

Pensieve header: Developing ρ_d .

Program

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\Oaxaca-2210"];
```

```
In[ ]:= Once[<< KnotTheory` ; << Rot.m];
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

Loading Rot.m from <http://drorbn.net/la22/ap> to compute rotation numbers.

```
In[ ]:= << "../../Projects/Profile/Profile.m"
```

This is Profile.m of <http://www.drorbn.net/AcademicPensieve/Projects/Profile/>.

This version: April 2020. Original version: July 1994.

```
In[ ]:= {p*, x*, pi*, xi*} = {pi, xi, p, x}; (u_{i_})* := (u*)_i;
```

```
In[ ]:= Zip_{ }[e_] := e;
```

```
Zip_{ {e_, e5_} }[e_] := (Collect[e // Zip_{ {e5_}, e5_ } /. f_ . e5^{d_} -> (D[f, {e5*, d}])] /. e5* -> 0
```

```
In[ ]:= V@gamma_{d, 0}[j_] := 0; V@gamma_{1, phi}[k_] := phi (1/2 - p_k x_k);
```

```
In[ ]:= V@gamma_{2, 1}[k_] := -1/2 p_k x_k; V@gamma_{2, -1}[k_] := -1/2 p_k x_k;
```

```
In[ ]:= V@r_{1, s}[i_, j_] :=
```

$$s \left(-\frac{1}{2} + p_i x_i - p_j x_j + \frac{1}{2} (-1 + T^s) p_i p_j x_i^2 + \frac{1}{2} (1 - T^s) p_j^2 x_i^2 - p_i p_j x_i x_j + p_j^2 x_i x_j \right);$$

```
In[ ]:= V@r_{2, 1}[i_, j_] := -1/2 p_i x_i + p_j x_j / 2 + 1/4 (1 - 3 T) p_i p_j x_i^2 + 1/4 (-1 + 3 T) p_j^2 x_i^2 + 1/3 (-1 + T) p_i^2 p_j x_i^3 -
```

$$\frac{1}{6} (-1 + T) (5 + T) p_i p_j^2 x_i^3 + \frac{1}{6} (-1 + T) (3 + T) p_j^3 x_i^3 + \frac{3}{2} p_i p_j x_i x_j - \frac{3}{2} p_j^2 x_i x_j -$$

$$\frac{1}{2} p_i^2 p_j x_i^2 x_j + \frac{1}{2} (2 + T) p_i p_j^2 x_i^2 x_j + \frac{1}{2} (-1 - T) p_j^3 x_i^2 x_j - \frac{1}{2} p_i p_j^2 x_i x_j^2 + \frac{1}{2} p_j^3 x_i x_j^2;$$

$$\begin{aligned}
 \text{In[*]:= } \mathbf{V@r_{2,-1}[i_-, j_-]} := & -\frac{1}{2} p_i x_i + \frac{p_j x_i}{2} + \frac{(-3+T) p_i p_j x_i^2}{4T} - \frac{(-3+T) p_j^2 x_i^2}{4T} - \frac{(-1+T) p_i^2 p_j x_i^3}{3T} + \\
 & \frac{(-1+T) (1+5T) p_i p_j^2 x_i^3}{6T^2} - \frac{(-1+T) (1+3T) p_j^3 x_i^3}{6T^2} + \frac{3}{2} p_i p_j x_i x_j - \frac{3}{2} p_j^2 x_i x_j - \\
 & \frac{1}{2} p_i^2 p_j x_i^2 x_j + \frac{(1+2T) p_i p_j^2 x_i^2 x_j}{2T} - \frac{(1+T) p_j^3 x_i^2 x_j}{2T} - \frac{1}{2} p_i p_j^2 x_i x_j^2 + \frac{1}{2} p_j^3 x_i x_j^2;
 \end{aligned}$$

```

In[*]:= gPair[1] = 1;
gPair[εd·Bs_] := εd gPair[Bs];
gPair[c_?NumberQ * Bs_] := c gPair[Bs];
gPair[ε_Plus] := gPair /@ ε;
gPair[rd,s[i_, j_] p] := gPair[{rd,s[i, j]p};
gPair[γd,φ[k_] p] := gPair[{γd,φ[k]p};
gPair[Bs_Times] := gPair[List@@Bs];
gPair[Bs_List] := Module[{es, BBs},
  BBs = Bs /. e_<sup>p_</sup> -> Sequence@@Table[e, {p}];
  es = Union@@(List@@@BBs);
  Do[v@i = es[[i]], {i, Length@es}];
  gpair[Replace[BBs, Thread[es -> Range@Length@es], {2}]] /. gα,β -> gv@α, v@β
]

