Pensieve header: Developing ρ_d.

Program

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\Oaxaca-2210"];
```

In[ ]:=
```
Once[<< KnotTheory`; << Rot.m];
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.

Loading Rot.m from http://drorbn.net/la22/ap to compute rotation numbers.
```

In[ ]:=
```
<< "../../Projects/Profile/Profile.m"
```

```
This is Profile.m of http://www.drorbn.net/AcademicPensieve/Projects/Profile/.

This version: April 2020. Original version: July 1994.
```

In[ ]:=
```
{p*, x*, π*, ξ*, p̄*, x̄*, π̄*, ξ̄*} = {π, ξ, p, x, π̄, ξ̄, p̄, x̄}; (u_i__)* := (u*)_i;
```

In[ ]:=
```
Zip_{}[ℰ_] := ℰ;
Zip_{ξ_,ξs___}[ℰ_] := (Collect[ℰ // Zip_{ξs}, ξ] /. f_. ξ^d_. :> (D[f, {ξ*, d}])) /. ξ* → 0
```

In[ ]:=
```
{ca_{1,2} = 1, ca_{1,10} = -1, ca_{2,1} = 0, cb_{2,10} = 3 / 2};
```

In[ ]:=
```
V@γ_{d_,0}[j_] := 0; V@γ_{1,φ_}[k_] := φ (1/2 - p̄_k x̄_k);
```

In[ ]:=
```
V@γ_{2,1}[k_] := -1/2 p̄_k x̄_k; V@γ_{2,-1}[k_] := -1/2 p̄_k x̄_k;
```

In[ ]:=
```
V@r_{1,s_}[i_, j_] :=
  s (-1/2 + p_i x_i - p_j x_i + 1/2 (-1 + T^s) p_i p_j x_i^2 + 1/2 (1 - T^s) p_j^2 x_i^2 - p_i p_j x_i x_j + p_j^2 x_i x_j);
```

In[ ]:=
```
V@r_{2,1}[i_, j_] := -1/2 p_i x_i + (p_j x_i)/2 + 1/4 (1 - 3 T) p_i p_j x_i^2 + 1/4 (-1 + 3 T) p_j^2 x_i^2 + 1/3 (-1 + T) p_i^2 p_j x_i^3 -
  1/6 (-1 + T) (5 + T) p_i p_j^2 x_i^3 + 1/6 (-1 + T) (3 + T) p_j^3 x_i^3 + 3/2 p_i p_j x_i x_j - 3/2 p_j^2 x_i x_j -
  1/2 p_i^2 p_j x_i^2 x_j + 1/2 (2 + T) p_i p_j^2 x_i^2 x_j + 1/2 (-1 - T) p_j^3 x_i^2 x_j - 1/2 p_i p_j^2 x_i x_j^2 + 1/2 p_j^3 x_i x_j^2;
```

In[ ]:=
```
V@r_{2,-1}[i_, j_] := -1/2 p_i x_i + p_j x_i/2 + (-3+T) p_i p_j x_i^2/(4 T) - (-3+T) p_j^2 x_i^2/(4 T) - (-1+T) p_i^2 p_j x_i^3/(3 T) +
  (-1+T) (1+5 T) p_i p_j^2 x_i^3/(6 T^2) - (-1+T) (1+3 T) p_j^3 x_i^3/(6 T^2) + 3/2 p_i p_j x_i x_j - 3/2 p_j^2 x_i x_j -
  1/2 p_i^2 p_j x_i^2 x_j + (1+2 T) p_i p_j^2 x_i^2 x_j/(2 T) - (1+T) p_j^3 x_i^2 x_j/(2 T) - 1/2 p_i p_j^2 x_i x_j^2 + 1/2 p_j^3 x_i x_j^2;
```

In[ ]:=
```
gPair[1] = 1;
gPair[ε^(d_)· Bs_] := ε^d gPair[Bs];
gPair[c_?NumberQ * Bs_] := c gPair[Bs];
gPair[ℰ_Plus] := gPair /@ ℰ;
gPair[r_{d_,s_}[i_, j_]^(p_)·] := gPair[{r_{d,s}[i, j]^p}];
gPair[γ_{d_,φ_}[k_]^(p_)·] := gPair[{γ_{d,φ}[k]^p}];
gPair[Bs_Times] := gPair[List @@ Bs];
gPair[Bs_List] := Module[{es, BBs},
  BBs = Bs /. e_^p_ :> Sequence @@ Table[e, {p}];
  es = Union @@ (List @@@ BBs);
  Do[ν@i = es[[i]], {i, Length@es}];
  gpair[Replace[BBs, Thread[es → Range@Length@es], {2}]] /. g_{α_,β_} :> g_{ν@α,ν@β}
 ]
```

In[ ]:=
```
gpair[Bs_List] := gpair[Bs] = Module[{es},
   Print["At gpair with Bs=", Bs];
   es = Union @@ (List @@@ Bs);
   Simplify@Zip_{Join@@Table[{p_α,p̄_α,x_α,x̄_α},{α,es}]}[Times[
      Times @@ (Bs /. {
          r_{d_,s_}[i_, j_] :> V[r_{d,s}[i, j]], γ_{d_,φ_}[k_] :> V[γ_{d,φ}[k]]
         }),
      Exp[Sum[g_{α,β} (π_α + π̄_α) (ξ_β + ξ̄_β), {α, es}, {β, es}] - Sum[ξ̄_α π_α, {α, es}]]
     ]]
  ]
```

