Pensieve header: A talk and a program about Archibald- and Γ-calculus and the Halacheva map between them. Continues pensieve://2021-03/

Title. I Still don't Understand the Alexander Polynomial

Abstract. As an algebraic knot theorist, I still don't understand the Alexander polynomial. There are two conventions as for how to present tangle theory in algebra: one may name the strands of a tangle, or one may name their ends. The distinction might seem too minor to matter, yet it leads to a completely different view of the set of tangles as an algebraic structure. There are lovely formulas for the Alexander polynomial as viewed from either perspective, and they even agree where they meet. But the "strands" formulas know about strand doubling while the "ends" ones don't, and the "ends" formulas know about skein relations while the "strands" ones don't. There ought to be a common generalization, but I don't know what it is.

tex

```
\def\nbpdfInput#1{\vskip 1mm\par\noindent\includegraphics{#1}}
\def\nbpdfPrint#1{\vskip 1mm\par\noindent\includegraphics{#1}}
\def\nbpdfEcho#1{\vskip 1mm\par\noindent\includegraphics{#1}}
\def\nbpdfOutput#1{\vskip 1mm\par\noindent\includegraphics{#1}}
\def\nbpdfText#1{\vskip 1mm\par\noindent\includegraphics{#1}}
\def\nbpdfSubsection#1{\vskip 1mm\par\noindent\includegraphics{#1}}
\def\nbpdfgraphInput#1{\vskip 1mm\par\noindent\includegraphics{#1}}
\def\nbpdfgraphOutput#1{\vskip 1mm\par\noindent\includegraphics[width=1.5in]{#1}}
```

pdf

In[ ]:=
```
<< KnotTheory`
```

pdf

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

$\mathcal{A}$-Calculus

pdf

In[ ]:=
```
WP[Wedge[u___], Wedge[v___]] := Signature[{u, v}] * Wedge @@ Sort[{u, v}];
WP[0, _] = WP[_, 0] = 0;
WP[A_, B_] :=
  Expand[Distribute[A ** B] /. (a_. * u_Wedge) ** (b_. * v_Wedge) :> a b WP[u, v]];
```

pdf

In[ ]:=
```
WExp[A_] := Module[{s = Wedge[], t = Wedge[], k = 0},
  While[t =!= 0, s += (t = Expand[WP[t, A] / (++k)])]; s]
```

pdf

In[∘]:=
```
c_{x_,y_}[w_Wedge] := Module[{i, j},
   {i} = FirstPosition[w, x, {0}]; {j} = FirstPosition[w, y, {0}];
   { w                                               (i == 0) ∧ (j == 0)
   { (-1)^(i+j+If[i>j,1,0]) Delete[w, {{i}, {j}}]    (i > 0) ∧ (j > 0)
  ];
c_{x_,y_}[ℰ_] := ℰ /. w_Wedge ⧴ c_{x,y}[w]
```

pdf

In[∘]:=
```
𝒜[Γ[ω_, _, λ_]] := Expand[ω WExp[Expand[λ] /. t_a_ h_b_ ⧴ ξ_a ∧ x_b]]
```

pdf

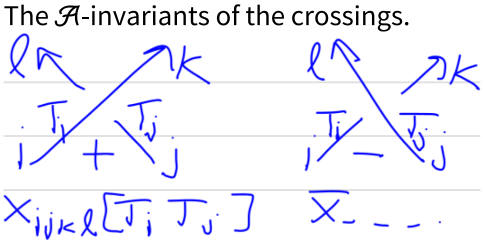
In[∘]:=
```
𝒜 /: 𝒜[is1_, os1_, cs1_, w1_] 𝒜[is2_, os2_, cs2_, w2_] :=
  𝒜[is1 ⋃ is2, os1 ⋃ os2, Join[cs1, cs2], WP[w1, w2]]
```

pdf

In[∘]:=
```
c_{h_,t_}@𝒜[is_, os_, cs_, w_] := 𝒜[
     DeleteCases[is, t], DeleteCases[os, h], KeyDrop[cs, {x_h, ξ_t}], c_{ξ_t,x_h}[w]
   ] /. If[cs[ξ_t]⟦1⟧ === τ, cs[ξ_t] → cs[x_h], cs[x_h] → cs[ξ_t]];
c@𝒜[is_, os_, cs_, w_] := Fold[c_{#2,#2}[#1] &, 𝒜[is, os, cs, w], is ⋂ os]
```

pdf

The 𝒜-invariants of the crossings.



