Total[φ] - Total[Cs[[All, 1]]])/2) Det@A, Inverse@A};
          Z1 = Exp[Total[Cases[Cs, {s_, i_, j_} :> Sum[ϵ^d1 r_{d1,s}[i, j], {d1, d}]]] +
               Sum[ϵ^d1 γ_{d1,φ[[k]]}[k], {k, 2 n}, {d1, d}] /. γ_,0[_] → 0];
          Z2 = Expand[F[{}, {}] × Normal@Series[Z1, {ϵ, 0, d}]] //. F[fs_, {es___}] ×
               (f : (r | γ)_ps__[is__])^p_. :> F[Join[fs, Table[f, p]], DeleteDuplicates@{es, is}];
          Z3 = Expand[Z2 /. F[fs_, es_] :> Expand[gPair[
                 Replace[fs, Thread[es → Range@Length@es], {2}], Length@es
               ] /. g_α_,β_ :> G[[es[[α]], es[[β]]]]]];
          Collect[{Δ, Z3 /. ϵ^p_. → p! Δ^(2 p) ϵ^p}, ϵ, T2z]];
```

```
In[•]:= Z_3[Knot[3, 1]] // Timing
```

KnotTheory: Loading precomputed data in PD4Knots`.

Out[•]=

$$\left\{49.9844,\ \left\{1+z^2,\ 1+\left(2z^2+z^4\right)\epsilon+\left(2-4z^2+3z^4+4z^6+z^8\right)\epsilon^2+\left(-12+74z^2-27z^4-20z^6+8z^8+6z^{10}+z^{12}\right)\epsilon^3\right\}\right\}$$

```
In[•]:= Z_3[Knot[3, 1]] // Timing
```

Out[•]=

$$\left\{1.26563,\ \left\{1+z^2,\ 1+\left(2z^2+z^4\right)\epsilon+\left(2-4z^2+3z^4+4z^6+z^8\right)\epsilon^2+\left(-12+74z^2-27z^4-20z^6+8z^8+6z^{10}+z^{12}\right)\epsilon^3\right\}\right\}$$

Demos

exec

```
nb2tex$PDFWidth = 8 / 0.75;
```

tex

```
\end{multicols}
\def\nbpdfInput#1{\vskip 1mm\par\noindent\includegraphics[scale=0.75]{#1}}
\def\nbpdfOutput#1{\vskip 1mm\par\noindent\includegraphics[scale=0.75]{#1}}
```

In[•]:=

```
GST48 = EPD[X14,1, X̄2,29, X3,40, X43,4, X̄26,5, X6,95, X96,7, X13,8, X̄9,28, X10,41, X42,11, X̄27,12,
   X30,15, X̄16,61, X̄17,72, X̄18,83, X19,34, X̄89,20, X̄21,92, X̄79,22, X̄68,23, X̄57,24, X̄25,56, X62,31,
   X73,32, X84,33, X̄50,35, X36,81, X37,70, X38,59, X̄39,54, X44,55, X58,45, X69,46, X80,47, X48,91,
   X90,49, X51,82, X52,71, X53,60, X̄63,74, X̄64,85, X̄76,65, X̄87,66, X̄67,94, X̄75,86, X̄88,77, X̄78,93];
Z2[GST48] // Timing
Z2[GST48] // Timing
```

Out[•]=

$$\{564.578,\ \{1-4\,z^2-61\,z^4-207\,z^6-296\,z^8-210\,z^{10}-77\,z^{12}-14\,z^{14}-z^{16},$$
$$1+\left(38\,z^2+255\,z^4+1696\,z^6+16\,281\,z^8+86\,952\,z^{10}+259\,994\,z^{12}+487\,372\,z^{14}+615\,066\,z^{16}+\right.$$
$$\left.543\,148\,z^{18}+341\,714\,z^{20}+153\,722\,z^{22}+48\,983\,z^{24}+10\,776\,z^{26}+1554\,z^{28}+132\,z^{30}+5\,z^{32}\right)\in+$$
$$\left(-8-484\,z^2+9709\,z^4+165\,952\,z^6+1\,590\,491\,z^8+16\,256\,508\,z^{10}+115\,341\,797\,z^{12}+432\,685\,748\,z^{14}+\right.$$
$$395\,838\,354\,z^{16}-4\,017\,557\,792\,z^{18}-23\,300\,064\,167\,z^{20}-70\,082\,264\,972\,z^{22}-142\,572\,271\,191\,z^{24}-$$
$$209\,475\,503\,700\,z^{26}-221\,616\,295\,209\,z^{28}-151\,502\,648\,428\,z^{30}-23\,700\,199\,243\,z^{32}+$$
$$99\,462\,146\,328\,z^{34}+164\,920\,463\,074\,z^{36}+162\,550\,825\,432\,z^{38}+119\,164\,552\,296\,z^{40}+$$
$$69\,153\,062\,608\,z^{42}+32\,547\,596\,611\,z^{44}+12\,541\,195\,448\,z^{46}+3\,961\,384\,155\,z^{48}+1\,021\,219\,696\,z^{50}+$$
$$\left.212\,773\,106\,z^{52}+35\,264\,208\,z^{54}+4\,537\,548\,z^{56}+436\,600\,z^{58}+29\,536\,z^{60}+1252\,z^{62}+25\,z^{64}\right)\in^2\}\}$$

Out[•]=

$$\{598.109,\ \{1-4\,z^2-61\,z^4-207\,z^6-296\,z^8-210\,z^{10}-77\,z^{12}-14\,z^{14}-z^{16},$$
$$1+\left(38\,z^2+255\,z^4+1696\,z^6+16\,281\,z^8+86\,952\,z^{10}+259\,994\,z^{12}+487\,372\,z^{14}+615\,066\,z^{16}+\right.$$
$$\left.543\,148\,z^{18}+341\,714\,z^{20}+153\,722\,z^{22}+48\,983\,z^{24}+10\,776\,z^{26}+1554\,z^{28}+132\,z^{30}+5\,z^{32}\right)\in+$$
$$\left(-8-484\,z^2+9709\,z^4+165\,952\,z^6+1\,590\,491\,z^8+16\,256\,508\,z^{10}+115\,341\,797\,z^{12}+432\,685\,748\,z^{14}+\right.$$
$$395\,838\,354\,z^{16}-4\,017\,557\,792\,z^{18}-23\,300\,064\,167\,z^{20}-70\,082\,264\,972\,z^{22}-142\,572\,271\,191\,z^{24}-$$
$$209\,475\,503\,700\,z^{26}-221\,616\,295\,209\,z^{28}-151\,502\,648\,428\,z^{30}-23\,700\,199\,243\,z^{32}+$$
$$99\,462\,146\,328\,z^{34}+164\,920\,463\,074\,z^{36}+162\,550\,825\,432\,z^{38}+119\,164\,552\,296\,z^{40}+$$
$$69\,153\,062\,608\,z^{42}+32\,547\,596\,611\,z^{44}+12\,541\,195\,448\,z^{46}+3\,961\,384\,155\,z^{48}+1\,021\,219\,696\,z^{50}+$$
$$\left.212\,773\,106\,z^{52}+35\,264\,208\,z^{54}+4\,537\,548\,z^{56}+436\,600\,z^{58}+29\,536\,z^{60}+1252\,z^{62}+25\,z^{64}\right)\in^2\}\}$$

pdf

```
Z₂[GST48] (* takes a few minutes *)
```

Out[•]=
pdf

$$\{1-4z^2-61z^4-207z^6-296z^8-210z^{10}-77z^{12}-14z^{14}-z^{16},$$
$$1+\left(38z^2+255z^4+1696z^6+16\,281z^8+86\,952z^{10}+259\,994z^{12}+487\,372z^{14}+615\,066z^{16}+543\,148z^{18}+341\,714z^{20}+153\,722z^{22}+48\,983z^{24}+10\,776z^{26}+1554z^{28}+132z^{30}+5z^{32}\right)\in+$$
$$\left(-8-484z^2+9709z^4+165\,952z^6+1\,590\,491z^8+16\,256\,508z^{10}+115\,341\,797z^{12}+432\,685\,748z^{14}+395\,838\,354z^{16}-4\,017\,557\,792z^{18}-23\,300\,064\,167z^{20}-70\,082\,264\,972z^{22}-142\,572\,271\,191z^{24}-209\,475\,503\,700z^{26}-221\,616\,295\,209z^{28}-151\,502\,648\,428z^{30}-23\,700\,199\,243z^{32}+99\,462\,146\,328z^{34}+164\,920\,463\,074z^{36}+162\,550\,825\,432z^{38}+119\,164\,552\,296z^{40}+69\,153\,062\,608z^{42}+32\,547\,596\,611z^{44}+12\,541\,195\,448z^{46}+3\,961\,384\,155z^{48}+1\,021\,219\,696z^{50}+212\,773\,106z^{52}+35\,264\,208z^{54}+4\,537\,548z^{56}+436\,600z^{58}+29\,536z^{60}+1252z^{62}+25z^{64}\right)\in^2\}$$

In[•]:=
```
Table[Join[{K⟦1⟧_K⟦2⟧}, Z₃[K]], {K, AllKnots[{3, 6}]}] // Timing
```

Out[•]=

$$\{256.063,\ \{\{3_1,\ 1+z^2,$$
$$1+\left(2z^2+z^4\right)\in+\left(2-4z^2+3z^4+4z^6+z^8\right)\in^2+\left(-12+74z^2-27z^4-20z^6+8z^8+6z^{10}+z^{12}\right)\in^3\},$$
$$\{4_1,\ 1-z^2,\ 1+\left(-2+2z^4\right)\in^2\},\ \{5_1,\ 1+3z^2+z^4,\ 1+\left(10z^2+21z^4+12z^6+2z^8\right)\in+$$
$$\left(6-28z^2+33z^4+364z^6+655z^8+536z^{10}+227z^{12}+48z^{14}+4z^{16}\right)\in^2+$$
$$\left(-60+970z^2+645z^4-3380z^6-3280z^8+7470z^{10}+19\,475z^{12}+20\,536z^{14}+12\,564z^{16}+4774z^{18}+1109z^{20}+144z^{22}+8z^{24}\right)\in^3\},$$
$$\{5_2,\ 1+2z^2,\ 1+\left(6z^2+5z^4\right)\in+\left(4-20z^2+43z^4+64z^6+26z^8\right)\in^2+$$
$$\left(-36+498z^2-883z^4+100z^6+816z^8+556z^{10}+146z^{12}\right)\in^3\},$$
$$\{6_1,\ 1-2z^2,\ 1+\left(-2z^2+z^4\right)\in+\left(-4+4z^2+25z^4-8z^6+2z^8\right)\in^2+$$
$$\left(12+154z^2-223z^4-608z^6+100z^8-52z^{10}+10z^{12}\right)\in^3\},$$
$$\{6_2,\ 1-z^2-z^4,\ 1+\left(-2z^2-3z^4+2z^6+z^8\right)\in+$$
$$\left(-2-4z^2+29z^4+28z^6+42z^8-8z^{10}-2z^{12}+4z^{14}+z^{16}\right)\in^2+\left(12+166z^2+155z^4-194z^6-2453z^8-1622z^{10}-1967z^{12}-258z^{14}+49z^{16}-30z^{18}+z^{20}+6z^{22}+z^{24}\right)\in^3\},$$
$$\{6_3,\ 1+z^2+z^4,\ 1+\left(2+8z^2-16z^6-24z^8-16z^{10}-2z^{12}\right)\in^2\}\}\}$$

In[•]:=
```
Table[Join[{K⟦1⟧_K⟦2⟧}, Z3[K]], {K, AllKnots[{3, 6}]}] // Timing
```

Out[•]=
$$\{143.641, \{\{3_1, 1+z^2, 1+\left(2z^2+z^4\right)\in+\left(2-4z^2+3z^4+4z^6+z^8\right)\in^2+\left(-12+74z^2-27z^4-20z^6+8z^8+6z^{10}+z^{12}\right)\in^3\},$$
$$\{4_1, 1-z^2, 1+\left(-2+2z^4\right)\in^2\},$$
$$\{5_1, 1+3z^2+z^4, 1+\left(10z^2+21z^4+12z^6+2z^8\right)\in+\left(6-28z^2+33z^4+364z^6+655z^8+536z^{10}+227z^{12}+48z^{14}+4z^{16}\right)\in^2+\left(-60+970z^2+645z^4-3380z^6-3280z^8+7470z^{10}+19\,475z^{12}+20\,536z^{14}+12\,564z^{16}+4774z^{18}+1109z^{20}+144z^{22}+8z^{24}\right)\in^3\},$$
$$\{5_2, 1+2z^2, 1+\left(6z^2+5z^4\right)\in+\left(4-20z^2+43z^4+64z^6+26z^8\right)\in^2+\left(-36+498z^2-883z^4+100z^6+816z^8+556z^{10}+146z^{12}\right)\in^3\},$$
$$\{6_1, 1-2z^2, 1+\left(-2z^2+z^4\right)\in+\left(-4+4z^2+25z^4-8z^6+2z^8\right)\in^2+\left(12+154z^2-223z^4-608z^6+100z^8-52z^{10}+10z^{12}\right)\in^3\},$$
$$\{6_2, 1-z^2-z^4, 1+\left(-2z^2-3z^4+2z^6+z^8\right)\in+\left(-2-4z^2+29z^4+28z^6+42z^8-8z^{10}-2z^{12}+4z^{14}+z^{16}\right)\in^2+\left(12+166z^2+155z^4-194z^6-2453z^8-1622z^{10}-1967z^{12}-258z^{14}+49z^{16}-30z^{18}+z^{20}+6z^{22}+z^{24}\right)\in^3\},$$
$$\{6_3, 1+z^2+z^4, 1+\left(2+8z^2-16z^6-24z^8-16z^{10}-2z^{12}\right)\in^2\}\}\}$$

tex
```
\def\nbpdfOutput#1{\vskip 1mm\par\noindent\includegraphics[width=\linewidth]{#1}}
```

pdf
```
TableForm[Table[Join[{K⟦1⟧_K⟦2⟧}, Z3[K]], {K, AllKnots[{3, 6}]}],
 TableAlignments → Center] (* takes a few minutes *)
```

pdf

KnotTheory: Loading precomputed data in PD4Knots`.