```
In[•]:= ρd_[K_] := ρd[K] = PPρd@Module[{Cs, φ, n, A, s, i, j, k, Δ, G, d1, ρd1, ρd2, ρd3, ρd4},
          PP"Green"[
           {Cs, φ} = Rot[K]; n = Length[Cs];
           A = IdentityMatrix[2 n + 1];
           Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ({{-T^s, T^s - 1}, {0, -1}}))];
           G = Inverse[A];
          ];
          ρd1 = PPMold@Exp[Total[Cases[Cs, {s_, i_, j_} :> Sum[ϵ^d1 r_{d1,s}[i, j], {d1, d}]]] +
              Sum[ϵ^d1 γ_{d1,φ[[k]]}[k], {k, 2 n}, {d1, d}]];
          ρd2 = PPExpandedMold@Table[Expand@SeriesCoefficient[ρd1, {ϵ, 0, d1}], {d1, d}];
          ρd3 = PPPairing[gPair /@ ρd2];
          ρd4 = PPSubstitution@Factor[Join[{Δ}, ρd3 * Δ^(2 Range[d])] /.
              {g_{α_,β_} :> G[[α, β]], Δ → T^(-Total[φ]-Total[Cs[[All,1]]])/2 Det[A]}]
         ];
```

Testing

```
In[•]:= gPair[r_{1,-1}[3, 4]]
```

```
At gpair with Bs={r_{1,-1}[1, 2]}
```

```
Out[•]=
```

$$\frac{1}{2}+\left(-1+\frac{1}{T}\right) g_{4,3}^2+g_{4,4}\left(1+g_{3,4}-2\,g_{4,4}\right)+g_{3,3}\left(-1+\frac{(-1+T)\,g_{4,3}}{T}+g_{4,4}\right)$$

```
In[•]:= gPair[r_{1,-1}[5, 6]]
```

```
Out[•]=
```

$$\frac{1}{2}+\left(-1+\frac{1}{T}\right) g_{6,5}^2+g_{6,5}\left(1+g_{5,6}-2\,g_{6,6}\right)+g_{5,5}\left(-1+\frac{(-1+T)\,g_{6,5}}{T}+g_{6,6}\right)$$

