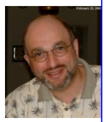
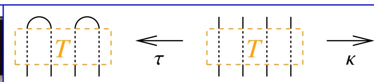
A Poly-Time Knot Polynomial Via Solvable Approximation

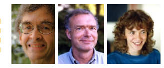
Work in Progress! Fluid! Help Needed!

Abstract. Rozansky [Ro2] and Overbay [Ov] described a spectacular knot polynomial that failed to attract the attention it deserved as the first poly-time-computable knot polynomial since Alexander's [Al, 1928] and (in my opinion) as the second most likely knot polynomial (after Alexander's) to carry topological information. With Roland van der Veen, I will explain how to compute the Rozansky polynomial using some new commutator-calculus techniques and a Lie algebra $\mathfrak{g}_1$ which is at the same time solvable and an approximation of the simple Lie algebra sl_2.


Rozansky


Overbay

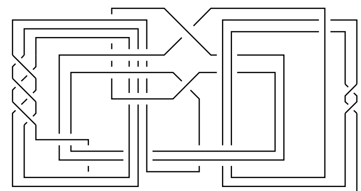



Gompf, Scharlemann, Thompson [GST]

$U \in \mathcal{T}_n \xrightarrow{\tau} \mathcal{T}_{2n}$, $1 \in \mathcal{A}_n \xrightarrow{\tau} \mathcal{A}_{2n}$, $\mathcal{T}_{2n} \xrightarrow{z} \mathcal{A}_{2n}$, with $\mathcal{R} := \kappa(\tau^{-1}(1))$

ribbon $K \in \mathcal{T}_1$ $\quad z(K) \in \mathcal{R} \subseteq \mathcal{A}_1$

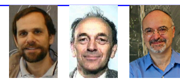
Faster is better, leaner is meaner!



$A^+ = -t^8 + 2t^7 - t^6 - 2t^4 + 5t^3 - 2t^2 - 7t + 13$

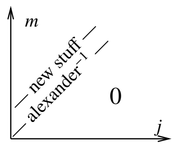
$\rho_1^+ = 5t^{15} - 18t^{14} + 33t^{13} - 32t^{12} + 2t^{11} + 42t^{10} - 62t^9 - 8t^8 + 166t^7 - 242t^6 + 108t^5 + 132t^4 - 226t^3 + 148t^2 - 11t - 36$

Theorem ([BNG], conjectured [MM], elucidated [Ro1]). Let $J_d(K)$ be the coloured Jones polynomial of K, in the d-dimensional representation of sl_2. Writing


Melvin, Morton, Garoufalidis

$$\left.\frac{(q^{1/2} - q^{-1/2})J_d(K)}{q^{d/2} - q^{-d/2}}\right|_{q=e^\hbar} = \sum_{j,m\geq 0} a_{jm}(K) d^j \hbar^m,$$

"below diagonal" coefficients vanish, $a_{jm}(K) = 0$ if $j > m$, and "on diagonal" coefficients give the inverse of the Alexander polynomial: $\left(\sum_{m=0}^\infty a_{mm}(K)\hbar^m\right) \cdot A(K)(e^\hbar) = 1$.



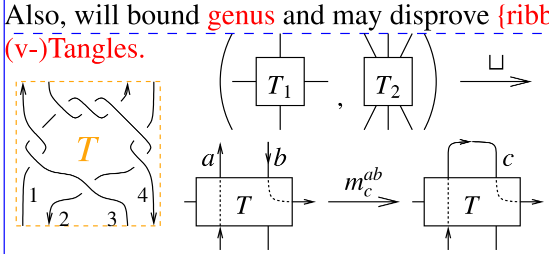
"Above diagonal" we have Rozansky's Theorem [Ro3, (1.2)]:

$$J_d(K)(q) = \frac{q^d - q^{-d}}{(q - q^{-1})A(K)(q^d)}\left(1 + \sum_{k=1}^\infty \frac{(q-1)^k R_k(K)(q^d)}{A^{2k}(K)(q^d)}\right).$$

Why "spectacular"? Foremost reason: *OBVIOUSLY.* Cf. proving (incomputable A)=(incomputable B), or categorifying (incomputable C).

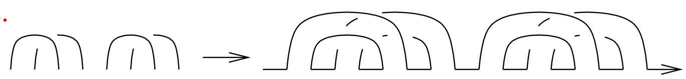
Also, will bound genus and may disprove {ribbon} = {slice}.

(v-)Tangles.

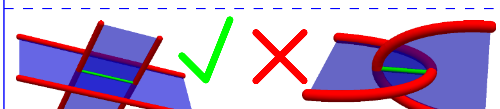


(meta-associativity: $m_x^{ab} /\!\!/ m_y^{xc} = m_x^{bc} /\!\!/ m_y^{ax}$) (tangles are generated by ↗↖ and ↖↗)

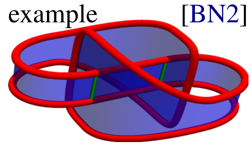
Genus.



example [BN2]



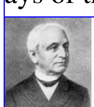
a ribbon singularity a clasp singularity



A bit about ribbon knots. A "ribbon knot" is a knot that can be presented as the boundary of a disk that has "ribbon singularities", but no "clasp singularities". A "slice knot" is a knot in $S^3 = \partial B^4$ which is the boundary of a non-singular disk in B^4. Every ribbon knots is clearly slice, yet,

Conjecture. Some slice knots are not ribbon.

Fox-Milnor. The Alexander polynomial of a ribbon knot is always of the form $A(t) = f(t)f(1/t)$. (also for slice)



"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

www.katlas.org

The Gold Standard is set by the "Γ-calculus" Alexander formulas [BNS, BN1]. An S-component tangle T has $\Gamma(T) \in R_S \times M_{S\times S}(R_S) = \left\{\begin{array}{c|c} \omega & S \\ \hline S & A\end{array}\right\}$ with $R_S := \mathbb{Z}(\{t_a \colon a \in S\})$:

$$\left({}_a\nearrow\!\!\nwarrow_b, {}_b\nwarrow\!\!\nearrow_a\right) \to \begin{array}{c|cc} 1 & a & b \\ \hline a & 1 & 1 - t_a^{\pm 1} \\ b & 0 & t_a^{\pm 1}\end{array} \qquad T_1 \sqcup T_2 \to \begin{array}{c|cc} \omega_1\omega_2 & S_1 & S_2 \\ \hline S_1 & A_1 & 0 \\ S_2 & 0 & A_2\end{array}$$

$$\begin{array}{c|ccc} \omega & a & b & S \\ \hline a & \alpha & \beta & \theta \\ b & \gamma & \delta & \epsilon \\ S & \phi & \psi & \Xi\end{array} \xrightarrow[t_a, t_b \to t_c]{m_c^{ab}} \left(\begin{array}{c|cc} (1-\beta)\omega & c & S \\ \hline c & \gamma + \frac{\alpha\delta}{1-\beta} & \epsilon + \frac{\delta\theta}{1-\beta} \\ S & \phi + \frac{\alpha\psi}{1-\beta} & \Xi + \frac{\psi\theta}{1-\beta}\end{array}\right)$$

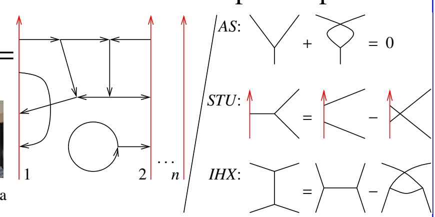
(Roland: "add to A the product of column b and row a, divide by $(1 - A_{ab})$, delete column b and row a".)