Out[•]//TableForm=
pdf

3_1	$1+z^2$	$1+\left(2z^2+z^4\right)$
4_1	$1-z^2$	
5_1	$1+3z^2+z^4$	$1+\left(10z^2+21z^4+12z^6+2z^8\right)\in+\left(6-28z^2+33z^4+364z^6+655z^8+536z^{10}+227$
5_2	$1+2z^2$	$1+\left(6z^2+5z^4\right)\in+\left(4-$
6_1	$1-2z^2$	$1+\left(-2z^2+z^4\right)\in+\left($
6_2	$1-z^2-z^4$	$1+\left(-2z^2-3z^4+2z^6+z^8\right)\in+\left(-2-4z^2+29z^4+28z^6+42z^8-$
6_3	$1+z^2+z^4$	

Finding the Duplicates for Z

In[•]:=
```
tab = Table[K → Z[K], {K, AllKnots[{3, 12}]}]
```

KnotTheory: Loading precomputed data in PD4Knots`.

KnotTheory: Loading precomputed data in DTCode4KnotsTo11`.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

KnotTheory: Loading precomputed data in KnotTheory/12A.dts.

General: Further output of KnotTheory::loading will be suppressed during this calculation.

Out[•]=

$$\left\{\text{Knot}[3,1] \to \left\{\frac{1-T+T^2}{T}, \frac{(-1+T)^2\,(1+T^2)}{T^2}\right\}, \text{Knot}[4,1] \to \left\{-\frac{1-3\,T+T^2}{T}, 0\right\}, \cdots 2974 \cdots, \text{Knot}[12, \text{NonAlternating}, 888] \to \left\{\frac{(1-T+T^2)^2\,(1+T-2\,T^2+T^3-2\,T^4+T^5+T^6)}{T^5}, -\frac{(-1+T)^2\,(1+T^2)\,(1-T+T^2)^2\,(5+10\,T-9\,T^2+2\,T^3+22\,T^4-30\,T^5+43\,T^6-30\,T^7+22\,T^8+2\,T^9-9\,T^{10}+10\,T^{11}+5\,T^{12})}{T^{10}}\right\}\right\}$$

Full expression not available (original memory size: 8.9 MB)

In[•]:=
```
DeleteCases[Gather[tab, (Last[#1] == Last[#2]) &], {_}]
```

Out[•]=

$$\left\{\left\{\text{Knot}[10,106] \to \left\{-\frac{\left(1-T+T^2\right)\left(-1+T-2\,T^2+T^3\right)\left(-1+2\,T-T^2+T^3\right)}{T^4}, -\frac{1}{T^8}(-1+T)^2\left(1-6\,T+20\,T^2-48\,T^3+82\,T^4-114\,T^5+134\,T^6-140\,T^7+134\,T^8-114\,T^9+82\,T^{10}-48\,T^{11}+20\,T^{12}-6\,T^{13}+T^{14}\right)\right\},\right.\right.$$

$$\text{Knot}[12, \text{NonAlternating}, 369] \to \left\{-\frac{\left(1-T+T^2\right)\left(-1+T-2\,T^2+T^3\right)\left(-1+2\,T-T^2+T^3\right)}{T^4}, -\frac{1}{T^8}(-1+T)^2\left(1-6\,T+20\,T^2-48\,T^3+82\,T^4-114\,T^5+134\,T^6-140\,T^7+134\,T^8-114\,T^9+82\,T^{10}-48\,T^{11}+20\,T^{12}-6\,T^{13}+T^{14}\right)\right\}\right\},$$

$$\left\{\text{Knot}[11, \text{Alternating}, 19] \to \left\{-\frac{1-6\,T+18\,T^2-33\,T^3+39\,T^4-33\,T^5+18\,T^6-6\,T^7+T^8}{T^4}, -\frac{(-1+T)^6\left(1-6\,T+20\,T^2-40\,T^3+57\,T^4-62\,T^5+57\,T^6-40\,T^7+20\,T^8-6\,T^9+T^{10}\right)}{T^8}\right\},\right.$$

$$\text{Knot}[11, \text{Alternating}, 25] \to \left\{-\frac{1-6\,T+18\,T^2-33\,T^3+39\,T^4-33\,T^5+18\,T^6-6\,T^7+T^8}{T^4}, -\frac{(-1+T)^6\left(1-6\,T+20\,T^2-40\,T^3+57\,T^4-62\,T^5+57\,T^6-40\,T^7+20\,T^8-6\,T^9+T^{10}\right)}{T^8}\right\}\right\},$$

$$\left\{\text{Knot}[11, \text{Alternating}, 24] \to \left\{\frac{1-6\,T+18\,T^2-33\,T^3+41\,T^4-33\,T^5+18\,T^6-6\,T^7+T^8}{T^4}, -\frac{(-1+T)^2\left(1-T+T^2\right)\left(1-9\,T+33\,T^2-70\,T^3+88\,T^4-70\,T^5+33\,T^6-9\,T^7+T^8\right)}{T^6}\right\},\right.$$

$$\text{Knot}[11, \text{Alternating}, 26] \to \left\{\frac{1-6T+18T^2-33T^3+41T^4-33T^5+18T^6-6T^7+T^8}{T^4},\right.$$
$$\left.-\frac{(-1+T)^2\left(1-T+T^2\right)\left(1-9T+33T^2-70T^3+88T^4-70T^5+33T^6-9T^7+T^8\right)}{T^6}\right\},$$
$$\left\{\text{Knot}[11, \text{Alternating}, 44] \to \left\{\frac{\left(1-T+T^2\right)^2\left(1-3T+5T^2-3T^3+T^4\right)}{T^4},\right.\right.$$
$$\left.\frac{4(-1+T)^2\left(1-T+T^2\right)^2\left(1-3T+5T^2-3T^3+T^4\right)}{T^5}\right\}, \text{Knot}[11, \text{Alternating}, 47] \to$$
$$\left.\left\{\frac{\left(1-T+T^2\right)^2\left(1-3T+5T^2-3T^3+T^4\right)}{T^4}, \frac{4(-1+T)^2\left(1-T+T^2\right)^2\left(1-3T+5T^2-3T^3+T^4\right)}{T^5}\right\}\right\},$$
$$\left\{\text{Knot}[11, \text{Alternating}, 57] \to \left\{-\frac{\left(1-T+T^2\right)^2\left(1-3T+3T^2-3T^3+T^4\right)}{T^4},\right.\right.$$
$$\left.\frac{(-1+T)^2\left(1-T+T^2\right)^2\left(1-6T+15T^2-22T^3+21T^4-24T^5+21T^6-22T^7+15T^8-6T^9+T^{10}\right)}{T^8}\right\},$$
$$\text{Knot}[11, \text{Alternating}, 231] \to \left\{-\frac{\left(1-T+T^2\right)^2\left(1-3T+3T^2-3T^3+T^4\right)}{T^4},\right.$$
$$\left.\left.\frac{(-1+T)^2\left(1-T+T^2\right)^2\left(1-6T+15T^2-22T^3+21T^4-24T^5+21T^6-22T^7+15T^8-6T^9+T^{10}\right)}{T^8}\right\}\right\},$$
$$\left\{\text{Knot}[11, \text{Alternating}, 251] \to \left\{\frac{1-6T+16T^2-27T^3+33T^4-27T^5+16T^6-6T^7+T^8}{T^4},\right.\right.$$
$$\left.\frac{(-1+T)^2\left(1-6T+24T^2-64T^3+107T^4-128T^5+107T^6-64T^7+24T^8-6T^9+T^{10}\right)}{T^6}\right\},$$
$$\text{Knot}[11, \text{Alternating}, 253] \to \left\{\frac{1-6T+16T^2-27T^3+33T^4-27T^5+16T^6-6T^7+T^8}{T^4},\right.$$
$$\left.\left.\frac{(-1+T)^2\left(1-6T+24T^2-64T^3+107T^4-128T^5+107T^6-64T^7+24T^8-6T^9+T^{10}\right)}{T^6}\right\}\right\},$$
$$\left\{\text{Knot}[11, \text{Alternating}, 252] \to \left\{-\frac{1-6T+16T^2-27T^3+31T^4-27T^5+16T^6-6T^7+T^8}{T^4},\right.\right.$$
$$\left.-\frac{(-1+T)^2\left(1-T+T^2\right)^2\left(1-8T+25T^2-40T^3+39T^4-36T^5+39T^6-40T^7+25T^8-8T^9+T^{10}\right)}{T^8}\right\},$$
$$\text{Knot}[11, \text{Alternating}, 254] \to \left\{-\frac{1-6T+16T^2-27T^3+31T^4-27T^5+16T^6-6T^7+T^8}{T^4},\right.$$
$$\left.\left.-\frac{(-1+T)^2\left(1-T+T^2\right)^2\left(1-8T+25T^2-40T^3+39T^4-36T^5+39T^6-40T^7+25T^8-8T^9+T^{10}\right)}{T^8}\right\}\right\},$$
$$\left\{\text{Knot}[11, \text{NonAlternating}, 34] \to \left\{1, -\frac{2(-1+T)^2\left(1+T^4\right)}{T^3}\right\},\right.$$
$$\left.\text{Knot}[11, \text{NonAlternating}, 42] \to \left\{1, -\frac{2(-1+T)^2\left(1+T^4\right)}{T^3}\right\}\right\},$$

$$\left\{\text{Knot}[11, \text{NonAlternating}, 35] \to \left\{-\frac{2-10\,T+20\,T^2-25\,T^3+20\,T^4-10\,T^5+2\,T^6}{T^3},\right.\right.$$
$$\left.-\frac{(-1+T)^2\left(9-74\,T+248\,T^2-514\,T^3+768\,T^4-868\,T^5+768\,T^6-514\,T^7+248\,T^8-74\,T^9+9\,T^{10}\right)}{T^6}\right\},$$
$$\text{Knot}[11, \text{NonAlternating}, 43] \to \left\{-\frac{2-10\,T+20\,T^2-25\,T^3+20\,T^4-10\,T^5+2\,T^6}{T^3},\right.$$
$$\left.\left.-\frac{(-1+T)^2\left(9-74\,T+248\,T^2-514\,T^3+768\,T^4-868\,T^5+768\,T^6-514\,T^7+248\,T^8-74\,T^9+9\,T^{10}\right)}{T^6}\right\}\right\},$$
$$\left\{\text{Knot}[11, \text{NonAlternating}, 36] \to \left\{-\frac{1-4\,T+8\,T^2-13\,T^3+15\,T^4-13\,T^5+8\,T^6-4\,T^7+T^8}{T^4},\right.\right.$$
$$-\frac{1}{T^8}(-1+T)^2\left(1-6\,T+19\,T^2-42\,T^3+64\,T^4-76\,T^5+85\,T^6-84\,T^7+85\,T^8-76\,T^9+\right.$$
$$\left.\left.64\,T^{10}-42\,T^{11}+19\,T^{12}-6\,T^{13}+T^{14}\right)\right\}, \text{Knot}[11, \text{NonAlternating}, 44] \to$$
$$\left\{-\frac{1-4\,T+8\,T^2-13\,T^3+15\,T^4-13\,T^5+8\,T^6-4\,T^7+T^8}{T^4}, -\frac{1}{T^8}(-1+T)^2\left(1-6\,T+19\,T^2-\right.\right.$$
$$\left.\left.42\,T^3+64\,T^4-76\,T^5+85\,T^6-84\,T^7+85\,T^8-76\,T^9+64\,T^{10}-42\,T^{11}+19\,T^{12}-6\,T^{13}+T^{14}\right)\right\}\right\},$$
$$\left\{\text{Knot}[11, \text{NonAlternating}, 39] \to \left\{\frac{\left(2-2\,T+T^2\right)\left(1-2\,T+2\,T^2\right)}{T^2},\right.\right.$$
$$\left.-\frac{(-1+T)^2\left(1-3\,T+T^2\right)\left(1-T+T^2\right)\left(3-4\,T+3\,T^2\right)}{T^4}\right\}, \text{Knot}[11, \text{NonAlternating}, 45] \to$$
$$\left\{\frac{\left(2-2\,T+T^2\right)\left(1-2\,T+2\,T^2\right)}{T^2}, -\frac{(-1+T)^2\left(1-3\,T+T^2\right)\left(1-T+T^2\right)\left(3-4\,T+3\,T^2\right)}{T^4}\right\}\right\},$$
$$\left\{\text{Knot}[11, \text{NonAlternating}, 40] \to \left\{\frac{2-8\,T+18\,T^2-23\,T^3+18\,T^4-8\,T^5+2\,T^6}{T^3},\right.\right.$$
$$\left.-\frac{(-1+T)^2\left(5-30\,T+113\,T^2-264\,T^3+438\,T^4-512\,T^5+438\,T^6-264\,T^7+113\,T^8-30\,T^9+5\,T^{10}\right)}{T^6}\right\},$$
$$\text{Knot}[11, \text{NonAlternating}, 46] \to \left\{\frac{2-8\,T+18\,T^2-23\,T^3+18\,T^4-8\,T^5+2\,T^6}{T^3},\right.$$
$$\left.\left.-\frac{(-1+T)^2\left(5-30\,T+113\,T^2-264\,T^3+438\,T^4-512\,T^5+438\,T^6-264\,T^7+113\,T^8-30\,T^9+5\,T^{10}\right)}{T^6}\right\}\right\},$$
