

Pensieve header: Developing ρ_d .

Program

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\Oaxaca-2210"];
```

(Alt) In[]:=

```
Once[<< KnotTheory` ; << Rot.m];
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

Loading Rot.m from <http://drorbn.net/la22/ap> to compute rotation numbers.

(Alt) In[]:=

```
<< "../..//Projects/Profile/Profile.m"
```

This is Profile.m of <http://www.drorbn.net/AcademicPensieve/Projects/Profile/>.

This version: April 2020. Original version: July 1994.

(Alt) In[]:=

```
{p*, x*, pi*, xi*, p_bar, x_bar, pi_bar, xi_bar} = {pi, xi, p, x, pi_bar, xi_bar, p_bar, x_bar}; (u_{i_})^* := (u^*)_i;
```

(Alt) In[]:=

```
Zip_{i}[e_] := e;
```

```
Zip_{e, s}[e_] := (Collect[e // Zip_{s}, s] /. f_ . s^d_ -> (D[f, {s*, d}])) /. s* -> 0
```

```
{ca_{1,2} = 1, ca_{1,10} = -1, ca_{2,1} = 0, cb_{2,10} = 3 / 2, cb_{3,10} = (7 - 12 ca_{3,1}) / 6};
```

(Alt) In[]:=

$$V@_{\gamma_{d,0}}[j_] := 0; V@_{\gamma_{1,\varphi}}[k_] := \varphi \left(\frac{\Delta^2}{2} - \Delta \bar{p}_k \bar{x}_k \right);$$

(Alt) In[]:=

$$V@_{\gamma_{2,1}}[k_] := -\frac{\Delta^3}{2} \bar{p}_k \bar{x}_k; V@_{\gamma_{2,-1}}[k_] := -\frac{\Delta^3}{2} \bar{p}_k \bar{x}_k;$$

(Alt) In[]:=

$$V@_{\gamma_{3,\varphi}}[k_] := -\frac{1}{6} \Delta^5 \varphi \left(\bar{p}_k \bar{x}_k (1 - 12 ca_{3,1}) + 6 \Delta ca_{3,1} \right);$$

(Alt) In[]:=

$$V@_{r_{1,s}}[i_, j_] := -\frac{1}{2} s \left(\Delta^2 + 2 \Delta p_j x_i - p_i x_i (2 \Delta + p_j ((-1 + T^s) x_i - 2 x_j)) + p_j^2 x_i ((-1 + T^s) x_i - 2 x_j) \right);$$

(Alt) In[]:=

$$\mathbf{V@r_{2,1}[i_, j_]} := \frac{1}{12} \Delta (\mathbf{p}_i - \mathbf{p}_j) \mathbf{x}_i \left(-6 \Delta^2 + \mathbf{p}_j \left(-2 (-1 + \mathbf{T}) \left(-2 \mathbf{p}_i + (3 + \mathbf{T}) \mathbf{p}_j \right) \mathbf{x}_i^2 - 6 \mathbf{x}_j \left(-3 \Delta + \mathbf{p}_j \mathbf{x}_j \right) + 3 \mathbf{x}_i \left(\Delta - 3 \mathbf{T} \Delta + 2 \left(-\mathbf{p}_i + (1 + \mathbf{T}) \mathbf{p}_j \right) \mathbf{x}_j \right) \right) \right);$$

(Alt) In[]:=

$$\mathbf{V@r_{2,-1}[i_, j_]} := -\frac{1}{12 \mathbf{T}^2} \Delta (\mathbf{p}_i - \mathbf{p}_j) \mathbf{x}_i \left(6 \mathbf{T}^2 \Delta^2 + \mathbf{p}_j \left(2 (-1 + \mathbf{T}) \left(2 \mathbf{T} \mathbf{p}_i - (1 + 3 \mathbf{T}) \mathbf{p}_j \right) \mathbf{x}_i^2 + 6 \mathbf{T}^2 \mathbf{x}_j \left(-3 \Delta + \mathbf{p}_j \mathbf{x}_j \right) - 3 \mathbf{T} \mathbf{x}_i \left((-3 + \mathbf{T}) \Delta + 2 \left(-\mathbf{T} \mathbf{p}_i + (1 + \mathbf{T}) \mathbf{p}_j \right) \mathbf{x}_j \right) \right) \right);$$

(Alt) In[]:=

$$\mathbf{V@r_{3,1}[i_, j_]} := \frac{1}{24} \Delta^2 \left(\mathbf{p}_i^3 \mathbf{p}_j \mathbf{x}_i^3 \left(3 (-1 + \mathbf{T}) \mathbf{x}_i - 4 \mathbf{x}_j \right) + 4 \Delta \mathbf{p}_j^3 \mathbf{x}_i \left((11 - \mathbf{T} (11 + 2 \mathbf{T})) \mathbf{x}_i^2 + 6 (3 + \mathbf{T}) \mathbf{x}_i \mathbf{x}_j - 6 \mathbf{x}_j^2 \right) + \mathbf{p}_j^4 \mathbf{x}_i \left(- \left((-1 + \mathbf{T}) (4 + \mathbf{T} (13 + \mathbf{T})) \mathbf{x}_i^3 \right) + 4 (-5 + \mathbf{T} (10 + \mathbf{T})) \mathbf{x}_i^2 \mathbf{x}_j - 6 (6 + \mathbf{T}) \mathbf{x}_i \mathbf{x}_j^2 + 4 \mathbf{x}_j^3 \right) + \mathbf{p}_i^2 \mathbf{p}_j \mathbf{x}_i^2 \left(-3 (-4 + \mathbf{T} + 3 \mathbf{T}^2) \mathbf{p}_j \mathbf{x}_i^2 + 12 \mathbf{x}_j \left(3 \Delta - 2 \mathbf{p}_j \mathbf{x}_j \right) + 4 \mathbf{x}_i \left(5 \Delta - 6 \mathbf{T} \Delta + 7 \mathbf{T} \mathbf{p}_j \mathbf{x}_j \right) \right) + 24 \Delta^4 \mathbf{ca}_{3,1} + 4 \Delta^3 \mathbf{p}_j \mathbf{x}_i \left(-1 + 12 \mathbf{ca}_{3,1} \right) + 2 \Delta^2 \mathbf{p}_j^2 \mathbf{x}_i \left(2 \mathbf{x}_j \left(7 - 12 \mathbf{ca}_{3,1} \right) + \mathbf{x}_i \left(-5 - 7 \mathbf{T} + 12 (-1 + \mathbf{T}) \mathbf{ca}_{3,1} \right) \right) + \mathbf{p}_i \mathbf{x}_i \left(4 \Delta \mathbf{p}_j^2 \left((-16 + \mathbf{T} (17 + 2 \mathbf{T})) \mathbf{x}_i^2 - 3 (9 + 2 \mathbf{T}) \mathbf{x}_i \mathbf{x}_j + 6 \mathbf{x}_j^2 \right) + \mathbf{p}_j^3 \left((-1 + \mathbf{T}) (13 + \mathbf{T} (22 + \mathbf{T})) \mathbf{x}_i^3 - 4 (-6 + \mathbf{T} (17 + \mathbf{T})) \mathbf{x}_i^2 \mathbf{x}_j + 6 (10 + \mathbf{T}) \mathbf{x}_i \mathbf{x}_j^2 - 4 \mathbf{x}_j^3 \right) + 4 \Delta^3 (1 - 12 \mathbf{ca}_{3,1}) + 2 \Delta^2 \mathbf{p}_j \left(2 \mathbf{x}_j \left(-7 + 12 \mathbf{ca}_{3,1} \right) + \mathbf{x}_i \left(5 + 7 \mathbf{T} - 12 (-1 + \mathbf{T}) \mathbf{ca}_{3,1} \right) \right) \right);$$