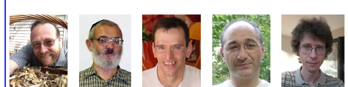
For long knots, ω is Alexander, and that's the fastest Alexander algorithm I know! Dunfield: 1000-crossing fast.

(There are also formulas for strand doubling and strand reversal).

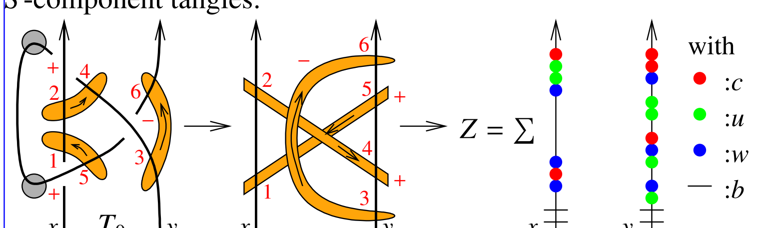
Theorem [EK, Ha, En, Se]. There is a "homomorphic expansion"

$$Z\colon \left\{\begin{array}{c} S\text{-component} \\ (v/b\text{-})\text{tangles}\end{array}\right\} \to \mathcal{A}_S^v :=$$




Etingof Kazhdan Haviv Enriquez Ševera

Algebras and Invariants. Given any unital algebra A (even better if A is Hopf; typically, $A \sim \hat{\mathcal{U}}(\mathfrak{g})$), appropriate orange $R \in A \otimes A$, and appropriate cuaps $\in A$, get an $A^{\otimes S}$-valued invariant of pure S-component tangles:



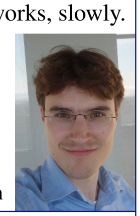
Good News. In theory, enough to know R, the cuaps, and stitching/multiplication $m_k^{ij}\colon A_i \otimes A_j \to A_k$.

Problem. Extract information out of Z.

Textbook Solution. Use representation theory ... works, slowly.

Today's Solution (with van der Veen). For some specific $\mathfrak{g}$'s, work in a space of "formulas of a specific type" for elements of $\hat{\mathcal{U}}(\mathfrak{g})^{\otimes S}$:

$$\left\{\begin{array}{c} \text{ordered perturbed} \\ \text{Gaussian formulas}\end{array}\right\} \to \hat{\mathcal{U}}(\mathfrak{g})^{\otimes S}$$


van der Veen

1-Smidgen sl_2 Let $\mathfrak{g}_1$ be the 4-dimensional Lie algebra $\mathfrak{g}_1 = \langle b, c, u, w\rangle$ over the ring $R = \mathbb{Q}[\epsilon]/(\epsilon^2 = 0)$, with b central and with $[w, c] = w$, $[c, u] = u$, and $[u, w] = b - 2\epsilon c$, with CYBE $r_{ij} = (b_i - \epsilon c_i)c_j + u_i w_j$ in $\mathcal{U}(\mathfrak{g}_1)^{\otimes\{i,j\}}$. Over $\mathbb{Q}$, $\mathfrak{g}_1$ is a solvable approximation of sl_2: $\mathfrak{g}_1 \supset \langle b, u, w, \epsilon b, \epsilon c, \epsilon u, \epsilon w\rangle \supset \langle b, \epsilon b, \epsilon c, \epsilon u, \epsilon w\rangle \supset 0$. (note: $\deg(b, c, u, w, \epsilon) = (1, 0, 1, 0, 1)$)

0-Smidgen sl_2 ☺. Let $\mathfrak{g}_0$ be $\mathfrak{g}_1$ at $\epsilon = 0$, or $\mathbb{Q}\langle b, c, u, w\rangle/([b, \cdot] = 0,\ [c, u] = u,\ [c, w] = -w,\ [u, w] = b$ with $r_{ij} = b_i c_j + u_i w_j$. It is $\mathfrak{b}^* \rtimes \mathfrak{b}$ where $\mathfrak{b}$ is the 2D Lie algebra $\mathbb{Q}\langle c, w\rangle$ and (b, u) is the dual basis of (c, w). For topology, it is more valuable than $\mathfrak{g}_1$ / sl_2, but topology already got by other means almost everything $\mathfrak{g}_0$ gives.

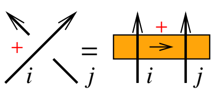
How did these arise? $sl_2 = \mathfrak{b}^+ \oplus \mathfrak{b}^-/\mathfrak{h} =: sl_2^+/\mathfrak{h}$, where $\mathfrak{b}^+ = \langle c, w\rangle/[w, c] = w$ is a Lie bialgebra with $\delta\colon \mathfrak{b}^+ \to \mathfrak{b}^+ \otimes \mathfrak{b}^+$ by $\delta\colon (c, w) \mapsto (0, c \wedge w)$. Going back, $sl_2^+ = \mathcal{D}(\mathfrak{b}^+) = (\mathfrak{b}^+)^* \oplus \mathfrak{b}^+ = \langle b, u, c, w\rangle/\cdots$. **Idea.** Replace $\delta \to \epsilon\delta$ over $\mathbb{Q}[\epsilon]/(\epsilon^{k+1} = 0)$. At $k = 0$, get $\mathfrak{g}_0$. At $k = 1$, get $[w, c] = w$, $[w, b'] = -\epsilon w$, $[c, u] = u$, $[b', u] = -\epsilon u$, $[b', c] = 0$, and $[u, w] = b' - \epsilon c$. Now note that $b' + \epsilon c$ is central, so switch to $b := b' + \epsilon c$. This is $\mathfrak{g}_1$.

Ordering Symbols. $\mathbb{O}\,(poly \mid specs)$ plants the variables of $poly$ in $\mathcal{S}(\oplus_i \mathfrak{g})$ on several tensor copies of $\mathcal{U}(\mathfrak{g})$ according to $specs$. E.g.,

$$\mathbb{O}\left(c_1^3 u_1 c_2 e^{u_3} w_3^9 | x\colon w_3 c_1,\ y\colon u_1 u_3 c_2\right) = w^9 c^3 \otimes u e^u c \in \mathcal{U}(\mathfrak{g})_x \otimes \mathcal{U}(\mathfrak{g})_y$$

This enables the description of elements of $\hat{\mathcal{U}}(\mathfrak{g})^{\otimes S}$ using commutative polynomials / power series.

