

Pensieve header: Developing  $\rho_d$ .

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\Oaxaca-2210"];
```

```
In[ ]:= Once[<< KnotTheory` ; << Rot.m];
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

Loading Rot.m from <http://drorbn.net/la22/ap> to compute rotation numbers.

```
In[ ]:= << "../..//Projects/Profile/Profile.m"
```

This is Profile.m of <http://www.drorbn.net/AcademicPensieve/Projects/Profile/>.

This version: April 2020. Original version: July 1994.

```
In[ ]:= CF[ $\mathcal{E}_-$ ] := Module[{vs = Union[{ $\epsilon$ }, Cases[ $\mathcal{E}$ , (g | p | x)_,  $\infty$ ]]}, Total[
  CoefficientRules[Expand@ $\mathcal{E}$ , vs] /. (ps_ -> c_) => Factor[c] (Times@@vsps)
]]
```

```
In[ ]:= {p*, x*,  $\pi^*$ ,  $\xi^*$ } = { $\pi$ ,  $\xi$ , p, x}; (u_{i_})^* := (u^*)_i;
```

```
In[ ]:= Zip_{ $\mathcal{E}$ } [ $\mathcal{E}_-$ ] :=  $\mathcal{E}$ ;
Zip_{ $\xi$ ,  $\xi$ ...} [ $\mathcal{E}_-$ ] := (Collect[ $\mathcal{E}$  // Zip_{ $\xi$ },  $\xi$ ] /. f_ .  $\xi^{d_}$  . => (D[f, { $\xi^*$ , d}])) /.  $\xi^* \rightarrow \theta$ 
```

```
In[ ]:= V@ $\gamma_{d, \theta}$  [ $j_-$ ] :=  $\theta$ ; V@ $\gamma_{1, \varphi}$  [ $k_-$ ] :=  $\varphi \left( \frac{1}{2} - p_k x_k \right)$ ;
```

```
In[ ]:= V@ $r_{1, s}$  [ $i_$ ,  $j_-$ ] :=
Expand[s ( - $\frac{1}{2} + p_i x_i - p_j x_j + \frac{1}{2} (-1 + T^s) p_i p_j x_i^2 + \frac{1}{2} (1 - T^s) p_j^2 x_i^2 - p_i p_j x_i x_j + p_j^2 x_i x_j ) ]$ ;
```

```
In[ ]:= V@ $\gamma_{2, 1}$  [ $k_-$ ] := - $\frac{1}{2} p_k x_k$ ; V@ $\gamma_{2, -1}$  [ $k_-$ ] := - $\frac{1}{2} p_k x_k$ ;
```

```
In[ ]:= V@ $r_{2, 1}$  [ $i_$ ,  $j_-$ ] := - $\frac{1}{2} p_i x_i + \frac{p_j x_i}{2} + \frac{1}{4} (1 - 3T) p_i p_j x_i^2 + \frac{1}{4} (-1 + 3T) p_j^2 x_i^2 + \frac{1}{3} (-1 + T) p_i^2 p_j x_i^3 -$ 
 $\frac{1}{6} (-1 + T) (5 + T) p_i p_j^2 x_i^3 + \frac{1}{6} (-1 + T) (3 + T) p_j^3 x_i^3 + \frac{3}{2} p_i p_j x_i x_j - \frac{3}{2} p_j^2 x_i x_j -$ 
 $\frac{1}{2} p_i^2 p_j x_i^2 x_j + \frac{1}{2} (2 + T) p_i p_j^2 x_i^2 x_j + \frac{1}{2} (-1 - T) p_j^3 x_i^2 x_j - \frac{1}{2} p_i p_j^2 x_i x_j^2 + \frac{1}{2} p_j^3 x_i x_j^2$ ;
```

