Pensieve header: Developing ρ_d.

Program

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\Oaxaca-2210"];
```

In[•]:=
```
Once[<< KnotTheory`; << Rot.m];
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
Loading Rot.m from http://drorbn.net/la22/ap to compute rotation numbers.
```

In[•]:=
```
<< "../../Projects/Profile/Profile.m"
```

```
This is Profile.m of http://www.drorbn.net/AcademicPensieve/Projects/Profile/.
This version: April 2020. Original version: July 1994.
```

In[•]:=
```
{p*, x*, π*, ξ*, p̄*, x̄*, π̄*, ξ̄*} = {π, ξ, p, x, π̄, ξ̄, p̄, x̄}; (u_i__)* := (u*)_i;
```

In[•]:=
```
Zip_{}[ℰ_] := ℰ;
Zip_{ξ_,ξs___}[ℰ_] := (Collect[ℰ // Zip_{ξs}, ξ] /. f_. ξ^d_. :> (D[f, {ξ*, d}])) /. ξ* → 0
```

In[•]:=
```
{ca_{1,2} = 1, ca_{1,10} = -1, ca_{2,1} = 0, cb_{2,10} = 3 / 2};
```

In[•]:=
```
V@γ_{d_,0}[j_] := 0; V@γ_{1,φ_}[k_] := φ (1/2 - p̄_k x̄_k);
```

In[•]:=
$$\texttt{V@}\gamma_{2,1}[k\_] := -\frac{1}{2}\,\bar{p}_k\,\bar{x}_k;\quad \texttt{V@}\gamma_{2,-1}[k\_] := -\frac{1}{2}\,\bar{p}_k\,\bar{x}_k;$$

In[•]:=
$$\texttt{V@}\gamma_{3,\varphi\_}[k\_] := \varphi\left(-ca_{3,1} + \bar{p}_k\,\bar{x}_k\,(1 - cb_{3,10})\right);$$

In[•]:=
$$\texttt{V@}r_{1,s\_}[i\_, j\_] := s\left(-\frac{1}{2} + p_i\,x_i - p_j\,x_i + \frac{1}{2}\left(-1 + T^s\right)p_i\,p_j\,x_i^2 + \frac{1}{2}\left(1 - T^s\right)p_j^2\,x_i^2 - p_i\,p_j\,x_i\,x_j + p_j^2\,x_i\,x_j\right);$$

In[•]:=
$$\texttt{V@}r_{2,1}[i\_, j\_] := -\frac{1}{2}\,p_i\,x_i + \frac{p_j\,x_i}{2} + \frac{1}{4}\,(1 - 3\,T)\,p_i\,p_j\,x_i^2 + \frac{1}{4}\,(-1 + 3\,T)\,p_j^2\,x_i^2 + \frac{1}{3}\,(-1 + T)\,p_i^2\,p_j\,x_i^3 - \frac{1}{6}\,(-1 + T)\,(5 + T)\,p_i\,p_j^2\,x_i^3 + \frac{1}{6}\,(-1 + T)\,(3 + T)\,p_j^3\,x_i^3 + \frac{3}{2}\,p_i\,p_j\,x_i\,x_j - \frac{3}{2}\,p_j^2\,x_i\,x_j - \frac{1}{2}\,p_i^2\,p_j\,x_i^2\,x_j + \frac{1}{2}\,(2 + T)\,p_i\,p_j^2\,x_i^2\,x_j + \frac{1}{2}\,(-1 - T)\,p_j^3\,x_i^2\,x_j - \frac{1}{2}\,p_i\,p_j^2\,x_i\,x_j^2 + \frac{1}{2}\,p_j^3\,x_i\,x_j^2;$$

In[ ]:=
$$V@r_{2,-1}[i\_,j\_] := -\frac{1}{2}p_i x_i + \frac{p_j x_i}{2} + \frac{(-3+T)\,p_i p_j x_i^2}{4T} - \frac{(-3+T)\,p_j^2 x_i^2}{4T} - \frac{(-1+T)\,p_i^2 p_j x_i^3}{3T} + \frac{(-1+T)\,(1+5T)\,p_i p_j^2 x_i^3}{6T^2} - \frac{(-1+T)\,(1+3T)\,p_j^3 x_i^3}{6T^2} + \frac{3}{2}p_i p_j x_i x_j - \frac{3}{2}p_j^2 x_i x_j - \frac{1}{2}p_i^2 p_j x_i^2 x_j + \frac{(1+2T)\,p_i p_j^2 x_i^2 x_j}{2T} - \frac{(1+T)\,p_j^3 x_i^2 x_j}{2T} - \frac{1}{2}p_i p_j^2 x_i x_j^2 + \frac{1}{2}p_j^3 x_i x_j^2;$$

In[ ]:=
$$V@r_{3,1}[i\_,j\_] := \frac{1}{6}(5-6T)\,p_i^2 p_j x_i^3 + \frac{1}{6}\left(-16+17T+2T^2\right)p_i p_j^2 x_i^3 + \frac{1}{6}\left(11-11T-2T^2\right)p_j^3 x_i^3 + \frac{1}{8}(-1+T)\,p_i^3 p_j x_i^4 - \frac{1}{8}(-1+T)\,(4+3T)\,p_i^2 p_j^2 x_i^4 + \frac{1}{24}(-1+T)\left(13+22T+T^2\right)p_i p_j^3 x_i^4 - \frac{1}{24}(-1+T)\left(4+13T+T^2\right)p_j^4 x_i^4 + \frac{3}{2}p_i^2 p_j x_i^2 x_j + \frac{1}{2}(-9-2T)\,p_i p_j^2 x_i^2 x_j + (3+T)\,p_j^3 x_i^2 x_j - \frac{1}{6}p_i^3 p_j x_i^3 x_j + \frac{7}{6}T\,p_i^2 p_j^2 x_i^3 x_j + \frac{1}{6}\left(6-17T-T^2\right)p_i p_j^3 x_i^3 x_j + \frac{1}{6}\left(-5+10T+T^2\right)p_j^4 x_i^3 x_j + p_i p_j^2 x_i x_j^2 - p_j^3 x_i x_j^2 - p_i^2 p_j^2 x_i^2 x_j^2 + \frac{1}{4}(10+T)\,p_i p_j^3 x_i^2 x_j^2 + \frac{1}{4}(-6-T)\,p_j^4 x_i^2 x_j^2 - \frac{1}{6}p_i p_j^3 x_i x_j^3 + \frac{1}{6}p_j^4 x_i x_j^3 + \frac{1}{6}p_i x_i\,(1-12\,ca_{3,1}) + ca_{3,1} + \frac{1}{6}p_j x_i\,(-1+12\,ca_{3,1}) - p_i p_j x_i x_j\,cb_{3,10} + p_j^2 x_i x_j\,cb_{3,10} + \frac{1}{2}p_j^2 x_i^2\,(-2+cb_{3,10}-T\,cb_{3,10}) + \frac{1}{2}p_i p_j x_i^2\,(2-cb_{3,10}+T\,cb_{3,10});$$

In[ ]:=