```

```

In[*]:= gpair[Bs_List] := gpair[Bs] = Module[{es},
  Print["At gpair with Bs=", Bs];
  es = Union@@(List@@@Bs);
  Simplify@ZipJoin@@Table[{p1,α, p2,α, x1,α, x2,α}, {α, es}] [Times [
    Times@@(Bs /. {
      rd,s[i_, j_] -> (V[rd,s[i, j]] /. {pi -> p2,i, pj -> p2,j, xi -> x2,i, xj -> x2,j}),
      γd,φ[k_] -> (V[γd,φ[k]] /. {pk -> p1,k, xk -> x1,k})
    }),
  Exp[Sum[gα,β(π1,α + π2,α)(ξ1,β + ξ2,β), {α, es}, {β, es}] - Sum[ξ1,α π2,α, {α, es}]]
]

```

```

In[ ]:=  $\rho_d$ [K_] := PP $\rho_d$ @Module[{Cs,  $\varphi$ , n, A, s, i, j, k,  $\Delta$ , G, d1,  $\rho_d$ },
  PP"Green"[
    {Cs,  $\varphi$ } = Rot[K]; n = Length[Cs];
    A = IdentityMatrix[2 n + 1];
    Cases[Cs, {s_, i_, j_}  $\Rightarrow$  (A[[{i, j}, {i + 1, j + 1}]] += ( $\begin{pmatrix} -T^s & T^s - 1 \\ 0 & -1 \end{pmatrix}$ ))];
     $\Delta$  = T(-Total[ $\varphi$ ]-Total[Cs[[All,1]])/2 Det[A];
    G = Inverse[A];
  ];
   $\rho_d$  = PPPairing@gPair[Series[Exp[
    Total[Cases[Cs, {s_, i_, j_}  $\Rightarrow$  Sum[ $e^{d1} r_{d1,s}$ [i, j], {d1, d}]]]
    + Sum[ $e^{d1} \gamma_{d1,\varphi[[k]]}$ [k], {k, 2 n}, {d1, d}]]
  ], { $\epsilon$ , 0, d}] // Normal // Expand];
  PPRenormalizing[
     $\rho_d$  = CoefficientList[ $\Delta$  Normal[Series[ $\rho_d$ , { $\epsilon$ , 0, d}]] /.  $\epsilon \rightarrow \Delta \epsilon$ ,  $\epsilon$ , d + 1]];
  PPSubstitution@Factor[ $\rho_d$  /.  $\alpha_-^+ \Rightarrow \alpha + 1$  /.  $g_{\alpha,\beta_-} \Rightarrow G[\alpha, \beta]$ 
];

```

Testing

In[]:= gPair[r_{1,-1}[3, 4]]

Out[]:=

$$\frac{1}{2} + \left(-1 + \frac{1}{T}\right) g_{4,3}^2 + g_{4,3} (1 + g_{3,4} - 2 g_{4,4}) + g_{3,3} \left(-1 + \frac{(-1+T) g_{4,3}}{T} + g_{4,4}\right)$$

In[]:= gPair[r_{1,-1}[5, 6]]