tex

The crossings: \input{figs/R3.pdf_t}

pdf

In[∘]:=
```
𝒜[X_{i_,j_,k_,l_}[Ti_, Tj_]] := 𝒜[{i, j}, {k, l}, ⟨|ξ_i → Ti, ξ_j → Tj, x_k → Ti, x_l → Tj|⟩,
    Expand[Ti^(-1/2) WExp[Expand[{ξ_i, ξ_j}.( 1  1-Ti ).{x_k, x_l}] /. ξ_a_ x_b_ ⧴ ξ_a ∧ x_b]]];
                                            ( 0   Ti  )
𝒜[X̄_{i_,j_,k_,l_}[Ti_, Tj_]] := 𝒜[{i, j}, {k, l}, ⟨|ξ_i → Ti, ξ_j → Tj, x_k → Ti, x_l → Tj|⟩,
    Expand[Tj^(1/2) WExp[Expand[{ξ_i, ξ_j}.(  Tj^-1    0 ).{x_k, x_l}] /. ξ_a_ x_b_ ⧴ ξ_a ∧ x_b]]];
                                           ( 1-Tj^-1  1 )
𝒜[X_{i_,j_,k_,l_}] := 𝒜[X_{i,j,k,l}[τ_i, τ_l]];
𝒜[X̄_{i_,j_,k_,l_}] := 𝒜[X̄_{i,j,k,l}[τ_i, τ_j]];
```

pdf

In[∘]:=
```
𝒜[P_{i_,j_}[T_]] := 𝒜[{i}, {j}, ⟨|ξ_i → T, x_j → T|⟩, WExp[ξ_i ∧ x_j]];
𝒜[P_{i_,j_}] := 𝒜[P_{i,j}[τ_i]];
```

pdf

In[∘]:=
```
c[{A_}] := c[A];
c[{𝒜[is_, os_, cs_, w_], As__}] := Module[{A},
  A = RandomChoice@MaximalBy[{As}, Length[is ⋂ #⟦2⟧] + Length[os ⋂ #⟦1⟧] &];
  c[Join[{c[𝒜[is, os, cs, w] A]}, DeleteCases[{As}, A]]] ]
```

In[∘]:=
```
mva = MultivariableAlexander[L = PD@Link["L11a401"]][t]
Z = c[List @@ 𝒜 /@ (L /. X[i_, j_, k_, l_] :> If[PositiveQ[X[i, j, k, l]], X_{l,i,j,k}, X̄_{i,j,k,l}] /.
      {x_{-i_,j_,k_,1} :> x_{i,j,k,0}, x_{-i_,j_,1,k_} :> x_{i,j,0,k}})];
Factor@Coefficient[Z⟦4⟧ / (τ_1 - 1), Wedge[]]
```

Out[∘]= $-\frac{(-1+t[1])\,(-1+t[2])^2\,(-1+t[3])^2\,\left(1-t[3]+t[3]^2\right)}{\sqrt{t[1]}\;t[2]\;t[3]^2}$

Out[∘]= $-\frac{(-1+\tau_1)\,\left(-1+\sqrt{\tau_{10}}\right)^2\,\left(1+\sqrt{\tau_{10}}\right)^2\,(-1+\tau_{22})^2\,\left(1-\tau_{22}+\tau_{22}^2\right)}{\tau_1\,\sqrt{\tau_{10}}\;\tau_{22}}$

In[∘]:=
```
{WP[Wedge[x], Wedge[y]], WP[Wedge[y], Wedge[x]]}
```

Out[∘]= $\{x\wedge y,\ -(x\wedge y)\}$

In[∘]:=
```
lhs = c_{x_4,y_4}[WExp[Sum[a_i (x_i ∧ y_i), {i, 4}]]]
rhs = Expand[(1 - a_4) WExp[Sum[a_i (x_i ∧ y_i), {i, 3}]]]
lhs == rhs
```

Out[∘]= $\text{Wedge}[\,]-a_4\,\text{Wedge}[\,]+a_1\,x_1\wedge y_1-a_1\,a_4\,x_1\wedge y_1+a_2\,x_2\wedge y_2-a_2\,a_4\,x_2\wedge y_2+a_3\,x_3\wedge y_3-a_3\,a_4\,x_3\wedge y_3-a_1\,a_2\,x_1\wedge x_2\wedge y_1\wedge y_2+a_1\,a_2\,a_4\,x_1\wedge x_2\wedge y_1\wedge y_2-a_1\,a_3\,x_1\wedge x_3\wedge y_1\wedge y_3+a_1\,a_3\,a_4\,x_1\wedge x_3\wedge y_1\wedge y_3-a_2\,a_3\,x_2\wedge x_3\wedge y_2\wedge y_3+a_2\,a_3\,a_4\,x_2\wedge x_3\wedge y_2\wedge y_3-a_1\,a_2\,a_3\,x_1\wedge x_2\wedge x_3\wedge y_1\wedge y_2\wedge y_3+a_1\,a_2\,a_3\,a_4\,x_1\wedge x_2\wedge x_3\wedge y_1\wedge y_2\wedge y_3$