$$V@r_{3,-1}[i\_,j\_] := -\frac{(-6+5T)\,p_i^2 p_j x_i^3}{6T} + \frac{\left(-2-17T+16T^2\right)p_i p_j^2 x_i^3}{6T^2} - \frac{\left(-2-11T+11T^2\right)p_j^3 x_i^3}{6T^2} + \frac{(-1+T)\,p_i^3 p_j x_i^4}{8T} - \frac{(-1+T)\,(3+4T)\,p_i^2 p_j^2 x_i^4}{8T^2} + \frac{(-1+T)\left(1+22T+13T^2\right)p_i p_j^3 x_i^4}{24T^3} - \frac{(-1+T)\left(1+13T+4T^2\right)p_j^4 x_i^4}{24T^3} - \frac{3}{2}p_i^2 p_j x_i^2 x_j + \frac{(2+9T)\,p_i p_j^2 x_i^2 x_j}{2T} - \frac{(1+3T)\,p_j^3 x_i^2 x_j}{T} + \frac{1}{6}p_i^3 p_j x_i^3 x_j - \frac{7\,p_i^2 p_j^2 x_i^3 x_j}{6T} - \frac{\left(-1-17T+6T^2\right)p_i p_j^3 x_i^3 x_j}{6T^2} + \frac{\left(-1-10T+5T^2\right)p_j^4 x_i^3 x_j}{6T^2} - p_i p_j^2 x_i x_j^2 + p_j^3 x_i x_j^2 + p_i^2 p_j^2 x_i^2 x_j^2 - \frac{(1+10T)\,p_i p_j^3 x_i^2 x_j^2}{4T} + \frac{(1+6T)\,p_j^4 x_i^2 x_j^2}{4T} + \frac{1}{6}p_i p_j^3 x_i x_j^3 - \frac{1}{6}p_j^4 x_i x_j^3 + \frac{1}{6}p_j x_i\,(1-12\,ca_{3,1}) - ca_{3,1} + \frac{1}{6}p_i x_i\,(-1+12\,ca_{3,1}) + p_i p_j x_i x_j\,cb_{3,10} - p_j^2 x_i x_j\,cb_{3,10} + \frac{p_i p_j x_i^2\,(-2T-cb_{3,10}+T\,cb_{3,10})}{2T} - \frac{p_j^2 x_i^2\,(-2T-cb_{3,10}+T\,cb_{3,10})}{2T};$$

In[•]:=
```
cb3,10 = (7 - 12 ca3,1) / 6
```

Out[•]=

$$\frac{1}{6}\,(7-12\,ca_{3,1})$$

In[•]:=
```
gPair[fs_, w_] := gPair[fs, w] = (
   Print["Running gPair on", {fs, w}];
   Zip_{Join@@Table[{p_α, p̄_α, x_α, x̄_α}, {α, w}]} [(Times @@ (V /@ fs))
     Exp[Sum[g_{α,β} (π_α + π̄_α) (ξ_β + ξ̄_β), {α, w}, {β, w}] - Sum[ξ̄_α π_α, {α, w}]]]
  )
```

In[•]:=
```
ρd_[K_] := PP_ρd@Module[{Cs, φ, n, A, s, i, j, k, Δ, G, d1, ρd1, ρd2, ρd3, ρd4},
    PP_"Green"[
     {Cs, φ} = Rot[K]; n = Length[Cs];
     A = IdentityMatrix[2 n + 1];
     Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ((-T^s  T^s - 1)
                                                                (0     -1   )))];
     G = Inverse[A];
    ];
    ρd1 = PP_Mold@Exp[Total[Cases[Cs, {s_, i_, j_} :> Sum[ϵ^d1 r_{d1,s}[i, j], {d1, d}]]] +
        Sum[ϵ^d1 γ_{d1,φ[[k]]}[k], {k, 2 n}, {d1, d}]];
    ρd2 = PP_ExpandedMold[
      Expand[F[{}, {}] × Normal@Series[ρd1, {ϵ, 0, d}]] //. F[fs_, {es___}] ×
          (f : (r | γ)_ps__[is__])^p_. :> F[Join[fs, Table[f, p]], DeleteDuplicates@{es, is}]
     ];
    ρd3 = PP_Pairing[
      ρd2 /. F[fs_, es_] :> (gPair[
            Replace[fs, Thread[es → Range@Length@es], {2}],
            Length@es
           ] /. g_{α_,β_} :> g_{es[[α]],es[[β]]})
     ];
    ρd4 = PP_Substitution@Collect[
       {Δ, ρd3 /. ϵ → ϵ Δ^2} /. {g_{α_,β_} :> G[[α, β]], Δ → T^((-Total[φ]-Total[Cs[[All,1]]])/2) Det[A]},
       ϵ, Factor]
   ];
```

Testing

In[ ]:=
```
ρ1[Knot[3, 1]]
```

» $1+\epsilon\left(-\frac{1}{2}+g_{4,4}\right)+\epsilon\left(\frac{1}{2}+g_{1,4}-2\,g_{1,1}\,g_{1,4}-\left(1-\frac{1}{T}\right)g_{1,4}^2+g_{1,4}\,g_{4,1}-g_{4,4}+g_{1,1}\,g_{4,4}+\left(1-\frac{1}{T}\right)g_{1,4}\,g_{4,4}\right)+$
$\epsilon\left(\frac{1}{2}-g_{2,2}+g_{5,2}+\left(1-\frac{1}{T}\right)g_{2,2}\,g_{5,2}+g_{2,5}\,g_{5,2}-\left(1-\frac{1}{T}\right)g_{5,2}^2+g_{2,2}\,g_{5,5}-2\,g_{5,2}\,g_{5,5}\right)+$
$\epsilon\left(\frac{1}{2}+g_{3,6}-2\,g_{3,3}\,g_{3,6}-\left(1-\frac{1}{T}\right)g_{3,6}^2+g_{3,6}\,g_{6,3}-g_{6,6}+g_{3,3}\,g_{6,6}+\left(1-\frac{1}{T}\right)g_{3,6}\,g_{6,6}\right)$

Out[ ]=
$$\left\{\frac{1-T+T^2}{T},\ 1+\frac{(-1+T)^2\left(1+T^2\right)\epsilon}{T^2}\right\}$$

In[ ]:=
```
TableForm[Table[Join[{K[[1]]_K[[2]]}, ρ1[K]], {K, AllKnots[{3, 6}]}], TableAlignments → Center]
```

Out[ ]//TableForm=

3_1	$\frac{1-T+T^2}{T}$	$1+\frac{(-1+T)^2\left(1+T^2\right)\epsilon}{T^2}$
4_1	$-\frac{1-3T+T^2}{T}$	1
5_1	$\frac{1-T+T^2-T^3+T^4}{T^2}$	$1+\frac{(-1+T)^2\left(1+T^2\right)\left(2+T^2+2T^4\right)\epsilon}{T^4}$
5_2	$\frac{2-3T+2T^2}{T}$	$1+\frac{(-1+T)^2\left(5-4T+5T^2\right)\epsilon}{T^2}$
6_1	$-\frac{(-2+T)(-1+2T)}{T}$	$1+\frac{(-1+T)^2\left(1-4T+T^2\right)\epsilon}{T^2}$
6_2	$-\frac{1-3T+3T^2-3T^3+T^4}{T^2}$	$1+\frac{(-1+T)^2\left(1-4T+4T^2-4T^3+4T^4-4T^5+T^6\right)\epsilon}{T^4}$