(Alt) In[]:=

$$\mathbf{V@r_{3,-1}[i_, j_]} := -\frac{1}{24 \mathbf{T}^3} \Delta^2 \left(\mathbf{T}^2 \mathbf{p}_i^3 \mathbf{p}_j \mathbf{x}_i^3 \left(-3 (-1 + \mathbf{T}) \mathbf{x}_i - 4 \mathbf{T} \mathbf{x}_j \right) + 4 \mathbf{T} \Delta \mathbf{p}_j^3 \mathbf{x}_i \left((-2 + 11 (-1 + \mathbf{T}) \mathbf{T}) \mathbf{x}_i^2 + 6 \mathbf{T} (1 + 3 \mathbf{T}) \mathbf{x}_i \mathbf{x}_j - 6 \mathbf{T}^2 \mathbf{x}_j^2 \right) + \mathbf{p}_j^4 \mathbf{x}_i \left((-1 + \mathbf{T}) (1 + \mathbf{T} (13 + 4 \mathbf{T})) \mathbf{x}_i^3 + 4 \mathbf{T} (1 - 5 (-2 + \mathbf{T}) \mathbf{T}) \mathbf{x}_i^2 \mathbf{x}_j - 6 \mathbf{T}^2 (1 + 6 \mathbf{T}) \mathbf{x}_i \mathbf{x}_j^2 + 4 \mathbf{T}^3 \mathbf{x}_j^3 \right) + \mathbf{T} \mathbf{p}_i^2 \mathbf{p}_j \mathbf{x}_i^2 \left(3 (-1 + \mathbf{T}) (3 + 4 \mathbf{T}) \mathbf{p}_j \mathbf{x}_i^2 + 12 \mathbf{T}^2 \mathbf{x}_j \left(3 \Delta - 2 \mathbf{p}_j \mathbf{x}_j \right) + 4 \mathbf{T} \mathbf{x}_i \left(-6 \Delta + 5 \mathbf{T} \Delta + 7 \mathbf{p}_j \mathbf{x}_j \right) \right) + 24 \mathbf{T}^3 \Delta^4 \mathbf{ca}_{3,1} + 4 \mathbf{T}^3 \Delta^3 \mathbf{p}_j \mathbf{x}_i \left(-1 + 12 \mathbf{ca}_{3,1} \right) - 2 \mathbf{T}^2 \Delta^2 \mathbf{p}_j^2 \mathbf{x}_i \left(2 \mathbf{T} \mathbf{x}_j \left(-7 + 12 \mathbf{ca}_{3,1} \right) + \mathbf{x}_i \left(7 + 5 \mathbf{T} + 12 (-1 + \mathbf{T}) \mathbf{ca}_{3,1} \right) \right) + \mathbf{p}_i \mathbf{x}_i \left(4 \mathbf{T} \Delta \mathbf{p}_j^2 \left((2 + (17 - 16 \mathbf{T}) \mathbf{T}) \mathbf{x}_i^2 - 3 \mathbf{T} (2 + 9 \mathbf{T}) \mathbf{x}_i \mathbf{x}_j + 6 \mathbf{T}^2 \mathbf{x}_j^2 \right) + \mathbf{p}_j^3 \left(- \left((-1 + \mathbf{T}) (1 + \mathbf{T} (22 + 13 \mathbf{T})) \mathbf{x}_i^3 \right) + 4 \mathbf{T} (-1 + \mathbf{T} (-17 + 6 \mathbf{T})) \mathbf{x}_i^2 \mathbf{x}_j + 6 \mathbf{T}^2 (1 + 10 \mathbf{T}) \mathbf{x}_i \mathbf{x}_j^2 - 4 \mathbf{T}^3 \mathbf{x}_j^3 \right) + 4 \mathbf{T}^3 \Delta^3 (1 - 12 \mathbf{ca}_{3,1}) + 2 \mathbf{T}^2 \Delta^2 \mathbf{p}_j \left(2 \mathbf{T} \mathbf{x}_j \left(-7 + 12 \mathbf{ca}_{3,1} \right) + \mathbf{x}_i \left(7 + 5 \mathbf{T} + 12 (-1 + \mathbf{T}) \mathbf{ca}_{3,1} \right) \right) \right);$$

(Alt) In[]:=

```
gPair[fs_, w_] := gPair[fs, w] = (
  Print["Running gPair[" , fs, ", ", w, "] ..."];
  Collect[ZipJoin@@Table[{pα, p̄α, xα, x̄α}, {α, w}] [(Times@@(V/@fs))
    Exp[Sum[gα,β(πα + π̄α)(ξβ + ξ̄β), {α, w}, {β, w}] - Sum[Δ ξ̄α πα, {α, w}]]],
  g_, Factor]
)
```

(Alt) In[]:=

```
ρd_[K_] := PPpd@Module[{Cs, φ, n, A, s, i, j, k, G, d1, ρd1, ρd2, ρd3, ρd4},
  PPGreen[
    {Cs, φ} = Rot[K]; n = Length[Cs];
    A = IdentityMatrix[2 n + 1];
    Cases[Cs, {s_, i_, j_} => (A[[{i, j}, {i + 1, j + 1}]] += (
      -Ts Ts - 1
      0 -1
    ))];
    G = Factor[T(-Total[φ]-Total[Cs[[All,1]])/2} Det[A] × Inverse[A]];
  ];
  ρd1 = PPMold@Exp[Total[Cases[Cs, {s_, i_, j_} => Sum[ed1 rd1,s[i, j], {d1, d}]]] +
    Sum[ed1 γd1,φ[[k]][k], {k, 2 n}, {d1, d}]];
  ρd2 = PPExpandedMold[
    Expand[F[{}, {}] × Normal@Series[ρd1, {ε, 0, d}]] /. F[fs_, {es___}] ×
    (f : (r | γ)ps_[is___])p_ => F[Join[fs, Table[f, p]], DeleteDuplicates@{es, is}]
  ];
  ρd3 = PPPairing[
    ρd2 /. F[fs_, es_] => (gPair[
      Replace[fs, Thread[es → Range@Length@es], {2}],
      Length@es
    ] /. gα,β_ => ges[[α],es[[β]])
  ];
  ρd4 = PPSubstitution@Collect[
    {Δ, ρd3} /. {gα,β_ => G[[α, β]], Δ → T(-Total[φ]-Total[Cs[[All,1]])/2} Det[A]},
    ε, Factor]
  ];
];
```

Testing

(Alt) In[]:=

```
ρ1[Knot[3, 1]]
```

KnotTheory: Loading precomputed data in PD4Knots`.

Running gPair[{}, 0] ...

Running gPair[{r1, -1 [1, 2]}, 2] ...

Running gPair[{γ1, -1 [1]}, 1] ...

Running gPair[{γ1, 0 [1]}, 1] ...