Out[]:=

$$\frac{1}{2} + \left(-1 + \frac{1}{T}\right) g_{6,5}^2 + g_{6,5} (1 + g_{5,6} - 2 g_{6,6}) + g_{5,5} \left(-1 + \frac{(-1+T) g_{6,5}}{T} + g_{6,6}\right)$$

In[]:= gPair[$\gamma_{1,-1}$ [3]]

Out[]:=

$$-\frac{1}{2} + g_{3,3}$$

In[]:= gPair[$\gamma_{1,0}$ [6]]

Out[]:=

$$0$$

In[]:= gPair[$\gamma_{1,-1}$ [3]²]

Out[]:=

$$\frac{1}{4} - g_{3,3} + 2 g_{3,3}^2$$

In[*]:= **gPair**[$r_{2,1}[3, 4]$ $\gamma_{1,-1}[3]$]

Out[*]=

$$\frac{1}{4} \left(4 g_{3,3}^3 (8 (-1 + T) g_{4,3} - 3 g_{4,4}) - g_{4,3} (1 + 8 g_{3,4}^2 g_{4,3} + 2 (-3 + 2 T + T^2) g_{4,3}^2 + 3 g_{3,4} (1 + 2 g_{4,3}) (3 + 2 (1 + T) g_{4,3} - 4 g_{4,4}) - 6 g_{4,4} + 6 g_{4,4}^2 - g_{4,3} (1 - 3 T + 6 (1 + T) g_{4,4})) - 2 g_{3,3}^2 (2 + 8 (-5 + 4 T + T^2) g_{4,3}^2 - 11 g_{4,4} + 4 g_{4,4}^2 + g_{4,3} (-13 + 19 T + 18 g_{3,4} - 12 (2 + T) g_{4,4})) + g_{3,3} (3 + 16 (-3 + 2 T + T^2) g_{4,3}^3 - 9 g_{4,4} + 6 g_{4,4}^2 + 6 g_{4,3}^2 (-6 + 7 T + T^2 + 4 (2 + T) g_{3,4} - 6 (1 + T) g_{4,4}) + g_{4,3} (1 + 9 T + g_{3,4} (44 - 32 g_{4,4}) - 12 (4 + T) g_{4,4} + 24 g_{4,4}^2)) \right)$$

In[*]:= **ρ_1** [**Knot**[3, 1]]

KnotTheory: Loading precomputed data in PD4Knots`.

At gpair with Bs={ $r_{1,-1}[1, 2]$ }

At gpair with Bs={ $r_{1,-1}[2, 1]$ }

At gpair with Bs={ $\gamma_{1,-1}[1]$ }

At gpair with Bs={ $\gamma_{1,0}[1]$ }

Out[*]=

$$\left\{ \frac{1 - T + T^2}{T}, \frac{(-1 + T)^2 (1 + T^2)}{T^2} \right\}$$

In[*]:= **Table**[$K \rightarrow \rho_1[K]$, {**K**, **AllKnots**[{3, 6}]}]

At gpair with Bs={ $r_{1,1}[1, 2]$ }

At gpair with Bs={ $\gamma_{1,1}[1]$ }

At gpair with Bs={ $r_{1,1}[2, 1]$ }

Out[*]=

$$\left\{ \text{Knot}[3, 1] \rightarrow \left\{ \frac{1 - T + T^2}{T}, \frac{(-1 + T)^2 (1 + T^2)}{T^2} \right\}, \text{Knot}[4, 1] \rightarrow \left\{ -\frac{1 - 3 T + T^2}{T}, \emptyset \right\}, \right.$$

$$\text{Knot}[5, 1] \rightarrow \left\{ \frac{1 - T + T^2 - T^3 + T^4}{T^2}, \frac{(-1 + T)^2 (1 + T^2) (2 + T^2 + 2 T^4)}{T^4} \right\},$$

$$\text{Knot}[5, 2] \rightarrow \left\{ \frac{2 - 3 T + 2 T^2}{T}, \frac{(-1 + T)^2 (5 - 4 T + 5 T^2)}{T^2} \right\},$$