6_3	$\frac{1-3T+5T^2-3T^3+T^4}{T^2}$	1

In[ ]:=
```
ρ2[Knot[3, 1]]
```

```
Running gPair on{{r2,-1[1, 2]}, 2}
Running gPair on{{γ2,-1[1]}, 1}
Running gPair on{{γ2,0[1]}, 1}
Running gPair on{{r1,-1[1, 2], r1,-1[1, 2]}, 2}
Running gPair on{{r1,-1[1, 2], r1,-1[3, 4]}, 4}
Running gPair on{{r1,-1[1, 2], γ1,-1[3]}, 3}
Running gPair on{{r1,-1[1, 2], γ1,0[3]}, 3}
Running gPair on{{r1,-1[1, 2], γ1,0[1]}, 2}
Running gPair on{{r1,-1[1, 2], γ1,0[2]}, 2}
Running gPair on{{r1,-1[1, 2], γ1,-1[1]}, 2}
Running gPair on{{γ1,-1[1], γ1,-1[1]}, 1}
Running gPair on{{γ1,-1[1], γ1,0[2]}, 2}
Running gPair on{{γ1,0[1], γ1,0[1]}, 1}
Running gPair on{{γ1,0[1], γ1,0[2]}, 2}
```

Out[ ]=

$$\left\{\frac{1-T+T^2}{T},\ 1+\frac{(-1+T)^2\left(1+T^2\right)\in}{T^2}+\frac{\left(1-4T+7T^2-12T^3+18T^4-12T^5+7T^6-4T^7+T^8\right)\in^2}{2T^4}\right\}$$

In[•]:=
```
BeginProfile[]
Timing[z1 = ρ2[Knot[10, 106]]]
PrintProfile[]
```

Out[•]=
```
ProfileRoot
```

Out[•]=
$$\left\{1.78125,\ \left\{-\frac{\left(1-T+T^2\right)\left(-1+T-2T^2+T^3\right)\left(-1+2T-T^2+T^3\right)}{T^4},\ 1-\frac{1}{T^8}(-1+T)^2\left(1-6T+20T^2-48T^3+82T^4-114T^5+134T^6-140T^7+134T^8-114T^9+82T^{10}-48T^{11}+20T^{12}-6T^{13}+T^{14}\right)\in+\frac{1}{2T^{16}}\left(1-16T+127T^2-676T^3+2735T^4-8980T^5+24938T^6-60420T^7+131072T^8-259992T^9+477614T^{10}-814576T^{11}+1282448T^{12}-1846716T^{13}+2411126T^{14}-2836312T^{15}+2995252T^{16}-2836312T^{17}+2411126T^{18}-1846716T^{19}+1282448T^{20}-814576T^{21}+477614T^{22}-259992T^{23}+131072T^{24}-60420T^{25}+24938T^{26}-8980T^{27}+2735T^{28}-676T^{29}+127T^{30}-16T^{31}+T^{32}\right)\in^2\right\}\right\}$$

Out[•]=
```
ProfileRoot is root. Profiled time: 1.781
  ( 1)  0.015/ 1.780 above ρd
Substitution: called 1 times, time in 1.031/1.031
  ( 1)  1.030/ 1.030 under ρd
Green: called 1 times, time in 0.594/0.594
  ( 1)  0.594/ 0.594 under ρd
Pairing: called 1 times, time in 0.079/0.079
  ( 1)  0.079/ 0.079 under ρd
ExpandedMold: called 1 times, time in 0.062/0.062
  ( 1)  0.062/ 0.062 under ρd
ρd: called 1 times, time in 0.015/1.781
  ( 1)  0.015/ 1.780 under ProfileRoot
  ( 1)  0.594/ 0.594 above Green
  ( 1)  0.062/ 0.062 above ExpandedMold
  ( 1)    0/   0 above Mold
  ( 1)  0.079/ 0.079 above Pairing
  ( 1)  1.030/ 1.030 above Substitution
Mold: called 1 times, time in 0./0.
  ( 1)    0/   0 under ρd
```

In[•]:=
```
BeginProfile[]
Timing[z2 = ρ2[Knot[12, NonAlternating, 369]]]
PrintProfile[]
```

Out[•]=
```
ProfileRoot
```

Out[•]=
$$\left\{5.17188,\ \left\{-\frac{\left(1-T+T^2\right)\left(-1+T-2T^2+T^3\right)\left(-1+2T-T^2+T^3\right)}{T^4},\ 1-\frac{1}{T^8}(-1+T)^2\left(1-6T+20T^2-48T^3+82T^4-114T^5+134T^6-140T^7+134T^8-114T^9+82T^{10}-48T^{11}+20T^{12}-6T^{13}+T^{14}\right)\in+\frac{1}{2T^{16}}\left(1-16T+127T^2-668T^3+2631T^4-8324T^5+22282T^6-52780T^7+114992T^8-236376T^9+460598T^{10}-839688T^{11}+1404696T^{12}-2121524T^{13}+2862782T^{14}-3432312T^{15}+3647156T^{16}-3432312T^{17}+2862782T^{18}-2121524T^{19}+1404696T^{20}-839688T^{21}+460598T^{22}-236376T^{23}+114992T^{24}-52780T^{25}+22282T^{26}-8324T^{27}+2631T^{28}-668T^{29}+127T^{30}-16T^{31}+T^{32}\right)\in^2\right\}\right\}$$

Out[•]=
```
ProfileRoot is root. Profiled time: 5.172
  ( 1)     0/ 5.172 above ρd
Substitution: called 1 times, time in 4.078/4.078
  ( 1)  4.078/ 4.078 under ρd
Green: called 1 times, time in 0.828/0.828
  ( 1)  0.828/ 0.828 under ρd
Pairing: called 1 times, time in 0.156/0.156
  ( 1)  0.156/ 0.156 under ρd
ExpandedMold: called 1 times, time in 0.11/0.11
  ( 1)  0.110/ 0.110 under ρd
ρd: called 1 times, time in 0./5.172
  ( 1)     0/ 5.172 under ProfileRoot
  ( 1)  0.828/ 0.828 above Green
  ( 1)  0.110/ 0.110 above ExpandedMold
  ( 1)     0/    0 above Mold
  ( 1)  0.156/ 0.156 above Pairing
  ( 1)  4.078/ 4.078 above Substitution
Mold: called 1 times, time in 0./0.