$$\begin{aligned}
 \text{In}[*]:= \mathbf{V@r}_{2,-1}[\mathbf{i}_-, \mathbf{j}_-] := & -\frac{1}{2} p_i x_i + \frac{p_j x_i}{2} + \frac{(-3+T) p_i p_j x_i^2}{4T} - \frac{(-3+T) p_j^2 x_i^2}{4T} - \frac{(-1+T) p_i^2 p_j x_i^3}{3T} + \\
 & \frac{(-1+T) (1+5T) p_i p_j^2 x_i^3}{6T^2} - \frac{(-1+T) (1+3T) p_j^3 x_i^3}{6T^2} + \frac{3}{2} p_i p_j x_i x_j - \frac{3}{2} p_j^2 x_i x_j - \\
 & \frac{1}{2} p_i^2 p_j x_i^2 x_j + \frac{(1+2T) p_i p_j^2 x_i^2 x_j}{2T} - \frac{(1+T) p_j^3 x_i^2 x_j}{2T} - \frac{1}{2} p_i p_j^2 x_i x_j^2 + \frac{1}{2} p_j^3 x_i x_j^2;
 \end{aligned}$$

```

In[*]:= gPair[1] = 1;
gPair[εd·Bs_] := εd gPair[Bs];
gPair[c_?NumberQ * Bs_] := c gPair[Bs];
gPair[ε_Plus] := gPair /@ ε;
gPair[rd,s[i_, j_] p] := gPair[{rd,s[i, j]p};
gPair[γd,φ[k_] p] := gPair[{γd,φ[k]p};
gPair[Bs_Times] := gPair[List@@Bs];
gPair[Bs_List] := Module[{es, BBs},
  BBs = Bs /. e_<sup>p</sup> -> Sequence@@Table[e, {p}];
  es = Union@@(List@@@BBs);
  Do[v@i = es[[i]], {i, Length@es}];
  gpair[Replace[BBs, Thread[es -> Range@Length@es], {2}]] /. gα,β -> gv@α,v@β
]

```

```

In[*]:= gpair[Bs_List] := gpair[Bs] = Module[{es},
  Print["At gpair with Bs=", Bs];
  es = Union@@(List@@@Bs);
  Simplify@ZipJoin@@Table[{p1,α,p2,α,x1,α,x2,α}, {α,es}][Times[
    Times@@(Bs /. {
      rd,s[i_, j_] -> (V[rd,s[i, j]] /. {pi -> p2,i, pj -> p2,j, xi -> x2,i, xj -> x2,j}),
      γd,φ[k_] -> (V[γd,φ[k]] /. {pk -> p1,k, xk -> x1,k})
    }),
  Exp[Sum[gα,β(π1,α + π2,α)(ξ1,β + ξ2,β), {α,es}, {β,es}] - Sum[ξ1,α π2,α, {α,es}]]
]

```

```

In[*]:= gPair[r1,-1[3, 4]]

```

$$\text{Out}[*]:= \frac{1}{2} + \left(-1 + \frac{1}{T}\right) g_{4,3}^2 + g_{4,3} (1 + g_{3,4} - 2 g_{4,4}) + g_{3,3} \left(-1 + \frac{(-1+T) g_{4,3}}{T} + g_{4,4}\right)$$

```

In[*]:= gPair[r1,-1[5, 6]]

```

$$\text{Out}[*]:= \frac{1}{2} + \left(-1 + \frac{1}{T}\right) g_{6,5}^2 + g_{6,5} (1 + g_{5,6} - 2 g_{6,6}) + g_{5,5} \left(-1 + \frac{(-1+T) g_{6,5}}{T} + g_{6,6}\right)$$