0-Smidgen Invariants. $r = Id \in \mathfrak{b}^- \otimes \mathfrak{b}^+$ solves the CYBE $[r_{12}, r_{13}] + [r_{12}, r_{23}] + [r_{13}, r_{23}] = 0$ in $\mathcal{U}(\mathfrak{g}_0)^{\otimes 3}$ and, by luck,



$= R_{ij} = e^{r_{ij}} = e^{b_i c_j + u_i w_j} \in \mathcal{U}(\mathfrak{g}_{0,i} \oplus \mathfrak{g}_{0,j})$ solves YB/R3.

Lemma. $R_{ij} = e^{b_i c_j + u_i w_j} = \mathbb{O}\left(\exp\left(b_i c_j + \frac{e^{b_i} - 1}{b_i} u_i w_j\right) \middle| i\colon u_i,\ j\colon c_j w_j\right)$

Example. $Z(T_0) =$ $= \sum_{m,n} \frac{b_i^{m-n}(e^{b_i} - 1)^n}{m!n!} u^n \otimes c^m w^n$.

$$\mathbb{O}\left(\exp\left(b_5 c_1 + \frac{e^{b_5} - 1}{b_5} u_5 w_1 + b_2 c_4 + \frac{e^{b_2} - 1}{b_2} u_2 w_4 - b_3 c_6 + \frac{e^{-b_3} - 1}{b_3} u_3 w_6\right) \middle| x\colon c_1 w_1 u_2,\ y\colon u_3 c_4 w_4 u_5 c_6 w_6\right) = \mathbb{O}\left(\zeta | x\colon u_x c_x w_x,\ y\colon u_y c_y w_y\right)$$

"ucw form"

Goal. Write ζ as a Gaussian: ωe^{L+Q} where L bilinear in b_i and c_i with integer coefficients, Q a balanced quadratic in u_i and w_i with coefficients in $R_S := \mathbb{Q}(b_i, e^{b_i})$, and $\omega \in R_S$.

The Big $\mathfrak{g}_0$ Lemma. Under $[c, u] = u$, $[c, w] = -w$, and $[u, w] = b$:

1a. $N^{cu} := \mathbb{O}(e^{\gamma c + \beta u}|uc) \overset{\rightarrow}{=} \mathbb{O}(e^{\gamma c + e^{\gamma}\beta u}|cu)$ (means $e^{\beta u}e^{\gamma c} = e^{\gamma c}e^{e^{\gamma}\beta u}$)

1b. $N^{wc} := \mathbb{O}(e^{\gamma c + \alpha w}|wc) \overset{\rightarrow}{=} \mathbb{O}(e^{\gamma c + e^{\gamma}\alpha w}|cw)$ ...in the $\{ax + b\}$ group)

2. $\mathbb{O}(e^{\alpha w + \beta u}|wu) = \mathbb{O}(e^{-b\alpha\beta + \alpha w + \beta u}|uw)$ (the Weyl relations)

3. $\mathbb{O}(e^{\delta uw}|wu)e^{\beta u} = e^{\nu\beta u}\mathbb{O}(e^{\delta uw}|wu)$, with $\nu = (1 + b\delta)^{-1}$

(a. expand and crunch. b. use $w = b\hat{x}$, $u = \partial_x$. c. use "scatter and glow".)

4. $\mathbb{O}(e^{\delta uw}|wu) = \mathbb{O}(\nu e^{\nu\delta uw}|uw)$ (same techniques)

5. $N^{wu} := \mathbb{O}(e^{\beta u + \alpha w + \delta uw}|wu) \overset{\rightarrow}{=} \mathbb{O}(\nu e^{-b\nu\alpha\beta + \nu\alpha w + \nu\beta u + \nu\delta uw}|uw)$

6. $N_k^{c_i c_j} := \mathbb{O}(\zeta|c_i c_j) \overset{\rightarrow}{=} \mathbb{O}(\zeta/(c_i, c_j \to c_k)|c_k)$

Sneaky. α may contain (other) u's, β may contain (other) w's.

Strand Stitching, m_k^{ij}, is defined as the composition

$$u_i c_i \overline{w_i u_j} c_j w_j \xrightarrow{N_x^{w_i u_j}} u_i \overline{c_i u_x}\, \overline{w_x c_j} w_j \xrightarrow{N_x^{c_i u_x} /\!/ N_x^{w_x c_j}} \overline{u_i u_x}\, \overline{c_x c_x}\, \overline{w_x w_j} \xrightarrow{i,j,x \to k} u_k c_k w_k$$

On to 1-smidgen invariants, where much is the same...

The Big $\mathfrak{g}_1$ Lemma. Parts 1 and 6 are the same, yet

5. $\mathbb{O}\left(e^{\alpha w + \beta u + \delta uw}|wu\right) = \mathbb{O}\left(\nu(1 + \epsilon\nu\Lambda)e^{\nu(-b\alpha\beta + \alpha w + \beta u + \delta uw)}|ucw\right)$

Here Λ is for Λόγος, "a principle of order and knowledge", a balanced quartic in α, β, u, c, and w:

$$\begin{aligned}\Lambda = &- b\nu(\alpha^2\beta^2\nu^2 + 4\alpha\beta\delta\nu + 2\delta^2)/2 + \beta^2\delta\nu^3(b\delta + 2)u^2/2 \\ &+ \delta^3\nu^3(3b\delta + 4)u^2w^2/2 + \beta\delta^2\nu^3(2b\delta + 3)u^2w \\ &+ \alpha\delta^2\nu^3(2b\delta + 3)uw^2 + 2\delta\nu^2(b\delta + 2)(\alpha\beta\nu + \delta)uw \\ &+ \alpha^2\delta\nu^3(b\delta + 2)w^2/2 + 2(\alpha\beta\nu + \delta)c + 2\beta\delta\nu uc + 2\delta^2\nu ucw \\ &+ 2\alpha\delta\nu cw + \beta\nu^2(\alpha\beta\nu + 2\delta)u + \alpha\nu^2(\alpha\beta\nu + 2\delta)w.\end{aligned}$$

Proof. A lengthy computation. (Verification: ωεβ/Big)

Problem. We now need to normal-order perturbed Gaussians!

Solution. Borrow some tactics from QFT:

$$\mathbb{O}(\epsilon P(c, u)e^{\gamma c + \beta u}|uc) = \mathbb{O}(\epsilon P(\partial_\gamma, \partial_\beta)e^{\gamma c + \beta u}|uc) = \mathbb{O}(\epsilon P(\partial_\gamma, \partial_\beta)e^{\gamma c + e^{-\gamma}\beta u}|cu),$$

and likewise

$$\mathbb{O}\left(\epsilon P(u, w)e^{\alpha w + \beta u + \delta uw}|wu\right) = \mathbb{O}\left(\epsilon P(\partial_\beta, \partial_\alpha)\nu e^{\nu(-b\alpha\beta + \alpha w + \beta u + \delta uw)}|ucw\right)$$

Finally, the values of the generators ⤯, ⤮, $\overrightarrow{n}$, and $\underset{\rightarrow}{u}$, are set by solving many equations, non-uniquely.