  ( 1)     0/    0 under ρd
```

In[•]:=
```
Simplify[Thread[z1 == z2]]
```

Out[•]=
$$\left\{\text{True},\ \frac{1}{T}(-1+T)\left(1-T+T^2\right)\left(1-6T+16T^2-23T^3+9T^4+47T^5-141T^6+231T^7-272T^8+231T^9-141T^{10}+47T^{11}+9T^{12}-23T^{13}+16T^{14}-6T^{15}+T^{16}\right)\in==0\right\}$$

```
In[ ]:= TableForm[Table[Join[{K⟦1⟧_K⟦2⟧}, ρ2[K]], {K, AllKnots[{3, 7}]}], TableAlignments → Center]
```

Out[ ]//TableForm=

3_1	$\frac{1-T+T^2}{T}$	$1+\frac{(-1+T)^2\,(1+T^2)\,\epsilon}{T^2}+$
4_1	$-\frac{1-3\,T+T^2}{T}$	$1+$
5_1	$\frac{1-T+T^2-T^3+T^4}{T^2}$	$1+\frac{(-1+T)^2\,(1+T^2)\,(2+T^2+2\,T^4)\,\epsilon}{T^4}+\frac{(4-16\,T+35\,T^2-60\,T^3+85\,T^4-$
5_2	$\frac{2-3\,T+2\,T^2}{T}$	$1+\frac{(-1+T)^2\,(5-4\,T+5\,T^2)\,\epsilon}{T^2}+\frac{(26-}$
6_1	$-\frac{(-2+T)\,(-1+2\,T)}{T}$	$1+\frac{(-1+T)^2\,(1-4\,T+T^2)\,\epsilon}{T^2}+\frac{(2-}$
6_2	$-\frac{1-3\,T+3\,T^2-3\,T^3+T^4}{T^2}$	$1+\frac{(-1+T)^2\,(1-4\,T+4\,T^2-4\,T^3+4\,T^4-4\,T^5+T^6)\,\epsilon}{T^4}+\frac{(1-12\,T+62\,T^2-180\,T^3+354$
6_3	$\frac{1-3\,T+5\,T^2-3\,T^3+T^4}{T^2}$	$1-\frac{(1-T+T^2)\,(1-3\,T$
7_1	$\frac{1-T+T^2-T^3+T^4-T^5+T^6}{T^3}$	$1+\frac{(-1+T)^2\,(1+T^2)\,(3+2\,T^2+4\,T^4+2\,T^6+3\,T^8)\,\epsilon}{T^6}+\frac{(9-36\,T+83\,T^2-152\,T^3+238\,T^4-336\,T^5+434\,T^6-556\,T^7+719\,T^8-$
7_2	$\frac{3-5\,T+3\,T^2}{T}$	$1+\frac{2\,(-1+T)^2\,(7-8\,T+7\,T^2)\,\epsilon}{T^2}+\frac{(105-65}$
7_3	$\frac{2-3\,T+3\,T^2-3\,T^3+2\,T^4}{T^2}$	$1-\frac{(-1+T)^2\,(9-8\,T+16\,T^2-12\,T^3+16\,T^4-8\,T^5+9\,T^6)\,\epsilon}{T^4}+\frac{(82-472\,T+1409\,T^2-2996\,T^3+5190\,T^4-$
7_4	$\frac{4-7\,T+4\,T^2}{T}$	$1-\frac{8\,(-1+T)^2\,(3-4\,T+3\,T^2)\,\epsilon}{T^2}+\frac{(304-2032}$
7_5	$\frac{2-4\,T+5\,T^2-4\,T^3+2\,T^4}{T^2}$	$1+\frac{(-1+T)^2\,(9-16\,T+29\,T^2-28\,T^3+29\,T^4-16\,T^5+9\,T^6)\,\epsilon}{T^4}+\frac{(82-616\,T+2412\,T^2-6560\,T^3+13\,875\,T^4-$
7_6	$-\frac{1-5\,T+7\,T^2-5\,T^3+T^4}{T^2}$	$1+\frac{(-1+T)^2\,(1-8\,T+19\,T^2-20\,T^3+19\,T^4-8\,T^5+T^6)\,\epsilon}{T^4}+\frac{(1-20\,T+175\,T^2-880\,T^3+2923\,T^4-$
7_7	$\frac{1-5\,T+9\,T^2-5\,T^3+T^4}{T^2}$	$1-\frac{(-1+T)^2\,(3-8\,T+3\,T^2)\,\epsilon}{T^2}+\frac{(1-20\,T+199\,T^2-1064$

```
In[ ]:= GST48 = EPD[X14,1, X̄2,29, X3,40, X43,4, X̄26,5, X6,95, X96,7, X13,8, X̄9,28, X10,41, X42,11, X̄27,12,
          X30,15, X̄16,61, X̄17,72, X̄18,83, X19,34, X̄89,20, X̄21,92, X̄79,22, X̄68,23, X̄57,24, X̄25,56, X62,31,
          X73,32, X84,33, X̄50,35, X36,81, X37,70, X38,59, X̄39,54, X44,55, X58,45, X69,46, X80,47, X48,91,
          X90,49, X51,82, X52,71, X53,60, X̄63,74, X̄64,85, X̄76,65, X̄87,66, X̄67,94, X̄75,86, X̄88,77, X̄78,93];
        BeginProfile[]
        Timing[z3 = ρ2[GST48]]
        PrintProfile[]
```

Out[ ]=

```
ProfileRoot
```

Out[ ]=

$$\left\{289.766,\ \left\{-\frac{\left(-1+2\,T-T^2-T^3+2\,T^4-T^5+T^8\right)\ \left(-1+T^3-2\,T^4+T^5+T^6-2\,T^7+T^8\right)}{T^8},\right.\right.$$

$$1+\frac{1}{T^{16}}\ (-1+T)^2\ \left(5-18\,T+33\,T^2-32\,T^3+2\,T^4+42\,T^5-62\,T^6-8\,T^7+166\,T^8-242\,T^9+108\,T^{10}+\right.$$

$$132\,T^{11}-226\,T^{12}+148\,T^{13}-11\,T^{14}-36\,T^{15}-11\,T^{16}+148\,T^{17}-226\,T^{18}+132\,T^{19}+108\,T^{20}-$$

$$\left.242\,T^{21}+166\,T^{22}-8\,T^{23}-62\,T^{24}+42\,T^{25}+2\,T^{26}-32\,T^{27}+33\,T^{28}-18\,T^{29}+5\,T^{30}\right)\ \epsilon+$$

$$\frac{1}{2\,T^{32}}\ \left(25-348\,T+2312\,T^2-9628\,T^3+27\,228\,T^4-51\,460\,T^5+52\,250\,T^6+25\,828\,T^7-\right.$$

$$197\,145\,T^8+313\,268\,T^9-36\,579\,T^{10}-887\,864\,T^{11}+2\,118\,398\,T^{12}-2\,494\,152\,T^{13}+772\,387\,T^{14}+$$

$$2\,785\,204\,T^{15}-5\,477\,089\,T^{16}+3\,765\,568\,T^{17}+2\,886\,710\,T^{18}-9\,712\,796\,T^{19}+9\,746\,285\,T^{20}-$$

$$708\,568\,T^{21}-11\,443\,177\,T^{22}+17\,013\,304\,T^{23}-11\,217\,405\,T^{24}-1\,334\,300\,T^{25}+10\,332\,369\,T^{26}-$$

$$8\,571\,752\,T^{27}-1\,186\,874\,T^{28}+8\,007\,252\,T^{29}-3\,568\,015\,T^{30}-8\,148\,860\,T^{31}+14\,395\,240\,T^{32}-$$

$$8\,148\,860\,T^{33}-3\,568\,015\,T^{34}+8\,007\,252\,T^{35}-1\,186\,874\,T^{36}-8\,571\,752\,T^{37}+10\,332\,369\,T^{38}-$$

$$1\,334\,300\,T^{39}-11\,217\,405\,T^{40}+17\,013\,304\,T^{41}-11\,443\,177\,T^{42}-708\,568\,T^{43}+9\,746\,285\,T^{44}-$$

$$9\,712\,796\,T^{45}+2\,886\,710\,T^{46}+3\,765\,568\,T^{47}-5\,477\,089\,T^{48}+2\,785\,204\,T^{49}+772\,387\,T^{50}-$$

$$2\,494\,152\,T^{51}+2\,118\,398\,T^{52}-887\,864\,T^{53}-36\,579\,T^{54}+313\,268\,T^{55}-197\,145\,T^{56}+$$

$$\left.\left.25\,828\,T^{57}+52\,250\,T^{58}-51\,460\,T^{59}+27\,228\,T^{60}-9628\,T^{61}+2312\,T^{62}-348\,T^{63}+25\,T^{64}\right)\ \epsilon^2\right\}\Big\}$$

Out[ ]=

```
ProfileRoot is root. Profiled time: 289.766
  ( 1)   0.219/ 289.770 above ρd
Substitution: called 1 times, time in 169./169.