In[\*]:= `gPair[γ1,-1[3]]`

Out[\*]= 
$$-\frac{1}{2} + g_{3,3}$$

In[\*]:= `gPair[γ1,0[6]]`

Out[\*]= 
$$0$$

In[\*]:= `gPair[γ1,-1[3]2]`

Out[\*]= 
$$\frac{1}{4} - g_{3,3} + 2 g_{3,3}^2$$

In[\*]:= `gPair[r2,1[3, 4] γ1,-1[3]]`

Out[\*]= 
$$\begin{aligned} & \frac{1}{4} \left( 4 g_{3,3}^3 (8 (-1 + T) g_{4,3} - 3 g_{4,4}) - g_{4,3} (1 + 8 g_{3,4}^2 g_{4,3} + 2 (-3 + 2 T + T^2) g_{4,3}^2 + \right. \\ & \quad \left. 3 g_{3,4} (1 + 2 g_{4,3}) (3 + 2 (1 + T) g_{4,3} - 4 g_{4,4}) - 6 g_{4,4} + 6 g_{4,4}^2 - g_{4,3} (1 - 3 T + 6 (1 + T) g_{4,4}) \right) - \\ & \quad 2 g_{3,3}^2 (2 + 8 (-5 + 4 T + T^2) g_{4,3}^2 - 11 g_{4,4} + 4 g_{4,4}^2 + g_{4,3} (-13 + 19 T + 18 g_{3,4} - 12 (2 + T) g_{4,4})) + \\ & \quad g_{3,3} (3 + 16 (-3 + 2 T + T^2) g_{4,3}^3 - 9 g_{4,4} + 6 g_{4,4}^2 + 6 g_{4,3}^2 (-6 + 7 T + T^2 + 4 (2 + T) g_{3,4} - 6 (1 + T) g_{4,4})) + \\ & \quad \left. g_{4,3} (1 + 9 T + g_{3,4} (44 - 32 g_{4,4}) - 12 (4 + T) g_{4,4} + 24 g_{4,4}^2) \right) \end{aligned}$$

```
In[*]:= ρd[K_] := PPρd@Module[{Cs, φ, n, A, s, i, j, k, Δ, G, d1, ρd},
  PPGreen[
    {Cs, φ} = Rot[K]; n = Length[Cs];
    A = IdentityMatrix[2 n + 1];
    Cases[Cs, {s_, i_, j_} >=> (A[[{i, j}, {i + 1, j + 1}]] += (
      -Ts Ts - 1
    ))];
    Δ = T(-Total[φ] - Total[Cs[[All, 1]])/2} Det[A];
    G = Inverse[A];
  ];
  ρd = PPPairing@gPair[Series[Exp[
    Total[Cases[Cs, {s_, i_, j_} >=> Sum[ed1 rd1,s[i, j], {d1, d}]]]
    + Sum[ed1 γd1,φ[[k]][k], {k, 2 n}, {d1, d}]]
  ], {ε, 0, d}] // Normal // Expand];
  PPRenormalizing[
    ρd = CoefficientList[Δ Normal[Series[ρd, {ε, 0, d}]] /. ε >=> Δ ε, ε, d + 1];
  PPSubstitution@Factor[ρd /. α-+ >=> α + 1 /. gα,β >=> G[[α, β]]
  ];
];
```

In[\*]:=  $\rho_1[\text{Knot}[3, 1]]$

KnotTheory: Loading precomputed data in PD4Knots`.

At gpair with Bs={r<sub>1,-1</sub>[2, 1]}

Out[\*]=

$$\left\{ \frac{1 - T + T^2}{T}, \frac{(-1 + T)^2 (1 + T^2)}{T^2} \right\}$$

In[\*]:=  $\text{Table}[K \rightarrow \rho_1[K], \{K, \text{AllKnots}[\{3, 6\}]\}]$

At gpair with Bs={r<sub>1,1</sub>[1, 2]}

At gpair with Bs={\gamma<sub>1,1</sub>[1]}

At gpair with Bs={r<sub>1,1</sub>[2, 1]}

Out[\*]=

$$\left\{ \text{Knot}[3, 1] \rightarrow \left\{ \frac{1 - T + T^2}{T}, \frac{(-1 + T)^2 (1 + T^2)}{T^2} \right\}, \text{Knot}[4, 1] \rightarrow \left\{ -\frac{1 - 3T + T^2}{T}, \emptyset \right\}, \right.$$

$$\text{Knot}[5, 1] \rightarrow \left\{ \frac{1 - T + T^2 - T^3 + T^4}{T^2}, \frac{(-1 + T)^2 (1 + T^2) (2 + T^2 + 2T^4)}{T^4} \right\},$$

$$\text{Knot}[5, 2] \rightarrow \left\{ \frac{2 - 3T + 2T^2}{T}, \frac{(-1 + T)^2 (5 - 4T + 5T^2)}{T^2} \right\},$$

$$\text{Knot}[6, 1] \rightarrow \left\{ -\frac{(-2 + T)(-1 + 2T)}{T}, \frac{(-1 + T)^2 (1 - 4T + T^2)}{T^2} \right\},$$

$$\text{Knot}[6, 2] \rightarrow \left\{ -\frac{1 - 3T + 3T^2 - 3T^3 + T^4}{T^2}, \frac{(-1 + T)^2 (1 - 4T + 4T^2 - 4T^3 + 4T^4 - 4T^5 + T^6)}{T^4} \right\},$$

$$\text{Knot}[6, 3] \rightarrow \left\{ \frac{1 - 3T + 5T^2 - 3T^3 + T^4}{T^2}, \emptyset \right\} \left. \right\}$$