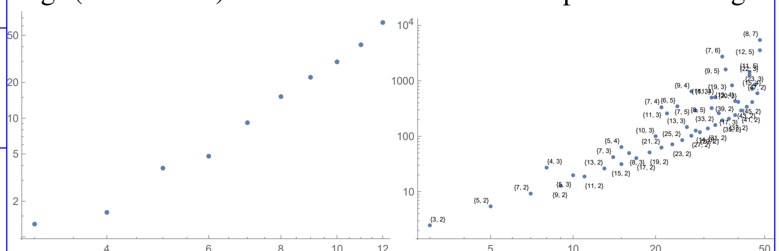
Pragmatic Simplifications. Set $t := e^b$, work with $v := (t - 1)u/b$, and set $\mathbb{E}(\omega, L, Q, P) := \mathbb{O}\left(\omega^{-1}e^{L + Q/\omega}(1 + \epsilon\omega^{-4}P)\colon (i\colon v_i c_i w_i)\right)$. Now $\omega \in R_S := \mathbb{Z}[t_i, t_i^{-1}]$ is Laurent, $L = \sum l_{ij}\log(t_i)c_j$ with $l_{ij} \in \mathbb{Z}$, $Q = \sum q_{ij}v_i w_j$ with $q_{ij} \in R_S$, and P is a quartic polynomial in v_i, c_j, w_k with coefficients in R_S. The operations are lightly modified, and the Λόγος and the values of the generators become somewhat simpler, as in the implementation below.

Rough complexity estimate, after $t_k \to t$. n: xing number; w: width, maybe $\sim \sqrt{n}$.

$$\underbrace{n}_{A}\overbrace{\sum_{d=0}^{4}\underbrace{w^{4-d}}_{E}\underbrace{w^d}_{F}\underbrace{n^2}_{G}}^{B} = n^3w^4 \in [n^5, n^7]$$

A: go over stitchings in order. B: multiplication ops per $N^{u_i w_j}$. d: deg of u_i, w_j in P. E: #terms of deg d in P. F: ops per term. G: cost per polynomial multiplication op.

Experimental Analysis (ωεβ/Exp). Log-log plots of computation time (sec) vs. crossing number, for all knots with up to 12 crossings (mean times) and for all torus knots with up to 48 crossings:



Conjecture (checked on the same collections). Given a knot K with Alexander polynomial A, there is a polynomial ρ_1 such that

$$P = A^2\frac{(t - 1)^3\rho_1 + t^2(2vw + (1 - t)(1 - 2c))AA'}{(1 - t)t}.$$

Furthermore, A and ρ_1 are symmetric under $t \to t^{-1}$, so let A^+ and ρ_1^+ be their "positive parts", so e.g., $\rho_1(t) = \rho_1^+(t) + \rho_1^+(t^{-1}) - \rho_1^+(0)$.

Power. On the 250 knots with at most 10 crossings, the pair (A, ρ_1) attains 250 distinct values, while (Khovanov, HOMFLY-PT) attains only 249 distinct values. To 11 crossings the numbers are (802, 788, 772) and to 12 they are (2978, 2883, 2786).

Genus. Up to 12 xings, always $\deg\rho_1^+ \leq 2g - 1$, where g is the 3-genus of K (equallity for 2530 knots). This gives a lower bound on g in terms of ρ_1 (conjectural, but undoubtedly true). This bound is often weaker than the Alexander bound, yet for 10 of the 12-xing Alexander failures it does give the right answer.

Demo Programs for 0-Co. ωεβ/Demo

The R-matrices

$$R^+_{0,i\_,j\_} := \mathbb{E}\left[b_i c_j + b_i^{-1}\left(e^{b_i}-1\right) u_i w_j\right];$$
$$R^-_{0,i\_,j\_} := \mathbb{E}\left[-b_i c_j + b_i^{-1}\left(e^{-b_i}-1\right) u_i w_j\right];$$

Utilities

$$\mathrm{CF}[\omega\_ .\ \mathbb{E}[Q\_]] := \mathrm{Simplify}[\omega]\ \mathbb{E}[\mathrm{Simplify}[Q]];$$
$$\mathbb{E}\ /:\ \mathbb{E}[Q1\_]\ \mathbb{E}[Q2\_] := \mathrm{CF}@\mathbb{E}[Q1+Q2];$$
$$\omega1\_ .\ \mathbb{E}[Q1\_] \equiv \omega2\_ .\ \mathbb{E}[Q2\_] := \mathrm{Simplify}[\omega1 == \omega2 \land Q1 == Q2];$$

Normal Ordering Operators

$$N_{(x:w|u)_{i\_}\ c_{j\_}\to k\_}[\omega\_ .\ \mathbb{E}[Q\_]] := \mathrm{CF}\Big[\omega\,\mathbb{E}[e^{\gamma}\alpha x_k + \gamma c_k + (Q\ /.\ c_j\ |\ x_i \to 0)]\ /.\ \{\gamma \to \partial_{c_j}Q,\ \alpha \to \partial_{x_i}Q\}\Big];$$
$$N_{w_{i\_}\ u_{j\_}\to k\_}[\omega\_ .\ \mathbb{E}[Q\_]] := \mathrm{CF}\Big[\nu\,\omega\,\mathbb{E}[-b_k\nu\alpha\beta + \nu\beta u_k + \nu\alpha w_k + \nu\delta u_k w_k + (Q\ /.\ w_i\ |\ u_j \to 0)]\ /.\ \nu \to (1+b_k\delta)^{-1}\ /.$$
$$\{\alpha \to \partial_{w_i}Q\ /.\ u_j\to 0,\ \beta \to \partial_{u_j}Q\ /.\ w_i \to 0,\ \delta \to \partial_{w_i,u_j}Q\}\Big];$$

Stitching

$$m_{i\_,j\_\to k\_}[Z\_] := \mathrm{Module}\Big[\{x, z\},\ \mathrm{CF}\Big[\left(Z\ //\ N_{w_i u_j \to x}\ //\ N_{c_i u_x \to x}\ //\ N_{w_x c_j \to x}\right)\ /.\ z\_{i|j|x} \to z_k\Big]\Big]$$

Some calculations for T_0

$$T_0 = R^+_{0,5,1}\, R^+_{0,2,4}\, R^-_{0,3,6}$$
$$\mathbb{E}\left[b_5 c_1 + b_2 c_4 - b_3 c_6 + \frac{(-1+e^{b_5})\,u_5 w_1}{b_5} + \frac{(-1+e^{b_2})\,u_2 w_4}{b_2} + \frac{(-1+e^{-b_3})\,u_3 w_6}{b_3}\right]$$
$$T_0\ //\ m_{1,2\to1}\ //\ m_{3,4\to3}\ //\ m_{3,5\to3}\ //\ m_{3,6\to3}$$
$$\frac{1}{1-(-1+e^{b_1})(-1+e^{b_3})}\ \mathbb{E}\Big[b_3 c_1 + b_1 c_3 - b_3 c_3 + \frac{e^{b_3}(-1+e^{b_1})(-1+e^{b_3})\,u_1 w_1}{(-e^{b_1}-e^{b_3}+e^{b_1+b_3})\,b_1} - \frac{e^{b_1}(-1+e^{b_3})\,u_3 w_1}{(-1+(-1+e^{b_1})(-1+e^{b_3}))\,b_3} -$$
$$\frac{e^{-b_3}(-1+e^{b_3})\,u_3 w_3}{b_3} - \frac{e^{-b_3}(-1+e^{b_1})\left(-e^{b_3} b_3 u_1 + e^{b_1}(-1+e^{b_3})\,b_1 u_3\right) w_3}{b_1\left(b_3 - (-1+e^{b_1})(-1+e^{b_3})\,b_3\right)}\Big]$$