  ( 1)  169.000/ 169.000 under ρd
Green: called 1 times, time in 112.438/112.438
  ( 1)  112.440/ 112.440 under ρd
ExpandedMold: called 1 times, time in 5.515/5.515
  ( 1)   5.515/  5.515 under ρd
Pairing: called 1 times, time in 2.594/2.594
  ( 1)   2.594/  2.594 under ρd
ρd: called 1 times, time in 0.219/289.766
  ( 1)   0.219/ 289.770 under ProfileRoot
  ( 1)  112.440/ 112.440 above Green
  ( 1)   5.515/  5.515 above ExpandedMold
  ( 1)      0/     0 above Mold
  ( 1)   2.594/  2.594 above Pairing
  ( 1)  169.000/ 169.000 above Substitution
Mold: called 1 times, time in 0./0.
  ( 1)      0/     0 under ρd
```

In[•]:=
```
BeginProfile[]
Timing[ρ3[Knot[3, 1]]]
PrintProfile[]
```

Out[•]=

ProfileRoot

Out[•]=

$$\left\{6.23438,\ \left\{\frac{1-T+T^2}{T},\ 1+\frac{(-1+T)^2\left(1+T^2\right)\epsilon}{T^2}+\frac{\left(1-4T+7T^2-12T^3+18T^4-12T^5+7T^6-4T^7+T^8\right)\epsilon^2}{2T^4}-\right.\right.$$
$$\frac{1}{6T^6}\epsilon^3\left(-1+6T-14T^2+34T^3-92T^4+98T^5-50T^6+98T^7-92T^8+34T^9-14T^{10}+6T^{11}-T^{12}+\right.$$
$$\left.\left.\left.12\,ca_{3,1}-72T\,ca_{3,1}+240T^2\,ca_{3,1}-552T^3\,ca_{3,1}+960T^4\,ca_{3,1}-1320T^5\,ca_{3,1}+1464T^6\,ca_{3,1}-\right.\right.\right.$$
$$\left.\left.\left.1320T^7\,ca_{3,1}+960T^8\,ca_{3,1}-552T^9\,ca_{3,1}+240T^{10}\,ca_{3,1}-72T^{11}\,ca_{3,1}+12T^{12}\,ca_{3,1}\right)\right\}\right\}$$

Out[•]=
```
ProfileRoot is root. Profiled time: 6.235
  ( 1)  0.016/ 6.235 above ρd
Substitution: called 1 times, time in 5.516/5.516
  ( 1)  5.516/ 5.516 under ρd
Pairing: called 1 times, time in 0.64/0.64
  ( 1)  0.640/ 0.640 under ρd
ExpandedMold: called 1 times, time in 0.063/0.063
  ( 1)  0.063/ 0.063 under ρd
ρd: called 1 times, time in 0.016/6.235
  ( 1)  0.016/ 6.235 under ProfileRoot
  ( 1)     0/    0 above Green
  ( 1)  0.063/ 0.063 above ExpandedMold
  ( 1)     0/    0 above Mold
  ( 1)  0.640/ 0.640 above Pairing
  ( 1)  5.516/ 5.516 above Substitution
Mold: called 1 times, time in 0./0.
  ( 1)     0/    0 under ρd
Green: called 1 times, time in 0./0.
  ( 1)     0/    0 under ρd
```

In[•]:=
```
BeginProfile[]
Timing[ρ3[Knot[4, 1]]]
PrintProfile[]
```

Out[•]=
```
ProfileRoot
```

Out[•]=
$$\left\{5.39063,\ \left\{-\frac{1-3\,T+T^2}{T},\ 1+\frac{\left(1-3\,T+T^2\right)\,\left(1-T+T^2\right)\,\epsilon^2}{T^2}\right\}\right\}$$

Out[•]=
```
ProfileRoot is root. Profiled time: 5.391
  ( 1)  0.110/ 5.391 above ρd
Substitution: called 1 times, time in 3.422/3.422
  ( 1)  3.422/ 3.422 under ρd
Pairing: called 1 times, time in 1.703/1.703
  ( 1)  1.703/ 1.703 under ρd
ExpandedMold: called 1 times, time in 0.156/0.156
  ( 1)  0.156/ 0.156 under ρd
ρd: called 1 times, time in 0.11/5.391
  ( 1)  0.110/ 5.391 under ProfileRoot
  ( 1)     0/    0 above Green
  ( 1)  0.156/ 0.156 above ExpandedMold
  ( 1)     0/    0 above Mold
  ( 1)  1.703/ 1.703 above Pairing
  ( 1)  3.422/ 3.422 above Substitution
Mold: called 1 times, time in 0./0.
  ( 1)     0/    0 under ρd
Green: called 1 times, time in 0./0.