(Alt) Out[] =

$$\left\{ \frac{1 - T + T^2}{T}, 1 + \frac{(-1 + T)^2 (1 + T^2)}{T^2} \in \right\}$$

(Alt) In[] :=

TableForm[Table[Join[{K[[1]]_{K[[2]]}}, ρ1[K]], {K, AllKnots[{3, 6]}}], TableAlignments → Center]

Running gPair[{r1, 1 [1, 2]}, 2] ...

Running gPair[{γ1, 1 [1]}, 1] ...

(Alt) Out[] // TableForm =

3 ₁	$\frac{1 - T + T^2}{T}$	1 +	$\frac{(-1 + T)^2 (1 + T^2)}{T^2} \in$
4 ₁	$-\frac{1 - 3T + T^2}{T}$		1
5 ₁	$\frac{1 - T + T^2 - T^3 + T^4}{T^2}$	1 +	$\frac{(-1 + T)^2 (1 + T^2) (2 + T^2 + 2T^4)}{T^4} \in$
5 ₂	$\frac{2 - 3T + 2T^2}{T}$	1 +	$\frac{(-1 + T)^2 (5 - 4T + 5T^2)}{T^2} \in$
6 ₁	$-\frac{(-2 + T)(-1 + 2T)}{T}$	1 +	$\frac{(-1 + T)^2 (1 - 4T + T^2)}{T^2} \in$
6 ₂	$-\frac{1 - 3T + 3T^2 - 3T^3 + T^4}{T^2}$	1 +	$\frac{(-1 + T)^2 (1 - 4T + 4T^2 - 4T^3 + 4T^4 - 4T^5 + T^6)}{T^4} \in$
6 ₃	$\frac{1 - 3T + 5T^2 - 3T^3 + T^4}{T^2}$		1

(Alt) In[] :=

ρ2[Knot[3, 1]]

```

Running gPair[{r2,-1[1,2]},2]...
Running gPair[{\gamma2,-1[1]},1]...
Running gPair[{\gamma2,0[1]},1]...
Running gPair[{r1,-1[1,2], r1,-1[1,2]},2]...
Running gPair[{r1,-1[1,2], r1,-1[3,4]},4]...
Running gPair[{r1,-1[1,2], \gamma1,-1[3]},3]...
Running gPair[{r1,-1[1,2], \gamma1,0[3]},3]...
Running gPair[{r1,-1[1,2], \gamma1,0[1]},2]...
Running gPair[{r1,-1[1,2], \gamma1,0[2]},2]...
Running gPair[{r1,-1[1,2], \gamma1,-1[1]},2]...
Running gPair[{\gamma1,-1[1], \gamma1,-1[1]},1]...
Running gPair[{\gamma1,-1[1], \gamma1,0[2]},2]...
Running gPair[{\gamma1,0[1], \gamma1,0[1]},1]...
Running gPair[{\gamma1,0[1], \gamma1,0[2]},2]...

```

(Alt) Out[] =

$$\left\{ \frac{1 - T + T^2}{T}, 1 + \frac{(-1 + T)^2 (1 + T^2)}{T^2} + \frac{(1 - 4T + 7T^2 - 12T^3 + 18T^4 - 12T^5 + 7T^6 - 4T^7 + T^8) \epsilon^2}{2T^4} \right\}$$

(Alt) In[]:=

```

BeginProfile[]
Timing[z1 = ρ₂[Knot[10, 106]]]
PrintProfile[]

```

(Alt) Out[]:=

ProfileRoot

(Alt) Out[]:=

$$\left\{ 3.90625, \left\{ -\frac{(1-T+T^2)(-1+T-2T^2+T^3)(-1+2T-T^2+T^3)}{T^4}, 1 - \frac{1}{T^8}(-1+T)^2(1-6T+20T^2-48T^3+82T^4-114T^5+134T^6-140T^7+134T^8-114T^9+82T^{10}-48T^{11}+20T^{12}-6T^{13}+T^{14}) \in + \frac{1}{2T^{16}}(1-16T+127T^2-676T^3+2735T^4-8980T^5+24938T^6-60420T^7+131072T^8-259992T^9+477614T^{10}-814576T^{11}+1282448T^{12}-1846716T^{13}+2411126T^{14}-2836312T^{15}+2995252T^{16}-2836312T^{17}+2411126T^{18}-1846716T^{19}+1282448T^{20}-814576T^{21}+477614T^{22}-259992T^{23}+131072T^{24}-60420T^{25}+24938T^{26}-8980T^{27}+2735T^{28}-676T^{29}+127T^{30}-16T^{31}+T^{32}) \in^2 \right\} \right\}$$

(Alt) Out[]:=

```

ProfileRoot is root. Profiled time: 3.906
( 1) 0/ 3.910 above ρd
Substitution: called 1 times, time in 2.25/2.25
( 1) 2.250/ 2.250 under ρd
Green: called 1 times, time in 1.437/1.437
( 1) 1.440/ 1.440 under ρd
ExpandedMold: called 1 times, time in 0.11/0.11
( 1) 0.110/ 0.110 under ρd
Pairing: called 1 times, time in 0.109/0.109
( 1) 0.109/ 0.109 under ρd
ρd: called 1 times, time in 0./3.906
( 1) 0/ 3.910 under ProfileRoot
( 1) 1.440/ 1.440 above Green
( 1) 0.110/ 0.110 above ExpandedMold
( 1) 0/ 0 above Mold
( 1) 0.109/ 0.109 above Pairing
( 1) 2.250/ 2.250 above Substitution
Mold: called 1 times, time in 0./0.
( 1) 0/ 0 under ρd

```

(Alt) In[]:=

```
BeginProfile[]
Timing[z2 = ρ2[Knot[12, NonAlternating, 369]]]
PrintProfile[]
```

(Alt) Out[]:=

ProfileRoot

(Alt) Out[]:=

$$\left\{ 8.5625, \left\{ -\frac{(1 - T + T^2)(-1 + T - 2T^2 + T^3)(-1 + 2T - T^2 + T^3)}{T^4}, 1 - \frac{1}{T^8}(-1 + T)^2(1 - 6T + 20T^2 - 48T^3 + 82T^4 - 114T^5 + 134T^6 - 140T^7 + 134T^8 - 114T^9 + 82T^{10} - 48T^{11} + 20T^{12} - 6T^{13} + T^{14}) \in + \frac{1}{2T^{16}}(1 - 16T + 127T^2 - 668T^3 + 2631T^4 - 8324T^5 + 22282T^6 - 52780T^7 + 114992T^8 - 236376T^9 + 460598T^{10} - 839688T^{11} + 1404696T^{12} - 2121524T^{13} + 2862782T^{14} - 3432312T^{15} + 3647156T^{16} - 3432312T^{17} + 2862782T^{18} - 2121524T^{19} + 1404696T^{20} - 839688T^{21} + 460598T^{22} - 236376T^{23} + 114992T^{24} - 52780T^{25} + 22282T^{26} - 8324T^{27} + 2631T^{28} - 668T^{29} + 127T^{30} - 16T^{31} + T^{32}) \in^2 \right\} \right\}$$