$$\text{Knot}[6, 1] \rightarrow \left\{ -\frac{(-2 + T) (-1 + 2 T)}{T}, \frac{(-1 + T)^2 (1 - 4 T + T^2)}{T^2} \right\},$$

$$\text{Knot}[6, 2] \rightarrow \left\{ -\frac{1 - 3 T + 3 T^2 - 3 T^3 + T^4}{T^2}, \frac{(-1 + T)^2 (1 - 4 T + 4 T^2 - 4 T^3 + 4 T^4 - 4 T^5 + T^6)}{T^4} \right\},$$

$$\text{Knot}[6, 3] \rightarrow \left\{ \frac{1 - 3 T + 5 T^2 - 3 T^3 + T^4}{T^2}, \emptyset \right\} \left. \right\}$$

In[*]:= **ρ_2** [**Knot**[3, 1]]

At gpair with Bs={r_{1,-1}[1, 2], r_{1,-1}[1, 2]}

At gpair with Bs={r_{1,-1}[2, 4], r_{1,-1}[3, 1]}

At gpair with Bs={r_{1,-1}[2, 1], r_{1,-1}[2, 1]}

At gpair with Bs={r_{1,-1}[1, 3], r_{1,-1}[4, 2]}

At gpair with Bs={r_{1,-1}[3, 1], r_{1,-1}[4, 2]}

At gpair with Bs={r_{2,-1}[1, 2]}

At gpair with Bs={r_{2,-1}[2, 1]}

At gpair with Bs={r_{1,-1}[1, 3], γ_{1,-1}[2]}

At gpair with Bs={r_{1,-1}[2, 1], γ_{1,-1}[2]}

At gpair with Bs={r_{1,-1}[3, 1], γ_{1,-1}[2]}

At gpair with Bs={γ_{1,-1}[1], γ_{1,-1}[1]}

At gpair with Bs={r_{1,-1}[2, 3], γ_{1,0}[1]}

At gpair with Bs={r_{1,-1}[2, 1], γ_{1,0}[1]}

At gpair with Bs={r_{1,-1}[3, 2], γ_{1,0}[1]}

At gpair with Bs={γ_{1,-1}[2], γ_{1,0}[1]}

At gpair with Bs={γ_{1,0}[1], γ_{1,0}[1]}

At gpair with Bs={r_{1,-1}[1, 2], γ_{1,0}[1]}

At gpair with Bs={r_{1,-1}[3, 1], γ_{1,0}[2]}

At gpair with Bs={γ_{1,0}[1], γ_{1,0}[2]}

At gpair with Bs={r_{1,-1}[1, 3], γ_{1,0}[2]}

At gpair with Bs={r_{1,-1}[1, 2], γ_{1,0}[2]}

At gpair with Bs={r_{1,-1}[2, 1], γ_{1,0}[3]}

At gpair with Bs={γ_{1,-1}[1], γ_{1,0}[2]}

At gpair with Bs={r_{1,-1}[1, 2], γ_{1,0}[3]}

At gpair with Bs={r_{1,-1}[2, 1], γ_{1,0}[2]}

At gpair with Bs={γ_{2,-1}[1]}

At gpair with Bs={γ_{2,0}[1]}

Out[]=

$$\left\{ \frac{1 - T + T^2}{T}, \frac{(-1 + T)^2 (1 + T^2)}{T^2}, \frac{1 - 4 T + 7 T^2 - 12 T^3 + 18 T^4 - 12 T^5 + 7 T^6 - 4 T^7 + T^8}{2 T^3 (1 - T + T^2)} \right\}$$

```
In[*]:= BeginProfile []
Timing[z1 = ρ2[Knot[10, 106]]]
PrintProfile []
```