  ( 1)     0/    0 under ρd
```

In[•]:=
```
TableForm[Table[Echo@Join[{K[[1]]_K[[2]]}, ρ3[K]], {K, AllKnots[{3, 7}]}],
 TableAlignments → Center]
```

» $$\left\{3_1,\ \frac{1-T+T^2}{T},\ 1+\frac{(-1+T)^2\,\left(1+T^2\right)\,\epsilon}{T^2}+\frac{\left(1-4\,T+7\,T^2-12\,T^3+18\,T^4-12\,T^5+7\,T^6-4\,T^7+T^8\right)\,\epsilon^2}{2\,T^4}-\right.$$
$$\left.\frac{1}{6\,T^6}\,\epsilon^3\,\left(-1+6\,T-14\,T^2+34\,T^3-92\,T^4+98\,T^5-50\,T^6+98\,T^7-92\,T^8+34\,T^9-14\,T^{10}+6\,T^{11}-T^{12}+\right.\right.$$
$$\left.\left.12\,ca_{3,1}-72\,T\,ca_{3,1}+240\,T^2\,ca_{3,1}-552\,T^3\,ca_{3,1}+960\,T^4\,ca_{3,1}-1320\,T^5\,ca_{3,1}+1464\,T^6\,ca_{3,1}-\right.\right.$$
$$\left.\left.1320\,T^7\,ca_{3,1}+960\,T^8\,ca_{3,1}-552\,T^9\,ca_{3,1}+240\,T^{10}\,ca_{3,1}-72\,T^{11}\,ca_{3,1}+12\,T^{12}\,ca_{3,1}\right)\right\}$$

» $$\left\{4_1,\ -\frac{1-3\,T+T^2}{T},\ 1+\frac{\left(1-3\,T+T^2\right)\,\left(1-T+T^2\right)\,\epsilon^2}{T^2}\right\}$$

» $$\left\{5_1,\ \frac{1-T+T^2-T^3+T^4}{T^2},\right.$$

$$1+\frac{(-1+T)^2\left(1+T^2\right)\left(2+T^2+2T^4\right)\epsilon}{T^4}+\frac{1}{2T^8}\left(4-16T+35T^2-60T^3+85T^4-120T^5+170T^6-\right.$$

$$220T^7+250T^8-220T^9+170T^{10}-120T^{11}+85T^{12}-60T^{13}+35T^{14}-16T^{15}+4T^{16}\left.\right)\epsilon^2-$$

$$\frac{1}{6T^{12}}\epsilon^3\left(-8+48T-149T^2+334T^3-590T^4+998T^5-1844T^6+3350T^7-5386T^8+6802T^9-\right.$$

$$6772T^{10}+5758T^{11}-5022T^{12}+5758T^{13}-6772T^{14}+6802T^{15}-5386T^{16}+3350T^{17}-1844T^{18}+$$

$$998T^{19}-590T^{20}+334T^{21}-149T^{22}+48T^{23}-8T^{24}+24\,ca_{3,1}-144T\,ca_{3,1}+492T^2\,ca_{3,1}-$$

$$1272T^3\,ca_{3,1}+2760T^4\,ca_{3,1}-5208T^5\,ca_{3,1}+8736T^6\,ca_{3,1}-13272T^7\,ca_{3,1}+18480T^8\,ca_{3,1}-$$

$$23736T^9\,ca_{3,1}+28272T^{10}\,ca_{3,1}-31368T^{11}\,ca_{3,1}+32472T^{12}\,ca_{3,1}-31368T^{13}\,ca_{3,1}+$$

$$28272T^{14}\,ca_{3,1}-23736T^{15}\,ca_{3,1}+18480T^{16}\,ca_{3,1}-13272T^{17}\,ca_{3,1}+8736T^{18}\,ca_{3,1}-$$

$$\left.5208T^{19}\,ca_{3,1}+2760T^{20}\,ca_{3,1}-1272T^{21}\,ca_{3,1}+492T^{22}\,ca_{3,1}-144T^{23}\,ca_{3,1}+24T^{24}\,ca_{3,1}\right)\Big\}$$

Running gPair on{{$\gamma_{3,1}$[1]}, 1}

Running gPair on{{$r_{1,-1}$[1, 2], $\gamma_{2,1}$[3]}, 3}

Running gPair on{{$r_{1,-1}$[1, 2], $\gamma_{2,1}$[2]}, 2}

Running gPair on{{$r_{2,-1}$[1, 2], $\gamma_{1,1}$[3]}, 3}

Running gPair on{{$r_{2,-1}$[1, 2], $\gamma_{1,1}$[2]}, 2}

Running gPair on{{$\gamma_{1,-1}$[1], $\gamma_{2,1}$[2]}, 2}

Running gPair on{{$\gamma_{1,0}$[1], $\gamma_{2,1}$[2]}, 2}

Running gPair on{{$\gamma_{1,1}$[1], $\gamma_{2,-1}$[2]}, 2}

Running gPair on{{$\gamma_{1,1}$[1], $\gamma_{2,0}$[2]}, 2}

Running gPair on{{$\gamma_{1,1}$[1], $\gamma_{2,1}$[1]}, 1}

Running gPair on{{$r_{1,-1}$[1, 2], $r_{1,-1}$[1, 2], $\gamma_{1,1}$[3]}, 3}

Running gPair on{{$r_{1,-1}$[1, 2], $r_{1,-1}$[3, 4], $\gamma_{1,1}$[5]}, 5}

Running gPair on{{$r_{1,-1}$[1, 2], $r_{1,-1}$[3, 4], $\gamma_{1,1}$[4]}, 4}

Running gPair on{{$r_{1,-1}$[1, 2], $\gamma_{1,-1}$[3], $\gamma_{1,1}$[4]}, 4}

Running gPair on{{$r_{1,-1}$[1, 2], $\gamma_{1,0}$[3], $\gamma_{1,1}$[4]}, 4}

Running gPair on{{$r_{1,-1}$[1, 2], $\gamma_{1,0}$[1], $\gamma_{1,1}$[3]}, 3}

Running gPair on{{$r_{1,-1}$[1, 2], $\gamma_{1,0}$[2], $\gamma_{1,1}$[3]}, 3}

Running gPair on{{$r_{1,-1}$[1, 2], $\gamma_{1,1}$[3], $\gamma_{1,1}$[3]}, 3}

Running gPair on{{$r_{1,-1}$[1, 2], $\gamma_{1,-1}$[1], $\gamma_{1,-1}$[3]}, 3}

Running gPair on{{$r_{1,-1}$[1, 2], $\gamma_{1,-1}$[1], $\gamma_{1,1}$[3]}, 3}

Running gPair on{{$r_{1,-1}$[1, 2], $r_{1,-1}$[1, 2], $\gamma_{1,1}$[2]}, 2}

Running gPair on{{$r_{1,-1}$[1, 2], $r_{1,-1}$[3, 4], $\gamma_{1,1}$[2]}, 4}

Running gPair on{{$r_{1,-1}$[1, 2], $\gamma_{1,-1}$[3], $\gamma_{1,1}$[2]}, 3}

Running gPair on{{$r_{1,-1}$[1, 2], $\gamma_{1,0}$[3], $\gamma_{1,1}$[2]}, 3}

Running gPair on{{$r_{1,-1}$[1, 2], $\gamma_{1,0}$[1], $\gamma_{1,1}$[2]}, 2}

Running gPair on$\{\{r_{1,-1}[1, 2], \gamma_{1,1}[2], \gamma_{1,1}[2]\}, 2\}$