$$\left\{\text{Knot}[11, \text{NonAlternating}, 41] \to \left\{\frac{1-4\,T+8\,T^2-9\,T^3+9\,T^4-9\,T^5+8\,T^6-4\,T^7+T^8}{T^4},\right.\right.$$
$$-\frac{1}{T^8}(-1+T)^2\left(2-12\,T+36\,T^2-64\,T^3+83\,T^4-88\,T^5+93\,T^6-92\,T^7+93\,T^8-88\,T^9+\right.$$
$$\left.\left.83\,T^{10}-64\,T^{11}+36\,T^{12}-12\,T^{13}+2\,T^{14}\right)\right\}, \text{Knot}[11, \text{NonAlternating}, 47] \to$$
$$\left\{\frac{1-4\,T+8\,T^2-9\,T^3+9\,T^4-9\,T^5+8\,T^6-4\,T^7+T^8}{T^4}, -\frac{1}{T^8}(-1+T)^2\left(2-12\,T+36\,T^2-64\,T^3+\right.\right.$$
$$\left.\left.83\,T^4-88\,T^5+93\,T^6-92\,T^7+93\,T^8-88\,T^9+83\,T^{10}-64\,T^{11}+36\,T^{12}-12\,T^{13}+2\,T^{14}\right)\right\}\right\},$$

$$\left\{\text{Knot}[11, \text{NonAlternating}, 73] \to \left\{\frac{\left(1-T+T^2\right)^2}{T^2}, \frac{4\,(-1+T)^2\left(1-T+T^2\right)^2}{T^3}\right\},\right.$$
$$\left.\text{Knot}[11, \text{NonAlternating}, 74] \to \left\{\frac{\left(1-T+T^2\right)^2}{T^2}, \frac{4\,(-1+T)^2\left(1-T+T^2\right)^2}{T^3}\right\}\right\},$$
$$\left\{\text{Knot}[11, \text{NonAlternating}, 151] \to \left\{-\frac{2-6T+7T^2-6T^3+2T^4}{T^2},\right.\right.$$
$$\left.-\frac{(-1+T)^2\left(7-28T+50T^2-60T^3+50T^4-28T^5+7T^6\right)}{T^4}\right\}, \text{Knot}[11, \text{NonAlternating}, 152] \to$$
$$\left.\left\{-\frac{2-6T+7T^2-6T^3+2T^4}{T^2}, -\frac{(-1+T)^2\left(7-28T+50T^2-60T^3+50T^4-28T^5+7T^6\right)}{T^4}\right\}\right\},$$
$$\left\{\text{Knot}[12, \text{Alternating}, 7] \to \left\{-\frac{1-8T+28T^2-55T^3+67T^4-55T^5+28T^6-8T^7+T^8}{T^4},\right.\right.$$
$$\frac{1}{T^8}(-1+T)^2\left(1-3T+T^2\right)\left(1-11T+59T^2-186T^3+381T^4-555T^5+626T^6-\right.$$
$$\left.\left.555T^7+381T^8-186T^9+59T^{10}-11T^{11}+T^{12}\right)\right\}, \text{Knot}[12, \text{Alternating}, 14] \to$$
$$\left.\left\{-\frac{1-8T+28T^2-55T^3+67T^4-55T^5+28T^6-8T^7+T^8}{T^4}, \frac{1}{T^8}(-1+T)^2\left(1-3T+T^2\right)\left(1-11T+\right.\right.\right.$$
$$\left.\left.\left.59T^2-186T^3+381T^4-555T^5+626T^6-555T^7+381T^8-186T^9+59T^{10}-11T^{11}+T^{12}\right)\right\}\right\},$$
$$\left\{\text{Knot}[12, \text{Alternating}, 13] \to \left\{\frac{1-8T+28T^2-55T^3+69T^4-55T^5+28T^6-8T^7+T^8}{T^4},\right.\right.$$
$$\left.\left.\frac{(-2+T)(-1+T)^2(-1+2T)\left(2-15T+54T^2-112T^3+140T^4-112T^5+54T^6-15T^7+2T^8\right)}{T^6}\right\},\right.$$
$$\left.\text{Knot}[12, \text{Alternating}, 15] \to \left\{\frac{1-8T+28T^2-55T^3+69T^4-55T^5+28T^6-8T^7+T^8}{T^4},\right.\right.$$
$$\left.\left.\frac{(-2+T)(-1+T)^2(-1+2T)\left(2-15T+54T^2-112T^3+140T^4-112T^5+54T^6-15T^7+2T^8\right)}{T^6}\right\}\right\},$$
$$\left\{\text{Knot}[12, \text{Alternating}, 30] \to \left\{\frac{\left(1-T+T^2\right)^2\left(1-5T+9T^2-5T^3+T^4\right)}{T^4},\right.\right.$$
$$\left.\left.\frac{(-1+T)^2\left(1-T+T^2\right)\left(3-21T+77T^2-178T^3+240T^4-178T^5+77T^6-21T^7+3T^8\right)}{T^6}\right\},\right.$$
$$\left.\text{Knot}[12, \text{Alternating}, 33] \to \left\{\frac{\left(1-T+T^2\right)^2\left(1-5T+9T^2-5T^3+T^4\right)}{T^4},\right.\right.$$
$$\left.\left.\frac{(-1+T)^2\left(1-T+T^2\right)\left(3-21T+77T^2-178T^3+240T^4-178T^5+77T^6-21T^7+3T^8\right)}{T^6}\right\}\right\},$$
$$\left\{\text{Knot}[12, \text{Alternating}, 36] \to \left\{-\frac{\left(1-T+T^2\right)\left(2-8T+11T^2-11T^3+11T^4-8T^5+2T^6\right)}{T^4},\right.\right.$$
$$\left.-\frac{1}{T^8}(-1+T)^2\left(13-100T+340T^2-752T^3+1285T^4-1812T^5+2209T^6-\right.\right.$$

$$2348\,T^7+2209\,T^8-1812\,T^9+1285\,T^{10}-752\,T^{11}+340\,T^{12}-100\,T^{13}+13\,T^{14}\big)\Big\},$$

$$\text{Knot}[12,\text{Alternating},694]\to\Big\{-\frac{\left(1-T+T^2\right)\,\left(2-8\,T+11\,T^2-11\,T^3+11\,T^4-8\,T^5+2\,T^6\right)}{T^4},$$

$$-\frac{1}{T^8}\,(-1+T)^2\,\big(13-100\,T+340\,T^2-752\,T^3+1285\,T^4-1812\,T^5+2209\,T^6-$$

$$2348\,T^7+2209\,T^8-1812\,T^9+1285\,T^{10}-752\,T^{11}+340\,T^{12}-100\,T^{13}+13\,T^{14}\big)\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{Alternating},44]\to\Big\{-\frac{\left(2-3\,T+2\,T^2\right)\,\left(2-8\,T+11\,T^2-8\,T^3+2\,T^4\right)}{T^3},-\frac{1}{T^6}\,(-1+T)^2\,\big(38-$$

$$332\,T+1265\,T^2-2924\,T^3+4666\,T^4-5420\,T^5+4666\,T^6-2924\,T^7+1265\,T^8-332\,T^9+38\,T^{10}\big)\Big\},$$

$$\text{Knot}[12,\text{Alternating},64]\to\Big\{-\frac{\left(2-3\,T+2\,T^2\right)\,\left(2-8\,T+11\,T^2-8\,T^3+2\,T^4\right)}{T^3},-\frac{1}{T^6}\,(-1+T)^2\,\big(38-$$

$$332\,T+1265\,T^2-2924\,T^3+4666\,T^4-5420\,T^5+4666\,T^6-2924\,T^7+1265\,T^8-332\,T^9+38\,T^{10}\big)\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{Alternating},45]\to\Big\{-\frac{1-8\,T+22\,T^2-37\,T^3+43\,T^4-37\,T^5+22\,T^6-8\,T^7+T^8}{T^4},$$

$$\frac{1}{T^8}\,(-1+T)^2\,\big(1-14\,T+77\,T^2-230\,T^3+459\,T^4-672\,T^5+808\,T^6-$$

$$848\,T^7+808\,T^8-672\,T^9+459\,T^{10}-230\,T^{11}+77\,T^{12}-14\,T^{13}+T^{14}\big)\Big\},$$

$$\text{Knot}[12,\text{Alternating},65]\to\Big\{-\frac{1-8\,T+22\,T^2-37\,T^3+43\,T^4-37\,T^5+22\,T^6-8\,T^7+T^8}{T^4},$$

$$\frac{1}{T^8}\,(-1+T)^2\,\big(1-14\,T+77\,T^2-230\,T^3+459\,T^4-672\,T^5+808\,T^6-$$

$$848\,T^7+808\,T^8-672\,T^9+459\,T^{10}-230\,T^{11}+77\,T^{12}-14\,T^{13}+T^{14}\big)\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{Alternating},48]\to\Big\{\frac{1-8\,T+28\,T^2-57\,T^3+73\,T^4-57\,T^5+28\,T^6-8\,T^7+T^8}{T^4},$$

$$\frac{(-1+T)^2\,\left(2-13\,T+40\,T^2-75\,T^3+88\,T^4-75\,T^5+40\,T^6-13\,T^7+2\,T^8\right)}{T^5}\Big\},$$

$$\text{Knot}[12,\text{Alternating},60]\to\Big\{\frac{1-8\,T+28\,T^2-57\,T^3+73\,T^4-57\,T^5+28\,T^6-8\,T^7+T^8}{T^4},$$

$$\frac{(-1+T)^2\,\left(2-13\,T+40\,T^2-75\,T^3+88\,T^4-75\,T^5+40\,T^6-13\,T^7+2\,T^8\right)}{T^5}\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{Alternating},59]\to\Big\{-\frac{1-8\,T+28\,T^2-57\,T^3+71\,T^4-57\,T^5+28\,T^6-8\,T^7+T^8}{T^4},$$

$$-\frac{1}{T^8}\,(-1+T)^2\,\big(1-14\,T+91\,T^2-364\,T^3+996\,T^4-1972\,T^5+2927\,T^6-$$

$$3328\,T^7+2927\,T^8-1972\,T^9+996\,T^{10}-364\,T^{11}+91\,T^{12}-14\,T^{13}+T^{14}\big)\Big\},$$

$$\text{Knot}[12,\text{Alternating},63]\to\Big\{-\frac{1-8\,T+28\,T^2-57\,T^3+71\,T^4-57\,T^5+28\,T^6-8\,T^7+T^8}{T^4},$$

$$-\frac{1}{T^8}\,(-1+T)^2\,\big(1-14\,T+91\,T^2-364\,T^3+996\,T^4-1972\,T^5+2927\,T^6-$$

$$3328\,T^7+2927\,T^8-1972\,T^9+996\,T^{10}-364\,T^{11}+91\,T^{12}-14\,T^{13}+T^{14}\Big)\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{Alternating},67]\to\Big\{-\frac{1-8\,T+26\,T^2-47\,T^3+55\,T^4-47\,T^5+26\,T^6-8\,T^7+T^8}{T^4},$$

$$\frac{1}{T^8}(-1+T)^4\Big(1-12\,T+60\,T^2-164\,T^3+277\,T^4-334\,T^5+348\,T^6-334\,T^7+$$

$$277\,T^8-164\,T^9+60\,T^{10}-12\,T^{11}+T^{12}\Big)\Big\},\ \text{Knot}[12,\text{Alternating},136]\to$$

$$\Big\{-\frac{1-8\,T+26\,T^2-47\,T^3+55\,T^4-47\,T^5+26\,T^6-8\,T^7+T^8}{T^4},\ \frac{1}{T^8}(-1+T)^4\Big(1-12\,T+60\,T^2-$$

$$164\,T^3+277\,T^4-334\,T^5+348\,T^6-334\,T^7+277\,T^8-164\,T^9+60\,T^{10}-12\,T^{11}+T^{12}\Big)\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{Alternating},91]\to\Big\{\frac{1-8\,T+26\,T^2-47\,T^3+57\,T^4-47\,T^5+26\,T^6-8\,T^7+T^8}{T^4},$$

$$-\frac{1}{T^8}(-1+T)^2\Big(2-28\,T+170\,T^2-600\,T^3+1418\,T^4-2476\,T^5+3383\,T^6-$$

$$3736\,T^7+3383\,T^8-2476\,T^9+1418\,T^{10}-600\,T^{11}+170\,T^{12}-28\,T^{13}+2\,T^{14}\Big)\Big\},$$

$$\text{Knot}[12,\text{Alternating},111]\to\Big\{\frac{1-8\,T+26\,T^2-47\,T^3+57\,T^4-47\,T^5+26\,T^6-8\,T^7+T^8}{T^4},$$

$$-\frac{1}{T^8}(-1+T)^2\Big(2-28\,T+170\,T^2-600\,T^3+1418\,T^4-2476\,T^5+3383\,T^6-$$

$$3736\,T^7+3383\,T^8-2476\,T^9+1418\,T^{10}-600\,T^{11}+170\,T^{12}-28\,T^{13}+2\,T^{14}\Big)\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{Alternating},101]\to\Big\{\frac{1-7\,T+25\,T^2-50\,T^3+63\,T^4-50\,T^5+25\,T^6-7\,T^7+T^8}{T^4},$$