Out[∘]= $\text{Wedge}[\,]-a_4\,\text{Wedge}[\,]+a_1\,x_1\wedge y_1-a_1\,a_4\,x_1\wedge y_1+a_2\,x_2\wedge y_2-a_2\,a_4\,x_2\wedge y_2+a_3\,x_3\wedge y_3-a_3\,a_4\,x_3\wedge y_3-a_1\,a_2\,x_1\wedge x_2\wedge y_1\wedge y_2+a_1\,a_2\,a_4\,x_1\wedge x_2\wedge y_1\wedge y_2-a_1\,a_3\,x_1\wedge x_3\wedge y_1\wedge y_3+a_1\,a_3\,a_4\,x_1\wedge x_3\wedge y_1\wedge y_3-a_2\,a_3\,x_2\wedge x_3\wedge y_2\wedge y_3+a_2\,a_3\,a_4\,x_2\wedge x_3\wedge y_2\wedge y_3-a_1\,a_2\,a_3\,x_1\wedge x_2\wedge x_3\wedge y_1\wedge y_2\wedge y_3+a_1\,a_2\,a_3\,a_4\,x_1\wedge x_2\wedge x_3\wedge y_1\wedge y_2\wedge y_3$

Out[∘]= True

In[∘]:=
```
lhs = c_{x_3,y_3}[c_{x_4,y_4}[WExp[Sum[a_i (x_i ∧ y_i), {i, 4}]]]]
```

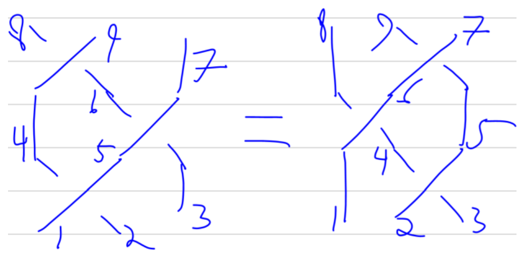
Out[∘]= $\text{Wedge}[\,]-a_3\,\text{Wedge}[\,]-a_4\,\text{Wedge}[\,]+a_3\,a_4\,\text{Wedge}[\,]+a_1\,x_1\wedge y_1-a_1\,a_3\,x_1\wedge y_1-a_1\,a_4\,x_1\wedge y_1+a_1\,a_3\,a_4\,x_1\wedge y_1+a_2\,x_2\wedge y_2-a_2\,a_3\,x_2\wedge y_2-a_2\,a_4\,x_2\wedge y_2+a_2\,a_3\,a_4\,x_2\wedge y_2-a_1\,a_2\,x_1\wedge x_2\wedge y_1\wedge y_2+a_1\,a_2\,a_3\,x_1\wedge x_2\wedge y_1\wedge y_2+a_1\,a_2\,a_4\,x_1\wedge x_2\wedge y_1\wedge y_2-a_1\,a_2\,a_3\,a_4\,x_1\wedge x_2\wedge y_1\wedge y_2$

In[∘]:= **n = 4; $\gamma 0 = \Gamma\left[\omega,\ \sum_{a=1}^{n} h_a\, \sigma_a,\ \sum_{a=1}^{n}\sum_{b=1}^{n} t_a\, h_b\, \alpha_{10\,a+b}\right]$**

$\gamma 0$ // tr[2]

Out[∘]= $\Gamma[\omega,\ h_1\,\sigma_1 + h_2\,\sigma_2 + h_3\,\sigma_3 + h_4\,\sigma_4,$
$h_1\,t_1\,\alpha_{11} + h_2\,t_1\,\alpha_{12} + h_3\,t_1\,\alpha_{13} + h_4\,t_1\,\alpha_{14} + h_1\,t_2\,\alpha_{21} + h_2\,t_2\,\alpha_{22} + h_3\,t_2\,\alpha_{23} + h_4\,t_2\,\alpha_{24} +$
$h_1\,t_3\,\alpha_{31} + h_2\,t_3\,\alpha_{32} + h_3\,t_3\,\alpha_{33} + h_4\,t_3\,\alpha_{34} + h_1\,t_4\,\alpha_{41} + h_2\,t_4\,\alpha_{42} + h_3\,t_4\,\alpha_{43} + h_4\,t_4\,\alpha_{44}]$

