Doodle on this page!

Better vaxxed!

The 17 Worlds of Planar Ants

Dror Bar-Natan, http://drorbn.net/mc21

MathCamp by Web, July 2021

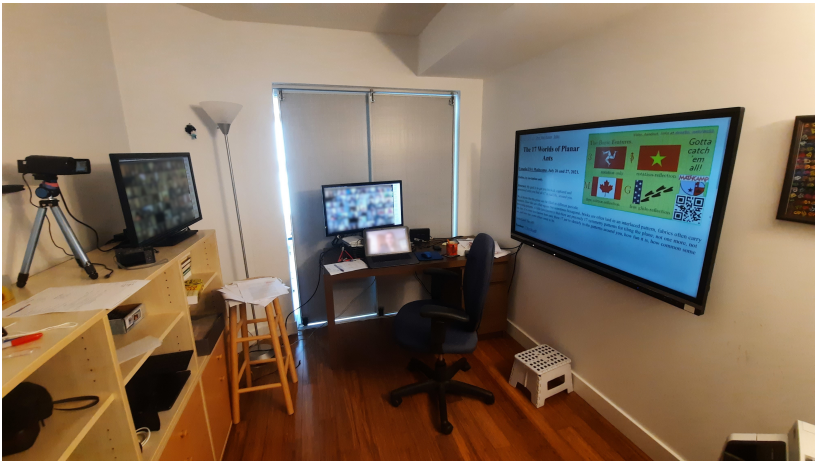
Abstract. My goal is to get you hooked, captured and unreleased until you find all 17 in real life, around you.

We all know that the plane can be filled in different periodic manners: floor tiles are often square but sometimes hexagonal, bricks are often laid in an interlaced pattern, fabrics often carry interesting patterns. A little less known is that there are precisely 17 symmetry patterns for tiling the plane; not one more, not one less. It is even less known how easy these 17 are to identify in the patterns around you, how fun it is, how common some are, and how rare some others seem to be.

Gotta Catch 'Em All!

Better vaxxed!

Thanks for inviting me to MathCamp! Just to feel a little closer, here's a picture of the lecture room:

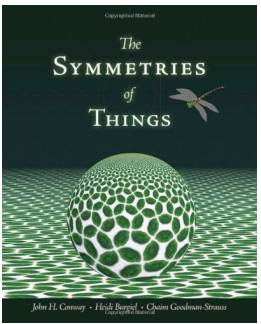


If you can, please turn your video on! (And mic, whenever needed).

Reading. An excellent book on the subject is *The Symmetries of Things* by J. H. Conway, H. Burgiel, and C. Goodman-Strauss, CRC Press, 2008.

Another nice text is *Classical Tessellations and Three-Manifolds* by J. M. Montesinos, Springer-Verlag, 1987.

And another is *Tilings & Patterns* by B. Grünbaum and G. C. Shephard, Dover, 2016.

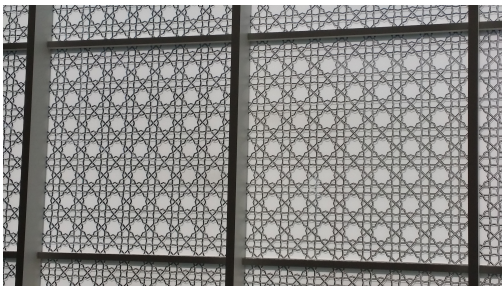


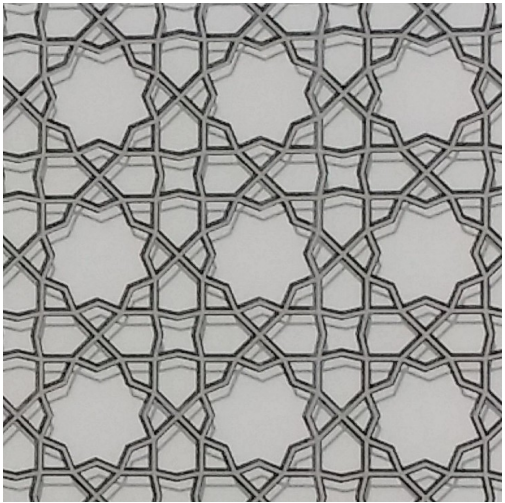
drorbn.net/mc21

(Easy) Question 1. In what ways can you make \$2 change, using coins denominated $\$\frac{1}{2}$, $\$\frac{2}{3}$, $\$\frac{3}{4}$, $\$\frac{4}{5}$, $\$\frac{5}{6}$, etc.?

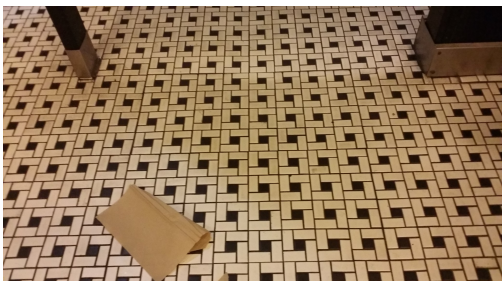
$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$

(Harder) Question 2. Why am I asking?

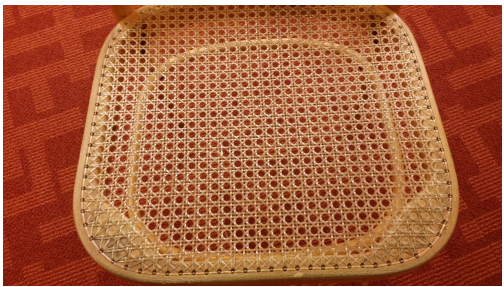


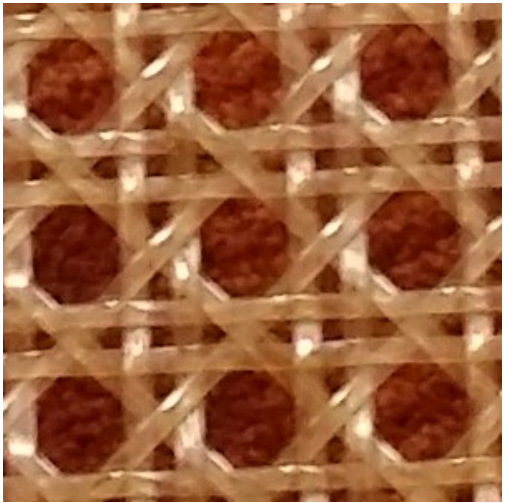


A decorated window at the Aga Khan Gallery, Toronto 2014



Floor tiles at Fran's Restaurant & Bar, Toronto 2014





A chair at the Toronto Public Library, 2014

A box of tissues

A living room sofa at the Karshon's, 2012

Theorem. There are precisely 17 patterns with which to tile the plane, no more, no less. They are all made of combinations of the 10 basic features, 2, 3, 4, 6, ✌, ✋, ✍, ✊, M, and G, as follows:

✓	Dror's	Conway's	crystallo-graphic	✓	Dror's	Conway's	crystallo-graphic
	2222	2222	p2		3✋	3*3	p31m
	333	333	p3		2✌✌	2*22	cmm
✓	442	442	p4		22M	22*	pmg
✓	632	632	p6		MM	**	pm
	2✌✌✌	*2222	pmm		MG	*o	cm
	3✋✋	*333	p3m1		GG	oo	pg
	4✍✌	*442	p4m		22G	22o	pgg
	6✊✋	*632	p6m		∅	0	p1
	4✌	4*2	p4g				

Video, handout, links at drorbn.net/mc21

The Basic Features.

Gotta catch 'em all!

3

rotation only

$\mathfrak{H}$

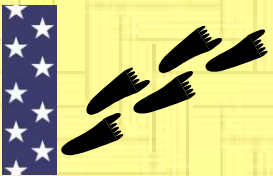
rotation-reflection

MATHCAMP

M

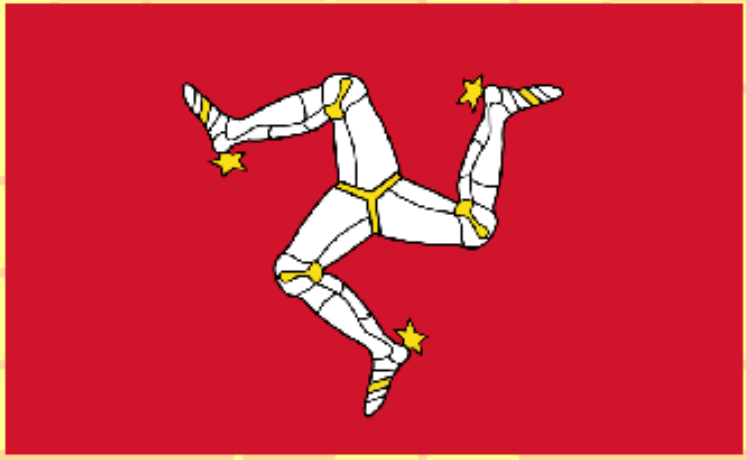
free mirror-reflection

G

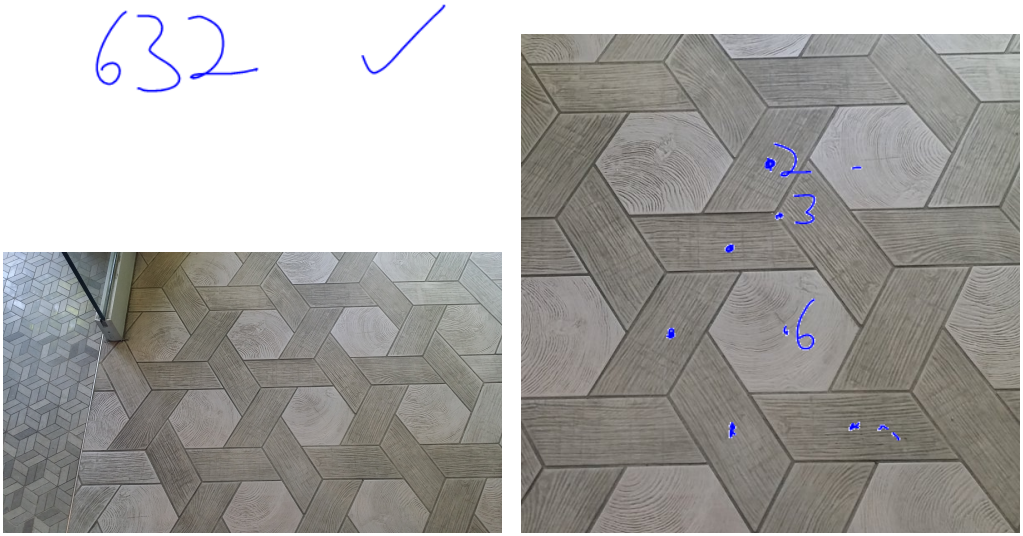


free glide-reflection

3



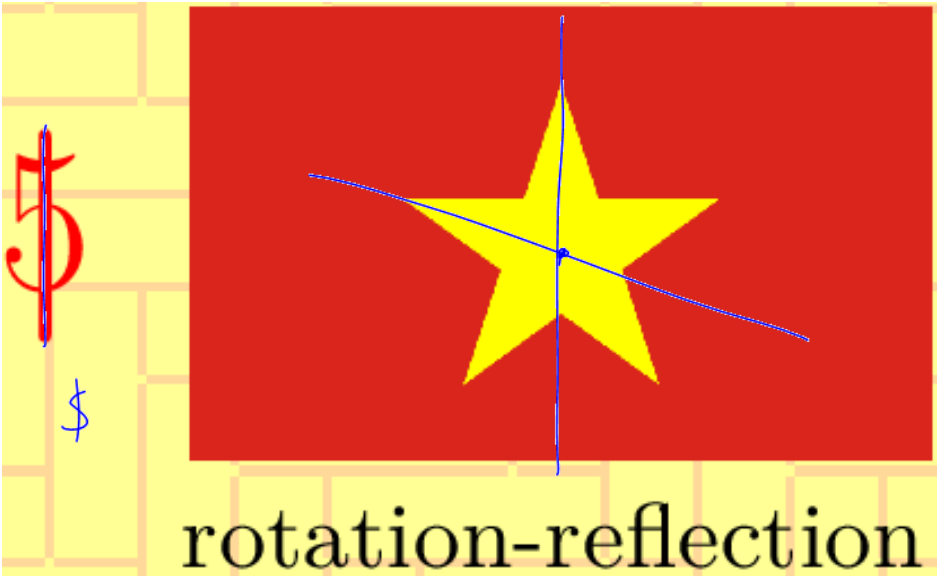
rotation only

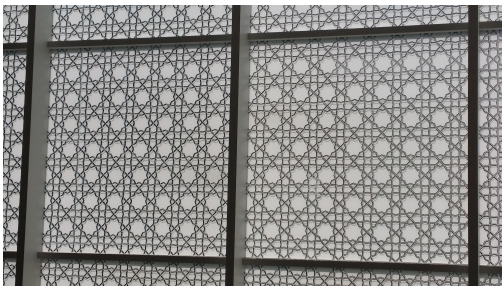


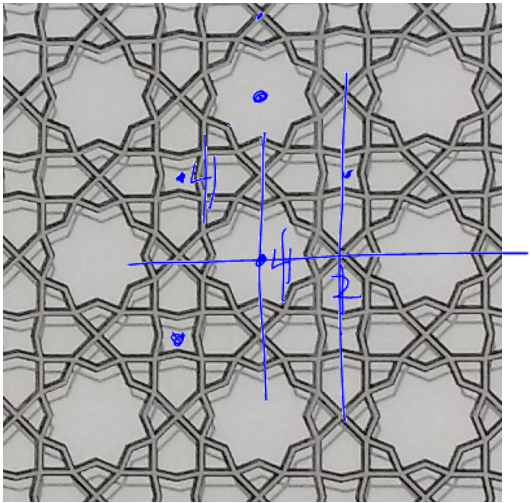