$$-\frac{(-1+T)^2\Big(6-46\,T+187\,T^2-482\,T^3+838\,T^4-1004\,T^5+838\,T^6-482\,T^7+187\,T^8-46\,T^9+6\,T^{10}\Big)}{T^6}\Big\},$$

$$\text{Knot}[12,\text{Alternating},115]\to\Big\{\frac{1-7\,T+25\,T^2-50\,T^3+63\,T^4-50\,T^5+25\,T^6-7\,T^7+T^8}{T^4},$$

$$-\frac{(-1+T)^2\Big(6-46\,T+187\,T^2-482\,T^3+838\,T^4-1004\,T^5+838\,T^6-482\,T^7+187\,T^8-46\,T^9+6\,T^{10}\Big)}{T^6}\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{Alternating},102]\to\Big\{\frac{7-27\,T+53\,T^2-65\,T^3+53\,T^4-27\,T^5+7\,T^6}{T^3},$$

$$-\frac{1}{T^6}(-1+T)^2\Big(174-934\,T+2711\,T^2-5312\,T^3+7760\,T^4-$$

$$8760\,T^5+7760\,T^6-5312\,T^7+2711\,T^8-934\,T^9+174\,T^{10}\Big)\Big\},$$

$$\text{Knot}[12,\text{Alternating},107]\to\Big\{\frac{7-27\,T+53\,T^2-65\,T^3+53\,T^4-27\,T^5+7\,T^6}{T^3},$$

$$-\frac{1}{T^6}(-1+T)^2\Big(174-934\,T+2711\,T^2-5312\,T^3+7760\,T^4-$$

$$8760\,T^5+7760\,T^6-5312\,T^7+2711\,T^8-934\,T^9+174\,T^{10}\Big)\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{Alternating},108]\to\Big\{-\frac{1-8\,T+26\,T^2-51\,T^3+63\,T^4-51\,T^5+26\,T^6-8\,T^7+T^8}{T^4},$$

$$-\frac{1}{T^8}(-1+T)^2\left(1-6T+6T^2-6T^3+T^4\right)\left(1-8T+31T^2-78T^3+132T^4-158T^5+132T^6-78T^7+31T^8-8T^9+T^{10}\right)\Big\},$$

$$\text{Knot}[12,\text{Alternating},120]\to\Big\{-\frac{1-8T+26T^2-51T^3+63T^4-51T^5+26T^6-8T^7+T^8}{T^4},$$
$$-\frac{1}{T^8}(-1+T)^2\left(1-6T+6T^2-6T^3+T^4\right)\left(1-8T+31T^2-78T^3+132T^4-158T^5+132T^6-78T^7+31T^8-8T^9+T^{10}\right)\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{Alternating},114]\to\Big\{-\frac{\left(1-T+T^2\right)\left(4-17T+25T^2-17T^3+4T^4\right)}{T^3},-\frac{1}{T^6}2(-1+T)^2$$
$$\left(19-156T+565T^2-1258T^3+1963T^4-2262T^5+1963T^6-1258T^7+565T^8-156T^9+19T^{10}\right)\Big\},$$

$$\text{Knot}[12,\text{Alternating},117]\to\Big\{-\frac{\left(1-T+T^2\right)\left(4-17T+25T^2-17T^3+4T^4\right)}{T^3},-\frac{1}{T^6}2(-1+T)^2$$
$$\left(19-156T+565T^2-1258T^3+1963T^4-2262T^5+1963T^6-1258T^7+565T^8-156T^9+19T^{10}\right)\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{Alternating},116]\to\Big\{-\frac{\left(1-T+T^2\right)^2\left(1-5T+7T^2-5T^3+T^4\right)}{T^4},$$
$$\frac{1}{T^8}(-1+T)^2\left(1-T+T^2\right)\left(1-11T+49T^2-128T^3+223T^4-277T^5+294T^6-277T^7+223T^8-128T^9+49T^{10}-11T^{11}+T^{12}\right)\Big\},$$

$$\text{Knot}[12,\text{Alternating},122]\to\Big\{-\frac{\left(1-T+T^2\right)^2\left(1-5T+7T^2-5T^3+T^4\right)}{T^4},$$
$$\frac{1}{T^8}(-1+T)^2\left(1-T+T^2\right)\left(1-11T+49T^2-128T^3+223T^4-277T^5+294T^6-277T^7+223T^8-128T^9+49T^{10}-11T^{11}+T^{12}\right)\Big\},$$

$$\text{Knot}[12,\text{Alternating},182]\to\Big\{-\frac{\left(1-T+T^2\right)^2\left(1-5T+7T^2-5T^3+T^4\right)}{T^4},$$
$$\frac{1}{T^8}(-1+T)^2\left(1-T+T^2\right)\left(1-11T+49T^2-128T^3+223T^4-277T^5+294T^6-277T^7+223T^8-128T^9+49T^{10}-11T^{11}+T^{12}\right)\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{Alternating},126]\to\Big\{\frac{1-8T+26T^2-49T^3+61T^4-49T^5+26T^6-8T^7+T^8}{T^4},$$
$$-\frac{2(-1+T)^6(1+T)^2\left(1-3T+T^2\right)\left(1-T+T^2\right)}{T^6}\Big\},$$

$$\text{Knot}[12,\text{Alternating},132]\to\Big\{\frac{1-8T+26T^2-49T^3+61T^4-49T^5+26T^6-8T^7+T^8}{T^4},$$
$$-\frac{2(-1+T)^6(1+T)^2\left(1-3T+T^2\right)\left(1-T+T^2\right)}{T^6}\Big\}\Big\},$$

$$\left\{\text{Knot}[12,\text{Alternating},131]\to\left\{-\frac{1-8T+26T^2-49T^3+59T^4-49T^5+26T^6-8T^7+T^8}{T^4},\right.\right.$$
$$\left.\frac{1}{T^8}(-1+T)^2\left(1-14T+83T^2-286T^3+655T^4-1088T^5+1418T^6-\right.\right.$$
$$\left.\left.1536T^7+1418T^8-1088T^9+655T^{10}-286T^{11}+83T^{12}-14T^{13}+T^{14}\right)\right\},$$
$$\text{Knot}[12,\text{Alternating},133]\to\left\{-\frac{1-8T+26T^2-49T^3+59T^4-49T^5+26T^6-8T^7+T^8}{T^4},\right.$$
$$\left.\left.\frac{1}{T^8}(-1+T)^2\left(1-14T+83T^2-286T^3+655T^4-1088T^5+1418T^6-\right.\right.\right.$$
$$\left.\left.\left.1536T^7+1418T^8-1088T^9+655T^{10}-286T^{11}+83T^{12}-14T^{13}+T^{14}\right)\right\}\right\},$$
$$\left\{\text{Knot}[12,\text{Alternating},134]\to\left\{\frac{1-8T+26T^2-47T^3+57T^4-47T^5+26T^6-8T^7+T^8}{T^4},\right.\right.$$
$$\left.\left.\frac{(-1+T)^2\left(2-22T+114T^2-348T^3+659T^4-804T^5+659T^6-348T^7+114T^8-22T^9+2T^{10}\right)}{T^6}\right\},\right.$$
$$\text{Knot}[12,\text{Alternating},188]\to\left\{\frac{1-8T+26T^2-47T^3+57T^4-47T^5+26T^6-8T^7+T^8}{T^4},\right.$$
$$\left.\left.\frac{(-1+T)^2\left(2-22T+114T^2-348T^3+659T^4-804T^5+659T^6-348T^7+114T^8-22T^9+2T^{10}\right)}{T^6}\right\}\right\},$$
$$\left\{\text{Knot}[12,\text{Alternating},154]\to\left\{-\frac{4-24T+56T^2-73T^3+56T^4-24T^5+4T^6}{T^3},-\frac{1}{T^6}2(-1+T)^2\right.\right.$$
$$\left.\left.\left(19-183T+742T^2-1766T^3+2856T^4-3328T^5+2856T^6-1766T^7+742T^8-183T^9+19T^{10}\right)\right\},\right.$$
$$\text{Knot}[12,\text{Alternating},162]\to\left\{-\frac{4-24T+56T^2-73T^3+56T^4-24T^5+4T^6}{T^3},-\frac{1}{T^6}2(-1+T)^2\right.$$
$$\left.\left.\left(19-183T+742T^2-1766T^3+2856T^4-3328T^5+2856T^6-1766T^7+742T^8-183T^9+19T^{10}\right)\right\}\right\},$$
$$\left\{\text{Knot}[12,\text{Alternating},164]\to\left\{-\frac{\left(1-T+T^2\right)^2\left(1-5T+7T^2-5T^3+T^4\right)}{T^4},\right.\right.$$
$$\left.\left.-\frac{(-1+T)^2\left(1-3T+T^2\right)\left(1-T+T^2\right)^2\left(1-7T+16T^2-33T^3+38T^4-33T^5+16T^6-7T^7+T^8\right)}{T^8}\right\},\right.$$
$$\text{Knot}[12,\text{Alternating},166]\to\left\{-\frac{\left(1-T+T^2\right)^2\left(1-5T+7T^2-5T^3+T^4\right)}{T^4},\right.$$
$$\left.\left.-\frac{(-1+T)^2\left(1-3T+T^2\right)\left(1-T+T^2\right)^2\left(1-7T+16T^2-33T^3+38T^4-33T^5+16T^6-7T^7+T^8\right)}{T^8}\right\}\right\},$$
$$\left\{\text{Knot}[12,\text{Alternating},167]\to\left\{\frac{\left(1-T+T^2\right)^2\left(2-4T+5T^2-4T^3+2T^4\right)}{T^4},\frac{1}{T^8}(-1+T)^2\right.\right.$$
$$\left.\left.\left(1-T+T^2\right)^2\left(9-34T+88T^2-144T^3+197T^4-212T^5+197T^6-144T^7+88T^8-34T^9+9T^{10}\right)\right\},\right.$$
$$\text{Knot}[12,\text{Alternating},692]\to\left\{\frac{\left(1-T+T^2\right)^2\left(2-4T+5T^2-4T^3+2T^4\right)}{T^4},\frac{1}{T^8}(-1+T)^2\right.$$
$$\left.\left.\left(1-T+T^2\right)^2\left(9-34T+88T^2-144T^3+197T^4-212T^5+197T^6-144T^7+88T^8-34T^9+9T^{10}\right)\right\}\right\},$$

$$\left\{\text{Knot}[12,\text{Alternating},195]\to\left\{\frac{\left(1-T+T^2\right)\left(1-6T+11T^2-11T^3+11T^4-6T^5+T^6\right)}{T^4},\right.\right.$$
$$\frac{1}{T^8}(-1+T)^2\left(2-24T+115T^2-308T^3+571T^4-840T^5+1025T^6-\right.$$
$$\left.\left.1092T^7+1025T^8-840T^9+571T^{10}-308T^{11}+115T^{12}-24T^{13}+2T^{14}\right)\right\},$$
$$\text{Knot}[12,\text{Alternating},693]\to\left\{\frac{\left(1-T+T^2\right)\left(1-6T+11T^2-11T^3+11T^4-6T^5+T^6\right)}{T^4},\right.$$
$$\frac{1}{T^8}(-1+T)^2\left(2-24T+115T^2-308T^3+571T^4-840T^5+1025T^6-\right.$$
$$\left.\left.1092T^7+1025T^8-840T^9+571T^{10}-308T^{11}+115T^{12}-24T^{13}+2T^{14}\right)\right\}\Big\},$$
$$\left\{\text{Knot}[12,\text{Alternating},273]\to\left\{-\frac{4-21T+47T^2-61T^3+47T^4-21T^5+4T^6}{T^3},0\right\},\right.$$
$$\left.\text{Knot}[12,\text{Alternating},890]\to\left\{-\frac{4-21T+47T^2-61T^3+47T^4-21T^5+4T^6}{T^3},0\right\}\right\},$$
$$\left\{\text{Knot}[12,\text{Alternating},341]\to\left\{\frac{1-8T+26T^2-49T^3+61T^4-49T^5+26T^6-8T^7+T^8}{T^4},0\right\},\right.$$
$$\left.\text{Knot}[12,\text{Alternating},627]\to\left\{\frac{1-8T+26T^2-49T^3+61T^4-49T^5+26T^6-8T^7+T^8}{T^4},0\right\}\right\},$$
$$\left\{\text{Knot}[12,\text{Alternating},427]\to\left\{\frac{\left(1-3T+T^2\right)^2\left(1-T+T^2\right)^2}{T^4},0\right\},\right.$$
$$\left.\text{Knot}[12,\text{Alternating},435]\to\left\{\frac{\left(1-3T+T^2\right)^2\left(1-T+T^2\right)^2}{T^4},0\right\},\right.$$
$$\left.\text{Knot}[12,\text{Alternating},990]\to\left\{\frac{\left(1-3T+T^2\right)^2\left(1-T+T^2\right)^2}{T^4},0\right\}\right\},$$