Running gPair on$\{\{\gamma_{1,-1}[1], \gamma_{1,-1}[1], \gamma_{1,1}[2]\}, 2\}$

Running gPair on$\{\{\gamma_{1,-1}[1], \gamma_{1,-1}[2], \gamma_{1,1}[3]\}, 3\}$

Running gPair on$\{\{\gamma_{1,-1}[1], \gamma_{1,0}[2], \gamma_{1,1}[3]\}, 3\}$

Running gPair on$\{\{\gamma_{1,-1}[1], \gamma_{1,1}[2], \gamma_{1,1}[2]\}, 2\}$

Running gPair on$\{\{\gamma_{1,0}[1], \gamma_{1,0}[1], \gamma_{1,1}[2]\}, 2\}$

Running gPair on$\{\{\gamma_{1,0}[1], \gamma_{1,0}[2], \gamma_{1,1}[3]\}, 3\}$

Running gPair on$\{\{\gamma_{1,0}[1], \gamma_{1,1}[2], \gamma_{1,1}[2]\}, 2\}$

Running gPair on$\{\{\gamma_{1,1}[1], \gamma_{1,1}[1], \gamma_{1,1}[1]\}, 1\}$

» $\left\{5_2, \frac{2 - 3\,T + 2\,T^2}{T},\right.$

$$1 + \frac{(-1+T)^2\left(5 - 4\,T + 5\,T^2\right)\epsilon}{T^2} + \frac{\left(26 - 144\,T + 387\,T^2 - 688\,T^3 + 842\,T^4 - 688\,T^5 + 387\,T^6 - 144\,T^7 + 26\,T^8\right)\epsilon^2}{2\,T^4} -$$

$$\frac{1}{6\,T^6}\epsilon^3\left(-146 + 1196\,T - 4892\,T^2 + 13\,528\,T^3 - 26\,915\,T^4 + 39\,038\,T^5 - 43\,582\,T^6 + 39\,038\,T^7 - \right.$$

$$26\,915\,T^8 + 13\,528\,T^9 - 4892\,T^{10} + 1196\,T^{11} - 146\,T^{12} + 960\,ca_{3,1} - 8448\,T\,ca_{3,1} + 36\,384\,T^2\,ca_{3,1} -$$

$$100\,704\,T^3\,ca_{3,1} + 198\,780\,T^4\,ca_{3,1} - 294\,216\,T^5\,ca_{3,1} + 334\,488\,T^6\,ca_{3,1} - 294\,216\,T^7\,ca_{3,1} +$$

$$\left.\left.198\,780\,T^8\,ca_{3,1} - 100\,704\,T^9\,ca_{3,1} + 36\,384\,T^{10}\,ca_{3,1} - 8448\,T^{11}\,ca_{3,1} + 960\,T^{12}\,ca_{3,1}\right)\right\}$$

Running gPair on$\{\{r_{1,1}[1, 2], \gamma_{2,1}[3]\}, 3\}$

Running gPair on$\{\{r_{2,1}[1, 2], \gamma_{1,1}[3]\}, 3\}$

Running gPair on$\{\{r_{1,-1}[1, 2], r_{1,1}[3, 4], \gamma_{1,1}[5]\}, 5\}$

Running gPair on$\{\{r_{1,-1}[1, 2], r_{1,1}[3, 4], \gamma_{1,1}[2]\}, 4\}$

Running gPair on$\{\{r_{1,1}[1, 2], r_{1,1}[1, 2], \gamma_{1,1}[3]\}, 3\}$

Running gPair on$\{\{r_{1,1}[1, 2], r_{1,1}[3, 4], \gamma_{1,1}[5]\}, 5\}$

Running gPair on$\{\{r_{1,1}[1, 2], \gamma_{1,-1}[3], \gamma_{1,-1}[2]\}, 3\}$

Running gPair on$\{\{r_{1,1}[1, 2], \gamma_{1,-1}[3], \gamma_{1,1}[4]\}, 4\}$

Running gPair on$\{\{r_{1,1}[1, 2], \gamma_{1,-1}[2], \gamma_{1,1}[3]\}, 3\}$

Running gPair on$\{\{r_{1,1}[1, 2], \gamma_{1,0}[3], \gamma_{1,1}[4]\}, 4\}$

Running gPair on$\{\{r_{1,1}[1, 2], \gamma_{1,0}[1], \gamma_{1,1}[3]\}, 3\}$

Running gPair on$\{\{r_{1,1}[1, 2], \gamma_{1,1}[3], \gamma_{1,1}[3]\}, 3\}$

Running gPair on$\{\{r_{1,1}[1, 2], \gamma_{1,0}[2], \gamma_{1,0}[3]\}, 3\}$

Running gPair on$\{\{r_{1,1}[1, 2], \gamma_{1,0}[2], \gamma_{1,0}[1]\}, 2\}$

Running gPair on$\{\{r_{1,1}[1, 2], \gamma_{1,0}[2], \gamma_{1,1}[3]\}, 3\}$

Running gPair on$\{\{\gamma_{1,-1}[1], \gamma_{1,-1}[2], \gamma_{1,-1}[3]\}, 3\}$

» $\left\{6_1,\ -\frac{(-2+T)\ (-1+2T)}{T},\right.$

$1+\frac{(-1+T)^2\left(1-4T+T^2\right)\epsilon}{T^2}+\frac{\left(2-24T+129T^2-328T^3+438T^4-328T^5+129T^6-24T^7+2T^8\right)\epsilon^2}{2T^4}-$

$\frac{1}{6T^6}\epsilon^3\left(-10+172T-1280T^2+5948T^3-17415T^4+32514T^5-39870T^6+32514T^7-\right.$

$17415T^8+5948T^9-1280T^{10}+172T^{11}-10T^{12}+192\,ca_{3,1}-3072T\,ca_{3,1}+21408T^2\,ca_{3,1}-$

$85920T^3\,ca_{3,1}+221004T^4\,ca_{3,1}-383400T^5\,ca_{3,1}+459576T^6\,ca_{3,1}-383400T^7\,ca_{3,1}+$

$\left.\left.221004T^8\,ca_{3,1}-85920T^9\,ca_{3,1}+21408T^{10}\,ca_{3,1}-3072T^{11}\,ca_{3,1}+192T^{12}\,ca_{3,1}\right)\right\}$

» $\left\{6_2,\ -\frac{1-3T+3T^2-3T^3+T^4}{T^2},\right.$

$1+\frac{(-1+T)^2\left(1-4T+4T^2-4T^3+4T^4-4T^5+T^6\right)\epsilon}{T^4}+\frac{1}{2T^8}\left(1-12T+62T^2-180T^3+354T^4-592T^5+\right.$

$1007T^6-1576T^7+1870T^8-1576T^9+1007T^{10}-592T^{11}+354T^{12}-180T^{13}+62T^{14}-12T^{15}+\left.T^{16}\right)\epsilon^2-$

$\frac{1}{6T^{12}}\epsilon^3\left(-1+18T-145T^2+688T^3-2165T^4+5386T^5-13442T^6+34666T^7-75044T^8+116434T^9-\right.$

$119944T^{10}+81054T^{11}-55022T^{12}+81054T^{13}-119944T^{14}+116434T^{15}-75044T^{16}+34666T^{17}-$

$13442T^{18}+5386T^{19}-2165T^{20}+688T^{21}-145T^{22}+18T^{23}-T^{24}+12\,ca_{3,1}-216T\,ca_{3,1}+1812T^2\,ca_{3,1}-$