In[\*]:=  $\rho_2[\text{Knot}[3, 1]]$

Out[\*]=

$$\left\{ \frac{1 - T + T^2}{T}, \frac{(-1 + T)^2 (1 + T^2)}{T^2}, \frac{1 - 4T + 7T^2 - 12T^3 + 18T^4 - 12T^5 + 7T^6 - 4T^7 + T^8}{2T^3 (1 - T + T^2)} \right\}$$

```
In[*]:= BeginProfile []
Timing[z1 = ρ2[Knot[10, 106]]]
PrintProfile []
```

```
Out[*]= ProfileRoot
```

```
Out[*]= {8.51563,
{ -  $\frac{(1 - T + T^2) (-1 + T - 2 T^2 + T^3) (-1 + 2 T - T^2 + T^3)}{T^4}$ , -  $\frac{1}{T^8} (-1 + T)^2 (1 - 6 T + 20 T^2 - 48 T^3 + 82 T^4 - 114 T^5 + 134 T^6 - 140 T^7 + 134 T^8 - 114 T^9 + 82 T^{10} - 48 T^{11} + 20 T^{12} - 6 T^{13} + T^{14})$ ,
- ((1 - 16 T + 127 T^2 - 676 T^3 + 2735 T^4 - 8980 T^5 + 24 938 T^6 - 60 420 T^7 + 131 072 T^8 - 259 992 T^9 + 477 614 T^{10} - 814 576 T^{11} + 1 282 448 T^{12} - 1 846 716 T^{13} + 2 411 126 T^{14} - 2 836 312 T^{15} + 2 995 252 T^{16} - 2 836 312 T^{17} + 2 411 126 T^{18} - 1 846 716 T^{19} + 1 282 448 T^{20} - 814 576 T^{21} + 477 614 T^{22} - 259 992 T^{23} + 131 072 T^{24} - 60 420 T^{25} + 24 938 T^{26} - 8980 T^{27} + 2735 T^{28} - 676 T^{29} + 127 T^{30} - 16 T^{31} + T^{32}) / (2 T^{12} (1 - T + T^2) (-1 + T - 2 T^2 + T^3) (-1 + 2 T - T^2 + T^3)) )}}
```

```
Out[*]= ProfileRoot is root. Profiled time: 8.515
( 1) 0/ 8.515 above ρd
Renormalizing: called 1 times, time in 6.297/6.297
( 1) 6.297/ 6.297 under ρd
Substitution: called 1 times, time in 1.125/1.125
( 1) 1.125/ 1.125 under ρd
Green: called 1 times, time in 0.812/0.812
( 1) 0.812/ 0.812 under ρd
Pairing: called 1 times, time in 0.281/0.281
( 1) 0.281/ 0.281 under ρd
ρd: called 1 times, time in 0./8.515
( 1) 0/ 8.515 under ProfileRoot
( 1) 0.812/ 0.812 above Green
( 1) 0.281/ 0.281 above Pairing
( 1) 6.297/ 6.297 above Renormalizing
( 1) 1.125/ 1.125 above Substitution
```

```
In[ ]:= BeginProfile []
Timing[z2 =  $\rho_2$ [Knot[12, NonAlternating, 369]]]
PrintProfile []
```

Out[ ]:= ProfileRoot

Out[ ]:=

$$\left\{ 14.1719, \left[ -\frac{(1 - T + T^2) (-1 + T - 2 T^2 + T^3) (-1 + 2 T - T^2 + T^3)}{T^4}, -\frac{1}{T^8} (-1 + T)^2 (1 - 6 T + 20 T^2 - 48 T^3 + 82 T^4 - 114 T^5 + 134 T^6 - 140 T^7 + 134 T^8 - 114 T^9 + 82 T^{10} - 48 T^{11} + 20 T^{12} - 6 T^{13} + T^{14}), -\left( (1 - 16 T + 127 T^2 - 668 T^3 + 2631 T^4 - 8324 T^5 + 22282 T^6 - 52780 T^7 + 114992 T^8 - 236376 T^9 + 460598 T^{10} - 839688 T^{11} + 1404696 T^{12} - 2121524 T^{13} + 2862782 T^{14} - 3432312 T^{15} + 3647156 T^{16} - 3432312 T^{17} + 2862782 T^{18} - 2121524 T^{19} + 1404696 T^{20} - 839688 T^{21} + 460598 T^{22} - 236376 T^{23} + 114992 T^{24} - 52780 T^{25} + 22282 T^{26} - 8324 T^{27} + 2631 T^{28} - 668 T^{29} + 127 T^{30} - 16 T^{31} + T^{32}) / (2 T^{12} (1 - T + T^2) (-1 + T - 2 T^2 + T^3) (-1 + 2 T - T^2 + T^3)) \right] \right\}$$

