Pensieve header: Developing ρ_d.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\Oaxaca-2210"];
```

In[•]:=
```
Once[<< KnotTheory`; << Rot.m];
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.

Loading Rot.m from http://drorbn.net/la22/ap to compute rotation numbers.
```

In[•]:=
```
<< "../../Projects/Profile/Profile.m"
```

```
This is Profile.m of http://www.drorbn.net/AcademicPensieve/Projects/Profile/.

This version: April 2020. Original version: July 1994.
```

In[•]:=
```
CF[ℰ_] := Module[{vs = Union[{ϵ}, Cases[ℰ, (g | p | x)_, ∞]]}, Total[
   CoefficientRules[Expand@ℰ, vs] /. (ps_ → c_) :> Factor[c] (Times @@ vs^ps)
  ]]
```

In[•]:=
```
{p^*, x^*, π^*, ξ^*} = {π, ξ, p, x};  (u_i__)^* := (u^*)_i;
```

In[•]:=
```
Zip_{}[ℰ_] := ℰ;
Zip_{ξ_,ξs___}[ℰ_] := (Collect[ℰ // Zip_{ξs}, ξ] /. f_. ξ^d_. :> (D[f, {ξ^*, d}])) /. ξ^* → 0
```

In[•]:=
```
γ_d[0, j_] := 0; γ_1[1, k_] := ϵ (1/2 - p_k x_k); γ_1[-1, k_] := ϵ (-1/2 + p_k x_k);
```

In[•]:=
```
r_1[1, i_, j_] := ϵ (-1/2 + p_i x_i - p_j x_i + 1/2 (-1 + T) p_i p_j x_i^2 + 1/2 (1 - T) p_j^2 x_i^2 - p_i p_j x_i x_j + p_j^2 x_i x_j);
r_1[-1, i_, j_] := ϵ (1/2 - p_i x_i + p_j x_i + ((-1 + T) p_i p_j x_i^2)/(2 T) - ((-1 + T) p_j^2 x_i^2)/(2 T) + p_i p_j x_i x_j - p_j^2 x_i x_j);
```

In[•]:=
```
γ_2[1, k_] := -1/2 ϵ^2 p_k x_k + ϵ (1/2 - p_k x_k); γ_2[-1, k_] := -1/2 ϵ^2 p_k x_k + ϵ (-1/2 + p_k x_k);
```

In[•]:=
$$r_2[1, i\_, j\_] := \epsilon \left(-\frac{1}{2} + p_i x_i - p_j x_i + \frac{1}{2}(-1+T)\, p_i p_j x_i^2 + \frac{1}{2}(1-T)\, p_j^2 x_i^2 - p_i p_j x_i x_j + p_j^2 x_i x_j\right) +$$
$$\epsilon^2 \left(-\frac{1}{2} p_i x_i + \frac{p_j x_i}{2} + \frac{1}{4}(1-3T)\, p_i p_j x_i^2 + \frac{1}{4}(-1+3T)\, p_j^2 x_i^2 + \frac{1}{3}(-1+T)\, p_i^2 p_j x_i^3 - \right.$$
$$\frac{1}{6}(-1+T)(5+T)\, p_i p_j^2 x_i^3 + \frac{1}{6}(-1+T)(3+T)\, p_j^3 x_i^3 + \frac{3}{2} p_i p_j x_i x_j - \frac{3}{2} p_j^2 x_i x_j -$$
$$\left.\frac{1}{2} p_i^2 p_j x_i^2 x_j + \frac{1}{2}(2+T)\, p_i p_j^2 x_i^2 x_j + \frac{1}{2}(-1-T)\, p_j^3 x_i^2 x_j - \frac{1}{2} p_i p_j^2 x_i x_j^2 + \frac{1}{2} p_j^3 x_i x_j^2\right);$$

In[•]:=
$$r_2[-1, i\_, j\_] := \epsilon \left(\frac{1}{2} - p_i x_i + p_j x_i + \frac{(-1+T)\, p_i p_j x_i^2}{2T} - \frac{(-1+T)\, p_j^2 x_i^2}{2T} + p_i p_j x_i x_j - p_j^2 x_i x_j\right) +$$