Out[∘]= $\text{tr}[2][\Gamma[\omega,\ h_1\,\sigma_1 + h_2\,\sigma_2 + h_3\,\sigma_3 + h_4\,\sigma_4,$
$h_1\,t_1\,\alpha_{11} + h_2\,t_1\,\alpha_{12} + h_3\,t_1\,\alpha_{13} + h_4\,t_1\,\alpha_{14} + h_1\,t_2\,\alpha_{21} + h_2\,t_2\,\alpha_{22} + h_3\,t_2\,\alpha_{23} + h_4\,t_2\,\alpha_{24} +$
$h_1\,t_3\,\alpha_{31} + h_2\,t_3\,\alpha_{32} + h_3\,t_3\,\alpha_{33} + h_4\,t_3\,\alpha_{34} + h_1\,t_4\,\alpha_{41} + h_2\,t_4\,\alpha_{42} + h_3\,t_4\,\alpha_{43} + h_4\,t_4\,\alpha_{44}]]$

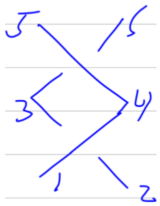


In[∘]:= **lhs = c[$\mathcal{A}$[$X_{1,2,5,4}$[T_1, T_2]] $\mathcal{A}$[$X_{5,3,7,6}$[T_1, T_3]] $\mathcal{A}$[$X_{4,6,9,8}$]]**
rhs = c[$\mathcal{A}$[$X_{2,3,5,4}$[T_2, T_3]] $\mathcal{A}$[$X_{1,4,6,8}$[T_1, T_3]] $\mathcal{A}$[$X_{6,5,7,9}$]]
lhs⟦4⟧ == rhs⟦4⟧

Out[∘]= $\mathcal{A}\Big[\{1, 2, 3\}, \{7, 8, 9\}, \langle| \xi_3 \to T_3, x_7 \to T_1, \xi_1 \to T_1, \xi_2 \to T_2, x_9 \to T_2, x_8 \to T_3 |\rangle,$
$\frac{\text{Wedge}[]}{T_1\sqrt{T_2}} - \frac{x_7 \wedge \xi_1}{T_1\sqrt{T_2}} + \frac{x_8 \wedge \xi_1}{\sqrt{T_2}} - \frac{x_8 \wedge \xi_1}{T_1\sqrt{T_2}} - \frac{x_8 \wedge \xi_2}{\sqrt{T_2}} + \sqrt{T_2}\, x_8 \wedge \xi_2 - \sqrt{T_2}\, x_8 \wedge \xi_3 + \frac{x_9 \wedge \xi_1}{\sqrt{T_2}} -$
$\frac{x_9 \wedge \xi_1}{T_1\sqrt{T_2}} - \frac{x_9 \wedge \xi_2}{\sqrt{T_2}} - \frac{x_7 \wedge x_8 \wedge \xi_1 \wedge \xi_2}{\sqrt{T_2}} + \sqrt{T_2}\, x_7 \wedge x_8 \wedge \xi_1 \wedge \xi_2 - \sqrt{T_2}\, x_7 \wedge x_8 \wedge \xi_1 \wedge \xi_3 -$
$\frac{x_7 \wedge x_9 \wedge \xi_1 \wedge \xi_2}{\sqrt{T_2}} - \sqrt{T_2}\, x_8 \wedge x_9 \wedge \xi_1 \wedge \xi_2 + T_1\sqrt{T_2}\, x_8 \wedge x_9 \wedge \xi_1 \wedge \xi_2 + \sqrt{T_2}\, x_8 \wedge x_9 \wedge \xi_1 \wedge \xi_3 -$
$T_1\sqrt{T_2}\, x_8 \wedge x_9 \wedge \xi_1 \wedge \xi_3 + T_1\sqrt{T_2}\, x_8 \wedge x_9 \wedge \xi_2 \wedge \xi_3 - T_1\sqrt{T_2}\, x_7 \wedge x_8 \wedge x_9 \wedge \xi_1 \wedge \xi_2 \wedge \xi_3\Big]$