Floor tiles in a restaurant in Toronto's Baldwin Street, 2018

442.

A box of tissues


rotation-reflection





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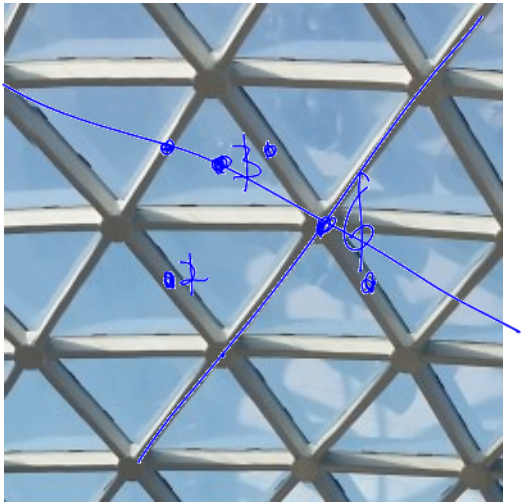
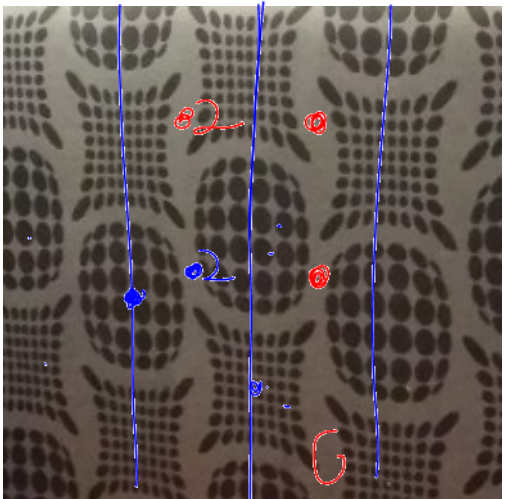


A food court at the Roma Fiumicino airport, 2017

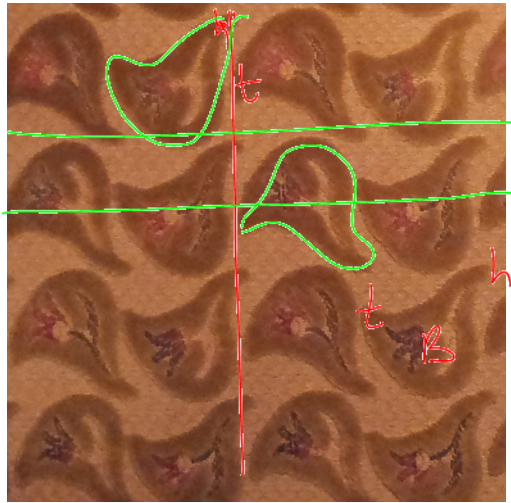
Better vaxxed!

A packet of tissues



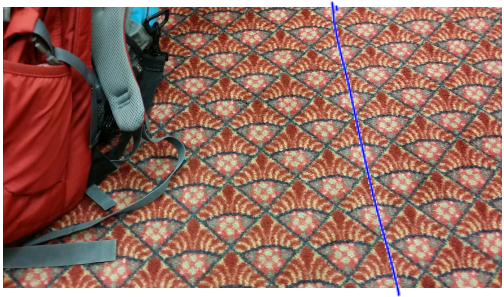
At Rick's Cafe in Toronto's Kensington Market, 2014

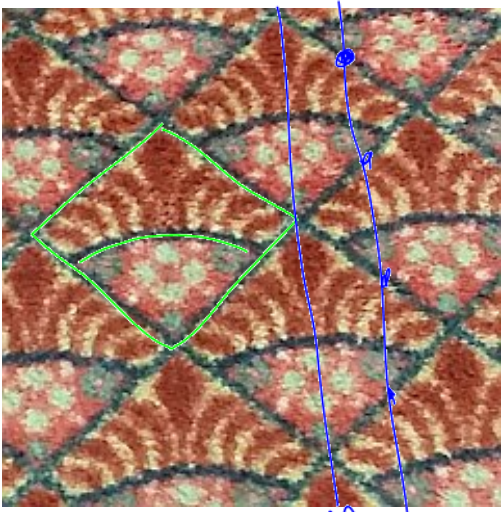

G
free glide-reflection



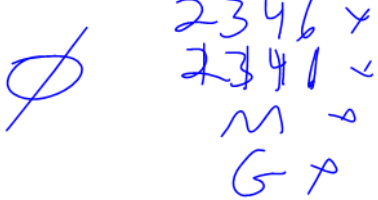
A living room sofa at the Karshon's, 2012

MG





A carpet seen at Indiana University, 2016



A bike parking in Groningen, 2020

Homework.