Verifying meta-associativity

$$Q0 = \mathbb{E}[\mathrm{Sum}[f_i c_i, \{i,3\}] + \mathrm{Sum}[f_{i,j} u_i w_j, \{i,3\}, \{j,3\}]]$$
$$\mathbb{E}[c_1 f_1 + c_2 f_2 + c_3 f_3 + u_1 w_1 f_{1,1} + u_1 w_2 f_{1,2} + u_1 w_3 f_{1,3} + u_2 w_1 f_{2,1} + u_2 w_2 f_{2,2} + u_2 w_3 f_{2,3} + u_3 w_1 f_{3,1} + u_3 w_2 f_{3,2} + u_3 w_3 f_{3,3}]$$
$$(Q0\ //\ m_{1,2\to1}\ //\ m_{1,3\to1}) \equiv (Q0\ //\ m_{2,3\to2}\ //\ m_{1,2\to1})$$

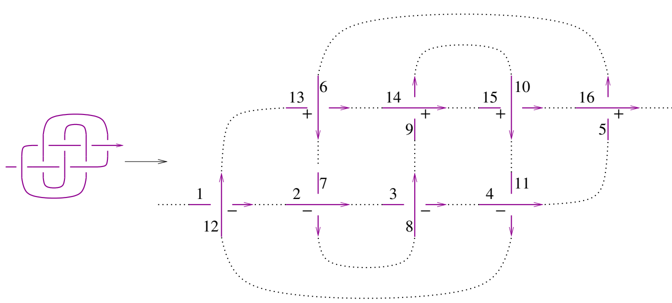
True

Testing R3

$$t1 = R^+_{0,1,2}\, R^+_{0,3,4}\, R^+_{0,5,6}\ //\ m_{3,5\to x}\ //\ m_{1,6\to y}\ //\ m_{2,4\to z}$$
$$\mathbb{E}\left[b_x c_y + b_x c_z + b_y c_z + \frac{e^{b_x}(-1+e^{b_y})\,u_y w_z}{b_y} + \frac{(-1+e^{b_x})\,u_x (w_y+w_z)}{b_x}\right]$$
$$t1 \equiv (R^+_{0,1,2}\, R^+_{0,3,4}\, R^+_{0,5,6}\ //\ m_{1,3\to x}\ //\ m_{2,5\to y}\ //\ m_{4,6\to z})$$

True

8_{17}



$$z1 = R^-_{0,12,1}\, R^-_{0,2,7}\, R^-_{0,8,3}\, R^-_{0,4,11}\, R^+_{0,16,5}\, R^+_{0,6,13}\, R^+_{0,14,9}\, R^+_{0,10,15};$$
$$\mathrm{Do}[z1 = (z1\ //\ m_{1,n\to1})\ /.\ b\_ \to b,\ \{n, 2, 16\}];$$
$$\{\mathrm{CF}@z1,\ \mathrm{KnotData}[\{8,17\}, \text{"AlexanderPolynomial"}][t]\}$$
$$\left\{-\frac{e^{3b}\,\mathbb{E}[0]}{1-4e^{b}+8e^{2b}-11e^{3b}+8e^{4b}-4e^{5b}+e^{6b}},\ 11-\frac{1}{t^3}+\frac{4}{t^2}-\frac{8}{t}-8t+4t^2-t^3\right\}$$

Demo Programs for 1-Co. ωεβ/Demo

The Λόγος

$$\Lambda[k\_] := \Big((t_k-1)\left(2(\alpha\beta+\delta\mu)^2-\alpha^2\beta^2\right) - 4 v_k c_k w_k \delta^2\mu^2 - \delta(1+\mu)\left(w_k^2\alpha^2+v_k^2\beta^2\right) - v_k^2 w_k^2 \delta^3 (1+3\mu) -$$
$$2\left(\alpha\beta + 2\delta\mu + v_k w_k \delta^2 (1+2\mu) + 2 c_k \delta \mu^2\right)(w_k\alpha + v_k\beta) - 4\left(c_k\mu^2 + v_k w_k \delta(1+\mu)\right)(\alpha\beta+\delta\mu)\Big)(1+t_k)/4;$$

The Generators

$$R^+_{i\_,j\_} := \mathbb{E}\left[1,\ \mathrm{Log}[t_i]\,c_j,\ v_i w_j,\ v_i c_i w_j + c_i c_j + v_i^2 w_j^2/4\right];$$
$$R^-_{i\_,j\_} := \mathbb{E}\left[1,\ -\mathrm{Log}[t_i]\,c_j,\ -t_i^{-1} v_i w_j,\ t_i^{-1} v_i c_j w_j - c_i c_j - t_i^{-2} v_i^2 w_j^2/4\right];$$
$$\left(ur_{i\_} := \mathbb{E}\left[t_i^{-1/2}, 0, 0, c_i t_i^{-2}\right];\ nr_{i\_} := \mathbb{E}\left[t_i^{1/2}, 0, 0, -c_i t_i^{2}\right];\right)$$

Differential Polynomials

$$DP_{x\_\to D_{\alpha\_},\,y\_\to D_{\beta\_}}[P\_][f\_] := \text{(* means } P[\partial_\alpha,\partial_\beta][f] \text{ *)}$$
$$\mathrm{Total}[\mathrm{CoefficientRules}[P, \{x, y\}]\ /.\ (\{m\_, n\_\} \to c\_) :\to c\, D[f, \{\alpha, m\}, \{\beta, n\}]]$$

Utilities

$$\mathrm{CF}[\mathcal{E}\_\mathbb{E}] := \mathrm{Expand}\ /@\ \mathrm{Together}\ /@\ \mathcal{E};$$
$$\mathbb{E}\ /:\ \mathbb{E}[\omega1\_, L1\_, Q1\_, P1\_]\ \mathbb{E}[\omega2\_, L2\_, Q2\_, P2\_] := \mathrm{CF}@\mathbb{E}\left[\omega1\,\omega2,\ L1+L2,\ \omega2\,Q1+\omega1\,Q2,\ \omega2^4\,P1+\omega1^4\,P2\right];$$