(Alt) Out[]:=

```
ProfileRoot is root. Profiled time: 8.562
( 1) 0.031/ 8.562 above ρd
Substitution: called 1 times, time in 6.047/6.047
( 1) 6.047/ 6.047 under ρd
Green: called 1 times, time in 2.094/2.094
( 1) 2.094/ 2.094 under ρd
ExpandedMold: called 1 times, time in 0.218/0.218
( 1) 0.218/ 0.218 under ρd
Pairing: called 1 times, time in 0.172/0.172
( 1) 0.172/ 0.172 under ρd
ρd: called 1 times, time in 0.031/8.562
( 1) 0.031/ 8.562 under ProfileRoot
( 1) 2.094/ 2.094 above Green
( 1) 0.218/ 0.218 above ExpandedMold
( 1) 0/ 0 above Mold
( 1) 0.172/ 0.172 above Pairing
( 1) 6.047/ 6.047 above Substitution
Mold: called 1 times, time in 0./0.
( 1) 0/ 0 under ρd
```

(Alt) In[]:=

Simplify[Thread[z1 == z2]]

(Alt) Out[]:=

$$\left\{ \text{True}, \frac{1}{T} (-1 + T) (1 - T + T^2) (1 - 6 T + 16 T^2 - 23 T^3 + 9 T^4 + 47 T^5 - 141 T^6 + 231 T^7 - 272 T^8 + 231 T^9 - 141 T^{10} + 47 T^{11} + 9 T^{12} - 23 T^{13} + 16 T^{14} - 6 T^{15} + T^{16}) \in == \emptyset \right\}$$

(Alt) In[]:=

TableForm[Table[Join[{K[[1]]K[[2]]}, ρ2[K]], {K, AllKnots[{3, 7}]}], TableAlignments → Center]

(Alt) Out[]//TableForm=

3₁	$\frac{1-T+T^2}{T}$	$1 + \frac{(-1+T)^2 (1+T^2)}{T^2} \in +$
4₁	$-\frac{1-3T+T^2}{T}$	$1 +$
5₁	$\frac{1-T+T^2-T^3+T^4}{T^2}$	$1 + \frac{(-1+T)^2 (1+T^2) (2+T^2+2T^4)}{T^4} \in + \frac{(4-16T+35T^2-60T^3+85T^4-}{T^4}$
5₂	$\frac{2-3T+2T^2}{T}$	$1 + \frac{(-1+T)^2 (5-4T+5T^2)}{T^2} \in + \frac{(26-}{T^2}$
6₁	$-\frac{(-2+T)(-1+2T)}{T}$	$1 + \frac{(-1+T)^2 (1-4T+T^2)}{T^2} \in + \frac{(2-}{T^2}$
6₂	$-\frac{1-3T+3T^2-3T^3+T^4}{T^2}$	$1 + \frac{(-1+T)^2 (1-4T+4T^2-4T^3+4T^4-4T^5+T^6)}{T^4} \in + \frac{(1-12T+62T^2-180T^3+354}{T^4}$
6₃	$\frac{1-3T+5T^2-3T^3+T^4}{T^2}$	$1 - \frac{(1-T+T^2)(1-3T}{T^2}$
7₁	$\frac{1-T+T^2-T^3+T^4-T^5+T^6}{T^3}$	$1 + \frac{(-1+T)^2 (1+T^2) (3+2T^2+4T^4+2T^6+3T^8)}{T^6} \in + \frac{(9-36T+83T^2-152T^3+238T^4-336T^5+434T^6-556T^7+719T^8-}{T^6}$
7₂	$\frac{3-5T+3T^2}{T}$	$1 + \frac{2(-1+T)^2 (7-8T+7T^2)}{T^2} \in + \frac{(105-65}{T^2}$
7₃	$\frac{2-3T+3T^2-3T^3+2T^4}{T^2}$	$1 - \frac{(-1+T)^2 (9-8T+16T^2-12T^3+16T^4-8T^5+9T^6)}{T^4} \in + \frac{(82-472T+1409T^2-2996T^3+5190T^4-}{T^4}$
7₄	$\frac{4-7T+4T^2}{T}$	$1 - \frac{8(-1+T)^2 (3-4T+3T^2)}{T^2} \in + \frac{(304-2032}{T^2}$
7₅	$\frac{2-4T+5T^2-4T^3+2T^4}{T^2}$	$1 + \frac{(-1+T)^2 (9-16T+29T^2-28T^3+29T^4-16T^5+9T^6)}{T^4} \in + \frac{(82-616T+2412T^2-6560T^3+13875T^4-}{T^4}$
7₆	$-\frac{1-5T+7T^2-5T^3+T^4}{T^2}$	$1 + \frac{(-1+T)^2 (1-8T+19T^2-20T^3+19T^4-8T^5+T^6)}{T^4} \in + \frac{(1-20T+175T^2-880T^3+2923T^4-}{T^4}$
7₇	$\frac{1-5T+9T^2-5T^3+T^4}{T^2}$	$1 - \frac{(-1+T)^2 (3-8T+3T^2)}{T^2} \in + \frac{(1-20T+199T^2-1064}{T^2}$

(Alt) In[]:=

GST48 = EPD[X_{14,1}, X̄_{2,29}, X_{3,40}, X_{43,4}, X̄_{26,5}, X_{6,95}, X_{96,7}, X_{13,8}, X̄_{9,28}, X_{10,41}, X_{42,11}, X̄_{27,12}, X_{30,15}, X̄_{16,61}, X̄_{17,72}, X̄_{18,83}, X_{19,34}, X̄_{89,20}, X̄_{21,92}, X̄_{79,22}, X̄_{68,23}, X̄_{57,24}, X̄_{25,56}, X_{62,31}, X_{73,32}, X_{84,33}, X̄_{50,35}, X_{36,81}, X_{37,70}, X_{38,59}, X̄_{39,54}, X_{44,55}, X_{58,45}, X_{69,46}, X_{80,47}, X_{48,91}, X_{90,49}, X_{51,82}, X_{52,71}, X_{53,60}, X̄_{63,74}, X̄_{64,85}, X̄_{76,65}, X̄_{87,66}, X̄_{67,94}, X̄_{75,86}, X̄_{88,77}, X̄_{78,93}];

BeginProfile[]

Timing[z3 = ρ2[GST48]]

PrintProfile[]

(Alt) Out[]:=

ProfileRoot

(Alt) Out[*]=

$$\left\{ 321.484, \left\{ -\frac{(-1 + 2T - T^2 - T^3 + 2T^4 - T^5 + T^8)(-1 + T^3 - 2T^4 + T^5 + T^6 - 2T^7 + T^8)}{T^8}, \right. \right.$$

$$1 + \frac{1}{T^{16}} (-1 + T)^2 (5 - 18T + 33T^2 - 32T^3 + 2T^4 + 42T^5 - 62T^6 - 8T^7 + 166T^8 - 242T^9 + 108T^{10} +$$

$$132T^{11} - 226T^{12} + 148T^{13} - 11T^{14} - 36T^{15} - 11T^{16} + 148T^{17} - 226T^{18} + 132T^{19} + 108T^{20} -$$

$$242T^{21} + 166T^{22} - 8T^{23} - 62T^{24} + 42T^{25} + 2T^{26} - 32T^{27} + 33T^{28} - 18T^{29} + 5T^{30}) \in +$$

$$\frac{1}{2T^{32}} (25 - 348T + 2312T^2 - 9628T^3 + 27228T^4 - 51460T^5 + 52250T^6 + 25828T^7 -$$