Out[*]= ProfileRoot

Out[*]=

$$\left\{ 9.4375, \right. \\ \left. \left\{ -\frac{(1 - T + T^2)(-1 + T - 2T^2 + T^3)(-1 + 2T - T^2 + T^3)}{T^4}, -\frac{1}{T^8}(-1 + T)^2(1 - 6T + 20T^2 - 48T^3 + 82T^4 - \right. \right. \\ \left. \left. 114T^5 + 134T^6 - 140T^7 + 134T^8 - 114T^9 + 82T^{10} - 48T^{11} + 20T^{12} - 6T^{13} + T^{14}), \right. \right. \\ \left. \left. - \left((1 - 16T + 127T^2 - 676T^3 + 2735T^4 - 8980T^5 + 24938T^6 - 60420T^7 + 131072T^8 - 259992T^9 + \right. \right. \right. \\ \left. \left. \left. 477614T^{10} - 814576T^{11} + 1282448T^{12} - 1846716T^{13} + 2411126T^{14} - 2836312T^{15} + \right. \right. \right. \\ \left. \left. \left. 2995252T^{16} - 2836312T^{17} + 2411126T^{18} - 1846716T^{19} + 1282448T^{20} - 814576T^{21} + \right. \right. \right. \\ \left. \left. \left. 477614T^{22} - 259992T^{23} + 131072T^{24} - 60420T^{25} + 24938T^{26} - 8980T^{27} + 2735T^{28} - 676T^{29} + \right. \right. \right. \\ \left. \left. \left. 127T^{30} - 16T^{31} + T^{32} \right) / (2T^{12}(1 - T + T^2)(-1 + T - 2T^2 + T^3)(-1 + 2T - T^2 + T^3)) \right\} \right\}$$

Out[*]= ProfileRoot is root. Profiled time: 9.437
 (1) 0/ 9.437 above ρ_d
 Renormalizing: called 1 times, time in 7.438/7.438
 (1) 7.438/ 7.438 under ρ_d
 Substitution: called 1 times, time in 0.906/0.906
 (1) 0.906/ 0.906 under ρ_d
 Green: called 1 times, time in 0.828/0.828
 (1) 0.828/ 0.828 under ρ_d
 Pairing: called 1 times, time in 0.265/0.265
 (1) 0.265/ 0.265 under ρ_d
 ρ_d: called 1 times, time in 0./9.437
 (1) 0/ 9.437 under ProfileRoot
 (1) 0.828/ 0.828 above Green
 (1) 0.265/ 0.265 above Pairing
 (1) 7.438/ 7.438 above Renormalizing
 (1) 0.906/ 0.906 above Substitution

```
In[ ]:= BeginProfile []
Timing[z2 =  $\rho_2$ [Knot[12, NonAlternating, 369]]]
PrintProfile []
```

Out[]:= ProfileRoot

Out[]:=

$$\left\{ 16.7031, \right. \\ \left. \left\{ -\frac{(1 - T + T^2) (-1 + T - 2 T^2 + T^3) (-1 + 2 T - T^2 + T^3)}{T^4}, -\frac{1}{T^8} (-1 + T)^2 (1 - 6 T + 20 T^2 - 48 T^3 + 82 T^4 - \right. \right. \\ \left. \left. 114 T^5 + 134 T^6 - 140 T^7 + 134 T^8 - 114 T^9 + 82 T^{10} - 48 T^{11} + 20 T^{12} - 6 T^{13} + T^{14}), \right. \right. \\ \left. \left. - \left((1 - 16 T + 127 T^2 - 668 T^3 + 2631 T^4 - 8324 T^5 + 22282 T^6 - 52780 T^7 + 114992 T^8 - 236376 T^9 + \right. \right. \right. \\ \left. \left. \left. 460598 T^{10} - 839688 T^{11} + 1404696 T^{12} - 2121524 T^{13} + 2862782 T^{14} - 3432312 T^{15} + \right. \right. \right. \\ \left. \left. \left. 3647156 T^{16} - 3432312 T^{17} + 2862782 T^{18} - 2121524 T^{19} + 1404696 T^{20} - 839688 T^{21} + \right. \right. \right. \\ \left. \left. \left. 460598 T^{22} - 236376 T^{23} + 114992 T^{24} - 52780 T^{25} + 22282 T^{26} - 8324 T^{27} + 2631 T^{28} - 668 T^{29} + \right. \right. \right. \\ \left. \left. \left. 127 T^{30} - 16 T^{31} + T^{32} \right) / (2 T^{12} (1 - T + T^2) (-1 + T - 2 T^2 + T^3) (-1 + 2 T - T^2 + T^3)) \right\} \right\}$$