Normal Ordering Operators

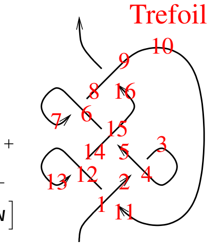
$$N_{c_{j\_}\ (x:v|w)_{i\_}\to k\_}[\mathbb{E}[\omega\_, L\_, Q\_, P\_]] := \mathrm{With}\Big[\{q = e^{\gamma}\beta x_k + \gamma c_k\},\ \mathrm{CF}\Big[\mathbb{E}\left[\omega,\ \gamma c_k + (L\ /.\ c_j \to 0),\ \omega e^{\gamma}\beta x_k + (Q\ /.\ x_i \to 0),\right.$$
$$\left. e^{-q}\, DP_{c_j\to D_\gamma,\,x_i\to D_\beta}[P][e^q]\right]\ /.\ \{\gamma \to \partial_{c_j}L,\ \beta \to \omega^{-1}\partial_{x_i}Q\}\Big]\Big];$$
$$N_{w_{i\_}\ v_{j\_}\to k\_}[\mathbb{E}[\omega\_, L\_, Q\_, P\_]] :=$$
$$\mathrm{With}\Big[\{q = ((1-t_k)\alpha\beta + \beta v_k + \alpha w_k + \delta v_k w_k)/\mu\},\ \mathrm{CF}\Big[\mathbb{E}\left[\mu\,\omega,\ L,\ \mu\,\omega\,q + \mu(Q\ /.\ w_i\ |\ v_j \to 0),\right.$$
$$\left.\mu^4 e^{-q}\, DP_{w_i\to D_\alpha,\,v_j\to D_\beta}[P][e^q] + \omega^4\Lambda[k]\right]\ /.\ \mu \to 1 + (t_k-1)\delta\ /.$$
$$\{\alpha \to \omega^{-1}(\partial_{w_i}Q\ /.\ v_j \to 0),\ \beta \to \omega^{-1}(\partial_{v_j}Q\ /.\ w_i \to 0),\ \delta \to \omega^{-1}\partial_{w_i,v_j}Q\}\Big]\Big];$$

Stitching

$$m_{i\_,j\_\to k\_}[Z\_\mathbb{E}] := \mathrm{Module}\Big[\{x, z\},\ \mathrm{CF}\Big[\left(Z\ //\ N_{w_i v_j \to x}\ //\ N_{c_i v_x \to x}\ //\ N_{w_x c_j \to x}\right)\ /.\ z\_{i|j|x} \to z_k\Big]\Big]$$

The 0-Framed Trefoil

$$z2 = R^+_{1,11}\, R^-_{4,2}\, nr_3\, R^+_{15,5}\, R^-_{6,8}\, ur_7\, R^+_{9,16}\, nr_{10}\, R^-_{12,14}\, ur_{13};$$
$$(\mathrm{Do}[z2 = z2\ //\ m_{1,k\to1},\ \{k, 2, 16\}];$$
$$z2 = z2\ /.\ a\_1 :\to a)$$



$$\mathbb{E}\Big[-1+\frac{1}{t}+t,\ 0,\ 0,\ 16+\frac{2c}{t^4}-\frac{1}{t^3}-\frac{6c}{t^3}+\frac{4}{t^2}+\frac{10c}{t^2}-\frac{10}{t}-\frac{8c}{t}-18t+8ct+14t^2-10ct^2-7t^3+6ct^3+2t^4-2ct^4+2vw-$$
$$\frac{2vw}{t^4}+\frac{4vw}{t^3}-\frac{6vw}{t^2}+\frac{2vw}{t}-6tvw+4t^2vw-2t^3vw\Big]$$

Questions and To Do List. • Clean up and write up. • Implement well, compute for everything in sight. • Why are our quantities polynomials rather than just rational functions? • Bounds on their degrees? • Their integrality ($\mathbb{Z}$) properties? • Can everything be re-stated using integrals ($\int$)? • Find the 2-variable version (for knots). How complex is it? • What about links / closed components? • Fully digest the "expansion" theorem; include cuaps. • Explore the (non-)dependence on R. • Is there a canonical R? • What does "group like" mean? • Strand removal? Strand doubling? Strand reversal? • Say something about knot genus. • Find the EK/AT/KV "vertex". • Use as a playground to study associators/braidors. • Restate in topological language. • Study the associated (v-)braid representations. • Study mirror images and the $\mathfrak{b}^+ \leftrightarrow \mathfrak{b}^-$ involution. • Study ribbon knots. • Make precise the relationship with Γ-calculus and Alexander. • Relate to the coloured Jones polynomial. • Relate with "ordinary" q-algebra. • k-smidgen sl_n, etc. • Are there "solvable" CYBE algebras not arising from semi-simple algebras? • Categorify and appease the Gods.

References.

[Al] J. W. Alexander, *Topological invariants of knots and link,* Trans. Amer. Math. Soc. **30** (1928) 275–306.

[BN1] D. Bar-Natan, *Balloons and Hoops and their Universal Finite Type Invariant, BF Theory, and an Ultimate Alexander Invariant,* ωεβ/KBH, arXiv:1308.1721.

[BN2] D. Bar-Natan, *Polynomial Time Knot Polynomial,* research proposal for the 2017 Killam Fellowship, ωεβ/K17.

[BNG] D. Bar-Natan and S. Garoufalidis, *On the Melvin-Morton-Rozansky conjecture,* Invent. Math. **125** (1996) 103–133.

[BNS] D. Bar-Natan and S. Selmani, *Meta-Monoids, Meta-Bicrossed Products, and the Alexander Polynomial,* J. of Knot Theory and its Ramifications **22-10** (2013), arXiv:1302.5689.

[En] B. Enriquez, *A Cohomological Construction of Quantization Functors of Lie Bialgebras,* Adv. in Math. **197-2** (2005) 430âĂŞ-479, arXiv:math/0212325.

[EK] P. Etingof and D. Kazhdan, *Quantization of Lie Bialgebras, I,* Selecta Mathematica **2** (1996) 1–41, arXiv:q-alg/9506005.

[GST] R. E. Gompf, M. Scharlemann, and A. Thompson, *Fibered Knots and Potential Counterexamples to the Property 2R and Slice-Ribbon Conjectures,* Geom. and Top. **14** (2010) 2305–2347, arXiv:1103.1601.

[Ha] A. Haviv, *Towards a diagrammatic analogue of the Reshetikhin-Turaev link invariants,* Hebrew University PhD thesis, Sep. 2002, arXiv:math.QA/0211031.

[MM] P. M. Melvin and H. R. Morton, *The coloured Jones function,* Commun. Math. Phys. **169** (1995) 501–520.

[Ov] A. Overbay, *Perturbative Expansion of the Colored Jones Polynomial,* University of North Carolina PhD thesis, ωεβ/Ov.

[Ro1] L. Rozansky, *A contribution of the trivial flat connection to the Jones polynomial and Witten's invariant of 3d manifolds, I,* Comm. Math. Phys. **175-2** (1996) 275–296, arXiv:hep-th/9401061.

[Ro2] L. Rozansky, *The Universal R-Matrix, Burau Representation and the Melvin-Morton Expansion of the Colored Jones Polynomial,* Adv. Math. **134-1** (1998) 1–31, arXiv:q-alg/9604005.