Go out and find them all! At home, around the corner, a mile away. Take pictures and upload them to http://drorbn.net/mc21/upload using the file format **name**-**type**-**description**.jpg, where

- **name** is your name or alias.
- **type** is the type of the tiling pattern, using the Conway conventions but with the "$*$" replaced with the English letter "s". In other words, **type** is one of 2222, 333, 442, 632, s2222, s333, s442, s632, 4s2, 3s3, 2s22, 22s, ss, so, oo, 22o, or 0.
- **description** is a short description.

As an example, see the file `Dror-4S2-StClairWSubway.jpg` there.

We will start our class tomorrow with a quick discussion of the patterns you will find — but I can only promise to look at whatever will be uploaded at least two hours ahead of class.

Privacy note. Whatever you upload I may post on my web site. So make sure the pictures you upload don't include anything personal.

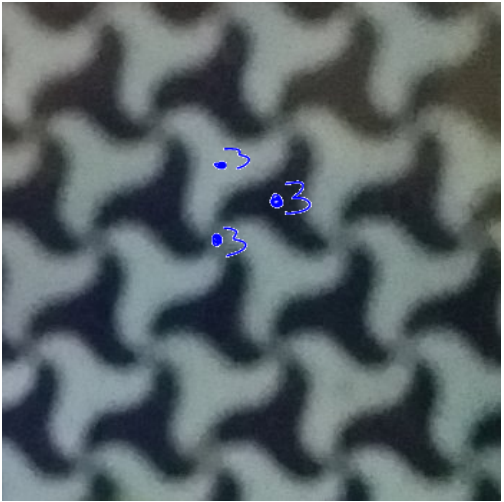
Prize.

333

US$50 if you find a 333 in time for our second meeting!

- You must find it "natural" — it can't be your own drawing, or within a book on symmetries, or in a museum that has an exhibit on symmetries (I think MoMath has one).
- In the unlikely event that more than one person will find a 333, I'll split the prize between all winners.
- I've paid the prize twice before, but it's really tough. I've only seen a 333 "in nature" three times in more than 20 years of looking for it!

333



_____ of the TV series _____

visiting _____

See you tomorrow!

Best with video on!

2, 3, 4, 6

Video, handout, links at drorbn.net/mc21

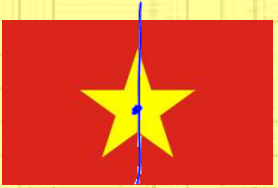
The Basic Features.

Gotta catch 'em all!

3

rotation only

$\mathfrak{H}$



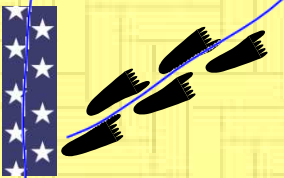
rotation-reflection

MATHCAMP

M

free mirror-reflection

G



free glide-reflection

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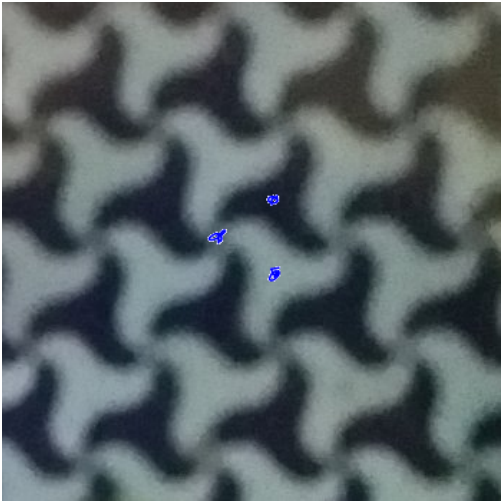
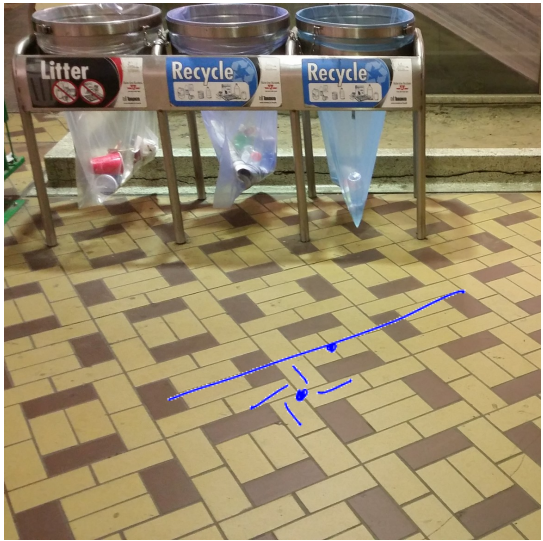
	✌✌✌✌	*2222	pmm	✓	MG	*o	cm
	✋✋✋	*333	p3m1	✓	GG	oo	pg
✓	44✌	*442	p4m		22G	22o	pgg
	63✌	*632	p6m	✓	∅	0	p1
	4✌	4*2	p4g				