Out[∘]= $\mathcal{A}\Big[\{1, 2, 3\}, \{7, 8, 9\}, \langle| x_7 \to T_1, x_9 \to T_2, \xi_1 \to T_1, x_8 \to T_3, \xi_2 \to T_2, \xi_3 \to T_3 |\rangle,$
$\frac{\text{Wedge}[]}{T_1\sqrt{T_2}} - \frac{x_7 \wedge \xi_1}{T_1\sqrt{T_2}} + \frac{x_8 \wedge \xi_1}{\sqrt{T_2}} - \frac{x_8 \wedge \xi_1}{T_1\sqrt{T_2}} - \frac{x_8 \wedge \xi_2}{\sqrt{T_2}} + \sqrt{T_2}\, x_8 \wedge \xi_2 - \sqrt{T_2}\, x_8 \wedge \xi_3 + \frac{x_9 \wedge \xi_1}{\sqrt{T_2}} -$
$\frac{x_9 \wedge \xi_1}{T_1\sqrt{T_2}} - \frac{x_9 \wedge \xi_2}{\sqrt{T_2}} - \frac{x_7 \wedge x_8 \wedge \xi_1 \wedge \xi_2}{\sqrt{T_2}} + \sqrt{T_2}\, x_7 \wedge x_8 \wedge \xi_1 \wedge \xi_2 - \sqrt{T_2}\, x_7 \wedge x_8 \wedge \xi_1 \wedge \xi_3 -$
$\frac{x_7 \wedge x_9 \wedge \xi_1 \wedge \xi_2}{\sqrt{T_2}} - \sqrt{T_2}\, x_8 \wedge x_9 \wedge \xi_1 \wedge \xi_2 + T_1\sqrt{T_2}\, x_8 \wedge x_9 \wedge \xi_1 \wedge \xi_2 + \sqrt{T_2}\, x_8 \wedge x_9 \wedge \xi_1 \wedge \xi_3 -$
$T_1\sqrt{T_2}\, x_8 \wedge x_9 \wedge \xi_1 \wedge \xi_3 + T_1\sqrt{T_2}\, x_8 \wedge x_9 \wedge \xi_2 \wedge \xi_3 - T_1\sqrt{T_2}\, x_7 \wedge x_8 \wedge x_9 \wedge \xi_1 \wedge \xi_2 \wedge \xi_3\Big]$

Out[∘]= True



In[∘]:= $\mathbf{c}\left[\mathcal{A}[X_{1,2,4,3}]\ \mathcal{A}[\overline{X}_{3,4,6,5}]\right]$

Out[∘]= $\mathcal{A}[\{1,2\},\{5,6\},\langle|\xi_1\to\tau_1,\ \xi_2\to\tau_3,\ x_6\to\tau_3,\ x_5\to\tau_1|\rangle,\ \text{Wedge}[\,]-x_5\wedge\xi_1-x_6\wedge\xi_2-x_5\wedge x_6\wedge\xi_1\wedge\xi_2]$

In[∘]:= $\{\mathbf{c}[\mathcal{A}[X_{1,2,2,3}]],\ \mathbf{c}[\mathcal{A}[X_{1,2,3,1}]],\ \mathbf{c}[\mathcal{A}[\overline{X}_{1,2,2,3}]],\ \mathbf{c}[\mathcal{A}[\overline{X}_{1,2,3,1}]]\}$

Out[∘]= $\left\{\mathcal{A}\left[\{1\},\{3\},\langle|\xi_1\to\tau_1,\ x_3\to\tau_1|\rangle,\ \frac{\text{Wedge}[\,]}{\sqrt{\tau_1}}-\frac{x_3\wedge\xi_1}{\sqrt{\tau_1}}\right],\right.$

$\mathcal{A}\left[\{2\},\{3\},\langle|\xi_2\to\tau_1,\ x_3\to\tau_1|\rangle,\ \sqrt{\tau_1}\ \text{Wedge}[\,]-\sqrt{\tau_1}\ x_3\wedge\xi_2\right],$

$\mathcal{A}\left[\{1\},\{3\},\langle|\xi_1\to\tau_1,\ x_3\to\tau_1|\rangle,\ \frac{\text{Wedge}[\,]}{\sqrt{\tau_1}}-\frac{x_3\wedge\xi_1}{\sqrt{\tau_1}}\right],$

$\left.\mathcal{A}\left[\{2\},\{3\},\langle|\xi_2\to\tau_2,\ x_3\to\tau_2|\rangle,\ \sqrt{\tau_2}\ \text{Wedge}[\,]-\sqrt{\tau_2}\ x_3\wedge\xi_2\right]\right\}$