```
In[•]:= gPair[γ_{1,-1}[3]]
```

```
Out[•]=
```

$$-\frac{1}{2}+g_{3,3}$$

```
In[•]:= gPair[γ_{1,0}[6]]
```

```
Out[•]=
```

$$0$$

```
In[•]:= gPair[γ_{1,-1}[3]^2]
```

```
Out[•]=
```

$$\frac{1}{4}-g_{3,3}+2\,g_{3,3}^2$$

In[•]:=
```
gPair[r2,1[3, 4] γ1,-1[3]]
```

Out[•]=

$$\frac{1}{4}\left(4\,g_{3,3}^3\,(8\,(-1+T)\,g_{4,3}-3\,g_{4,4})-g_{4,3}\left(1+8\,g_{3,4}^2\,g_{4,3}+2\left(-3+2\,T+T^2\right)g_{4,3}^2+\right.\right.$$
$$3\,g_{3,4}\,(1+2\,g_{4,3})\,(3+2\,(1+T)\,g_{4,3}-4\,g_{4,4})-6\,g_{4,4}+6\,g_{4,4}^2-g_{4,3}\,(1-3\,T+6\,(1+T)\,g_{4,4})\Big)-$$
$$2\,g_{3,3}^2\left(2+8\left(-5+4\,T+T^2\right)g_{4,3}^2-11\,g_{4,4}+4\,g_{4,4}^2+g_{4,3}\,(-13+19\,T+18\,g_{3,4}-12\,(2+T)\,g_{4,4})\right)+$$
$$g_{3,3}\left(3+16\left(-3+2\,T+T^2\right)g_{4,3}^3-9\,g_{4,4}+6\,g_{4,4}^2+6\,g_{4,3}^2\left(-6+7\,T+T^2+4\,(2+T)\,g_{3,4}-6\,(1+T)\,g_{4,4}\right)+\right.$$
$$\left.\left.g_{4,3}\left(1+9\,T+g_{3,4}\,(44-32\,g_{4,4})-12\,(4+T)\,g_{4,4}+24\,g_{4,4}^2\right)\right)\right)$$

In[•]:=
```
ρ1[Knot[3, 1]]
```

Out[•]=

$$\left\{\frac{1-T+T^2}{T},\ \frac{(-1+T)^2\left(1+T^2\right)}{T^2}\right\}$$

In[•]:=
```
TableForm[Table[Join[{K[[1]]_K[[2]]}, ρ1[K]], {K, AllKnots[{3, 6}]}], TableAlignments → Center]
```

Out[•]//TableForm=

3_1	$\frac{1-T+T^2}{T}$	$\frac{(-1+T)^2\left(1+T^2\right)}{T^2}$
4_1	$-\frac{1-3\,T+T^2}{T}$	0
5_1	$\frac{1-T+T^2-T^3+T^4}{T^2}$	$\frac{(-1+T)^2\left(1+T^2\right)\left(2+T^2+2\,T^4\right)}{T^4}$
5_2	$\frac{2-3\,T+2\,T^2}{T}$	$\frac{(-1+T)^2\left(5-4\,T+5\,T^2\right)}{T^2}$
6_1	$-\frac{(-2+T)\,(-1+2\,T)}{T}$	$\frac{(-1+T)^2\left(1-4\,T+T^2\right)}{T^2}$
6_2	$-\frac{1-3\,T+3\,T^2-3\,T^3+T^4}{T^2}$	$\frac{(-1+T)^2\left(1-4\,T+4\,T^2-4\,T^3+4\,T^4-4\,T^5+T^6\right)}{T^4}$
6_3	$\frac{1-3\,T+5\,T^2-3\,T^3+T^4}{T^2}$	0

In[•]:=
```
ρ2[Knot[3, 1]]
```

Out[•]=

$$\left\{\frac{1-T+T^2}{T},\ \frac{(-1+T)^2\left(1+T^2\right)}{T^2},\ \frac{1-4\,T+7\,T^2-12\,T^3+18\,T^4-12\,T^5+7\,T^6-4\,T^7+T^8}{2\,T^4}\right\}$$

In[ ]:=
```
BeginProfile[]
Timing[z1 = ρ2[Knot[10, 106]]]
PrintProfile[]
```

Out[ ]=
```
ProfileRoot
```

Out[ ]=
$$\left\{2.26563,\ \left\{-\frac{\left(1-T+T^2\right)\left(-1+T-2T^2+T^3\right)\left(-1+2T-T^2+T^3\right)}{T^4},\ -\frac{1}{T^8}(-1+T)^2\left(1-6T+20T^2-48T^3+82T^4-114T^5+134T^6-140T^7+134T^8-114T^9+82T^{10}-48T^{11}+20T^{12}-6T^{13}+T^{14}\right),\ \frac{1}{2T^{16}}\left(1-16T+127T^2-676T^3+2735T^4-8980T^5+24938T^6-60420T^7+131072T^8-259992T^9+477614T^{10}-814576T^{11}+1282448T^{12}-1846716T^{13}+2411126T^{14}-2836312T^{15}+2995252T^{16}-2836312T^{17}+2411126T^{18}-1846716T^{19}+1282448T^{20}-814576T^{21}+477614T^{22}-259992T^{23}+131072T^{24}-60420T^{25}+24938T^{26}-8980T^{27}+2735T^{28}-676T^{29}+127T^{30}-16T^{31}+T^{32}\right)\right\}\right\}$$