$$\left\{\text{Knot}[12,\text{Alternating},458]\to\left\{\frac{\left(1-4T+6T^2-5T^3+T^4\right)\left(1-5T+6T^2-4T^3+T^4\right)}{T^4},0\right\},\right.$$
$$\left.\text{Knot}[12,\text{Alternating},887]\to\left\{\frac{\left(1-4T+6T^2-5T^3+T^4\right)\left(1-5T+6T^2-4T^3+T^4\right)}{T^4},0\right\}\right\},$$
$$\left\{\text{Knot}[12,\text{Alternating},510]\to\left\{-\frac{4-20T+44T^2-57T^3+44T^4-20T^5+4T^6}{T^3},0\right\},\right.$$
$$\left.\text{Knot}[12,\text{Alternating},821]\to\left\{-\frac{4-20T+44T^2-57T^3+44T^4-20T^5+4T^6}{T^3},0\right\}\right\},$$
$$\left\{\text{Knot}[12,\text{Alternating},639]\to\left\{-\frac{2-10T+24T^2-40T^3+47T^4-40T^5+24T^6-10T^7+2T^8}{T^4},\right.\right.$$
$$-\frac{1}{T^8}(-1+T)^2\left(5-38T+148T^2-400T^3+814T^4-1306T^5+1722T^6-\right.$$
$$\left.\left.1880T^7+1722T^8-1306T^9+814T^{10}-400T^{11}+148T^{12}-38T^{13}+5T^{14}\right)\right\},$$
$$\text{Knot}[12,\text{Alternating},680]\to\left\{-\frac{2-10T+24T^2-40T^3+47T^4-40T^5+24T^6-10T^7+2T^8}{T^4},\right.$$

$$-\frac{1}{T^8}\,(-1+T)^2\,\left(5-38\,T+148\,T^2-400\,T^3+814\,T^4-1306\,T^5+1722\,T^6-\right.$$
$$\left.1880\,T^7+1722\,T^8-1306\,T^9+814\,T^{10}-400\,T^{11}+148\,T^{12}-38\,T^{13}+5\,T^{14}\right)\Big\}\Big\},$$

$$\left\{\text{Knot}[12,\text{Alternating},675]\to\left\{\frac{2-10\,T+26\,T^2-42\,T^3+49\,T^4-42\,T^5+26\,T^6-10\,T^7+2\,T^8}{T^4},\right.\right.$$
$$-\frac{1}{T^8}\,(-1+T)^2\,\left(9-70\,T+284\,T^2-756\,T^3+1497\,T^4-2340\,T^5+3020\,T^6-\right.$$
$$\left.3276\,T^7+3020\,T^8-2340\,T^9+1497\,T^{10}-756\,T^{11}+284\,T^{12}-70\,T^{13}+9\,T^{14}\right)\Big\},$$

$$\text{Knot}[12,\text{Alternating},688]\to\left\{\frac{2-10\,T+26\,T^2-42\,T^3+49\,T^4-42\,T^5+26\,T^6-10\,T^7+2\,T^8}{T^4},\right.$$
$$-\frac{1}{T^8}\,(-1+T)^2\,\left(9-70\,T+284\,T^2-756\,T^3+1497\,T^4-2340\,T^5+3020\,T^6-\right.$$
$$\left.3276\,T^7+3020\,T^8-2340\,T^9+1497\,T^{10}-756\,T^{11}+284\,T^{12}-70\,T^{13}+9\,T^{14}\right)\Big\}\Big\},$$

$$\left\{\text{Knot}[12,\text{Alternating},707]\to\left\{\frac{1-8\,T+29\,T^2-59\,T^3+75\,T^4-59\,T^5+29\,T^6-8\,T^7+T^8}{T^4},\right.\right.$$
$$\left.\frac{(-1+T)^2\,\left(5-48\,T+220\,T^2-614\,T^3+1121\,T^4-1364\,T^5+1121\,T^6-614\,T^7+220\,T^8-48\,T^9+5\,T^{10}\right)}{T^6}\right\},$$

$$\text{Knot}[12,\text{Alternating},935]\to\left\{\frac{1-8\,T+29\,T^2-59\,T^3+75\,T^4-59\,T^5+29\,T^6-8\,T^7+T^8}{T^4},\right.$$
$$\left.\left.\frac{(-1+T)^2\,\left(5-48\,T+220\,T^2-614\,T^3+1121\,T^4-1364\,T^5+1121\,T^6-614\,T^7+220\,T^8-48\,T^9+5\,T^{10}\right)}{T^6}\right\}\right\},$$

$$\left\{\text{Knot}[12,\text{Alternating},811]\to\left\{\frac{\left(1-T+T^2-T^3+T^4\right)\,\left(3-7\,T+9\,T^2-7\,T^3+3\,T^4\right)}{T^4},\right.\right.$$
$$-\frac{1}{T^8}\,(-1+T)^2\,\left(39-172\,T+465\,T^2-916\,T^3+1483\,T^4-2032\,T^5+2444\,T^6-\right.$$
$$\left.2588\,T^7+2444\,T^8-2032\,T^9+1483\,T^{10}-916\,T^{11}+465\,T^{12}-172\,T^{13}+39\,T^{14}\right)\Big\},$$

$$\text{Knot}[12,\text{Alternating},817]\to\left\{\frac{\left(1-T+T^2-T^3+T^4\right)\,\left(3-7\,T+9\,T^2-7\,T^3+3\,T^4\right)}{T^4},\right.$$
$$-\frac{1}{T^8}\,(-1+T)^2\,\left(39-172\,T+465\,T^2-916\,T^3+1483\,T^4-2032\,T^5+2444\,T^6-\right.$$
$$\left.2588\,T^7+2444\,T^8-2032\,T^9+1483\,T^{10}-916\,T^{11}+465\,T^{12}-172\,T^{13}+39\,T^{14}\right)\Big\}\Big\},$$

$$\left\{\text{Knot}[12,\text{Alternating},829]\to\left\{-\frac{2-11\,T+25\,T^2-37\,T^3+41\,T^4-37\,T^5+25\,T^6-11\,T^7+2\,T^8}{T^4},\right.\right.$$
$$-\frac{1}{T^8}\,(-1+T)^2\,\left(13-112\,T+421\,T^2-1002\,T^3+1793\,T^4-2604\,T^5+3220\,T^6-\right.$$
$$\left.3444\,T^7+3220\,T^8-2604\,T^9+1793\,T^{10}-1002\,T^{11}+421\,T^{12}-112\,T^{13}+13\,T^{14}\right)\Big\},$$

$$\text{Knot}[12,\text{Alternating},832]\to\left\{-\frac{2-11\,T+25\,T^2-37\,T^3+41\,T^4-37\,T^5+25\,T^6-11\,T^7+2\,T^8}{T^4},\right.$$

$$-\frac{1}{T^8}\,(-1+T)^2\,\left(13-112\,T+421\,T^2-1002\,T^3+1793\,T^4-2604\,T^5+3220\,T^6-\right.$$
$$\left.3444\,T^7+3220\,T^8-2604\,T^9+1793\,T^{10}-1002\,T^{11}+421\,T^{12}-112\,T^{13}+13\,T^{14}\right)\Big\}\Big\},$$

$$\left\{\text{Knot}[12,\text{Alternating},830]\to\left\{\frac{2-9\,T+24\,T^2-42\,T^3+51\,T^4-42\,T^5+24\,T^6-9\,T^7+2\,T^8}{T^4},\right.\right.$$
$$\frac{1}{T^8}\,(-1+T)^2\,\left(1-4\,T+17\,T^2-58\,T^3+163\,T^4-342\,T^5+538\,T^6-\right.$$
$$\left.624\,T^7+538\,T^8-342\,T^9+163\,T^{10}-58\,T^{11}+17\,T^{12}-4\,T^{13}+T^{14}\right)\Big\},$$

$$\text{Knot}[12,\text{Alternating},831]\to\left\{\frac{2-9\,T+24\,T^2-42\,T^3+51\,T^4-42\,T^5+24\,T^6-9\,T^7+2\,T^8}{T^4},\right.$$
$$\frac{1}{T^8}\,(-1+T)^2\,\left(1-4\,T+17\,T^2-58\,T^3+163\,T^4-342\,T^5+538\,T^6-\right.$$
$$\left.624\,T^7+538\,T^8-342\,T^9+163\,T^{10}-58\,T^{11}+17\,T^{12}-4\,T^{13}+T^{14}\right)\Big\}\Big\},$$

$$\left\{\text{Knot}[12,\text{Alternating},844]\to\left\{-\frac{1-8\,T+24\,T^2-43\,T^3+51\,T^4-43\,T^5+24\,T^6-8\,T^7+T^8}{T^4},\right.\right.$$
$$-\frac{1}{T^8}\,(-1+T)^2\,\left(1-T+T^2\right)\,\left(1-13\,T+69\,T^2-192\,T^3+331\,T^4-\right.$$
$$\left.403\,T^5+418\,T^6-403\,T^7+331\,T^8-192\,T^9+69\,T^{10}-13\,T^{11}+T^{12}\right)\Big\},$$

$$\text{Knot}[12,\text{Alternating},846]\to\left\{-\frac{1-8\,T+24\,T^2-43\,T^3+51\,T^4-43\,T^5+24\,T^6-8\,T^7+T^8}{T^4},\right.$$
$$-\frac{1}{T^8}\,(-1+T)^2\,\left(1-T+T^2\right)\,\left(1-13\,T+69\,T^2-192\,T^3+331\,T^4-\right.$$
$$\left.403\,T^5+418\,T^6-403\,T^7+331\,T^8-192\,T^9+69\,T^{10}-13\,T^{11}+T^{12}\right)\Big\}\Big\},$$

$$\left\{\text{Knot}[12,\text{Alternating},908]\to\left\{\frac{4-22\,T+55\,T^2-73\,T^3+55\,T^4-22\,T^5+4\,T^6}{T^3},\,-\frac{1}{T^6}\,(-1+T)^2\right.\right.$$
$$\left.\left(24-204\,T+845\,T^2-2128\,T^3+3599\,T^4-4260\,T^5+3599\,T^6-2128\,T^7+845\,T^8-204\,T^9+24\,T^{10}\right)\Big\},$$

$$\text{Knot}[12,\text{Alternating},1185]\to\left\{\frac{4-22\,T+55\,T^2-73\,T^3+55\,T^4-22\,T^5+4\,T^6}{T^3},\,-\frac{1}{T^6}\,(-1+T)^2\right.$$
$$\left.\left.\left(24-204\,T+845\,T^2-2128\,T^3+3599\,T^4-4260\,T^5+3599\,T^6-2128\,T^7+845\,T^8-204\,T^9+24\,T^{10}\right)\right\}\right\},$$

$$\left\{\text{Knot}[12,\text{NonAlternating},21]\to\left\{\frac{2-12\,T+28\,T^2-35\,T^3+28\,T^4-12\,T^5+2\,T^6}{T^3},\right.\right.$$
$$\left.\left.-\frac{(-1+T)^2\,\left(5-54\,T+235\,T^2-564\,T^3+889\,T^4-1028\,T^5+889\,T^6-564\,T^7+235\,T^8-54\,T^9+5\,T^{10}\right)}{T^6}\right\},\right.$$

$$\text{Knot}[12,\text{NonAlternating},29]\to\left\{\frac{2-12\,T+28\,T^2-35\,T^3+28\,T^4-12\,T^5+2\,T^6}{T^3},\right.$$
$$\left.\left.-\frac{(-1+T)^2\,\left(5-54\,T+235\,T^2-564\,T^3+889\,T^4-1028\,T^5+889\,T^6-564\,T^7+235\,T^8-54\,T^9+5\,T^{10}\right)}{T^6}\right\}\right\},$$

$$\left\{\text{Knot}[12,\text{NonAlternating},22]\to\left\{\frac{1-6\,T+16\,T^2-29\,T^3+37\,T^4-29\,T^5+16\,T^6-6\,T^7+T^8}{T^4},\right.\right.$$

$$\frac{(-1+T)^2\left(1-3T+3T^2-3T^3+T^4\right)\left(1-7T+22T^2-34T^3+22T^4-7T^5+T^6\right)}{T^6}\Big\},$$

$$\text{Knot}[12,\text{NonAlternating},30]\to\Big\{\frac{1-6T+16T^2-29T^3+37T^4-29T^5+16T^6-6T^7+T^8}{T^4},$$

$$\frac{(-1+T)^2\left(1-3T+3T^2-3T^3+T^4\right)\left(1-7T+22T^2-34T^3+22T^4-7T^5+T^6\right)}{T^6}\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{NonAlternating},23]\to\Big\{-\frac{(-2+T)(-1+2T)}{T},$$

$$-\frac{(-1+T)^2\left(2-10T+15T^2-12T^3+15T^4-10T^5+2T^6\right)}{T^4}\Big\},\ \text{Knot}[12,\text{NonAlternating},31]\to$$

$$\Big\{-\frac{(-2+T)(-1+2T)}{T},-\frac{(-1+T)^2\left(2-10T+15T^2-12T^3+15T^4-10T^5+2T^6\right)}{T^4}\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{NonAlternating},26]\to\Big\{-\frac{2-10T+22T^2-29T^3+22T^4-10T^5+2T^6}{T^3},$$

$$-\frac{(-1+T)^4\left(1-8T+35T^2-92T^3+116T^4-92T^5+35T^6-8T^7+T^8\right)}{T^6}\Big\},$$

$$\text{Knot}[12,\text{NonAlternating},32]\to\Big\{-\frac{2-10T+22T^2-29T^3+22T^4-10T^5+2T^6}{T^3},$$