Out[]:= ProfileRoot is root. Profiled time: 16.703
 (1) 0/ 16.700 above ρ_d
 Renormalizing: called 1 times, time in 11.594/11.594
 (1) 11.590/ 11.590 under ρ_d
 Substitution: called 1 times, time in 3.046/3.046
 (1) 3.046/ 3.046 under ρ_d
 Green: called 1 times, time in 1.453/1.453
 (1) 1.453/ 1.453 under ρ_d
 Pairing: called 1 times, time in 0.61/0.61
 (1) 0.610/ 0.610 under ρ_d
 ρ_d : called 1 times, time in 0./16.703
 (1) 0/ 16.700 under ProfileRoot
 (1) 1.453/ 1.453 above Green
 (1) 0.610/ 0.610 above Pairing
 (1) 11.590/ 11.590 above Renormalizing
 (1) 3.046/ 3.046 above Substitution

```
In[ ]:= Simplify[Thread[z1 == z2]]
```

Out[]:=

$$\left\{ \text{True, True,} \right. \\ \left. \frac{(-1 + T) (1 - T + T^2) (1 - 3 T + 2 T^2 + 5 T^3 - 12 T^4 + 18 T^5 - 12 T^6 + 5 T^7 + 2 T^8 - 3 T^9 + T^{10})}{T} == 0 \right\}$$

```
In[*]:= GST48 = EPD[X14,1, X̄2,29, X3,40, X43,4, X̄26,5, X6,95, X96,7, X13,8, X̄9,28, X10,41, X42,11, X̄27,12,
  X30,15, X̄16,61, X̄17,72, X̄18,83, X19,34, X̄89,20, X̄21,92, X̄79,22, X̄68,23, X̄57,24, X̄25,56, X62,31,
  X73,32, X84,33, X̄50,35, X36,81, X37,70, X38,59, X̄39,54, X44,55, X58,45, X69,46, X80,47, X48,91,
  X90,49, X51,82, X52,71, X53,60, X̄63,74, X̄64,85, X̄76,65, X̄87,66, X̄67,94, X̄75,86, X̄88,77, X̄78,93];
BeginProfile[]
Timing[z3 = ρ2[GST48]]
PrintProfile[]
```

Out[*]=

```
ProfileRoot

At gpair with Bs={r1,-1[1, 4], r1,-1[2, 3]}
At gpair with Bs={r1,-1[1, 2], r1,-1[3, 4]}
At gpair with Bs={r1,-1[2, 1], r1,-1[3, 4]}
At gpair with Bs={r1,-1[1, 2], r1,-1[4, 3]}
At gpair with Bs={r1,-1[2, 1], r1,-1[4, 3]}
At gpair with Bs={r1,-1[2, 3], r1,1[1, 4]}
At gpair with Bs={r1,-1[3, 2], r1,1[1, 4]}
At gpair with Bs={r1,1[1, 4], r1,1[2, 3]}
At gpair with Bs={r1,-1[1, 4], r1,1[3, 2]}
At gpair with Bs={r1,-1[4, 1], r1,1[3, 2]}
At gpair with Bs={r1,1[3, 2], r1,1[4, 1]}
At gpair with Bs={r1,-1[1, 4], r1,1[2, 3]}
At gpair with Bs={r1,-1[4, 1], r1,1[2, 3]}
At gpair with Bs={r1,-1[3, 2], r1,1[4, 1]}
At gpair with Bs={r1,1[2, 3], r1,1[4, 1]}
At gpair with Bs={r1,-1[2, 3], r1,1[4, 1]}
At gpair with Bs={r1,-1[2, 3], γ1,1[1]}
At gpair with Bs={r1,-1[3, 2], γ1,1[1]}
```