$$\epsilon^2 \left(-\frac{1}{2} p_i x_i + \frac{p_j x_i}{2} + \frac{(-3+T)\, p_i p_j x_i^2}{4T} - \frac{(-3+T)\, p_j^2 x_i^2}{4T} - \frac{(-1+T)\, p_i^2 p_j x_i^3}{3T} + \right.$$
$$\frac{(-1+T)(1+5T)\, p_i p_j^2 x_i^3}{6T^2} - \frac{(-1+T)(1+3T)\, p_j^3 x_i^3}{6T^2} + \frac{3}{2} p_i p_j x_i x_j - \frac{3}{2} p_j^2 x_i x_j -$$
$$\left.\frac{1}{2} p_i^2 p_j x_i^2 x_j + \frac{(1+2T)\, p_i p_j^2 x_i^2 x_j}{2T} - \frac{(1+T)\, p_j^3 x_i^2 x_j}{2T} - \frac{1}{2} p_i p_j^2 x_i x_j^2 + \frac{1}{2} p_j^3 x_i x_j^2\right);$$

In[ ]:=

```
ρd_[K_] := PPρd@Module[{Cs, φ, n, A, s, i, j, k, Δ, G, ρd},
  PP"Green"[
   {Cs, φ} = Rot[K]; n = Length[Cs];
   A = IdentityMatrix[2 n + 1];
   Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ({{-T^s, T^s - 1}, {0, -1}}))];
   Δ = T^((-Total[φ] - Total[Cs[[All, 1]]])/2) Det[A];
   G = Inverse[A];
  ];
  PPPairing[
   ρd = Times[
     Series[Exp[
       Total[Cases[Cs, {s_, i_, j_} :> rd[s, i, j]] /. {pα_ :> p2,α, xα_ :> x2,α}]
        + (Sum[γd[φ[[k]]], k], {k, 2 n}] /. {pα_ :> p1,α, xα_ :> x1,α})
       ], {ϵ, 0, d}] // Normal,
     Exp[
      Sum[gα,β (π1,α + π2,α) (ξ1,β + ξ2,β), {α, 2 n + 1}, {β, 2 n + 1}]
       - Sum[ξ1,α π2,α, {α, 2 n + 1}]
     ]
    ] // ZipJoin@@Table[{p1,k, p2,k, x1,k, x2,k}, {k, 2 n + 1}];
  ];
  PPRenormalizing[
   ρd = CoefficientList[Δ Normal[Series[ρd, {ϵ, 0, d}]] /. ϵ → Δ ϵ, ϵ, d + 1]];
  PPSubstitution@Factor[ρd /. α_^+ :> α + 1 /. gα_,β_ :> G[[α, β]]]
 ];
```

In[ ]:=
```
Table[K → ρ1[K], {K, AllKnots[{3, 6}]}]
```

Out[ ]=

$$\left\{\text{Knot}[3,1]\to\left\{\frac{1-T+T^2}{T},\frac{(-1+T)^2\left(1+T^2\right)}{T^2}\right\},\ \text{Knot}[4,1]\to\left\{-\frac{1-3T+T^2}{T},0\right\},\right.$$
$$\text{Knot}[5,1]\to\left\{\frac{1-T+T^2-T^3+T^4}{T^2},\frac{(-1+T)^2\left(1+T^2\right)\left(2+T^2+2T^4\right)}{T^4}\right\},$$
$$\text{Knot}[5,2]\to\left\{\frac{2-3T+2T^2}{T},\frac{(-1+T)^2\left(5-4T+5T^2\right)}{T^2}\right\},$$
$$\text{Knot}[6,1]\to\left\{-\frac{(-2+T)(-1+2T)}{T},\frac{(-1+T)^2\left(1-4T+T^2\right)}{T^2}\right\},$$
$$\text{Knot}[6,2]\to\left\{-\frac{1-3T+3T^2-3T^3+T^4}{T^2},\frac{(-1+T)^2\left(1-4T+4T^2-4T^3+4T^4-4T^5+T^6\right)}{T^4}\right\},$$
$$\left.\text{Knot}[6,3]\to\left\{\frac{1-3T+5T^2-3T^3+T^4}{T^2},0\right\}\right\}$$

In[ ]:=
```
ρ2[Knot[3, 1]]
```

Out[ ]=