In[∘]:= **L = Link["L4a1"] // PD**

Out[∘]= PD[X[6, 1, 7, 2], X[8, 3, 5, 4], X[2, 5, 3, 6], X[4, 7, 1, 8]]

In[∘]:= **Skeleton[L]**

Out[∘]= {Loop[1, 2, 3, 4], Loop[5, 6, 7, 8]}

In[∘]:= **Crossings[L]**

Out[∘]= 4

```
In[∘]:= MultivariableAlexander[L = PD@Link["L5a1"]][t]
        Z = c[Times @@ 𝒜 /@ (L /. X[i_, j_, k_, l_] :→ If[PositiveQ[X[i, j, k, l]], X_{l,i,j,k}, X̄_{i,j,k,l}] /.
              {x_{i_,j_,k_,1} :→ x_{i,j,k,0}, x_{i_,j_,1,k_} :→ x_{i,j,0,k}})]
        Coefficient[
         Z⟦
          4⟧,
         Wedge[]]
```

Out[∘]= $\frac{(-1+t[1])\,(-1+t[2])}{\sqrt{t[1]}\,\sqrt{t[2]}}$

Out[∘]= $\mathcal{A}\Big[\{1\},\ \{0\},\ \langle|\, x_0 \to \tau_1,\ \xi_1 \to \tau_1 \,|\rangle,\ -2\sqrt{\tau_{10}}\,\text{Wedge}[\,] + \frac{\sqrt{\tau_{10}}\,\text{Wedge}[\,]}{\tau_1} + \tau_1\sqrt{\tau_{10}}\,\text{Wedge}[\,] + 2\,\tau_{10}^{3/2}\,\text{Wedge}[\,] - \frac{\tau_{10}^{3/2}\,\text{Wedge}[\,]}{\tau_1} - \tau_1\,\tau_{10}^{3/2}\,\text{Wedge}[\,] + 2\sqrt{\tau_{10}}\,x_0\wedge\xi_1 - \frac{\sqrt{\tau_{10}}\,x_0\wedge\xi_1}{\tau_1} - \tau_1\sqrt{\tau_{10}}\,x_0\wedge\xi_1 - 2\,\tau_{10}^{3/2}\,x_0\wedge\xi_1 + \frac{\tau_{10}^{3/2}\,x_0\wedge\xi_1}{\tau_1} + \tau_1\,\tau_{10}^{3/2}\,x_0\wedge\xi_1\Big]$

Out[∘]= $-2\sqrt{\tau_{10}} + \frac{\sqrt{\tau_{10}}}{\tau_1} + \tau_1\sqrt{\tau_{10}} + 2\,\tau_{10}^{3/2} - \frac{\tau_{10}^{3/2}}{\tau_1} - \tau_1\,\tau_{10}^{3/2}$

```
In[∘]:= (L /. X[i_, j_, k_, l_] :→ If[PositiveQ[X[i, j, k, l]], X_{l,i,j,k}, X̄_{i,j,k,l}] /.
          {x_{i_,j_,k_,1} :→ x_{i,j,k,0}, x_{i_,j_,1,k_} :→ x_{i,j,0,k}})
```

Out[∘]= $\text{PD}\big[\bar{X}_{6,1,7,2},\ \bar{X}_{10,3,11,4},\ X_{7,12,8,5},\ X_{11,8,12,9},\ \bar{X}_{2,5,3,6},\ \bar{X}_{4,9,0,10}\big]$

```
In[∘]:= L
Out[∘]= PD[X[6, 1, 7, 2], X[10, 3, 11, 4], X[12, 8, 5, 7], X[8, 12, 9, 11], X[2, 5, 3, 6], X[4, 9, 1, 10]]

In[∘]:= Skeleton[L]
Out[∘]= {Loop[1, 2, 3, 4], Loop[5, 6, 7, 8, 9, 10, 11, 12]}
```

In[∘]:= $\text{Factor}\Big[\frac{-3 + \frac{1}{\tau_1} + 2\,\tau_1 + 3\,\tau_{10} - \frac{2\,\tau_{10}}{\tau_1} - \tau_1\,\tau_{10}}{\Big(\frac{1-2\,t[1]-2\,t[2]+t[1]\,t[2]}{\sqrt{t[1]}\,\sqrt{t[2]}}\ /.\ \{t[1]\to\tau_1,\ t[2]\to\tau_{10}\}\Big)}\Big]$

Out[∘]= $-\frac{\left(-1+\sqrt{\tau_1}\right)\left(1+\sqrt{\tau_1}\right)\sqrt{\tau_{10}}}{\sqrt{\tau_1}}$