[Ro3] L. Rozansky, *A Universal U(1)-RCC Invariant of Links and Rationality Conjecture,* arXiv:math/0201139.

[Se] P. Ševera, *Quantization of Lie Bialgebras Revisited,* Sel. Math., NS, to appear, arXiv:1401.6164.

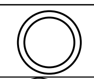
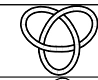
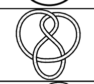
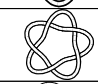
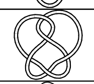
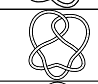
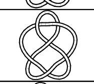
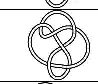
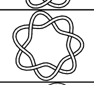
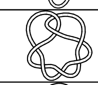
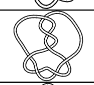
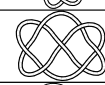
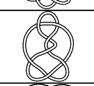
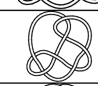
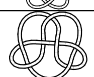
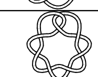
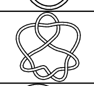
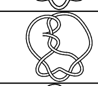
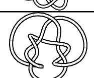
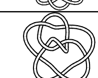
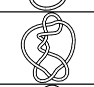
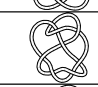
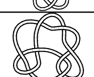
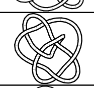
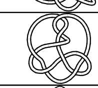
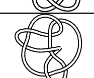
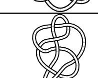
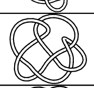
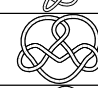
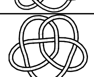
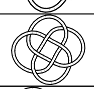
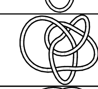
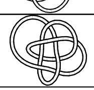
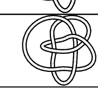
diagram	n_k^t Alexander's A_+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral	diagram	n_k^t Alexander's A_+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral
	0_1^a 1 0	0 / ✔ 0 / ✔		3_1^a $t-1$ t	1 / ✘ 1 / ✘
	4_1^a $3-t$ 0	1 / ✘ 1 / ✔		5_1^a t^2-t+1 $2t^3+3t$	2 / ✘ 2 / ✘
	5_2^a $2t-3$ $5t-4$	1 / ✘ 1 / ✘		6_1^a $5-2t$ $t-4$	1 / ✔ 1 / ✘
	6_2^a $-t^2+3t-3$ t^3-4t^2+4t-4	2 / ✘ 1 / ✘		6_3^a t^2-3t+5 0	2 / ✘ 1 / ✔
	7_1^a t^3-t^2+t-1 $3t^5+5t^3+6t$	3 / ✘ 3 / ✘		7_2^a $3t-5$ $14t-16$	1 / ✘ 1 / ✘
	7_3^a $2t^2-3t+3$ $-9t^3+8t^2-16t+12$	2 / ✘ 2 / ✘		7_4^a $4t-7$ $32-24t$	1 / ✘ 2 / ✘
	7_5^a $2t^2-4t+5$ $9t^3-16t^2+29t-28$	2 / ✘ 2 / ✘		7_6^a $-t^2+5t-7$ $t^3-8t^2+19t-20$	2 / ✘ 1 / ✘
	7_7^a t^2-5t+9 $8-3t$	2 / ✘ 1 / ✘		8_1^a $7-3t$ $5t-16$	1 / ✘ 1 / ✘
	8_2^a $-t^3+3t^2-3t+3$ $2t^5-8t^4+10t^3-12t^2+13t-12$	3 / ✘ 2 / ✘		8_3^a $9-4t$ 0	1 / ✘ 2 / ✔
	8_4^a $-2t^2+5t-5$ $3t^3-8t^2+6t-4$	2 / ✘ 2 / ✘		8_5^a $-t^3+3t^2-4t+5$ $-2t^5+8t^4-13t^3+20t^2-22t+24$	3 / ✘ 2 / ✘
	8_6^a $-2t^2+6t-7$ $5t^3-20t^2+28t-32$	2 / ✘ 2 / ✘		8_7^a t^3-3t^2+5t-5 $-t^5+4t^4-10t^3+12t^2-13t+12$	3 / ✘ 1 / ✘
	8_8^a $2t^2-6t+9$ $-t^3+4t^2-12t+16$	2 / ✔ 2 / ✘		8_9^a $-t^3+3t^2-5t+7$ 0	3 / ✔ 1 / ✔
	8_{10}^a t^3-3t^2+6t-7 $-t^5+4t^4-11t^3+16t^2-21t+20$	3 / ✘ 2 / ✘		8_{11}^a $-2t^2+7t-9$ $5t^3-24t^2+39t-44$	2 / ✘ 1 / ✘
	8_{12}^a $t^2-7t+13$ 0	2 / ✘ 2 / ✔		8_{13}^a $2t^2-7t+11$ $-t^3+4t^2-14t+20$	2 / ✘ 1 / ✘
	8_{14}^a $-2t^2+8t-11$ $5t^3-28t^2+57t-68$	2 / ✘ 1 / ✘		8_{15}^a $3t^2-8t+11$ $21t^3-64t^2+120t-140$	2 / ✘ 2 / ✘
	8_{16}^a t^3-4t^2+8t-9 $t^5-6t^4+17t^3-28t^2+35t-36$	3 / ✘ 2 / ✘		8_{17}^a $-t^3+4t^2-8t+11$ 0	3 / ✘ 1 / ✔
	8_{18}^a $-t^3+5t^2-10t+13$ 0	3 / ✘ 2 / ✔		8_{19}^n t^3-t^2+1 $-3t^5-4t^2-3t$	3 / ✘ 3 / ✘
	8_{20}^n t^2-2t+3 $4t-4$	2 / ✔ 1 / ✘		8_{21}^n $-t^2+4t-5$ $t^3-8t^2+16t-20$	2 / ✘ 1 / ✘