$9552T^3\,ca_{3,1}+36060T^4\,ca_{3,1}-105240T^5\,ca_{3,1}+249504T^6\,ca_{3,1}-496776T^7\,ca_{3,1}+850128T^8\,ca_{3,1}-$

$1271352T^9\,ca_{3,1}+1681320T^{10}\,ca_{3,1}-1982088T^{11}\,ca_{3,1}+2092776T^{12}\,ca_{3,1}-1982088T^{13}\,ca_{3,1}+$

$1681320T^{14}\,ca_{3,1}-1271352T^{15}\,ca_{3,1}+850128T^{16}\,ca_{3,1}-496776T^{17}\,ca_{3,1}+249504T^{18}\,ca_{3,1}-$

$\left.\left.105240T^{19}\,ca_{3,1}+36060T^{20}\,ca_{3,1}-9552T^{21}\,ca_{3,1}+1812T^{22}\,ca_{3,1}-216T^{23}\,ca_{3,1}+12T^{24}\,ca_{3,1}\right)\right\}$

Running gPair on$\{\{r_{1,1}[1,2],\ r_{1,1}[3,4],\ r_{1,1}[5,6]\},\ 6\}$

Running gPair on$\{\{r_{1,1}[1,2],\ r_{1,1}[3,4],\ \gamma_{1,0}[2]\},\ 4\}$

» $\left\{6_3,\ \frac{1-3T+5T^2-3T^3+T^4}{T^2},\ 1-\frac{\left(1-T+T^2\right)\left(1-3T+5T^2-3T^3+T^4\right)\left(1-11T^2+19T^3-11T^4+T^6\right)\epsilon^2}{T^6}\right\}$

Out[ ]=

$Aborted

In[ ]:=
```
BeginProfile[]
Timing[ρ3[Knot[11, NonAlternating, 34]]]
PrintProfile[]
```

Out[ ]=

ProfileRoot

KnotTheory: Loading precomputed data in DTCode4KnotsTo11`.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

```
Running gPair on{{γ1,1[1], γ2,1[2]}, 2}
Running gPair on{{r1,-1[1, 2], γ1,1[2], γ1,1[3]}, 3}
Running gPair on{{r1,-1[1, 2], γ1,1[3], γ1,1[4]}, 4}
Running gPair on{{r1,-1[1, 2], γ1,1[3], γ1,1[2]}, 3}
Running gPair on{{r1,1[1, 2], γ1,1[3], γ1,1[4]}, 4}
Running gPair on{{γ1,-1[1], γ1,1[2], γ1,1[3]}, 3}
Running gPair on{{γ1,0[1], γ1,1[2], γ1,1[3]}, 3}
Running gPair on{{γ1,1[1], γ1,1[1], γ1,1[2]}, 2}
Running gPair on{{γ1,1[1], γ1,1[2], γ1,1[2]}, 2}
```

Out[ ]=

$$\left\{5207.64,\ \left\{1,\ 1-\frac{2\ (-1+T)^2\ \left(1+T^4\right)\ \epsilon}{T^3}+\right.\right.$$
$$\frac{2\ (-1+T)^2\ \left(6-15\,T+12\,T^2+2\,T^3-3\,T^4-2\,T^5-3\,T^6+2\,T^7+12\,T^8-15\,T^9+6\,T^{10}\right)\ \epsilon^2}{T^6}-$$
$$\frac{1}{3\,T^9}\ \epsilon^3\ \left(360-2520\,T+7632\,T^2-12\,510\,T^3+10\,899\,T^4-2988\,T^5-2942\,T^6+2731\,T^7-695\,T^8+54\,T^9-\right.$$
$$\left.\left.\left.695\,T^{10}+2731\,T^{11}-2942\,T^{12}-2988\,T^{13}+10\,899\,T^{14}-12\,510\,T^{15}+7632\,T^{16}-2520\,T^{17}+\right.\right.\right.$$
$$\left.\left.\left.360\,T^{18}-12\,T^6\,ca_{3,1}+24\,T^7\,ca_{3,1}-12\,T^8\,ca_{3,1}-12\,T^{10}\,ca_{3,1}+24\,T^{11}\,ca_{3,1}-12\,T^{12}\,ca_{3,1}\right)\right\}\right\}$$

Out[ ]=

```
ProfileRoot is root. Profiled time: 5207.64
  ( 1)     8.110/ 5207.641 above ρd
Substitution: called 1 times, time in 5144.33/5144.33
  ( 1)  5144.328/ 5144.328 under ρd
Pairing: called 1 times, time in 51.156/51.156
  ( 1)    51.156/   51.156 under ρd
ρd: called 1 times, time in 8.11/5207.64
  ( 1)     8.110/ 5207.641 under ProfileRoot
  ( 1)     1.235/    1.235 above Green
  ( 1)     2.812/    2.812 above ExpandedMold
  ( 1)        0/       0 above Mold
  ( 1)    51.156/   51.156 above Pairing
  ( 1)  5144.328/ 5144.328 above Substitution
ExpandedMold: called 1 times, time in 2.812/2.812
  ( 1)     2.812/    2.812 under ρd
Green: called 1 times, time in 1.235/1.235
  ( 1)     1.235/    1.235 under ρd
Mold: called 1 times, time in 0./0.
  ( 1)        0/       0 under ρd
```

In[•]:=
```
BeginProfile[]
Timing[ρ3[Knot[11, NonAlternating, 42]]]
PrintProfile[]
```

Out[•]=
```
ProfileRoot
```

Out[•]=
```
$Aborted
```

Out[•]=
```
ProfileRoot is root. Profiled time: 53.734
  ( 1)     0/    0 above ρd
Pairing: called 1 times, time in 50.312/50.312
  ( 1)  50.310/ 50.310 under ρd
ExpandedMold: called 1 times, time in 2.719/2.719
  ( 1)  2.719/ 2.719 under ρd
Green: called 1 times, time in 0.703/0.703
  ( 1)  0.703/ 0.703 under ρd
ρd: called 1 times, time in 0./0.
  ( 1)     0/    0 under ProfileRoot
  ( 1)  0.703/ 0.703 above Green
  ( 1)  2.719/ 2.719 above ExpandedMold
  ( 1)     0/    0 above Mold
  ( 1)  50.310/ 50.310 above Pairing
  ( 1)     0/    0 above Substitution
Substitution: called 1 times, time in 0./0.
  ( 1)     0/    0 under ρd
Mold: called 1 times, time in 0./0.
  ( 1)     0/    0 under ρd
```