Out[*]=

$$\left\{ 621.75, \left\{ -\frac{(-1 + 2T - T^2 - T^3 + 2T^4 - T^5 + T^8)(-1 + T^3 - 2T^4 + T^5 + T^6 - 2T^7 + T^8)}{T^8}, \right. \right.$$

$$\frac{1}{T^{16}} (-1 + T)^2 (5 - 18T + 33T^2 - 32T^3 + 2T^4 + 42T^5 - 62T^6 - 8T^7 + 166T^8 - 242T^9 + 108T^{10} +$$

$$132T^{11} - 226T^{12} + 148T^{13} - 11T^{14} - 36T^{15} - 11T^{16} + 148T^{17} - 226T^{18} + 132T^{19} + 108T^{20} -$$

$$242T^{21} + 166T^{22} - 8T^{23} - 62T^{24} + 42T^{25} + 2T^{26} - 32T^{27} + 33T^{28} - 18T^{29} + 5T^{30}),$$

$$- \left((25 - 348T + 2312T^2 - 9628T^3 + 27228T^4 - 51460T^5 + 52250T^6 + 25828T^7 - 197145T^8 +$$

$$313268T^9 - 36579T^{10} - 887864T^{11} + 2118398T^{12} - 2494152T^{13} + 772387T^{14} +$$

$$2785204T^{15} - 5477089T^{16} + 3765568T^{17} + 2886710T^{18} - 9712796T^{19} + 9746285T^{20} -$$

$$708568T^{21} - 11443177T^{22} + 17013304T^{23} - 11217405T^{24} - 1334300T^{25} + 10332369T^{26} -$$

$$8571752T^{27} - 1186874T^{28} + 8007252T^{29} - 3568015T^{30} - 8148860T^{31} + 14395240T^{32} -$$

$$8148860T^{33} - 3568015T^{34} + 8007252T^{35} - 1186874T^{36} - 8571752T^{37} + 10332369T^{38} -$$

$$1334300T^{39} - 11217405T^{40} + 17013304T^{41} - 11443177T^{42} - 708568T^{43} + 9746285T^{44} -$$

$$9712796T^{45} + 2886710T^{46} + 3765568T^{47} - 5477089T^{48} + 2785204T^{49} + 772387T^{50} -$$

$$2494152T^{51} + 2118398T^{52} - 887864T^{53} - 36579T^{54} + 313268T^{55} - 197145T^{56} +$$

$$25828T^{57} + 52250T^{58} - 51460T^{59} + 27228T^{60} - 9628T^{61} + 2312T^{62} - 348T^{63} + 25T^{64}) /$$

$$\left. \left. \left(2T^{24} (-1 + 2T - T^2 - T^3 + 2T^4 - T^5 + T^8) (-1 + T^3 - 2T^4 + T^5 + T^6 - 2T^7 + T^8) \right) \right) \right\}$$

Out[*]=

ProfileRoot is root. Profiled time: 621.75
 (1) 0.204/ 621.750 above ρd
 Renormalizing: called 1 times, time in 282.468/282.468
 (1) 282.468/ 282.468 under ρd
 Substitution: called 1 times, time in 154.437/154.437
 (1) 154.437/ 154.437 under ρd
 Green: called 1 times, time in 121.188/121.188
 (1) 121.188/ 121.188 under ρd
 Pairing: called 1 times, time in 63.453/63.453
 (1) 63.453/ 63.453 under ρd
 ρd : called 1 times, time in 0.204/621.75
 (1) 0.204/ 621.750 under ProfileRoot
 (1) 121.188/ 121.188 above Green
 (1) 63.453/ 63.453 above Pairing
 (1) 282.468/ 282.468 above Renormalizing
 (1) 154.437/ 154.437 above Substitution