$$197145T^8 + 313268T^9 - 36579T^{10} - 887864T^{11} + 2118398T^{12} - 2494152T^{13} + 772387T^{14} +$$

$$2785204T^{15} - 5477089T^{16} + 3765568T^{17} + 2886710T^{18} - 9712796T^{19} + 9746285T^{20} -$$

$$708568T^{21} - 11443177T^{22} + 17013304T^{23} - 11217405T^{24} - 1334300T^{25} + 10332369T^{26} -$$

$$8571752T^{27} - 1186874T^{28} + 8007252T^{29} - 3568015T^{30} - 8148860T^{31} + 14395240T^{32} -$$

$$8148860T^{33} - 3568015T^{34} + 8007252T^{35} - 1186874T^{36} - 8571752T^{37} + 10332369T^{38} -$$

$$1334300T^{39} - 11217405T^{40} + 17013304T^{41} - 11443177T^{42} - 708568T^{43} + 9746285T^{44} -$$

$$9712796T^{45} + 2886710T^{46} + 3765568T^{47} - 5477089T^{48} + 2785204T^{49} + 772387T^{50} -$$

$$2494152T^{51} + 2118398T^{52} - 887864T^{53} - 36579T^{54} + 313268T^{55} - 197145T^{56} +$$

$$25828T^{57} + 52250T^{58} - 51460T^{59} + 27228T^{60} - 9628T^{61} + 2312T^{62} - 348T^{63} + 25T^{64}) \in^2 \left. \right\}$$

(Alt) Out[*]=

```
ProfileRoot is root. Profiled time: 321.484
( 1) 0.171/ 321.480 above ρd
Green: called 1 times, time in 163.891/163.891
( 1) 163.890/ 163.890 under ρd
Substitution: called 1 times, time in 148.922/148.922
( 1) 148.920/ 148.920 under ρd
ExpandedMold: called 1 times, time in 6.562/6.562
( 1) 6.562/ 6.562 under ρd
Pairing: called 1 times, time in 1.938/1.938
( 1) 1.938/ 1.938 under ρd
ρd: called 1 times, time in 0.171/321.484
( 1) 0.171/ 321.480 under ProfileRoot
( 1) 163.890/ 163.890 above Green
( 1) 6.562/ 6.562 above ExpandedMold
( 1) 0/ 0 above Mold
( 1) 1.938/ 1.938 above Pairing
( 1) 148.920/ 148.920 above Substitution
Mold: called 1 times, time in 0./0.
( 1) 0/ 0 under ρd
```

(Alt) In[]:=

```
BeginProfile[]
Timing[ρ₃[Knot[3, 1]]]
PrintProfile[]
```

(Alt) Out[]:=

ProfileRoot

(Alt) Out[]:=

$$\left\{ 3.75, \left\{ \frac{1 - T + T^2}{T}, 1 + \frac{(-1 + T)^2 (1 + T^2)}{T^2} + \frac{(1 - 4T + 7T^2 - 12T^3 + 18T^4 - 12T^5 + 7T^6 - 4T^7 + T^8) \epsilon^2}{2T^4} - \frac{1}{6T^6} \epsilon^3 (-1 + 6T - 14T^2 + 34T^3 - 92T^4 + 98T^5 - 50T^6 + 98T^7 - 92T^8 + 34T^9 - 14T^{10} + 6T^{11} - T^{12} + 12ca_{3,1} - 72Tca_{3,1} + 240T^2ca_{3,1} - 552T^3ca_{3,1} + 960T^4ca_{3,1} - 1320T^5ca_{3,1} + 1464T^6ca_{3,1} - 1320T^7ca_{3,1} + 960T^8ca_{3,1} - 552T^9ca_{3,1} + 240T^{10}ca_{3,1} - 72T^{11}ca_{3,1} + 12T^{12}ca_{3,1}) \right\} \right\}$$

(Alt) Out[]:=

```
ProfileRoot is root. Profiled time: 3.75
( 1) 0.016/ 3.750 above ρd
Substitution: called 1 times, time in 3.421/3.421
( 1) 3.420/ 3.420 under ρd
Pairing: called 1 times, time in 0.25/0.25
( 1) 0.250/ 0.250 under ρd
ExpandedMold: called 1 times, time in 0.063/0.063
( 1) 0.063/ 0.063 under ρd
ρd: called 1 times, time in 0.016/3.75
( 1) 0.016/ 3.750 under ProfileRoot
( 1) 0/ 0 above Green
( 1) 0.063/ 0.063 above ExpandedMold
( 1) 0/ 0 above Mold
( 1) 0.250/ 0.250 above Pairing
( 1) 3.420/ 3.420 above Substitution
Mold: called 1 times, time in 0./0.
( 1) 0/ 0 under ρd
Green: called 1 times, time in 0./0.
( 1) 0/ 0 under ρd
```

(Alt) In[]:=

```
BeginProfile[]
Timing[ρ3[Knot[4, 1]]]
PrintProfile[]
```

(Alt) Out[]:=

ProfileRoot

(Alt) Out[]:=

$$\left\{ 3.35938, \left\{ -\frac{1 - 3T + T^2}{T}, 1 + \frac{(1 - 3T + T^2)(1 - T + T^2)\epsilon^2}{T^2} \right\} \right\}$$

(Alt) Out[]:=

```
ProfileRoot is root. Profiled time: 3.359
( 1) 0.078/ 3.360 above ρd
Substitution: called 1 times, time in 2.469/2.469
( 1) 2.470/ 2.470 under ρd
Pairing: called 1 times, time in 0.719/0.719
( 1) 0.719/ 0.719 under ρd
ExpandedMold: called 1 times, time in 0.093/0.093
( 1) 0.093/ 0.093 under ρd
ρd: called 1 times, time in 0.078/3.359
( 1) 0.078/ 3.360 under ProfileRoot
( 1) 0/ 0 above Green
( 1) 0.093/ 0.093 above ExpandedMold
( 1) 0/ 0 above Mold
( 1) 0.719/ 0.719 above Pairing
( 1) 2.470/ 2.470 above Substitution
Mold: called 1 times, time in 0./0.
( 1) 0/ 0 under ρd
Green: called 1 times, time in 0./0.
( 1) 0/ 0 under ρd
```

(Alt) In[]:=

```
TableForm[Table[Echo@Join[{K[[1]]K[[2]]}, ρ3[K]], {K, AllKnots[{3, 6]}]],
TableAlignments -> Center]
```

$$\gg \left\{ 3_1, \frac{1 - T + T^2}{T}, 1 + \frac{(-1 + T)^2(1 + T^2)\epsilon}{T^2} + \frac{(1 - 4T + 7T^2 - 12T^3 + 18T^4 - 12T^5 + 7T^6 - 4T^7 + T^8)\epsilon^2}{2T^4} - \frac{1}{6T^6} \epsilon^3 (-1 + 6T - 14T^2 + 34T^3 - 92T^4 + 98T^5 - 50T^6 + 98T^7 - 92T^8 + 34T^9 - 14T^{10} + 6T^{11} - T^{12} + 12ca_{3,1} - 72Tca_{3,1} + 240T^2ca_{3,1} - 552T^3ca_{3,1} + 960T^4ca_{3,1} - 1320T^5ca_{3,1} + 1464T^6ca_{3,1} - 1320T^7ca_{3,1} + 960T^8ca_{3,1} - 552T^9ca_{3,1} + 240T^{10}ca_{3,1} - 72T^{11}ca_{3,1} + 12T^{12}ca_{3,1}) \right\}$$

$$\gg \left\{ 4_1, -\frac{1 - 3T + T^2}{T}, 1 + \frac{(1 - 3T + T^2)(1 - T + T^2)\epsilon^2}{T^2} \right\}$$