$$-\frac{(-1+T)^4\left(1-8T+35T^2-92T^3+116T^4-92T^5+35T^6-8T^7+T^8\right)}{T^6}\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{NonAlternating},27]\to\Big\{-\frac{1-6T+16T^2-25T^3+27T^4-25T^5+16T^6-6T^7+T^8}{T^4},$$

$$-\frac{(-1+T)^4\left(1-8T+27T^2-50T^3+54T^4-46T^5+38T^6-46T^7+54T^8-50T^9+27T^{10}-8T^{11}+T^{12}\right)}{T^8}\Big\},$$

$$\text{Knot}[12,\text{NonAlternating},33]\to\Big\{-\frac{1-6T+16T^2-25T^3+27T^4-25T^5+16T^6-6T^7+T^8}{T^4},$$

$$-\frac{(-1+T)^4\left(1-8T+27T^2-50T^3+54T^4-46T^5+38T^6-46T^7+54T^8-50T^9+27T^{10}-8T^{11}+T^{12}\right)}{T^8}\Big\}\Big\}$$

$$\Big\{\text{Knot}[12,\text{NonAlternating},28]\to\Big\{\frac{2-8T+13T^2-8T^3+2T^4}{T^2},\frac{(-1+T)^6\left(5-12T+5T^2\right)}{T^4}\Big\},$$

$$\text{Knot}[12,\text{NonAlternating},34]\to\Big\{\frac{2-8T+13T^2-8T^3+2T^4}{T^2},\frac{(-1+T)^6\left(5-12T+5T^2\right)}{T^4}\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{NonAlternating},55]\to\Big\{\frac{\left(1-T+T^2\right)\left(2-9T+15T^2-9T^3+2T^4\right)}{T^3},$$

$$-\frac{(-1+T)^2\left(5-48T+199T^2-480T^3+777T^4-904T^5+777T^6-480T^7+199T^8-48T^9+5T^{10}\right)}{T^6}\Big\},$$

$$\text{Knot}[12,\text{NonAlternating},223]\to\Big\{\frac{\left(1-T+T^2\right)\left(2-9T+15T^2-9T^3+2T^4\right)}{T^3},$$

$$-\frac{(-1+T)^2\left(5-48T+199T^2-480T^3+777T^4-904T^5+777T^6-480T^7+199T^8-48T^9+5T^{10}\right)}{T^6}\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{NonAlternating},56]\to\Big\{\frac{\left(1-T+T^2\right)^2}{T^2},-\frac{4(-1+T)^2\left(1-3T+T^2\right)\left(1-T+T^2\right)}{T^3}\Big\},$$

$$\text{Knot}[12,\text{NonAlternating},57]\to\Big\{\frac{\left(1-T+T^2\right)^2}{T^2},-\frac{4(-1+T)^2\left(1-3T+T^2\right)\left(1-T+T^2\right)}{T^3}\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{NonAlternating},59]\to$$

$$\Big\{-\frac{\left(1-T+T^2\right)\left(1-2T-3T^2+7T^3-3T^4-2T^5+T^6\right)}{T^4},-\frac{1}{T^8}(-1+T)^2\left(3-12T+T^2+48T^3-70T^4-8T^5+160T^6-232T^7+160T^8-8T^9-70T^{10}+48T^{11}+T^{12}-12T^{13}+3T^{14}\right)\Big\},$$

$$\text{Knot}[12,\text{NonAlternating},220]\to\Big\{-\frac{\left(1-T+T^2\right)\left(1-2T-3T^2+7T^3-3T^4-2T^5+T^6\right)}{T^4},$$

$$-\frac{1}{T^8}(-1+T)^2\left(3-12T+T^2+48T^3-70T^4-8T^5+160T^6-232T^7+160T^8-8T^9-70T^{10}+48T^{11}+T^{12}-12T^{13}+3T^{14}\right)\Big\}\Big\},\ \Big\{\text{Knot}[12,\text{NonAlternating},60]\to$$

$$\Big\{-\frac{\left(1-T+T^2\right)^2\left(1-3T+3T^2-3T^3+T^4\right)}{T^4},-\frac{1}{T^8}(-1+T)^2\left(1-T+T^2\right)\left(1-7T+22T^2-43T^3+50T^4-43T^5+42T^6-43T^7+50T^8-43T^9+22T^{10}-7T^{11}+T^{12}\right)\Big\},$$

$$\text{Knot}[12,\text{NonAlternating},61]\to\Big\{-\frac{\left(1-T+T^2\right)^2\left(1-3T+3T^2-3T^3+T^4\right)}{T^4},$$

$$-\frac{1}{T^8}(-1+T)^2\left(1-T+T^2\right)\left(1-7T+22T^2-43T^3+50T^4-43T^5+42T^6-43T^7+50T^8-43T^9+22T^{10}-7T^{11}+T^{12}\right)\Big\},$$

$$\text{Knot}[12,\text{NonAlternating},219]\to\Big\{-\frac{\left(1-T+T^2\right)^2\left(1-3T+3T^2-3T^3+T^4\right)}{T^4},$$

$$-\frac{1}{T^8}(-1+T)^2\left(1-T+T^2\right)\left(1-7T+22T^2-43T^3+50T^4-43T^5+42T^6-43T^7+50T^8-43T^9+22T^{10}-7T^{11}+T^{12}\right)\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{NonAlternating},62]\to\Big\{-\frac{(-2+T)(-1+2T)\left(1-T+T^2\right)^2}{T^3},$$

$$-\frac{(-1+T)^2\left(1-T+T^2\right)\left(1-7T+27T^2-78T^3+116T^4-78T^5+27T^6-7T^7+T^8\right)}{T^6}\Big\},$$

$$\text{Knot}[12,\text{NonAlternating},66]\to\Big\{-\frac{(-2+T)(-1+2T)\left(1-T+T^2\right)^2}{T^3},$$

$$-\frac{(-1+T)^2\left(1-T+T^2\right)\left(1-7T+27T^2-78T^3+116T^4-78T^5+27T^6-7T^7+T^8\right)}{T^6}\Big\}\Big\},$$

$$\left\{\text{Knot}[12,\text{NonAlternating},64]\to\left\{\frac{\left(1-T+T^2\right)\left(1-4T+5T^2-3T^3+5T^4-4T^5+T^6\right)}{T^4},\right.\right.$$
$$\left.-\frac{2(-1+T)^4\left(1-2T+T^2-2T^3+T^4\right)\left(1-4T+4T^2-2T^3+T^4-2T^5+4T^6-4T^7+T^8\right)}{T^8}\right\},$$
$$\text{Knot}[12,\text{NonAlternating},261]\to\left\{\frac{\left(1-T+T^2\right)\left(1-4T+5T^2-3T^3+5T^4-4T^5+T^6\right)}{T^4},\right.$$
$$\left.\left.-\frac{2(-1+T)^4\left(1-2T+T^2-2T^3+T^4\right)\left(1-4T+4T^2-2T^3+T^4-2T^5+4T^6-4T^7+T^8\right)}{T^8}\right\}\right\},$$
$$\left\{\text{Knot}[12,\text{NonAlternating},85]\to\left\{-\frac{2-8T+14T^2-17T^3+14T^4-8T^5+2T^6}{T^3},\right.\right.$$
$$\left.\frac{(-1+T)^2\left(1-6T+25T^2-84T^3+170T^4-208T^5+170T^6-84T^7+25T^8-6T^9+T^{10}\right)}{T^6}\right\},$$
$$\text{Knot}[12,\text{NonAlternating},130]\to\left\{-\frac{2-8T+14T^2-17T^3+14T^4-8T^5+2T^6}{T^3},\right.$$
$$\left.\left.\frac{(-1+T)^2\left(1-6T+25T^2-84T^3+170T^4-208T^5+170T^6-84T^7+25T^8-6T^9+T^{10}\right)}{T^6}\right\}\right\},$$
$$\left\{\text{Knot}[12,\text{NonAlternating},86]\to\left\{\frac{2-10T+24T^2-31T^3+24T^4-10T^5+2T^6}{T^3},\right.\right.$$
$$\left.-\frac{(-1+T)^2\left(5-42T+172T^2-422T^3+700T^4-820T^5+700T^6-422T^7+172T^8-42T^9+5T^{10}\right)}{T^6}\right\},$$
$$\text{Knot}[12,\text{NonAlternating},131]\to\left\{\frac{2-10T+24T^2-31T^3+24T^4-10T^5+2T^6}{T^3},\right.$$
$$\left.\left.-\frac{(-1+T)^2\left(5-42T+172T^2-422T^3+700T^4-820T^5+700T^6-422T^7+172T^8-42T^9+5T^{10}\right)}{T^6}\right\}\right\},$$
$$\left\{\text{Knot}[12,\text{NonAlternating},87]\to\left\{\frac{\left(2-3T+2T^2\right)^2}{T^2},\right.\right.$$
$$\left.-\frac{2(-1+T)^2\left(2-3T+2T^2\right)\left(3-10T+16T^2-10T^3+3T^4\right)}{T^4}\right\},\ \text{Knot}[12,\text{NonAlternating},132]\to$$
$$\left\{\frac{\left(2-3T+2T^2\right)^2}{T^2},-\frac{2(-1+T)^2\left(2-3T+2T^2\right)\left(3-10T+16T^2-10T^3+3T^4\right)}{T^4}\right\}\right\},$$
$$\left\{\text{Knot}[12,\text{NonAlternating},88]\to\left\{\frac{\left(2-3T+2T^2\right)\left(2-4T+5T^2-4T^3+2T^4\right)}{T^3},-\frac{1}{T^6}(-1+T)^2\right.\right.$$
$$\left.\left.\left(54-252T+669T^2-1224T^3+1722T^4-1916T^5+1722T^6-1224T^7+669T^8-252T^9+54T^{10}\right)\right\},\right.$$
$$\text{Knot}[12,\text{NonAlternating},133]\to\left\{\frac{\left(2-3T+2T^2\right)\left(2-4T+5T^2-4T^3+2T^4\right)}{T^3},-\frac{1}{T^6}(-1+T)^2\right.$$
$$\left.\left.\left.\left(54-252T+669T^2-1224T^3+1722T^4-1916T^5+1722T^6-1224T^7+669T^8-252T^9+54T^{10}\right)\right\}\right\},$$
$$\left\{\text{Knot}[12,\text{NonAlternating},89]\to\left\{-\frac{1-4T+6T^2-7T^3+7T^4-7T^5+6T^6-4T^7+T^8}{T^4},\right.\right.$$

$$-\frac{1}{T^8}\,(-1+T)^2\left(3-18\,T+41\,T^2-62\,T^3+79\,T^4-86\,T^5+95\,T^6-92\,T^7+95\,T^8-86\,T^9+79\,T^{10}-62\,T^{11}+41\,T^{12}-18\,T^{13}+3\,T^{14}\right)\Big\},\ \text{Knot}[12,\text{NonAlternating},134]\to\Big\{-\frac{1-4\,T+6\,T^2-7\,T^3+7\,T^4-7\,T^5+6\,T^6-4\,T^7+T^8}{T^4},\ -\frac{1}{T^8}\,(-1+T)^2\left(3-18\,T+41\,T^2-62\,T^3+79\,T^4-86\,T^5+95\,T^6-92\,T^7+95\,T^8-86\,T^9+79\,T^{10}-62\,T^{11}+41\,T^{12}-18\,T^{13}+3\,T^{14}\right)\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{NonAlternating},90]\to\Big\{\frac{1-4\,T+10\,T^2-15\,T^3+17\,T^4-15\,T^5+10\,T^6-4\,T^7+T^8}{T^4},\ \frac{1}{T^8}\,(-1+T)^2\left(2-12\,T+44\,T^2-100\,T^3+176\,T^4-238\,T^5+283\,T^6-292\,T^7+283\,T^8-238\,T^9+176\,T^{10}-100\,T^{11}+44\,T^{12}-12\,T^{13}+2\,T^{14}\right)\Big\},$$

$$\text{Knot}[12,\text{NonAlternating},135]\to\Big\{\frac{1-4\,T+10\,T^2-15\,T^3+17\,T^4-15\,T^5+10\,T^6-4\,T^7+T^8}{T^4},\ \frac{1}{T^8}\,(-1+T)^2\left(2-12\,T+44\,T^2-100\,T^3+176\,T^4-238\,T^5+283\,T^6-292\,T^7+283\,T^8-238\,T^9+176\,T^{10}-100\,T^{11}+44\,T^{12}-12\,T^{13}+2\,T^{14}\right)\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{NonAlternating},91]\to\Big\{\frac{1-6\,T^2+15\,T^3-19\,T^4+15\,T^5-6\,T^6+T^8}{T^4},\ -\frac{1}{T^8}2\,(-1+T)^2\left(1+T^2\right)\left(2+4\,T-18\,T^2+8\,T^3+76\,T^4-202\,T^5+267\,T^6-202\,T^7+76\,T^8+8\,T^9-18\,T^{10}+4\,T^{11}+2\,T^{12}\right)\Big\},$$