In[∘]:= Z

Out[∘]= $\mathcal{A}\Big[\{1\},\ \{0\},\ \langle|\, \xi_1 \to \tau_1,\ x_0 \to \tau_1 \,|\rangle,\ -3\,\text{Wedge}[\,] + \frac{\text{Wedge}[\,]}{\tau_1} + 2\,\tau_1\,\text{Wedge}[\,] + 3\,\tau_{10}\,\text{Wedge}[\,] - \frac{2\,\tau_{10}\,\text{Wedge}[\,]}{\tau_1} - \tau_1\,\tau_{10}\,\text{Wedge}[\,] + 3\,x_0\wedge\xi_1 - \frac{x_0\wedge\xi_1}{\tau_1} - 2\,\tau_1\,x_0\wedge\xi_1 - 3\,\tau_{10}\,x_0\wedge\xi_1 + \frac{2\,\tau_{10}\,x_0\wedge\xi_1}{\tau_1} + \tau_1\,\tau_{10}\,x_0\wedge\xi_1\Big]$

In[∘]:=
```
MultivariableAlexander[L = PD@Link["L5a1"]][t]
Timing[
 Z1 = c[Times @@ 𝒜 /@ (L /. X[i_, j_, k_, l_] :> If[PositiveQ[X[i, j, k, l]], X_{l,i,j,k}, X̄_{i,j,k,l}] /.
      {x_{-i_,j_,k_,1} :> x_{i,j,k,0}, x_{-i_,j_,1,k_} :> x_{i,j,0,k}})];]
Coefficient[Z1[[4]], Wedge[]]
Timing[
 Z2 = c[List @@ 𝒜 /@ (L /. X[i_, j_, k_, l_] :> If[PositiveQ[X[i, j, k, l]], X_{l,i,j,k}, X̄_{i,j,k,l}] /.
      {x_{-i_,j_,k_,1} :> x_{i,j,k,0}, x_{-i_,j_,1,k_} :> x_{i,j,0,k}})];]
Coefficient[
 Z2[[
  4]],
 Wedge[]]
```

Out[∘]= $\frac{(-1+t[1])\ (-1+t[2])}{\sqrt{t[1]}\ \sqrt{t[2]}}$

Out[∘]= {411.453, Null}

Out[∘]= $-2\sqrt{\tau_{10}}+\frac{\sqrt{\tau_{10}}}{\tau_1}+\tau_1\sqrt{\tau_{10}}+2\,\tau_{10}^{3/2}-\frac{\tau_{10}^{3/2}}{\tau_1}-\tau_1\,\tau_{10}^{3/2}$

Out[∘]= {0.125, Null}

Out[∘]= $-2\sqrt{\tau_6}+\frac{\sqrt{\tau_6}}{\tau_1}+\tau_1\sqrt{\tau_6}+2\,\tau_6^{3/2}-\frac{\tau_6^{3/2}}{\tau_1}-\tau_1\,\tau_6^{3/2}$

In[∘]:= $\text{Factor}\left[\frac{-2\sqrt{\tau_6}+\frac{\sqrt{\tau_6}}{\tau_1}+\tau_1\sqrt{\tau_6}+2\,\tau_6^{3/2}-\frac{\tau_6^{3/2}}{\tau_1}-\tau_1\,\tau_6^{3/2}}{(\tau_1-1)\ (\tau_6-1)}\right]$

Out[∘]= $-\frac{(-1+\tau_1)\ \sqrt{\tau_6}}{\tau_1}$

```
In[∘]:= MultivariableAlexander[L = PD@Link["L4a1"]][t]
Timing[
 Z1 = c[Times @@ 𝒜 /@ (L /. X[i_, j_, k_, l_] ⧴ If[PositiveQ[X[i, j, k, l]], X_{l,i,j,k}, X̄_{i,j,k,l}] /.
     {x_{i_,j_,k_,1} ⧴ x_{i,j,k,0}, x_{i_,j_,1,k_} ⧴ x_{i,j,0,k}})];]
Coefficient[Z1⟦4⟧, Wedge[]]
Timing[
 Z2 = c[List @@ 𝒜 /@ (L /. X[i_, j_, k_, l_] ⧴ If[PositiveQ[X[i, j, k, l]], X_{l,i,j,k}, X̄_{i,j,k,l}] /.
     {x_{i_,j_,k_,1} ⧴ x_{i,j,k,0}, x_{i_,j_,1,k_} ⧴ x_{i,j,0,k}})];]
Coefficient[
 Z2⟦
  4⟧,
 Wedge[]]
```