Out[ ]:= ProfileRoot is root. Profiled time: 14.172  
 ( 1) 0/ 14.170 above  $\rho d$   
 Renormalizing: called 1 times, time in 9.562/9.562  
 ( 1) 9.562/ 9.562 under  $\rho d$   
 Substitution: called 1 times, time in 3.125/3.125  
 ( 1) 3.125/ 3.125 under  $\rho d$   
 Green: called 1 times, time in 1.047/1.047  
 ( 1) 1.047/ 1.047 under  $\rho d$   
 Pairing: called 1 times, time in 0.438/0.438  
 ( 1) 0.438/ 0.438 under  $\rho d$   
 $\rho d$ : called 1 times, time in 0./14.172  
 ( 1) 0/ 14.170 under ProfileRoot  
 ( 1) 1.047/ 1.047 above Green  
 ( 1) 0.438/ 0.438 above Pairing  
 ( 1) 9.562/ 9.562 above Renormalizing  
 ( 1) 3.125/ 3.125 above Substitution

```
In[ ]:= Simplify[Thread[z1 == z2]]
```

Out[ ]:=

$$\left\{ \text{True, True, } \frac{(-1 + T) (1 - T + T^2) (1 - 3 T + 2 T^2 + 5 T^3 - 12 T^4 + 18 T^5 - 12 T^6 + 5 T^7 + 2 T^8 - 3 T^9 + T^{10})}{T} == 0 \right\}$$

```
In[*]:= GST48 = EPD[X14,1, X̄2,29, X3,40, X43,4, X̄26,5, X6,95, X96,7, X13,8, X̄9,28, X10,41, X42,11, X̄27,12,
    X30,15, X̄16,61, X̄17,72, X̄18,83, X19,34, X̄89,20, X̄21,92, X̄79,22, X̄68,23, X̄57,24, X̄25,56, X62,31,
    X73,32, X84,33, X̄50,35, X36,81, X37,70, X38,59, X̄39,54, X44,55, X58,45, X69,46, X80,47, X48,91,
    X90,49, X51,82, X52,71, X53,60, X̄63,74, X̄64,85, X̄76,65, X̄87,66, X̄67,94, X̄75,86, X̄88,77, X̄78,93];
BeginProfile[]
Timing[z3 = ρ2[GST48]]
PrintProfile[]
```

Out[\*]=

```
ProfileRoot

At gpair with Bs={r1,-1[1, 4], r1,-1[2, 3]}
At gpair with Bs={r1,-1[1, 2], r1,-1[3, 4]}
At gpair with Bs={r1,-1[2, 1], r1,-1[3, 4]}
At gpair with Bs={r1,-1[1, 2], r1,-1[4, 3]}
At gpair with Bs={r1,-1[2, 1], r1,-1[4, 3]}
At gpair with Bs={r1,-1[2, 3], r1,1[1, 4]}
At gpair with Bs={r1,-1[3, 2], r1,1[1, 4]}
At gpair with Bs={r1,1[1, 4], r1,1[2, 3]}
At gpair with Bs={r1,-1[1, 4], r1,1[3, 2]}
At gpair with Bs={r1,-1[4, 1], r1,1[3, 2]}
At gpair with Bs={r1,1[3, 2], r1,1[4, 1]}
At gpair with Bs={r1,-1[1, 4], r1,1[2, 3]}
At gpair with Bs={r1,-1[4, 1], r1,1[2, 3]}
At gpair with Bs={r1,-1[3, 2], r1,1[4, 1]}
At gpair with Bs={r1,1[2, 3], r1,1[4, 1]}
At gpair with Bs={r1,-1[2, 3], r1,1[4, 1]}
At gpair with Bs={r1,-1[2, 3], γ1,1[1]}
At gpair with Bs={r1,-1[3, 2], γ1,1[1]}
```