$$\text{Knot}[12,\text{NonAlternating},136]\to\Big\{\frac{1-6\,T^2+15\,T^3-19\,T^4+15\,T^5-6\,T^6+T^8}{T^4},\ -\frac{1}{T^8}2\,(-1+T)^2\left(1+T^2\right)\left(2+4\,T-18\,T^2+8\,T^3+76\,T^4-202\,T^5+267\,T^6-202\,T^7+76\,T^8+8\,T^9-18\,T^{10}+4\,T^{11}+2\,T^{12}\right)\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{NonAlternating},92]\to\Big\{\frac{1-4\,T+12\,T^2-23\,T^3+29\,T^4-23\,T^5+12\,T^6-4\,T^7+T^8}{T^4},\ -\frac{(-1+T)^2\left(5-22\,T+50\,T^2-64\,T^3+50\,T^4-22\,T^5+5\,T^6\right)}{T^4}\Big\},$$

$$\text{Knot}[12,\text{NonAlternating},137]\to\Big\{\frac{1-4\,T+12\,T^2-23\,T^3+29\,T^4-23\,T^5+12\,T^6-4\,T^7+T^8}{T^4},\ -\frac{(-1+T)^2\left(5-22\,T+50\,T^2-64\,T^3+50\,T^4-22\,T^5+5\,T^6\right)}{T^4}\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{NonAlternating},93]\to\Big\{-\frac{1-4\,T+4\,T^2+T^3-5\,T^4+T^5+4\,T^6-4\,T^7+T^8}{T^4},\ -\frac{1}{T^8}\,(-1+T)^2\,(1+T)^2\left(3-24\,T+76\,T^2-134\,T^3+152\,T^4-124\,T^5+104\,T^6-124\,T^7+152\,T^8-134\,T^9+76\,T^{10}-24\,T^{11}+3\,T^{12}\right)\Big\},\ \text{Knot}[12,\text{NonAlternating},138]\to$$

$$\left\{-\frac{1-4\,T+4\,T^2+T^3-5\,T^4+T^5+4\,T^6-4\,T^7+T^8}{T^4},\ -\frac{1}{T^8}\,(-1+T)^2\,(1+T)^2\,\left(3-24\,T+76\,T^2-134\,T^3+152\,T^4-124\,T^5+104\,T^6-124\,T^7+152\,T^8-134\,T^9+76\,T^{10}-24\,T^{11}+3\,T^{12}\right)\right\}\right\},$$

$$\left\{\text{Knot}[12,\text{NonAlternating},98]\to\left\{\frac{1-6\,T+14\,T^2-17\,T^3+17\,T^4-17\,T^5+14\,T^6-6\,T^7+T^8}{T^4},\ -\frac{1}{T^8}\,(-1+T)^2\,\left(2-20\,T+85\,T^2-196\,T^3+285\,T^4-328\,T^5+357\,T^6-368\,T^7+357\,T^8-328\,T^9+285\,T^{10}-196\,T^{11}+85\,T^{12}-20\,T^{13}+2\,T^{14}\right)\right\},\right.$$

$$\text{Knot}[12,\text{NonAlternating},125]\to\left\{\frac{1-6\,T+14\,T^2-17\,T^3+17\,T^4-17\,T^5+14\,T^6-6\,T^7+T^8}{T^4},\ -\frac{1}{T^8}\,(-1+T)^2\,\left(2-20\,T+85\,T^2-196\,T^3+285\,T^4-328\,T^5+357\,T^6-368\,T^7+357\,T^8-328\,T^9+285\,T^{10}-196\,T^{11}+85\,T^{12}-20\,T^{13}+2\,T^{14}\right)\right\}\right\},$$

$$\left\{\text{Knot}[12,\text{NonAlternating},99]\to\left\{\frac{2-12\,T+32\,T^2-43\,T^3+32\,T^4-12\,T^5+2\,T^6}{T^3},\ -\frac{(-1+T)^2\,\left(5-54\,T+255\,T^2-688\,T^3+1205\,T^4-1436\,T^5+1205\,T^6-688\,T^7+255\,T^8-54\,T^9+5\,T^{10}\right)}{T^6}\right\},\right.$$

$$\text{Knot}[12,\text{NonAlternating},126]\to\left\{\frac{2-12\,T+32\,T^2-43\,T^3+32\,T^4-12\,T^5+2\,T^6}{T^3},\ -\frac{(-1+T)^2\,\left(5-54\,T+255\,T^2-688\,T^3+1205\,T^4-1436\,T^5+1205\,T^6-688\,T^7+255\,T^8-54\,T^9+5\,T^{10}\right)}{T^6}\right\}\right\}$$

$$\left\{\text{Knot}[12,\text{NonAlternating},122]\to\left\{-\frac{1-6\,T+14\,T^2-21\,T^3+23\,T^4-21\,T^5+14\,T^6-6\,T^7+T^8}{T^4},\ -\frac{1}{T^8}\,(-1+T)^2\,\left(1-10\,T+42\,T^2-98\,T^3+146\,T^4-152\,T^5+133\,T^6-120\,T^7+133\,T^8-152\,T^9+146\,T^{10}-98\,T^{11}+42\,T^{12}-10\,T^{13}+T^{14}\right)\right\},\right.$$

$$\text{Knot}[12,\text{NonAlternating},127]\to\left\{-\frac{1-6\,T+14\,T^2-21\,T^3+23\,T^4-21\,T^5+14\,T^6-6\,T^7+T^8}{T^4},\ -\frac{1}{T^8}\,(-1+T)^2\,\left(1-10\,T+42\,T^2-98\,T^3+146\,T^4-152\,T^5+133\,T^6-120\,T^7+133\,T^8-152\,T^9+146\,T^{10}-98\,T^{11}+42\,T^{12}-10\,T^{13}+T^{14}\right)\right\}\right\},$$

$$\left\{\text{Knot}[12,\text{NonAlternating},123]\to\left\{-\frac{2-14\,T+34\,T^2-45\,T^3+34\,T^4-14\,T^5+2\,T^6}{T^3},\ -\frac{1}{T^6}\,(-1+T)^2\,\left(9-114\,T+542\,T^2-1394\,T^3+2353\,T^4-2776\,T^5+2353\,T^6-1394\,T^7+542\,T^8-114\,T^9+9\,T^{10}\right)\right\},\right.$$

$$\text{Knot}[12,\text{NonAlternating},128]\to\left\{-\frac{2-14\,T+34\,T^2-45\,T^3+34\,T^4-14\,T^5+2\,T^6}{T^3},\ -\frac{1}{T^6}\,(-1+T)^2\,\left(9-114\,T+542\,T^2-1394\,T^3+2353\,T^4-2776\,T^5+2353\,T^6-1394\,T^7+542\,T^8-114\,T^9+9\,T^{10}\right)\right\}\right\},$$

$$\left\{\text{Knot}[12,\text{NonAlternating},124]\to\left\{\frac{2-3\,T+2\,T^2}{T},\right.\right.$$

$$-\frac{(-1+T)^2\,\left(1+T^2\right)\,\left(2-6\,T+5\,T^2-6\,T^3+2\,T^4\right)}{T^4}\Big\},\ \text{Knot}[12,\ \text{NonAlternating},\ 129]\to$$
$$\Big\{\frac{2-3\,T+2\,T^2}{T},\ -\frac{(-1+T)^2\,\left(1+T^2\right)\,\left(2-6\,T+5\,T^2-6\,T^3+2\,T^4\right)}{T^4}\Big\}\Big\},$$
$$\Big\{\text{Knot}[12,\ \text{NonAlternating},\ 144]\to\Big\{\frac{\left(1-3\,T+T^2\right)^2\,\left(1-T+T^2\right)}{T^3},$$
$$\frac{(-1+T)^2\,\left(1+T^2\right)\,\left(1-3\,T+T^2\right)^2\,\left(1-10\,T+15\,T^2-10\,T^3+T^4\right)}{T^6}\Big\},$$
$$\text{Knot}[12,\ \text{NonAlternating},\ 507]\to\Big\{\frac{\left(1-3\,T+T^2\right)^2\,\left(1-T+T^2\right)}{T^3},$$
$$\frac{(-1+T)^2\,\left(1+T^2\right)\,\left(1-3\,T+T^2\right)^2\,\left(1-10\,T+15\,T^2-10\,T^3+T^4\right)}{T^6}\Big\}\Big\},$$
$$\Big\{\text{Knot}[12,\ \text{NonAlternating},\ 205]\to\Big\{\frac{1-5\,T+11\,T^2-14\,T^3+15\,T^4-14\,T^5+11\,T^6-5\,T^7+T^8}{T^4},$$
$$-\frac{1}{T^8}\,(-1+T)^2\,\left(2-16\,T+55\,T^2-110\,T^3+153\,T^4-172\,T^5+176\,T^6-\right.$$
$$\left.172\,T^7+176\,T^8-172\,T^9+153\,T^{10}-110\,T^{11}+55\,T^{12}-16\,T^{13}+2\,T^{14}\right)\Big\},$$
$$\text{Knot}[12,\ \text{NonAlternating},\ 226]\to\Big\{\frac{1-5\,T+11\,T^2-14\,T^3+15\,T^4-14\,T^5+11\,T^6-5\,T^7+T^8}{T^4},$$
$$-\frac{1}{T^8}\,(-1+T)^2\,\left(2-16\,T+55\,T^2-110\,T^3+153\,T^4-172\,T^5+176\,T^6-\right.$$
$$\left.172\,T^7+176\,T^8-172\,T^9+153\,T^{10}-110\,T^{11}+55\,T^{12}-16\,T^{13}+2\,T^{14}\right)\Big\}\Big\},$$
$$\Big\{\text{Knot}[12,\ \text{NonAlternating},\ 206]\to\Big\{-\frac{1-5\,T+13\,T^2-24\,T^3+29\,T^4-24\,T^5+13\,T^6-5\,T^7+T^8}{T^4},$$
$$\frac{1}{T^8}\,(-1+T)^2\,\left(1-8\,T+31\,T^2-78\,T^3+133\,T^4-170\,T^5+181\,T^6-\right.$$
$$\left.176\,T^7+181\,T^8-170\,T^9+133\,T^{10}-78\,T^{11}+31\,T^{12}-8\,T^{13}+T^{14}\right)\Big\},$$
$$\text{Knot}[12,\ \text{NonAlternating},\ 227]\to\Big\{-\frac{1-5\,T+13\,T^2-24\,T^3+29\,T^4-24\,T^5+13\,T^6-5\,T^7+T^8}{T^4},$$
$$\frac{1}{T^8}\,(-1+T)^2\,\left(1-8\,T+31\,T^2-78\,T^3+133\,T^4-170\,T^5+181\,T^6-\right.$$
$$\left.176\,T^7+181\,T^8-170\,T^9+133\,T^{10}-78\,T^{11}+31\,T^{12}-8\,T^{13}+T^{14}\right)\Big\}\Big\},$$
$$\Big\{\text{Knot}[12,\ \text{NonAlternating},\ 207]\to\Big\{-\frac{1-3\,T-T^2+12\,T^3-19\,T^4+12\,T^5-T^6-3\,T^7+T^8}{T^4},$$
$$-\frac{1}{T^8}\,(-1+T)^2\,\left(3-12\,T-4\,T^2+70\,T^3-91\,T^4-60\,T^5+335\,T^6-\right.$$
$$\left.472\,T^7+335\,T^8-60\,T^9-91\,T^{10}+70\,T^{11}-4\,T^{12}-12\,T^{13}+3\,T^{14}\right)\Big\},$$
$$\text{Knot}[12,\ \text{NonAlternating},\ 228]\to\Big\{-\frac{1-3\,T-T^2+12\,T^3-19\,T^4+12\,T^5-T^6-3\,T^7+T^8}{T^4},$$

$$-\frac{1}{T^8}\,(-1+T)^2\,\left(3-12\,T-4\,T^2+70\,T^3-91\,T^4-60\,T^5+335\,T^6-\right.$$
$$\left.472\,T^7+335\,T^8-60\,T^9-91\,T^{10}+70\,T^{11}-4\,T^{12}-12\,T^{13}+3\,T^{14}\right)\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{NonAlternating},208]\to\Big\{\frac{3-11\,T+23\,T^2-29\,T^3+23\,T^4-11\,T^5+3\,T^6}{T^3},$$
$$\frac{(-1+T)^2\,\left(12-62\,T+219\,T^2-494\,T^3+805\,T^4-936\,T^5+805\,T^6-494\,T^7+219\,T^8-62\,T^9+12\,T^{10}\right)}{T^6}\Big\},$$

$$\text{Knot}[12,\text{NonAlternating},212]\to\Big\{\frac{3-11\,T+23\,T^2-29\,T^3+23\,T^4-11\,T^5+3\,T^6}{T^3},$$
$$\frac{(-1+T)^2\,\left(12-62\,T+219\,T^2-494\,T^3+805\,T^4-936\,T^5+805\,T^6-494\,T^7+219\,T^8-62\,T^9+12\,T^{10}\right)}{T^6}\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{NonAlternating},209]\to\Big\{-\frac{3-15\,T+31\,T^2-39\,T^3+31\,T^4-15\,T^5+3\,T^6}{T^3},-\frac{1}{T^6}(-1+T)^2$$
$$\left(21-174\,T+603\,T^2-1296\,T^3+1980\,T^4-2264\,T^5+1980\,T^6-1296\,T^7+603\,T^8-174\,T^9+21\,T^{10}\right)\Big\},$$

$$\text{Knot}[12,\text{NonAlternating},213]\to\Big\{-\frac{3-15\,T+31\,T^2-39\,T^3+31\,T^4-15\,T^5+3\,T^6}{T^3},-\frac{1}{T^6}(-1+T)^2$$
$$\left(21-174\,T+603\,T^2-1296\,T^3+1980\,T^4-2264\,T^5+1980\,T^6-1296\,T^7+603\,T^8-174\,T^9+21\,T^{10}\right)\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{NonAlternating},210]\to\Big\{\frac{\left(1-T+T^3\right)\,\left(1-T^2+T^3\right)}{T^3},$$
