In[ ]:=
```
pol = 1 - 2 (-1 + T)^2 (1 + T^4) ϵ / T^3 +
  1/T^6 2 (-1 + T)^2 (6 - 15 T + 12 T^2 + 2 T^3 - 3 T^4 - 2 T^5 - 3 T^6 + 2 T^7 + 12 T^8 - 15 T^9 + 6 T^10) ϵ^2 -
  1/(3 T^9) ϵ^3 (360 - 2520 T + 7632 T^2 - 12510 T^3 + 10899 T^4 - 2988 T^5 - 2942 T^6 + 2731 T^7 - 695 T^8 +
    54 T^9 - 695 T^10 + 2731 T^11 - 2942 T^12 - 2988 T^13 + 10899 T^14 - 12510 T^15 + 7632 T^16 -
    2520 T^17 + 360 T^18 - 12 T^6 ca3 + 24 T^7 ca3 - 12 T^8 ca3 - 12 T^10 ca3 + 24 T^11 ca3 - 12 T^12 ca3)
```

Out[ ]=

$$1-\frac{2\ (-1+T)^2\ \left(1+T^4\right)\ \epsilon}{T^3}+\frac{2\ (-1+T)^2\ \left(6-15\ T+12\ T^2+2\ T^3-3\ T^4-2\ T^5-3\ T^6+2\ T^7+12\ T^8-15\ T^9+6\ T^{10}\right)\ \epsilon^2}{T^6}-\frac{1}{3\ T^9}\epsilon^3\ \left(360-2520\ T+7632\ T^2-12\,510\ T^3+10\,899\ T^4-2988\ T^5-2942\ T^6+2731\ T^7-695\ T^8+54\ T^9-695\ T^{10}+2731\ T^{11}-2942\ T^{12}-2988\ T^{13}+10\,899\ T^{14}-12\,510\ T^{15}+7632\ T^{16}-2520\ T^{17}+360\ T^{18}-12\ T^6\ ca_3+24\ T^7\ ca_3-12\ T^8\ ca_3-12\ T^{10}\ ca_3+24\ T^{11}\ ca_3-12\ T^{12}\ ca_3\right)$$

In[ ]:=
```
Solve[z == T^(1/2) - T^(-1/2), T]
```

Solve: There may be values of the parameters for which some or all solutions are not valid.

Out[ ]=

$$\left\{\left\{T\to\frac{1}{2}\ \left(2+z^2-z\ \sqrt{4+z^2}\right)\right\},\ \left\{T\to\frac{1}{2}\ \left(2+z^2+z\ \sqrt{4+z^2}\right)\right\}\right\}$$

In[ ]:=
```
Collect[
 ExpandAll[pol /. Solve[z == T^(1/2) - T^(-1/2), T][[1]]],
 ϵ, Together]
```

Solve: There may be values of the parameters for which some or all solutions are not valid.

Out[ ]=

$$1-2\ \left(2\ z^2+4\ z^4+z^6\right)\ \epsilon+2\ \left(2\ z^2+23\ z^4+74\ z^6+102\ z^8+45\ z^{10}+6\ z^{12}\right)\ \epsilon^2+\frac{1}{3}\epsilon^3\ \left(12+94\ z^2-271\ z^4-2827\ z^6-11\,502\ z^8-24\,723\ z^{10}-28\,818\ z^{12}-15\,912\ z^{14}-3960\ z^{16}-360\ z^{18}+24\ z^2\ ca_3+48\ z^4\ ca_3+12\ z^6\ ca_3\right)$$

In[•]:=
```
pol1 = Series[Log[pol], {ϵ, 0, Exponent[pol, ϵ]}]
Collect[
 ExpandAll[pol1 /. Solve[z == T^(1/2) - T^(-1/2), T][[1]]],
 ϵ, Together]
```

Out[•]=

$$\left(-\frac{2\,(-1+T)^2}{T^3}-2\,(-1+T)^2\,T\right)\epsilon+\frac{2\,(-1+T)^2\,\left(5-13\,T+11\,T^2+2\,T^3-5\,T^4+2\,T^5-5\,T^6+2\,T^7+11\,T^8-13\,T^9+5\,T^{10}\right)\epsilon^2}{T^6}+\frac{1}{3\,T^9}\left(-296+2100\,T-6456\,T^2+10\,750\,T^3-9447\,T^4+2220\,T^5+3762\,T^6-3979\,T^7+1727\,T^8-750\,T^9+1727\,T^{10}-3979\,T^{11}+3762\,T^{12}+2220\,T^{13}-9447\,T^{14}+10\,750\,T^{15}-6456\,T^{16}+2100\,T^{17}-296\,T^{18}+12\,T^6\,ca_3-24\,T^7\,ca_3+12\,T^8\,ca_3+12\,T^{10}\,ca_3-24\,T^{11}\,ca_3+12\,T^{12}\,ca_3\right)\epsilon^3+O[\epsilon]^4$$

Solve: There may be values of the parameters for which some or all solutions are not valid.

Out[•]=

$$-2\left(2\,z^2+4\,z^4+z^6\right)\epsilon+2\left(2\,z^2+19\,z^4+58\,z^6+82\,z^8+37\,z^{10}+5\,z^{12}\right)\epsilon^2+\frac{1}{3}\epsilon^3\left(12+94\,z^2-223\,z^4-2243\,z^6-8982\,z^8-19\,311\,z^{10}-22\,850\,z^{12}-12\,816\,z^{14}-3228\,z^{16}-296\,z^{18}+24\,z^2\,ca_3+48\,z^4\,ca_3+12\,z^6\,ca_3\right)$$

In[•]:=
```
Collect[
 Expand[pol] /. {T^p_. /; p > 0 :> ((2 + z^2 - z Sqrt[4 + z^2])/2)^p,
                 T^p_. /; p < 0 :> ((2 + z^2 + z Sqrt[4 + z^2])/2)^(-p)},
 ϵ, Expand]
```

Out[•]=

$$1+\left(-4\,z^2-8\,z^4-2\,z^6\right)\epsilon+\left(4\,z^2+46\,z^4+148\,z^6+204\,z^8+90\,z^{10}+12\,z^{12}\right)\epsilon^2+\epsilon^3\left(4+\frac{94\,z^2}{3}-\frac{271\,z^4}{3}-\frac{2827\,z^6}{3}-3834\,z^8-8241\,z^{10}-9606\,z^{12}-5304\,z^{14}-1320\,z^{16}-120\,z^{18}+8\,z^2\,ca_3+16\,z^4\,ca_3+4\,z^6\,ca_3\right)$$

```
T2z[p_] := Module[{P = Expand[p], n, c},
   If[P === 0, 0, c = Coefficient[P, T, n = Exponent[P, T]];
    c z^(2 n) + T2z[P - c (T^(1/2) - T^(-1/2))^(2 n)]]];
T2z[1/T^16 (-1 + T)^2 (5 - 18 T + 33 T^2 - 32 T^3 + 2 T^4 + 42 T^5 - 62 T^6 - 8 T^7 + 166 T^8 - 242 T^9 +
    108 T^10 + 132 T^11 - 226 T^12 + 148 T^13 - 11 T^14 - 36 T^15 - 11 T^16 + 148 T^17 - 226 T^18 + 132 T^19 +
    108 T^20 - 242 T^21 + 166 T^22 - 8 T^23 - 62 T^24 + 42 T^25 + 2 T^26 - 32 T^27 + 33 T^28 - 18 T^29 + 5 T^30)]
```