Out[∘]= $\frac{-t[1]-t[2]}{\sqrt{t[1]}\ \sqrt{t[2]}}$

Out[∘]= {2.53125, Null}

Out[∘]= $-1+\tau_1+\tau_8-\frac{\tau_8}{\tau_1}$

Out[∘]= {0.03125, Null}

Out[∘]= $-1+\tau_1+\tau_8-\frac{\tau_8}{\tau_1}$

In[∘]:= Factor$\left[\frac{-1+\tau_1+\tau_8-\frac{\tau_8}{\tau_1}}{-\tau_1-\tau_8}\right]$

Out[∘]= $-\frac{-1+\tau_1}{\tau_1}$

In[∘]:= 𝒜[$P_{1,4}$]

Out[∘]= 𝒜[{1}, {4}, ⟨| $\xi_1 \to \tau_1$, $x_4 \to \tau_1$ |⟩, Wedge[] − $x_4 \wedge \xi_1$]

In[∘]:= c[c[𝒜[$P_{1,2}$] 𝒜[$P_{2,3}$]] 𝒜[$P_{3,4}$]]

Out[∘]= 𝒜[{1}, {4}, ⟨| $\xi_1 \to \tau_1$, $x_4 \to \tau_1$ |⟩, Wedge[] − $x_4 \wedge \xi_1$]

In[∘]:= c[c[𝒜[$P_{2,3}$] 𝒜[$P_{3,4}$]] 𝒜[$P_{1,2}$]]

Out[∘]= 𝒜[{1}, {4}, ⟨| $\xi_1 \to \tau_1$, $x_4 \to \tau_1$ |⟩, Wedge[] − $x_4 \wedge \xi_1$]

nb2tex

As in http://drorbn.net/AcademicPensieve/Projects/nb2tex/.

```
In[∘]:= SetOptions[$FrontEndSession, PrintingStyleEnvironment → "Working"];
nb2tex[nb_String, opts___Rule] := nb2tex[nb, nb, opts];
```

In[∘]:=
```
nb2tex[nb_String, tex_String, opts___Rule] := Module[
  {notebook, PDFCounter = 0, lines, type, tag, pdfname, cell, c, cl,
   PDFFolder = PDFFolder /. {opts} /. PDFFolder → nb
  },
  nb2tex$PDFWidth = PDFWidth /. {opts} /. PDFWidth → 6.5;
  notebook = NotebookGet[NotebookOpen@FileNameJoin[{Directory[], nb <> ".nb"}]];
  If[FileType[PDFFolder] === None, CreateDirectory[PDFFolder]];
  DeleteFile /@ FileNames["*.pdf", PDFFolder];
  lines = Table[
    type = cell〚2〛;
    tag = CellTags /. Cases[cell, _Rule] /. CellTags → "";
    Which[
     type == "Text" ∧ tag == "tex", StringReplace[cell〚1〛, {"’" → "'", "”" → "\""}],
     StringMatchQ[tag, "pdf" ~~ ___], (
      pdfname = PDFFolder <> "/" <> ToString[++PDFCounter] <> ".pdf";
      Export[pdfname, Join[cell, Cell[PageWidth → 80 nb2tex$PDFWidth / 0.75]]];
      cl = "c:\\drorbn\\bin\\cpdf.exe -scale-page \"0.75 0.75\" " <>
        pdfname <> " -o " <> pdfname;
      Close@OpenRead["!" <> cl];
      StringReplace[
       "\\noindent\\nbpdfXXXType{pdfname}",
       {"XXX" → StringDrop[tag, 3], "Type" → type, "pdfname" → pdfname}
      ]
     ),
     type == "Text" ∧ tag == "exec", ToExpression[cell〚1〛]; "",
     True, ""
    ],
    {cell, Cases[notebook, c_Cell /; Length[c] ≥ 2(*∧FreeQ[c, _Cell, {1,∞}]*), ∞]}
   ];
  lines = StringJoin @@ Riffle[DeleteCases[lines, ""], "\n\n"];
  WriteString[tex <> ".tex", lines]; Close[tex <> ".tex"]; ]
```

Run

In[∘]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\MoscowByWeb-2104"];
nb2tex["AnG-Programs", PDFFolder → "Snips", PDFWidth → 154.3 / 10 / 2.54];
Run@"\"C:\\Program Files\\MiKTeX 2.9\\miktex\\bin\\x64\\pdflatex.exe\" AnG.tex"
```

Out[∘]= 0