$$\frac{3\,(-1+T)^2\,\left(1-2\,T+T^2+2\,T^3-T^4-T^6+2\,T^7+T^8-2\,T^9+T^{10}\right)}{T^6}\Big\},$$

$$\text{Knot}[12,\text{NonAlternating},214]\to\Big\{\frac{\left(1-T+T^3\right)\,\left(1-T^2+T^3\right)}{T^3},$$
$$\frac{3\,(-1+T)^2\,\left(1-2\,T+T^2+2\,T^3-T^4-T^6+2\,T^7+T^8-2\,T^9+T^{10}\right)}{T^6}\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{NonAlternating},231]\to\Big\{-\frac{2-8\,T+11\,T^2-8\,T^3+2\,T^4}{T^2},$$
$$\frac{(-1+T)^4\,\left(9-34\,T+38\,T^2-34\,T^3+9\,T^4\right)}{T^4}\Big\},\text{Knot}[12,\text{NonAlternating},232]\to$$
$$\Big\{-\frac{2-8\,T+11\,T^2-8\,T^3+2\,T^4}{T^2},\frac{(-1+T)^4\,\left(9-34\,T+38\,T^2-34\,T^3+9\,T^4\right)}{T^4}\Big\}\Big\},$$

$$\Big\{\text{Knot}[12,\text{NonAlternating},252]\to\Big\{-\frac{2-10\,T+20\,T^2-25\,T^3+20\,T^4-10\,T^5+2\,T^6}{T^3},$$
$$\frac{(-1+T)^2\,\left(1-8\,T+36\,T^2-128\,T^3+276\,T^4-348\,T^5+276\,T^6-128\,T^7+36\,T^8-8\,T^9+T^{10}\right)}{T^6}\Big\},$$

$$\text{Knot}[12,\text{NonAlternating},262]\to\Big\{-\frac{2-10\,T+20\,T^2-25\,T^3+20\,T^4-10\,T^5+2\,T^6}{T^3},$$
$$\frac{(-1+T)^2\,\left(1-8\,T+36\,T^2-128\,T^3+276\,T^4-348\,T^5+276\,T^6-128\,T^7+36\,T^8-8\,T^9+T^{10}\right)}{T^6}\Big\}\Big\},$$

$$\left\{\text{Knot}[12, \text{NonAlternating}, 255] \to \left\{\frac{2-12\,T+30\,T^2-39\,T^3+30\,T^4-12\,T^5+2\,T^6}{T^3},\right.\right.$$
$$\left.-\frac{(-1+T)^4\left(5-42\,T+146\,T^2-276\,T^3+332\,T^4-276\,T^5+146\,T^6-42\,T^7+5\,T^8\right)}{T^6}\right\},$$
$$\text{Knot}[12, \text{NonAlternating}, 263] \to \left\{\frac{2-12\,T+30\,T^2-39\,T^3+30\,T^4-12\,T^5+2\,T^6}{T^3},\right.$$
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$$\left\{\text{Knot}[12, \text{NonAlternating}, 256] \to \left\{\frac{\left(2-2\,T+T^2\right)\left(1-2\,T+2\,T^2\right)}{T^2},\right.\right.$$
$$\left.-\frac{(-1+T)^2\left(2-3\,T-18\,T^2+66\,T^3-92\,T^4+66\,T^5-18\,T^6-3\,T^7+2\,T^8\right)}{T^5}\right\},$$
$$\text{Knot}[12, \text{NonAlternating}, 264] \to \left\{\frac{\left(2-2\,T+T^2\right)\left(1-2\,T+2\,T^2\right)}{T^2},\right.$$
$$\left.\left.-\frac{(-1+T)^2\left(2-3\,T-18\,T^2+66\,T^3-92\,T^4+66\,T^5-18\,T^6-3\,T^7+2\,T^8\right)}{T^5}\right\}\right\},$$
$$\left\{\text{Knot}[12, \text{NonAlternating}, 313] \to \left\{1, -\frac{2\,(-1+T)^2\left(1-T-2\,T^2-T^3+T^4\right)}{T^3}\right\},\right.$$
$$\left.\text{Knot}[12, \text{NonAlternating}, 430] \to \left\{1, -\frac{2\,(-1+T)^2\left(1-T-2\,T^2-T^3+T^4\right)}{T^3}\right\}\right\},$$
$$\left\{\text{Knot}[12, \text{NonAlternating}, 670] \to \left\{\frac{\left(1-T+T^2-T^3+T^4\right)^2}{T^4},\right.\right.$$
$$\left.\frac{2\,(-1+T)^2\left(1-T+T^2-T^3+T^4\right)\left(1-T-T^2+7\,T^3-8\,T^4+7\,T^5-T^6-T^7+T^8\right)}{T^7}\right\},$$
$$\text{Knot}[12, \text{NonAlternating}, 681] \to \left\{\frac{\left(1-T+T^2-T^3+T^4\right)^2}{T^4},\right.$$
$$\left.\left.\frac{2\,(-1+T)^2\left(1-T+T^2-T^3+T^4\right)\left(1-T-T^2+7\,T^3-8\,T^4+7\,T^5-T^6-T^7+T^8\right)}{T^7}\right\}\right\},$$
$$\left\{\text{Knot}[12, \text{NonAlternating}, 671] \to \left\{-\frac{\left(1-5\,T+7\,T^2-5\,T^3+T^4\right)\left(1-T+T^2-T^3+T^4\right)}{T^4},\right.\right.$$
$$-\frac{1}{T^8}(-1+T)^2\left(3-32\,T+125\,T^2-288\,T^3+495\,T^4-696\,T^5+852\,T^6-\right.$$
$$\left.\left.908\,T^7+852\,T^8-696\,T^9+495\,T^{10}-288\,T^{11}+125\,T^{12}-32\,T^{13}+3\,T^{14}\right)\right\},$$
$$\text{Knot}[12, \text{NonAlternating}, 682] \to \left\{-\frac{\left(1-5\,T+7\,T^2-5\,T^3+T^4\right)\left(1-T+T^2-T^3+T^4\right)}{T^4},\right.$$
$$-\frac{1}{T^8}(-1+T)^2\left(3-32\,T+125\,T^2-288\,T^3+495\,T^4-696\,T^5+852\,T^6-\right.$$
$$\left.\left.\left.908\,T^7+852\,T^8-696\,T^9+495\,T^{10}-288\,T^{11}+125\,T^{12}-32\,T^{13}+3\,T^{14}\right)\right\}\right\},$$

$$\left\{\text{Knot}[12, \text{NonAlternating}, 691] \to \left\{\frac{\left(2-3\,T+2\,T^2\right)\,\left(1-2\,T^2+3\,T^3-2\,T^4+T^6\right)}{T^4},\right.\right.$$
$$-\frac{1}{T^8}\,(-1+T)^2\,\left(17-16\,T-40\,T^2+138\,T^3-113\,T^4-112\,T^5+445\,T^6-\right.$$
$$\left.\left.592\,T^7+445\,T^8-112\,T^9-113\,T^{10}+138\,T^{11}-40\,T^{12}-16\,T^{13}+17\,T^{14}\right)\right\},$$
$$\text{Knot}[12, \text{NonAlternating}, 692] \to \left\{\frac{\left(2-3\,T+2\,T^2\right)\,\left(1-2\,T^2+3\,T^3-2\,T^4+T^6\right)}{T^4},\right.$$
$$-\frac{1}{T^8}\,(-1+T)^2\,\left(17-16\,T-40\,T^2+138\,T^3-113\,T^4-112\,T^5+445\,T^6-\right.$$
$$\left.\left.\left.592\,T^7+445\,T^8-112\,T^9-113\,T^{10}+138\,T^{11}-40\,T^{12}-16\,T^{13}+17\,T^{14}\right)\right\}\right\},$$
$$\left\{\text{Knot}[12, \text{NonAlternating}, 693] \to \left\{-\frac{1-4\,T+2\,T^2+7\,T^3-13\,T^4+7\,T^5+2\,T^6-4\,T^7+T^8}{T^4},\right.\right.$$
$$-\frac{1}{T^8}\,(-1+T)^2\,\left(3-18\,T+21\,T^2+42\,T^3-105\,T^4+48\,T^5+132\,T^6-\right.$$
$$\left.\left.224\,T^7+132\,T^8+48\,T^9-105\,T^{10}+42\,T^{11}+21\,T^{12}-18\,T^{13}+3\,T^{14}\right)\right\},$$
$$\text{Knot}[12, \text{NonAlternating}, 696] \to \left\{-\frac{1-4\,T+2\,T^2+7\,T^3-13\,T^4+7\,T^5+2\,T^6-4\,T^7+T^8}{T^4},\right.$$
$$-\frac{1}{T^8}\,(-1+T)^2\,\left(3-18\,T+21\,T^2+42\,T^3-105\,T^4+48\,T^5+132\,T^6-\right.$$
$$\left.\left.\left.224\,T^7+132\,T^8+48\,T^9-105\,T^{10}+42\,T^{11}+21\,T^{12}-18\,T^{13}+3\,T^{14}\right)\right\}\right\}\right\}$$

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In[•]:=
```
PD /@ {Knot[10, 106], Knot[12, NonAlternating, 369]}
```

Out[•]=
```
{PD[X[6, 2, 7, 1], X[16, 8, 17, 7], X[10, 3, 11, 4], X[2, 15, 3, 16], X[14, 5, 15, 6],
  X[4, 11, 5, 12], X[18, 10, 19, 9], X[20, 14, 1, 13], X[8, 18, 9, 17], X[12, 20, 13, 19]],
 PD[X[4, 2, 5, 1], X[10, 4, 11, 3], X[14, 6, 15, 5], X[18, 7, 19, 8],
  X[2, 10, 3, 9], X[20, 11, 21, 12], X[6, 14, 7, 13], X[15, 23, 16, 22],
  X[12, 17, 13, 18], X[8, 19, 9, 20], X[21, 1, 22, 24], X[23, 17, 24, 16]]}
```

Finding the Duplicates for HOMFLYPT

In[•]:=
```
tabH = Table[K → HOMFLYPT[K][a, z], {K, AllKnots[{3, 12}]}]
```

KnotTheory: The HOMFLYPT program was written by Scott Morrison.

Out[•]=
```
{Knot[3, 1] → 2 a^2 - a^4 + a^2 z^2, Knot[4, 1] → -1 + 1/a^2 + a^2 - z^2,
 Knot[5, 1] → 3 a^4 - 2 a^6 + 4 a^4 z^2 - a^6 z^2 + a^4 z^4, Knot[5, 2] → a^2 + a^4 - a^6 + a^2 z^2 + a^4 z^2, ··· 2970 ···,
 Knot[12, NonAlternating, 886] → -2/a^8 + 3/a^6 - 1/a^4 + 1/a^2 + (3 z^2)/a^6 - z^2/a^4 + (2 z^2)/a^2 - z^4 - z^4/a^6 + z^4/a^2 + z^6/a^4 + z^6/a^2,
 Knot[12, NonAlternating, 887] → 2/a^8 - 6/a^6 + 5/a^4 + z^2/a^8 - (5 z^2)/a^6 + (5 z^2)/a^4 - (2 z^4)/a^8 + (3 z^4)/a^6 - z^4/a^4 - z^6/a^8 + (4 z^6)/a^6 - z^6/a^4 + z^8/a^6,
 Knot[12, NonAlternating, 888] → 2/a^14 - 9/a^12 + 8/a^10 + (5 z^2)/a^14 - (32 z^2)/a^12 + (40 z^2)/a^10 + (2 z^4)/a^14 - (30 z^4)/a^12 + (58 z^4)/a^10 - (10 z^6)/a^12 + (36 z^6)/a^10 - z^8/a^12 + (10 z^8)/a^10 + z^10/a^10}
```

Full expression not available (original memory size: 8.6 MB)

In[•]:=
```
DeleteCases[Gather[tabH, (Last[#1] == Last[#2]) &], {_}]
```

Out[•]=
```
{{Knot[5, 1] → 3 a^4 - 2 a^6 + 4 a^4 z^2 - a^6 z^2 + a^4 z^4, Knot[10, 132] → 3 a^4 - 2 a^6 + 4 a^4 z^2 - a^6 z^2 + a^4 z^4}, ··· 202 ···,
 {Knot[12, NonAlternating, 693] → 1/a^12 - 4/a^10 + 4/a^8 - 2/a^6 + 2/a^4 - (4 z^2)/a^10 + (10 z^2)/a^8 - (7 z^2)/a^6 + (6 z^2)/a^4 + (7 z^4)/a^8 - (10 z^4)/a^6 + (5 z^4)/a^4 + z^6/a^8 - (6 z^6)/a^6 + z^6/a^4 - z^8/a^6,
  Knot[12, NonAlternating, 696] → 1/a^12 - 4/a^10 + 4/a^8 - 2/a^6 + 2/a^4 - (4 z^2)/a^10 + (10 z^2)/a^8 - (7 z^2)/a^6 + (6 z^2)/a^4 + (7 z^4)/a^8 - (10 z^4)/a^6 + (5 z^4)/a^4 + z^6/a^8 - (6 z^6)/a^6 + z^6/a^4 - z^8/a^6}}
```

Full expression not available (original memory size: 1.3 MB)

```
In[•]:= Column[DeleteCases[Gather[tabH, (Last[#1] == Last[#2]) &], {_}] /. {(K_Knot → _) :> K}]
```

Out[•]=

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