$$\gg \left\{ 5_1, \frac{1 - T + T^2 - T^3 + T^4}{T^2}, \right. \\ \left. 1 + \frac{(-1 + T)^2 (1 + T^2) (2 + T^2 + 2 T^4) \epsilon}{T^4} + \frac{1}{2 T^8} (4 - 16 T + 35 T^2 - 60 T^3 + 85 T^4 - 120 T^5 + 170 T^6 - \right. \\ \left. 220 T^7 + 250 T^8 - 220 T^9 + 170 T^{10} - 120 T^{11} + 85 T^{12} - 60 T^{13} + 35 T^{14} - 16 T^{15} + 4 T^{16}) \epsilon^2 - \right. \\ \left. \frac{1}{6 T^{12}} \epsilon^3 (-8 + 48 T - 149 T^2 + 334 T^3 - 590 T^4 + 998 T^5 - 1844 T^6 + 3350 T^7 - 5386 T^8 + 6802 T^9 - \right. \\ \left. 6772 T^{10} + 5758 T^{11} - 5022 T^{12} + 5758 T^{13} - 6772 T^{14} + 6802 T^{15} - 5386 T^{16} + 3350 T^{17} - 1844 T^{18} + \right. \\ \left. 998 T^{19} - 590 T^{20} + 334 T^{21} - 149 T^{22} + 48 T^{23} - 8 T^{24} + 24 ca_{3,1} - 144 T ca_{3,1} + 492 T^2 ca_{3,1} - \right. \\ \left. 1272 T^3 ca_{3,1} + 2760 T^4 ca_{3,1} - 5208 T^5 ca_{3,1} + 8736 T^6 ca_{3,1} - 13272 T^7 ca_{3,1} + 18480 T^8 ca_{3,1} - \right. \\ \left. 23736 T^9 ca_{3,1} + 28272 T^{10} ca_{3,1} - 31368 T^{11} ca_{3,1} + 32472 T^{12} ca_{3,1} - 31368 T^{13} ca_{3,1} + \right. \\ \left. 28272 T^{14} ca_{3,1} - 23736 T^{15} ca_{3,1} + 18480 T^{16} ca_{3,1} - 13272 T^{17} ca_{3,1} + 8736 T^{18} ca_{3,1} - \right. \\ \left. 5208 T^{19} ca_{3,1} + 2760 T^{20} ca_{3,1} - 1272 T^{21} ca_{3,1} + 492 T^{22} ca_{3,1} - 144 T^{23} ca_{3,1} + 24 T^{24} ca_{3,1} \right) \left. \right\}$$

Running gPair[{γ_{3,1}[1]},1]...

Running gPair[{r_{1,-1}[1,2], γ_{2,1}[3]},3]...

Running gPair[{r_{1,-1}[1,2], γ_{2,1}[2]},2]...

Running gPair[{r_{2,-1}[1,2], γ_{1,1}[3]},3]...

Running gPair[{r_{2,-1}[1,2], γ_{1,1}[2]},2]...

Running gPair[{γ_{1,-1}[1], γ_{2,1}[2]},2]...

Running gPair[{γ_{1,0}[1], γ_{2,1}[2]},2]...

Running gPair[{γ_{1,1}[1], γ_{2,-1}[2]},2]...

Running gPair[{γ_{1,1}[1], γ_{2,0}[2]},2]...

Running gPair[{γ_{1,1}[1], γ_{2,1}[1]},1]...

Running gPair[{r_{1,-1}[1,2], r_{1,-1}[1,2], γ_{1,1}[3]},3]...

Running gPair[{r_{1,-1}[1,2], r_{1,-1}[3,4], γ_{1,1}[5]},5]...

Running gPair[{r_{1,-1}[1,2], r_{1,-1}[3,4], γ_{1,1}[4]},4]...

Running gPair[{r_{1,-1}[1,2], γ_{1,-1}[3], γ_{1,1}[4]},4]...

Running gPair[{r_{1,-1}[1,2], γ_{1,0}[3], γ_{1,1}[4]},4]...

Running gPair[{r_{1,-1}[1,2], γ_{1,0}[1], γ_{1,1}[3]},3]...

Running gPair[{r_{1,-1}[1,2], γ_{1,0}[2], γ_{1,1}[3]},3]...

Running gPair[{r_{1,-1}[1,2], γ_{1,1}[3], γ_{1,1}[3]},3]...

Running gPair[{r_{1,-1}[1,2], γ_{1,-1}[1], γ_{1,-1}[3]},3]...

Running gPair[{r_{1,-1}[1,2], γ_{1,-1}[1], γ_{1,1}[3]},3]...

Running gPair[{r_{1,-1}[1,2], r_{1,-1}[1,2], γ_{1,1}[2]},2]...

Running gPair[{r_{1,-1}[1,2], r_{1,-1}[3,4], γ_{1,1}[2]},4]...

Running gPair[{r_{1,-1}[1,2], γ_{1,-1}[3], γ_{1,1}[2]},3]...

Running gPair[{r_{1,-1}[1,2], γ_{1,0}[3], γ_{1,1}[2]},3]...

Running gPair[{r_{1,-1}[1,2], γ_{1,0}[1], γ_{1,1}[2]},2]...

Running gPair[{r1,-1[1, 2], γ1,1[2], γ1,1[2]},2]...
 Running gPair[{γ1,-1[1], γ1,-1[1], γ1,1[2]},2]...
 Running gPair[{γ1,-1[1], γ1,-1[2], γ1,1[3]},3]...
 Running gPair[{γ1,-1[1], γ1,0[2], γ1,1[3]},3]...
 Running gPair[{γ1,-1[1], γ1,1[2], γ1,1[2]},2]...
 Running gPair[{γ1,0[1], γ1,0[1], γ1,1[2]},2]...
 Running gPair[{γ1,0[1], γ1,0[2], γ1,1[3]},3]...
 Running gPair[{γ1,0[1], γ1,1[2], γ1,1[2]},2]...
 Running gPair[{γ1,1[1], γ1,1[1], γ1,1[1]},1]...

$$\gg \left\{ 5_2, \frac{2 - 3T + 2T^2}{T}, \right. \\
 1 + \frac{(-1 + T)^2 (5 - 4T + 5T^2)}{T^2} + \frac{(26 - 144T + 387T^2 - 688T^3 + 842T^4 - 688T^5 + 387T^6 - 144T^7 + 26T^8) \epsilon^2}{2T^4} - \\
 \left. \frac{1}{6T^6} \epsilon^3 (-146 + 1196T - 4892T^2 + 13528T^3 - 26915T^4 + 39038T^5 - 43582T^6 + 39038T^7 - \right. \\
 26915T^8 + 13528T^9 - 4892T^{10} + 1196T^{11} - 146T^{12} + 960ca_{3,1} - 8448Tca_{3,1} + 36384T^2ca_{3,1} - \\
 100704T^3ca_{3,1} + 198780T^4ca_{3,1} - 294216T^5ca_{3,1} + 334488T^6ca_{3,1} - 294216T^7ca_{3,1} + \\
 \left. 198780T^8ca_{3,1} - 100704T^9ca_{3,1} + 36384T^{10}ca_{3,1} - 8448T^{11}ca_{3,1} + 960T^{12}ca_{3,1}) \right\}$$