Out[ ]=
```
ProfileRoot is root. Profiled time: 2.266
  ( 1)    0/ 2.270 above ρd
ExpandedMold: called 1 times, time in 0.844/0.844
  ( 1)  0.844/ 0.844 under ρd
Green: called 1 times, time in 0.656/0.656
  ( 1)  0.656/ 0.656 under ρd
Substitution: called 1 times, time in 0.641/0.641
  ( 1)  0.641/ 0.641 under ρd
Pairing: called 1 times, time in 0.125/0.125
  ( 1)  0.125/ 0.125 under ρd
ρd: called 1 times, time in 0./2.266
  ( 1)    0/ 2.270 under ProfileRoot
  ( 1)  0.656/ 0.656 above Green
  ( 1)  0.844/ 0.844 above ExpandedMold
  ( 1)    0/   0 above Mold
  ( 1)  0.125/ 0.125 above Pairing
  ( 1)  0.641/ 0.641 above Substitution
Mold: called 1 times, time in 0./0.
  ( 1)    0/   0 under ρd
```

In[•]:=
```
BeginProfile[]
Timing[z2 = ρ2[Knot[12, NonAlternating, 369]]]
PrintProfile[]
```

Out[•]=
```
ProfileRoot
```

Out[•]=
$$\left\{4.6875,\ \left\{-\frac{\left(1-T+T^2\right)\left(-1+T-2T^2+T^3\right)\left(-1+2T-T^2+T^3\right)}{T^4},\ -\frac{1}{T^8}(-1+T)^2\left(1-6T+20T^2-48T^3+82T^4-114T^5+134T^6-140T^7+134T^8-114T^9+82T^{10}-48T^{11}+20T^{12}-6T^{13}+T^{14}\right),\right.\right.$$
$$\left.\left.\frac{1}{2T^{16}}\left(1-16T+127T^2-668T^3+2631T^4-8324T^5+22\,282T^6-52\,780T^7+114\,992T^8-236\,376T^9+460\,598T^{10}-839\,688T^{11}+1\,404\,696T^{12}-2\,121\,524T^{13}+2\,862\,782T^{14}-3\,432\,312T^{15}+3\,647\,156T^{16}-3\,432\,312T^{17}+2\,862\,782T^{18}-2\,121\,524T^{19}+1\,404\,696T^{20}-839\,688T^{21}+460\,598T^{22}-236\,376T^{23}+114\,992T^{24}-52\,780T^{25}+22\,282T^{26}-8324T^{27}+2631T^{28}-668T^{29}+127T^{30}-16T^{31}+T^{32}\right)\right\}\right\}$$

Out[•]=
```
ProfileRoot is root. Profiled time: 4.687
  ( 1)    0/ 4.690 above ρd
Substitution: called 1 times, time in 1.812/1.812
  ( 1)  1.810/ 1.810 under ρd
ExpandedMold: called 1 times, time in 1.766/1.766
  ( 1)  1.770/ 1.770 under ρd
Green: called 1 times, time in 0.937/0.937
  ( 1)  0.937/ 0.937 under ρd
Pairing: called 1 times, time in 0.172/0.172
  ( 1)  0.172/ 0.172 under ρd
ρd: called 1 times, time in 0./4.687
  ( 1)    0/ 4.690 under ProfileRoot
  ( 1)  0.937/ 0.937 above Green
  ( 1)  1.770/ 1.770 above ExpandedMold
  ( 1)    0/   0 above Mold
  ( 1)  0.172/ 0.172 above Pairing
  ( 1)  1.810/ 1.810 above Substitution
Mold: called 1 times, time in 0./0.
  ( 1)    0/   0 under ρd
```

In[•]:=
```
Simplify[Thread[z1 == z2]]
```

Out[•]=
$$\left\{\text{True},\ \text{True},\ \frac{1}{T}(-1+T)\left(1-T+T^2\right)\left(1-6T+16T^2-23T^3+9T^4+47T^5-141T^6+231T^7-272T^8+231T^9-141T^{10}+47T^{11}+9T^{12}-23T^{13}+16T^{14}-6T^{15}+T^{16}\right)==0\right\}$$