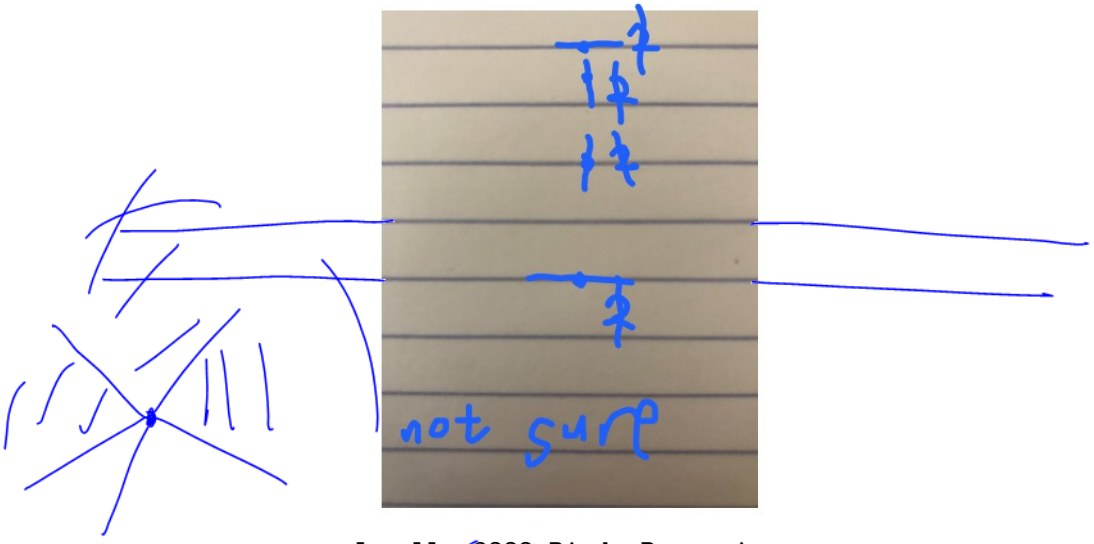
Jaime of the TV series GOT
visiting Dane