$$\left\{\frac{1-T+T^2}{T},\frac{(-1+T)^2\left(1+T^2\right)}{T^2},\frac{1-4T+7T^2-12T^3+18T^4-12T^5+7T^6-4T^7+T^8}{2T^3\left(1-T+T^2\right)}\right\}$$

In[•]:=
```
BeginProfile[]
Timing[z1 = ρ2[Knot[10, 106]]]
PrintProfile[]
```

Out[•]=

ProfileRoot

Out[•]=

$$\Big\{991.141,\ \Big\{-\frac{\left(1-T+T^2\right)\left(-1+T-2T^2+T^3\right)\left(-1+2T-T^2+T^3\right)}{T^4},\ -\frac{1}{T^8}(-1+T)^2\left(1-6T+20T^2-48T^3+82T^4-114T^5+134T^6-140T^7+134T^8-114T^9+82T^{10}-48T^{11}+20T^{12}-6T^{13}+T^{14}\right),\ -\Big(\Big(1-16T+127T^2-676T^3+2735T^4-8980T^5+24\,938T^6-60\,420T^7+131\,072T^8-259\,992T^9+477\,614T^{10}-814\,576T^{11}+1\,282\,448T^{12}-1\,846\,716T^{13}+2\,411\,126T^{14}-2\,836\,312T^{15}+2\,995\,252T^{16}-2\,836\,312T^{17}+2\,411\,126T^{18}-1\,846\,716T^{19}+1\,282\,448T^{20}-814\,576T^{21}+477\,614T^{22}-259\,992T^{23}+131\,072T^{24}-60\,420T^{25}+24\,938T^{26}-8980T^{27}+2735T^{28}-676T^{29}+127T^{30}-16T^{31}+T^{32}\Big)\Big/\Big(2T^{12}\left(1-T+T^2\right)\left(-1+T-2T^2+T^3\right)\left(-1+2T-T^2+T^3\right)\Big)\Big)\Big\}\Big\}$$

Out[•]=
```
ProfileRoot is root. Profiled time: 991.141
  ( 1)       0/ 991.141 above ρd
Pairing: called 1 times, time in 962.812/962.812
  ( 1)  962.812/ 962.812 under ρd
Renormalizing: called 1 times, time in 26.672/26.672
  ( 1)   26.672/  26.672 under ρd
Green: called 1 times, time in 0.938/0.938
  ( 1)    0.938/   0.938 under ρd
Substitution: called 1 times, time in 0.719/0.719
  ( 1)    0.719/   0.719 under ρd
ρd: called 1 times, time in 0./991.141
  ( 1)       0/ 991.141 under ProfileRoot
  ( 1)    0.938/   0.938 above Green
  ( 1)  962.812/ 962.812 above Pairing
  ( 1)   26.672/  26.672 above Renormalizing
  ( 1)    0.719/   0.719 above Substitution
```

In[•]:=
```
BeginProfile[]
Timing[z2 = ρ2[Knot[12, NonAlternating, 369]]]
PrintProfile[]
```

Out[•]=
```
ProfileRoot
```

KnotTheory: Loading precomputed data in KnotTheory/12N.dts.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

Out[•]=
```
$Aborted
```

Out[•]=
```
ProfileRoot is root. Profiled time: 1.407
  ( 1)    0/   0 above ρd
Green: called 1 times, time in 1.407/1.407
  ( 1)  1.410/ 1.410 under ρd
ρd: called 1 times, time in 0./0.
  ( 1)    0/   0 under ProfileRoot
  ( 1)  1.410/ 1.410 above Green
  ( 1)    0/   0 above Pairing
Pairing: called 1 times, time in 0./0.
  ( 1)    0/   0 under ρd
```

In[•]:=
```
Simplify[Thread[z1 == z2]]
```