Out[\*]=

$$\left\{ 621.75, \left\{ -\frac{(-1 + 2 T - T^2 - T^3 + 2 T^4 - T^5 + T^8) (-1 + T^3 - 2 T^4 + T^5 + T^6 - 2 T^7 + T^8)}{T^8}, \right. \right.$$

$$\frac{1}{T^{16}} (-1 + T)^2 (5 - 18 T + 33 T^2 - 32 T^3 + 2 T^4 + 42 T^5 - 62 T^6 - 8 T^7 + 166 T^8 - 242 T^9 + 108 T^{10} +$$

$$132 T^{11} - 226 T^{12} + 148 T^{13} - 11 T^{14} - 36 T^{15} - 11 T^{16} + 148 T^{17} - 226 T^{18} + 132 T^{19} + 108 T^{20} -$$

$$242 T^{21} + 166 T^{22} - 8 T^{23} - 62 T^{24} + 42 T^{25} + 2 T^{26} - 32 T^{27} + 33 T^{28} - 18 T^{29} + 5 T^{30}),$$

$$- \left( (25 - 348 T + 2312 T^2 - 9628 T^3 + 27228 T^4 - 51460 T^5 + 52250 T^6 + 25828 T^7 - 197145 T^8 +$$

$$313268 T^9 - 36579 T^{10} - 887864 T^{11} + 2118398 T^{12} - 2494152 T^{13} + 772387 T^{14} +$$

$$2785204 T^{15} - 5477089 T^{16} + 3765568 T^{17} + 2886710 T^{18} - 9712796 T^{19} + 9746285 T^{20} -$$

$$708568 T^{21} - 11443177 T^{22} + 17013304 T^{23} - 11217405 T^{24} - 1334300 T^{25} + 10332369 T^{26} -$$

$$8571752 T^{27} - 1186874 T^{28} + 8007252 T^{29} - 3568015 T^{30} - 8148860 T^{31} + 14395240 T^{32} -$$

$$8148860 T^{33} - 3568015 T^{34} + 8007252 T^{35} - 1186874 T^{36} - 8571752 T^{37} + 10332369 T^{38} -$$

$$1334300 T^{39} - 11217405 T^{40} + 17013304 T^{41} - 11443177 T^{42} - 708568 T^{43} + 9746285 T^{44} -$$

$$9712796 T^{45} + 2886710 T^{46} + 3765568 T^{47} - 5477089 T^{48} + 2785204 T^{49} + 772387 T^{50} -$$

$$2494152 T^{51} + 2118398 T^{52} - 887864 T^{53} - 36579 T^{54} + 313268 T^{55} - 197145 T^{56} +$$

$$25828 T^{57} + 52250 T^{58} - 51460 T^{59} + 27228 T^{60} - 9628 T^{61} + 2312 T^{62} - 348 T^{63} + 25 T^{64}) /$$

$$\left. \left. \left( 2 T^{24} (-1 + 2 T - T^2 - T^3 + 2 T^4 - T^5 + T^8) (-1 + T^3 - 2 T^4 + T^5 + T^6 - 2 T^7 + T^8) \right) \right) \right\}$$

Out[\*]=

```
ProfileRoot is root. Profiled time: 621.75
( 1) 0.204/ 621.750 above ρd
Renormalizing: called 1 times, time in 282.468/282.468
( 1) 282.468/ 282.468 under ρd
Substitution: called 1 times, time in 154.437/154.437
( 1) 154.437/ 154.437 under ρd
Green: called 1 times, time in 121.188/121.188
( 1) 121.188/ 121.188 under ρd
Pairing: called 1 times, time in 63.453/63.453
( 1) 63.453/ 63.453 under ρd
ρd: called 1 times, time in 0.204/621.75
( 1) 0.204/ 621.750 under ProfileRoot
( 1) 121.188/ 121.188 above Green
( 1) 63.453/ 63.453 above Pairing
( 1) 282.468/ 282.468 above Renormalizing
( 1) 154.437/ 154.437 above Substitution
```