Let's look at what you found... http://drorbn.net/mc21/upload

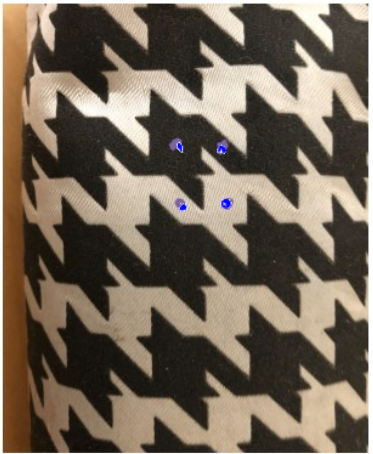


Dror-4S2-StClairWSubway.jpg

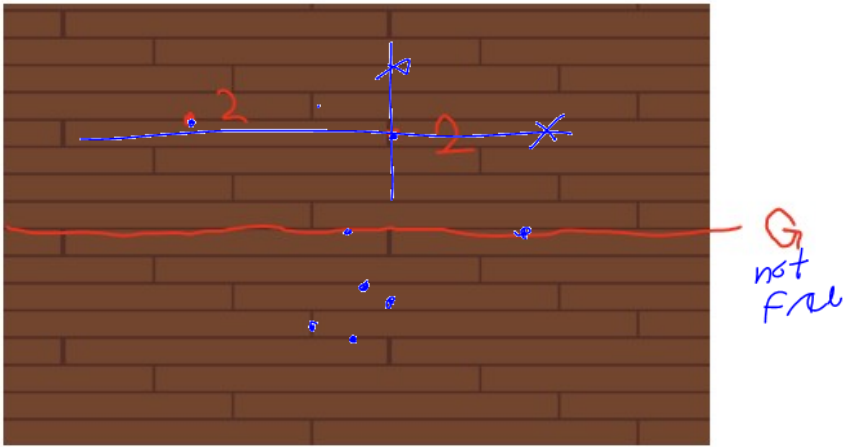
4½



Janelle-2222-BinderPaper.jpg

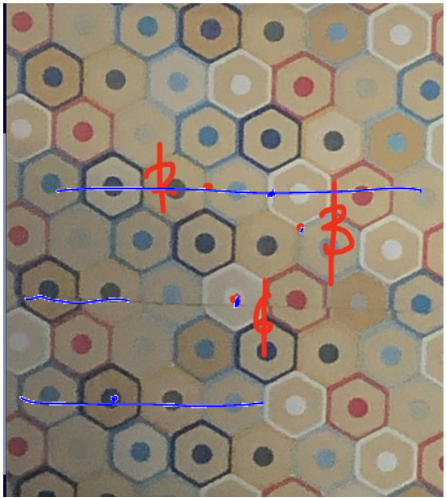


Janelle-2222-PencilCase.jpg



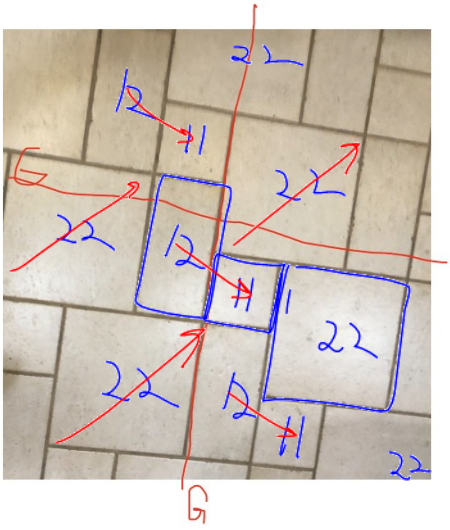
Janelle-220-MathtownFloorAtTau.jpg

2222



ignore colour

Janelle-632-TissueBox.jpg

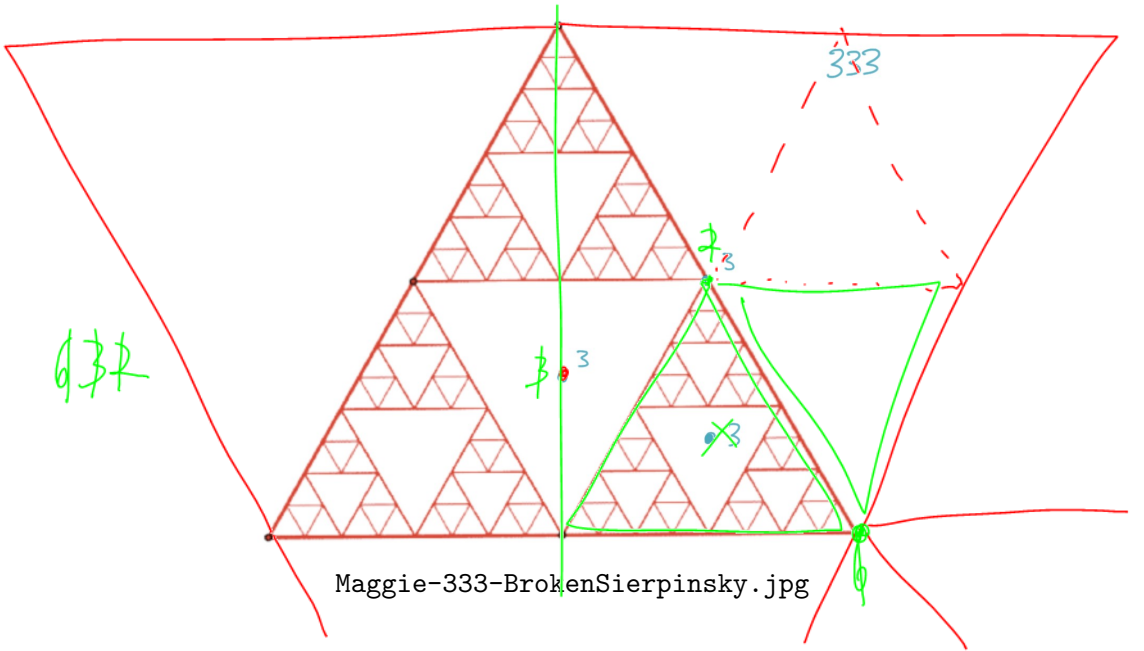


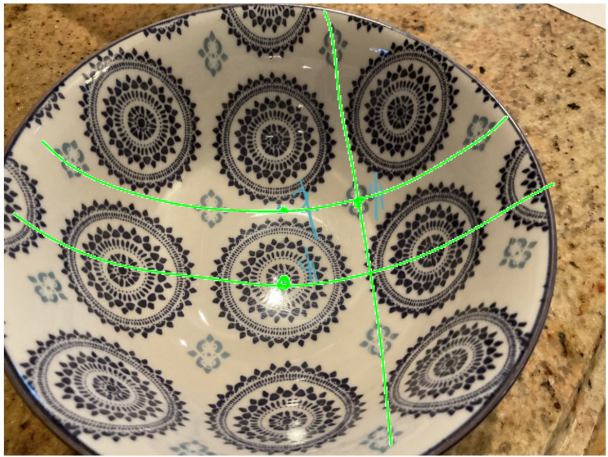
1×1 11
2×1 12
2×2 22

no rotations.
no reflections.

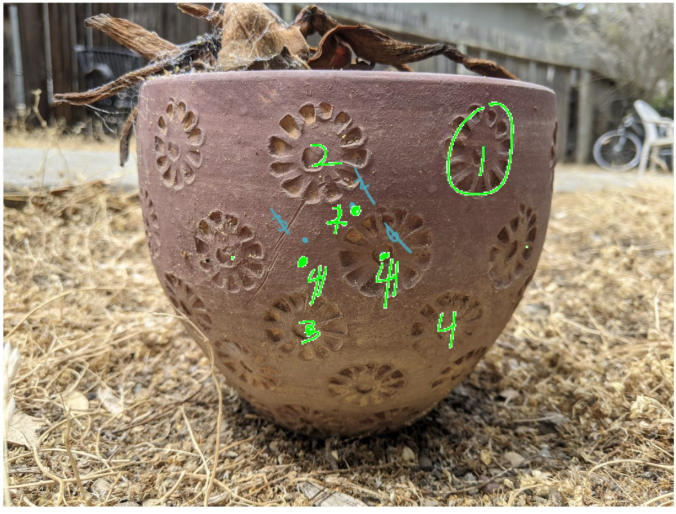
$\emptyset$

Janelle-oo-floor.jpg


333
Maggie-333-BrokenSierpinsky.jpg



Maggie-442-BowlPattern.jpg

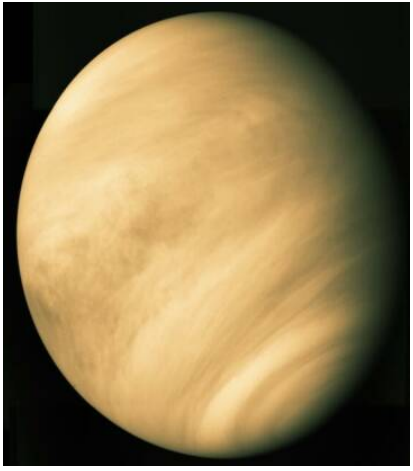


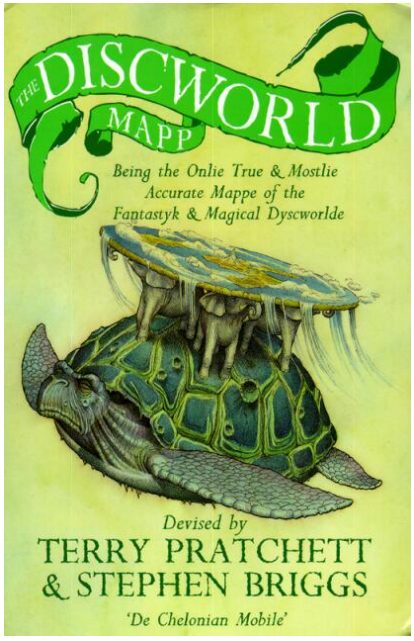
Maggie-632-JLo'sPot.jpg

Better vaxxed!

Maggie-GG-JLo'sBag.jpg

What if we lived on Venus?