```
In[ ]:= TableForm[Table[Join[{K[[1]]_K[[2]]}, ρ2[K]], {K, AllKnots[{3, 7}]}], TableAlignments → Center]
```

Out[ ]//TableForm=

3_1	$\frac{1-T+T^2}{T}$	$\frac{(-1+T)^2\,(1+T^2)}{T^2}$	
4_1	$-\frac{1-3\,T+T^2}{T}$	0	
5_1	$\frac{1-T+T^2-T^3+T^4}{T^2}$	$\frac{(-1+T)^2\,(1+T^2)\,(2+T^2+2\,T^4)}{T^4}$	$4-16\,T+35\,T^2-60\,T$
5_2	$\frac{2-3\,T+2\,T^2}{T}$	$\frac{(-1+T)^2\,(5-4\,T+5\,T^2)}{T^2}$	
6_1	$-\frac{(-2+T)\,(-1+2\,T)}{T}$	$\frac{(-1+T)^2\,(1-4\,T+T^2)}{T^2}$	
6_2	$-\frac{1-3\,T+3\,T^2-3\,T^3+T^4}{T^2}$	$\frac{(-1+T)^2\,(1-4\,T+4\,T^2-4\,T^3+4\,T^4-4\,T^5+T^6)}{T^4}$	$1-12\,T+62\,T^2-180\,T^3+3$
6_3	$\frac{1-3\,T+5\,T^2-3\,T^3+T^4}{T^2}$	0	
7_1	$\frac{1-T+T^2-T^3+T^4-T^5+T^6}{T^3}$	$\frac{(-1+T)^2\,(1+T^2)\,(3+2\,T^2+4\,T^4+2\,T^6+3\,T^8)}{T^6}$	$9-36\,T+83\,T^2-152\,T^3+238\,T^4-336\,T^5+434\,T^6-556\,T^7+719\,T$
7_2	$\frac{3-5\,T+3\,T^2}{T}$	$\frac{2\,(-1+T)^2\,(7-8\,T+7\,T^2)}{T^2}$	
7_3	$\frac{2-3\,T+3\,T^2-3\,T^3+2\,T^4}{T^2}$	$-\frac{(-1+T)^2\,(9-8\,T+16\,T^2-12\,T^3+16\,T^4-8\,T^5+9\,T^6)}{T^4}$	$82-472\,T+1409\,T^2-2996\,T^3+5190\,T^4-$
7_4	$\frac{4-7\,T+4\,T^2}{T}$	$-\frac{8\,(-1+T)^2\,(3-4\,T+3\,T^2)}{T^2}$	
7_5	$\frac{2-4\,T+5\,T^2-4\,T^3+2\,T^4}{T^2}$	$\frac{(-1+T)^2\,(9-16\,T+29\,T^2-28\,T^3+29\,T^4-16\,T^5+9\,T^6)}{T^4}$	$82-616\,T+2412\,T^2-6560\,T^3+13\,875\,T^4-2$
7_6	$-\frac{1-5\,T+7\,T^2-5\,T^3+T^4}{T^2}$	$\frac{(-1+T)^2\,(1-8\,T+19\,T^2-20\,T^3+19\,T^4-8\,T^5+T^6)}{T^4}$	$1-20\,T+175\,T^2-880\,T^3+2923\,T^4$
7_7	$\frac{1-5\,T+9\,T^2-5\,T^3+T^4}{T^2}$	$-\frac{(-1+T)^2\,(3-8\,T+3\,T^2)}{T^2}$	$1-20\,T+$