Running gPair[{r1,1[1, 2], γ2,1[3]},3]...
 Running gPair[{r2,1[1, 2], γ1,1[3]},3]...
 Running gPair[{r1,-1[1, 2], r1,1[3, 4], γ1,1[5]},5]...
 Running gPair[{r1,-1[1, 2], r1,1[3, 4], γ1,1[2]},4]...
 Running gPair[{r1,1[1, 2], r1,1[1, 2], γ1,1[3]},3]...
 Running gPair[{r1,1[1, 2], r1,1[3, 4], γ1,1[5]},5]...
 Running gPair[{r1,1[1, 2], γ1,-1[3], γ1,-1[2]},3]...
 Running gPair[{r1,1[1, 2], γ1,-1[3], γ1,1[4]},4]...
 Running gPair[{r1,1[1, 2], γ1,-1[2], γ1,1[3]},3]...
 Running gPair[{r1,1[1, 2], γ1,0[3], γ1,1[4]},4]...
 Running gPair[{r1,1[1, 2], γ1,0[1], γ1,1[3]},3]...
 Running gPair[{r1,1[1, 2], γ1,1[3], γ1,1[3]},3]...
 Running gPair[{r1,1[1, 2], γ1,0[2], γ1,0[3]},3]...
 Running gPair[{r1,1[1, 2], γ1,0[2], γ1,0[1]},2]...
 Running gPair[{r1,1[1, 2], γ1,0[2], γ1,1[3]},3]...
 Running gPair[{γ1,-1[1], γ1,-1[2], γ1,-1[3]},3]...

$$\gg \left\{ 6_1, -\frac{(-2+T)(-1+2T)}{T}, \right.$$

$$1 + \frac{(-1+T)^2(1-4T+T^2)}{T^2} \in \frac{(2-24T+129T^2-328T^3+438T^4-328T^5+129T^6-24T^7+2T^8)\epsilon^2}{2T^4} -$$

$$\frac{1}{6T^6} \epsilon^3 (-10+172T-1280T^2+5948T^3-17415T^4+32514T^5-39870T^6+32514T^7 -$$

$$17415T^8+5948T^9-1280T^{10}+172T^{11}-10T^{12}+192ca_{3,1}-3072Tca_{3,1}+21408T^2ca_{3,1}-$$

$$85920T^3ca_{3,1}+221004T^4ca_{3,1}-383400T^5ca_{3,1}+459576T^6ca_{3,1}-383400T^7ca_{3,1}+$$

$$221004T^8ca_{3,1}-85920T^9ca_{3,1}+21408T^{10}ca_{3,1}-3072T^{11}ca_{3,1}+192T^{12}ca_{3,1}) \left. \right\}$$

$$\gg \left\{ 6_2, -\frac{1-3T+3T^2-3T^3+T^4}{T^2}, \right.$$

$$1 + \frac{(-1+T)^2(1-4T+4T^2-4T^3+4T^4-4T^5+T^6)}{T^4} \in \frac{1}{2T^8} (1-12T+62T^2-180T^3+354T^4-592T^5+1007T^6 -$$

$$1576T^7+1870T^8-1576T^9+1007T^{10}-592T^{11}+354T^{12}-180T^{13}+62T^{14}-12T^{15}+T^{16}) \epsilon^2 -$$

$$\frac{1}{6T^{12}} \epsilon^3 (-1+18T-145T^2+688T^3-2165T^4+5386T^5-13442T^6+34666T^7-75044T^8+116434T^9 -$$

$$119944T^{10}+81054T^{11}-55022T^{12}+81054T^{13}-119944T^{14}+116434T^{15}-75044T^{16}+34666T^{17}-$$

$$13442T^{18}+5386T^{19}-2165T^{20}+688T^{21}-145T^{22}+18T^{23}-T^{24}+12ca_{3,1}-216Tca_{3,1}+1812T^2ca_{3,1}-$$

$$9552T^3ca_{3,1}+36060T^4ca_{3,1}-105240T^5ca_{3,1}+249504T^6ca_{3,1}-496776T^7ca_{3,1}+850128T^8ca_{3,1}-$$

$$1271352T^9ca_{3,1}+1681320T^{10}ca_{3,1}-1982088T^{11}ca_{3,1}+2092776T^{12}ca_{3,1}-1982088T^{13}ca_{3,1}+$$

$$1681320T^{14}ca_{3,1}-1271352T^{15}ca_{3,1}+850128T^{16}ca_{3,1}-496776T^{17}ca_{3,1}+249504T^{18}ca_{3,1}-$$

$$105240T^{19}ca_{3,1}+36060T^{20}ca_{3,1}-9552T^{21}ca_{3,1}+1812T^{22}ca_{3,1}-216T^{23}ca_{3,1}+12T^{24}ca_{3,1}) \left. \right\}$$

Running gPair[{r1,1[1, 2], r1,1[3, 4], r1,1[5, 6]},6]...

Running gPair[{r1,1[1, 2], r1,1[3, 4], \gamma1,0[2]},4]...

$$\gg \left\{ 6_3, \frac{1-3T+5T^2-3T^3+T^4}{T^2}, 1 - \frac{(1-T+T^2)(1-3T+5T^2-3T^3+T^4)(1-11T^2+19T^3-11T^4+T^6)\epsilon^2}{T^6} \right\}$$

(Alt) Out[]:=

3 ₁	$\frac{1-T+T^2}{T}$	
4 ₁	$-\frac{1-3T+T^2}{T}$	
5 ₁	$\frac{1-T+T^2-T^3+T^4}{T^2}$	$1 + \frac{(-1+T)^2(1+T^2)(2+T^2+2T^4)\epsilon}{T^4} + \frac{(4-16T+35T^2-60T^3+85T^4-1}{T^6}$
5 ₂	$\frac{2-3T+2T^2}{T}$	
6 ₁	$-\frac{(-2+T)(-1+2T)}{T}$	
6 ₂	$-\frac{1-3T+3T^2-3T^3+T^4}{T^2}$	$1 + \frac{(-1+T)^2(1-4T+4T^2-4T^3+4T^4-4T^5+T^6)\epsilon}{T^4} + \frac{(1-12T+62T^2-180T^3+354T^4-592T^5+1007T^6-1576T^7+1870T^8-1576T^9+1007T^{10}-592T^{11}+354T^{12}-180T^{13}+62T^{14}-12T^{15}+T^{16})\epsilon^2}{2T^8}$
6 ₃	$\frac{1-3T+5T^2-3T^3+T^4}{T^2}$	

(Alt) In[]:=

```
BeginProfile []
Timing[z1 = \rho3[Knot[11, NonAlternating, 34]]]
PrintProfile []
```

(Alt) Out[]=

ProfileRoot

Running gPair[{ $\gamma_{1,1}[1]$, $\gamma_{2,1}[2]$ },2]...
 Running gPair[{ $r_{1,-1}[1, 2]$, $\gamma_{1,1}[2]$, $\gamma_{1,1}[3]$ },3]...
 Running gPair[{ $r_{1,-1}[1, 2]$, $\gamma_{1,1}[3]$, $\gamma_{1,1}[4]$ },4]...
 Running gPair[{ $r_{1,-1}[1, 2]$, $\gamma_{1,1}[3]$, $\gamma_{1,1}[2]$ },3]...
 Running gPair[{ $r_{1,1}[1, 2]$, $\gamma_{1,1}[3]$, $\gamma_{1,1}[4]$ },4]...
 Running gPair[{ $\gamma_{1,-1}[1]$, $\gamma_{1,1}[2]$, $\gamma_{1,1}[3]$ },3]...
 Running gPair[{ $\gamma_{1,0}[1]$, $\gamma_{1,1}[2]$, $\gamma_{1,1}[3]$ },3]...
 Running gPair[{ $\gamma_{1,1}[1]$, $\gamma_{1,1}[1]$, $\gamma_{1,1}[2]$ },2]...
 Running gPair[{ $\gamma_{1,1}[1]$, $\gamma_{1,1}[2]$, $\gamma_{1,1}[2]$ },2]...