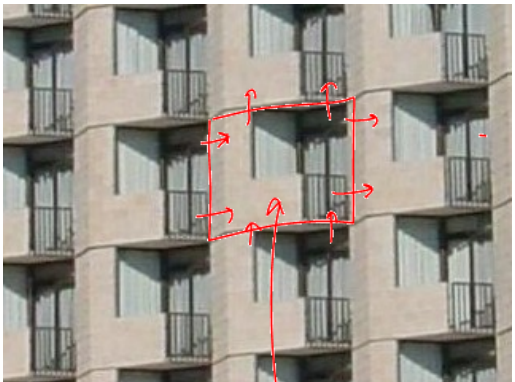

THE DISCWORLD MAPP
Being the Onlie True & Mostlie Accurate Mappe of the Fantastyk & Magical Dyscworlde
Devised by
TERRY PRATCHETT & STEPHEN BRIGGS
'De Chelonian Mobile'

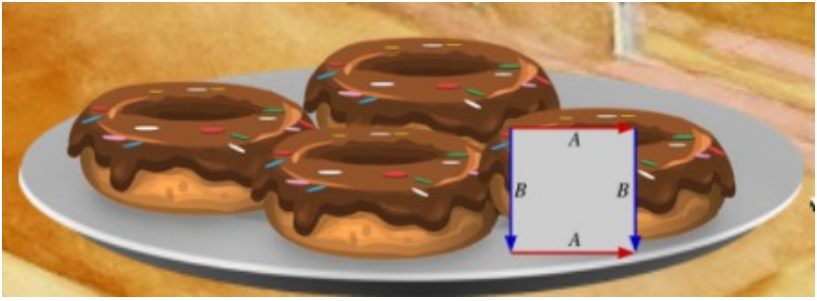
Better vaxxed!

Better vaxxed!

Better vaxxed!