```
In[ ]:= GST48 = EPD[X14,1, X̄2,29, X3,40, X43,4, X̄26,5, X6,95, X96,7, X13,8, X̄9,28, X10,41, X42,11, X̄27,12,
          X30,15, X̄16,61, X̄17,72, X̄18,83, X19,34, X̄89,20, X̄21,92, X̄79,22, X̄68,23, X̄57,24, X̄25,56, X62,31,
          X73,32, X84,33, X̄50,35, X36,81, X37,70, X38,59, X̄39,54, X44,55, X58,45, X69,46, X80,47, X48,91,
          X90,49, X51,82, X52,71, X53,60, X̄63,74, X̄64,85, X̄76,65, X̄87,66, X̄67,94, X̄75,86, X̄88,77, X̄78,93];
        BeginProfile[]
        Timing[z3 = ρ2[GST48]]
        PrintProfile[]
```

Out[ ]=

```
ProfileRoot
```

Out[ ]=

$$\left\{270.141,\ \left\{-\frac{\left(-1+2\,T-T^2-T^3+2\,T^4-T^5+T^8\right)\,\left(-1+T^3-2\,T^4+T^5+T^6-2\,T^7+T^8\right)}{T^8},\right.\right.$$

$$\frac{1}{T^{16}}\,(-1+T)^2\,\left(5-18\,T+33\,T^2-32\,T^3+2\,T^4+42\,T^5-62\,T^6-8\,T^7+166\,T^8-242\,T^9+108\,T^{10}+\right.$$
$$\left.132\,T^{11}-226\,T^{12}+148\,T^{13}-11\,T^{14}-36\,T^{15}-11\,T^{16}+148\,T^{17}-226\,T^{18}+132\,T^{19}+108\,T^{20}-\right.$$
$$\left.242\,T^{21}+166\,T^{22}-8\,T^{23}-62\,T^{24}+42\,T^{25}+2\,T^{26}-32\,T^{27}+33\,T^{28}-18\,T^{29}+5\,T^{30}\right),$$

$$\frac{1}{2\,T^{32}}\,\left(25-348\,T+2312\,T^2-9628\,T^3+27\,228\,T^4-51\,460\,T^5+52\,250\,T^6+25\,828\,T^7-\right.$$
$$197\,145\,T^8+313\,268\,T^9-36\,579\,T^{10}-887\,864\,T^{11}+2\,118\,398\,T^{12}-2\,494\,152\,T^{13}+772\,387\,T^{14}+$$
$$2\,785\,204\,T^{15}-5\,477\,089\,T^{16}+3\,765\,568\,T^{17}+2\,886\,710\,T^{18}-9\,712\,796\,T^{19}+9\,746\,285\,T^{20}-$$
$$708\,568\,T^{21}-11\,443\,177\,T^{22}+17\,013\,304\,T^{23}-11\,217\,405\,T^{24}-1\,334\,300\,T^{25}+10\,332\,369\,T^{26}-$$
$$8\,571\,752\,T^{27}-1\,186\,874\,T^{28}+8\,007\,252\,T^{29}-3\,568\,015\,T^{30}-8\,148\,860\,T^{31}+14\,395\,240\,T^{32}-$$
$$8\,148\,860\,T^{33}-3\,568\,015\,T^{34}+8\,007\,252\,T^{35}-1\,186\,874\,T^{36}-8\,571\,752\,T^{37}+10\,332\,369\,T^{38}-$$
$$1\,334\,300\,T^{39}-11\,217\,405\,T^{40}+17\,013\,304\,T^{41}-11\,443\,177\,T^{42}-708\,568\,T^{43}+9\,746\,285\,T^{44}-$$
$$9\,712\,796\,T^{45}+2\,886\,710\,T^{46}+3\,765\,568\,T^{47}-5\,477\,089\,T^{48}+2\,785\,204\,T^{49}+772\,387\,T^{50}-$$
$$2\,494\,152\,T^{51}+2\,118\,398\,T^{52}-887\,864\,T^{53}-36\,579\,T^{54}+313\,268\,T^{55}-197\,145\,T^{56}+$$
$$\left.\left.25\,828\,T^{57}+52\,250\,T^{58}-51\,460\,T^{59}+27\,228\,T^{60}-9628\,T^{61}+2312\,T^{62}-348\,T^{63}+25\,T^{64}\right)\right\}\right\}$$

Out[ ]=