(Alt) Out[]=

$$\left\{ 550.219, \left\{ 1, 1 - \frac{2(-1+T)^2(1+T^4)}{T^3} + \frac{2(-1+T)^2(6-15T+12T^2+2T^3-3T^4-2T^5-3T^6+2T^7+12T^8-15T^9+6T^{10})}{T^6} \epsilon^2 - \frac{1}{3T^9} \epsilon^3 (360-2520T+7632T^2-12510T^3+10899T^4-2988T^5-2942T^6+2731T^7-695T^8+54T^9-695T^{10}+2731T^{11}-2942T^{12}-2988T^{13}+10899T^{14}-12510T^{15}+7632T^{16}-2520T^{17}+360T^{18}-12T^6 ca_{3,1}+24T^7 ca_{3,1}-12T^8 ca_{3,1}-12T^{10} ca_{3,1}+24T^{11} ca_{3,1}-12T^{12} ca_{3,1}) \right\} \right\}$$

(Alt) Out[]=

ProfileRoot is root. Profiled time: 550.219
 (1) 0.735/ 550.219 above ρd
 Substitution: called 1 times, time in 522.171/522.171
 (1) 522.171/ 522.171 under ρd
 Pairing: called 1 times, time in 23.813/23.813
 (1) 23.813/ 23.813 under ρd
 ExpandedMold: called 1 times, time in 2.594/2.594
 (1) 2.594/ 2.594 under ρd
 Green: called 1 times, time in 0.906/0.906
 (1) 0.906/ 0.906 under ρd
 ρd : called 1 times, time in 0.735/550.219
 (1) 0.735/ 550.219 under ProfileRoot
 (1) 0.906/ 0.906 above Green
 (1) 2.594/ 2.594 above ExpandedMold
 (1) 0/ 0 above Mold
 (1) 23.813/ 23.813 above Pairing
 (1) 522.171/ 522.171 above Substitution
 Mold: called 1 times, time in 0./0.
 (1) 0/ 0 under ρd

(Alt) In[]:=

```
BeginProfile[]
Timing[z2 = ρ3[Knot[11, NonAlternating, 42]]]
PrintProfile[]
```

(Alt) Out[]:=

ProfileRoot

KnotTheory: Loading precomputed data in DTCode4KnotsTo11`.

(Alt) Out[]:=

$$\left\{ 573.203, \left\{ 1, 1 - \frac{2(-1+T)^2(1+T^4)}{T^3} \epsilon + \frac{2(-1+T)^2(6-15T+12T^2+2T^3-3T^4-2T^5-3T^6+2T^7+12T^8-15T^9+6T^{10})}{T^6} \epsilon^2 - \frac{1}{3T^9} \epsilon^3 (360-2520T+7632T^2-12510T^3+10899T^4-2988T^5-2942T^6+2731T^7-695T^8+54T^9-695T^{10}+2731T^{11}-2942T^{12}-2988T^{13}+10899T^{14}-12510T^{15}+7632T^{16}-2520T^{17}+360T^{18}-12T^6 ca_{3,1}+24T^7 ca_{3,1}-12T^8 ca_{3,1}-12T^{10} ca_{3,1}+24T^{11} ca_{3,1}-12T^{12} ca_{3,1}) \right\} \right\}$$

(Alt) Out[]:=

```
ProfileRoot is root. Profiled time: 573.203
( 1) 1.750/ 573.203 above ρd
Substitution: called 1 times, time in 547./547.
( 1) 547.000/ 547.000 under ρd
Pairing: called 1 times, time in 21.11/21.11
( 1) 21.110/ 21.110 under ρd
ExpandedMold: called 1 times, time in 2.609/2.609
( 1) 2.609/ 2.609 under ρd
ρd: called 1 times, time in 1.75/573.203
( 1) 1.750/ 573.203 under ProfileRoot
( 1) 0.734/ 0.734 above Green
( 1) 2.609/ 2.609 above ExpandedMold
( 1) 0/ 0 above Mold
( 1) 21.110/ 21.110 above Pairing
( 1) 547.000/ 547.000 above Substitution
Green: called 1 times, time in 0.734/0.734
( 1) 0.734/ 0.734 under ρd
Mold: called 1 times, time in 0./0.
( 1) 0/ 0 under ρd
```

(Alt) In[]:=

```
z1 - z2
```

(Alt) Out[]:=

```
{0, 0}
```


(Alt) In[]:=

```

BeginProfile[]
Timing[ $\rho_3$ [GST48]]
PrintProfile[]

```

(Alt) Out[]:=

```

ProfileRoot

Running gPair[{ $r_{1,1}$ [1, 2],  $\gamma_{2,1}$ [1]}, 2]...
Running gPair[{ $r_{2,1}$ [1, 2],  $\gamma_{1,1}$ [1]}, 2]...
Running gPair[{ $r_{1,-1}$ [1, 2],  $r_{1,1}$ [3, 4],  $\gamma_{1,1}$ [3]}, 4]...
Running gPair[{ $r_{1,1}$ [1, 2],  $r_{1,1}$ [3, 4],  $\gamma_{1,-1}$ [4]}, 4]...
Running gPair[{ $r_{1,1}$ [1, 2],  $r_{1,1}$ [3, 4],  $\gamma_{1,1}$ [3]}, 4]...
Running gPair[{ $r_{1,1}$ [1, 2],  $r_{1,1}$ [1, 2],  $\gamma_{1,1}$ [1]}, 2]...
Running gPair[{ $r_{1,1}$ [1, 2],  $r_{1,1}$ [3, 4],  $\gamma_{1,1}$ [1]}, 4]...
Running gPair[{ $r_{1,1}$ [1, 2],  $\gamma_{1,-1}$ [3],  $\gamma_{1,1}$ [1]}, 3]...
Running gPair[{ $r_{1,1}$ [1, 2],  $\gamma_{1,0}$ [3],  $\gamma_{1,1}$ [1]}, 3]...
Running gPair[{ $r_{1,1}$ [1, 2],  $\gamma_{1,0}$ [2],  $\gamma_{1,1}$ [1]}, 2]...
Running gPair[{ $r_{1,1}$ [1, 2],  $\gamma_{1,1}$ [1],  $\gamma_{1,1}$ [1]}, 2]...
Running gPair[{ $r_{1,1}$ [1, 2],  $\gamma_{1,1}$ [1],  $\gamma_{1,1}$ [3]}, 3]...
Running gPair[{ $r_{1,1}$ [1, 2],  $\gamma_{1,1}$ [3],  $\gamma_{1,1}$ [1]}, 3]...
Running gPair[{ $\gamma_{1,1}$ [1],  $\gamma_{1,1}$ [2],  $\gamma_{1,1}$ [3]}, 3]...

```