The Jerusalem Renaissance Hotel

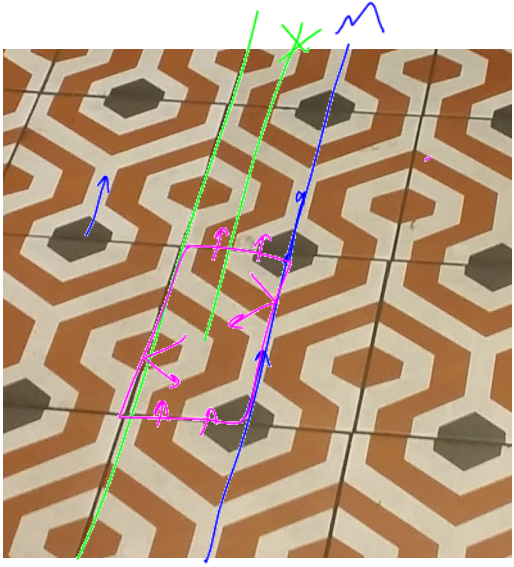

Fundamental
domain



Doughnuts stolen from the MathCamp Kitchen

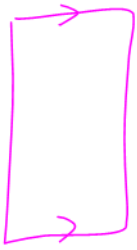
A truck's tire inner tube and Itai

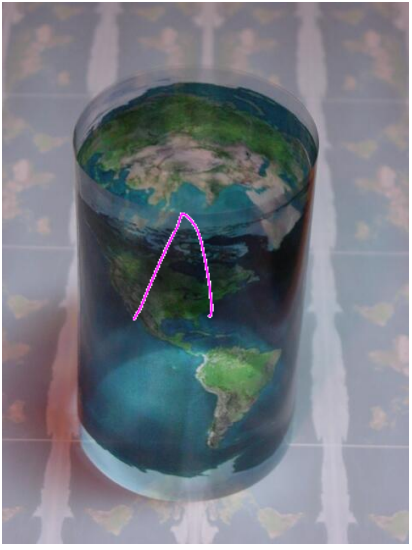
Floor of La Tortilleria, Toronto 2018



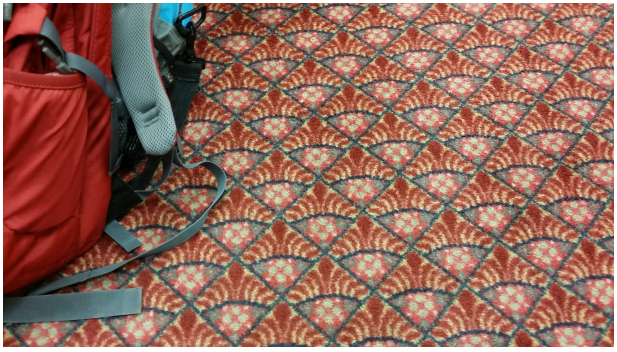
respect colour

MM

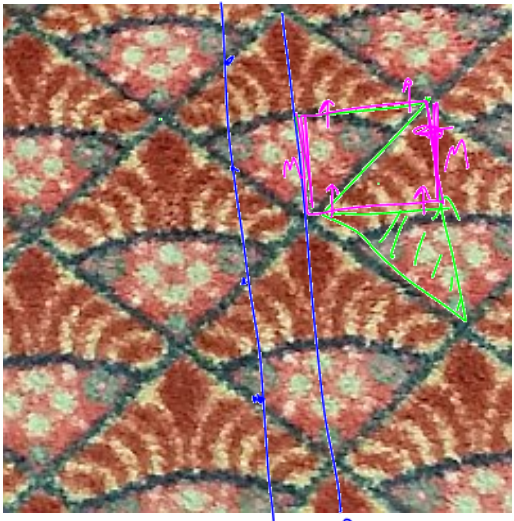


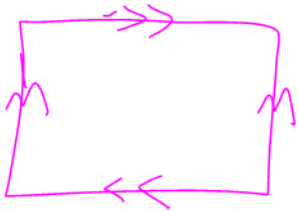


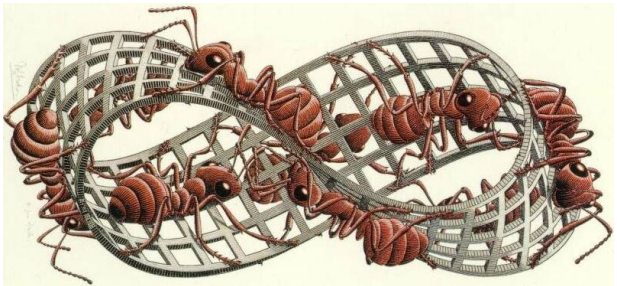
Earth on a cylinder



A carpet seen at Indiana University, 2016

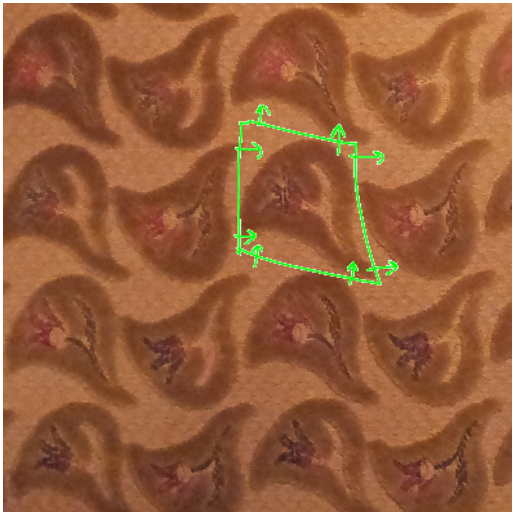

G
M



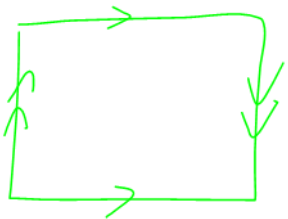


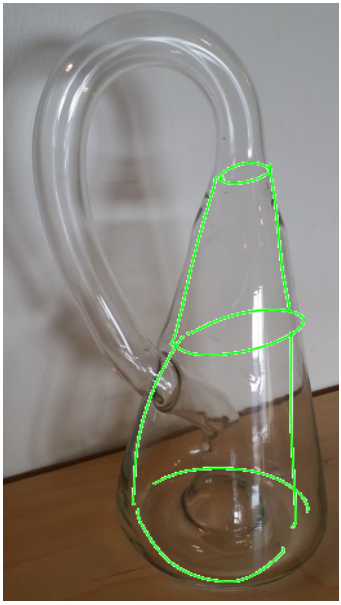
A Möbius band by M. C. Escher

A living room sofa at the Karshon's, 2012



GG





A Klein bottle from https://www.kleinbottle.com/

A bed cover and Annie, 2000

a $90°90°90°90°$ pillow

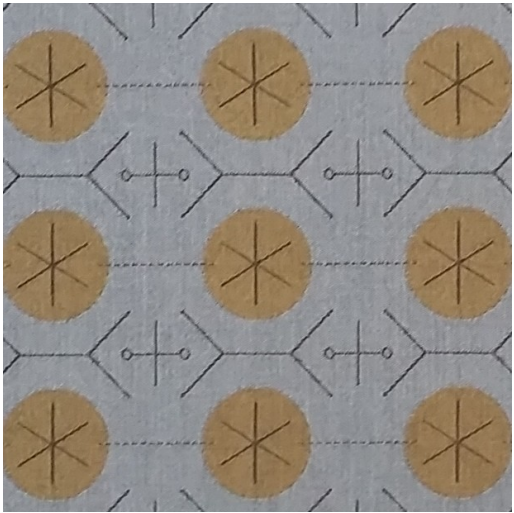
This one earned Angela Wu C$50 a few years ago

a $60^\circ 60^\circ 60^\circ$ Hamantashan

Similarly, 442 $\rightarrow$ a $90^\circ 45^\circ 45^\circ$ pillow or cookie, and 632 $\rightarrow$ a $90^\circ 60^\circ 30^\circ$ pillow or cookie.

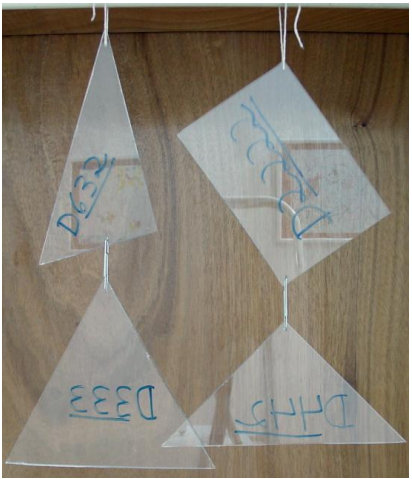
Wallpaper at Bridgehead Coffee in Ottawa

Better vaxxed!



a $90°90°90°90°$ rectangle

Similarly, 442 $\rightarrow$ a $90°45°45°$ triangle, 333 $\rightarrow$ a $60°60°60°$ triangle, and 632 $\rightarrow$ a $90°60°30°$ triangle.



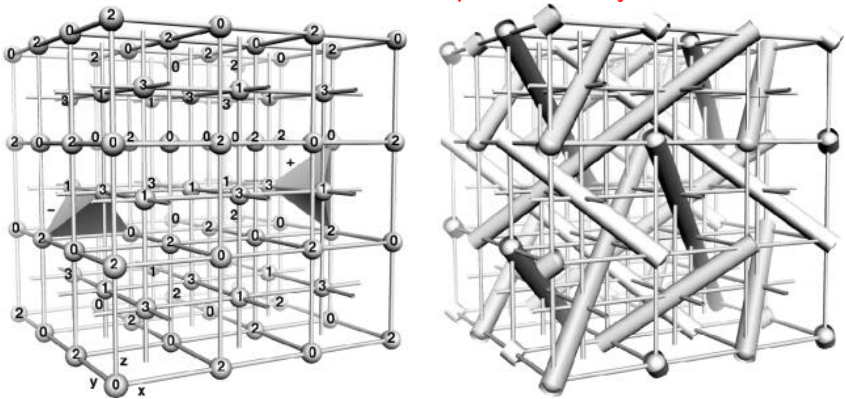
The powder room at the Kuperberg-Zieve's

A plastic bag

Homework.

What about **42**, **33**, **222**, and **22G**?

The 230 Worlds of Spacial Monkeys



The 219 Worlds of Monkeys that Can't Tell Left From Right

(Numbers and pictures from arXiv:math/9911185 by Conway, Friedrichs, Huson, and Thurston; see also http://webmineral.com/crystall.shtml)

Thank You!