```
ProfileRoot is root. Profiled time: 270.14
  ( 1)   0.203/ 270.140 above ρd
Substitution: called 1 times, time in 155.437/155.437
  ( 1)  155.440/ 155.440 under ρd
Green: called 1 times, time in 102.25/102.25
  ( 1)  102.250/ 102.250 under ρd
ExpandedMold: called 1 times, time in 9.062/9.062
  ( 1)   9.062/  9.062 under ρd
Pairing: called 1 times, time in 3.188/3.188
  ( 1)   3.188/  3.188 under ρd
ρd: called 1 times, time in 0.203/270.14
  ( 1)   0.203/ 270.140 under ProfileRoot
  ( 1)  102.250/ 102.250 above Green
  ( 1)   9.062/  9.062 above ExpandedMold
  ( 1)      0/     0 above Mold
  ( 1)   3.188/  3.188 above Pairing
  ( 1)  155.440/ 155.440 above Substitution
Mold: called 1 times, time in 0./0.
  ( 1)      0/     0 under ρd
```

According to C:\drorbn\AcademicPensieve\Projects\SL2Invariant\k=2\12XingStats.nb, expect 2893 below:

In[•]:=
```
BeginProfile[]
Monitor[{NumberOfKnots[{3, 12}],
  Length@Union@Table[ρ2[K], {K, AllKnots[{3, 12}]}],
  Length@Union@Table[{HOMFLYPT[K], Kh[K]}, {K, AllKnots[{3, 12}]}]},
 K]
PrintProfile[]
```

Out[•]=
```
ProfileRoot
```

KnotTheory: Loading precomputed data in KnotTheory/12A.dts.

KnotTheory: The HOMFLYPT program was written by Scott Morrison.

KnotTheory: Loading precomputed data in Kh4Knots`.

KnotTheory: Loading precomputed data in Kh4Knots11`.

General: Further output of KnotTheory::loading will be suppressed during this calculation.

KnotTheory: The Khovanov homology program JavaKh–v2 is an update of Jeremy Green's program JavaKh–v1, written by Scott Morrison in 2008 at Microsoft Station Q.

Out[•]=
```
{2977, 2893, 2785}
```

Out[•]=
```
ProfileRoot is root. Profiled time: 18724.7
  ( 2590)    36.759/ 18724.750 above ρd
ExpandedMold: called 2590 times, time in 6902.73/6902.73
  ( 2590)  6902.730/ 6902.730 under ρd
Substitution: called 2590 times, time in 6350.66/6350.66
  ( 2590)  6350.660/ 6350.660 under ρd
Green: called 2590 times, time in 4654.33/4654.33
  ( 2590)  4654.332/ 4654.332 under ρd
Pairing: called 2590 times, time in 778.931/778.931
  ( 2590)   778.931/  778.931 under ρd
ρd: called 2590 times, time in 36.759/18724.7
  ( 2590)    36.759/ 18724.750 under ProfileRoot
  ( 2590)  4654.332/ 4654.332 above Green
  ( 2590)  6902.730/ 6902.730 above ExpandedMold
  ( 2590)     1.336/    1.336 above Mold
  ( 2590)   778.931/  778.931 above Pairing
  ( 2590)  6350.660/ 6350.660 above Substitution
Mold: called 2590 times, time in 1.336/1.336
  ( 2590)     1.336/    1.336 under ρd
```

In[•]:=
```
Table[K → ρ2[K], {K, AllKnots[{3, 12}]}] >> Rho2Data.m
```