diagram	n_k^t	Alexander's A_+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral
	9_1^a	$t^4 - t^3 + t^2 - t + 1$ $4t^7 + 7t^5 + 9t^3 + 10t$	4 / ✘ 4 / ✘
	9_2^a	$4t - 7$ $30t - 40$	1 / ✘ 1 / ✘
	9_3^a	$2t^3 - 3t^2 + 3t - 3$ $-13t^5 + 12t^4 - 25t^3 + 20t^2 - 32t + 24$	3 / ✘ 3 / ✘
	9_4^a	$3t^2 - 5t + 5$ $23t^3 - 28t^2 + 46t - 44$	2 / ✘ 2 / ✘
	9_5^a	$6t - 11$ $100 - 65t$	1 / ✘ 2 / ✘
	9_6^a	$2t^3 - 4t^2 + 5t - 5$ $13t^5 - 24t^4 + 45t^3 - 52t^2 + 68t - 64$	3 / ✘ 3 / ✘
	9_7^a	$3t^2 - 7t + 9$ $23t^3 - 56t^2 + 99t - 108$	2 / ✘ 2 / ✘
	9_8^a	$-2t^2 + 8t - 11$ $3t^3 - 16t^2 + 29t - 28$	2 / ✘ 2 / ✘
	9_9^a	$2t^3 - 4t^2 + 6t - 7$ $13t^5 - 24t^4 + 55t^3 - 72t^2 + 98t - 96$	3 / ✘ 3 / ✘
	9_{10}^a	$4t^2 - 8t + 9$ $-40t^3 + 72t^2 - 114t + 120$	2 / ✘ 2, 3 / ✘
	9_{11}^a	$-t^3 + 5t^2 - 7t + 7$ $-2t^5 + 16t^4 - 41t^3 + 52t^2 - 66t + 64$	3 / ✘ 2 / ✘
	9_{12}^a	$-2t^2 + 9t - 13$ $5t^3 - 36t^2 + 84t - 100$	2 / ✘ 1 / ✘
	9_{13}^a	$4t^2 - 9t + 11$ $-40t^3 + 92t^2 - 154t + 168$	2 / ✘ 2, 3 / ✘
	9_{14}^a	$2t^2 - 9t + 15$ $-t^3 + 8t^2 - 35t + 60$	2 / ✘ 1 / ✘
	9_{15}^a	$-2t^2 + 10t - 15$ $-5t^3 + 40t^2 - 108t + 136$	2 / ✘ 2 / ✘
	9_{16}^a	$2t^3 - 5t^2 + 8t - 9$ $-13t^5 + 36t^4 - 80t^3 + 120t^2 - 161t + 168$	3 / ✘ 3 / ✘
	9_{17}^a	$t^3 - 5t^2 + 9t - 9$ $t^5 - 8t^4 + 23t^3 - 32t^2 + 28t - 24$	3 / ✘ 2 / ✘
	9_{18}^a	$4t^2 - 10t + 13$ $40t^3 - 108t^2 + 193t - 220$	2 / ✘ 2 / ✘
	9_{19}^a	$2t^2 - 10t + 17$ $t^3 - 8t^2 + 20t - 24$	2 / ✘ 1 / ✘
	9_{20}^a	$-t^3 + 5t^2 - 9t + 11$ $2t^5 - 16t^4 + 47t^3 - 84t^2 + 117t - 124$	3 / ✘ 2 / ✘
	9_{21}^a	$-2t^2 + 11t - 17$ $-5t^3 + 44t^2 - 127t + 164$	2 / ✘ 1 / ✘
	9_{22}^a	$t^3 - 5t^2 + 10t - 11$ $-t^5 + 8t^4 - 24t^3 + 38t^2 - 40t + 36$	3 / ✘ 1 / ✘
	9_{23}^a	$4t^2 - 11t + 15$ $40t^3 - 128t^2 + 243t - 288$	2 / ✘ 2 / ✘
	9_{24}^a	$-t^3 + 5t^2 - 10t + 13$ $-4t^2 + 16t - 20$	3 / ✘ 1 / ✘
	9_{25}^a	$-3t^2 + 12t - 17$ $12t^3 - 70t^2 + 153t - 188$	2 / ✘ 2 / ✘
	9_{26}^a	$t^3 - 5t^2 + 11t - 13$ $-t^5 + 8t^4 - 31t^3 + 64t^2 - 85t + 92$	3 / ✘ 1 / ✘
	9_{27}^a	$-t^3 + 5t^2 - 11t + 15$ $t^3 - 8t^2 + 24t - 32$	3 / ✔ 1 / ✘
	9_{28}^a	$t^3 - 5t^2 + 12t - 15$ $t^5 - 8t^4 + 30t^3 - 68t^2 + 105t - 120$	3 / ✘ 1 / ✘
	9_{29}^a	$t^3 - 5t^2 + 12t - 15$ $t^5 - 8t^4 + 26t^3 - 48t^2 + 59t - 56$	3 / ✘ 2 / ✘
	9_{30}^a	$-t^3 + 5t^2 - 12t + 17$ $2t^3 - 10t^2 + 25t - 32$	3 / ✘ 1 / ✘
	9_{31}^a	$t^3 - 5t^2 + 13t - 17$ $t^5 - 8t^4 + 33t^3 - 80t^2 + 132t - 152$	3 / ✘ 2 / ✘
	9_{32}^a	$t^3 - 6t^2 + 14t - 17$ $-t^5 + 10t^4 - 42t^3 + 94t^2 - 133t + 148$	3 / ✘ 2 / ✘
	9_{33}^a	$-t^3 + 6t^2 - 14t + 19$ $t^3 - 10t^2 + 30t - 40$	3 / ✘ 1 / ✘
	9_{34}^a	$-t^3 + 6t^2 - 16t + 23$ $3t^3 - 18t^2 + 43t - 56$	3 / ✘ 1 / ✘
	9_{35}^a	$7t - 13$ $90t - 144$	1 / ✘ 2, 3 / ✘
	9_{36}^a	$-t^3 + 5t^2 - 8t + 9$ $-2t^5 + 16t^4 - 44t^3 + 66t^2 - 87t + 88$	3 / ✘ 2 / ✘
	9_{37}^a	$2t^2 - 11t + 19$ $t^3 - 8t^2 + 22t - 28$	2 / ✘ 2 / ✘
	9_{38}^a	$5t^2 - 14t + 19$ $62t^3 - 204t^2 + 382t - 452$	2 / ✘ 2, 3 / ✘
	9_{39}^a	$-3t^2 + 14t - 21$ $-12t^3 + 84t^2 - 210t + 268$	2 / ✘ 1 / ✘
	9_{40}^a	$t^3 - 7t^2 + 18t - 23$ $t^5 - 12t^4 + 57t^3 - 144t^2 + 229t - 264$	3 / ✘ 2 / ✘
	9_{41}^a	$3t^2 - 12t + 19$ $3t^3 - 20t^2 + 70t - 108$	2 / ✔ 2 / ✘
	9_{42}^n	$-t^2 + 2t - 1$ $-t^3 + 2t^2 + t - 4$	2 / ✘ 1 / ✘
	9_{43}^n	$-t^3 + 3t^2 - 2t + 1$ $-2t^5 + 8t^4 - 7t^3 + 2t^2 - 5t + 4$	3 / ✘ 2 / ✘
	9_{44}^n	$t^2 - 4t + 7$ $-2t^2 + 9t - 12$	2 / ✘ 1 / ✘
	9_{45}^n	$-t^2 + 6t - 9$ $t^3 - 14t^2 + 47t - 60$	2 / ✘ 1 / ✘
	9_{46}^n	$5 - 2t$ $3t - 12$	1 / ✔ 2 / ✘
	9_{47}^n	$t^3 - 4t^2 + 6t - 5$ $-t^5 + 6t^4 - 15t^3 + 16t^2 - 10t + 12$	3 / ✘ 2 / ✘
	9_{48}^n	$-t^2 + 7t - 11$ $-t^3 + 12t^2 - 42t + 52$	2 / ✘ 2 / ✘
	9_{49}^n	$3t^2 - 6t + 7$ $-21t^3 + 38t^2 - 61t + 60$	2 / ✘ 3 / ✘
	10_1^a	$9 - 4t$ $14t - 40$	1 / ✘ 1 / ✘
	10_2^a	$-t^4 + 3t^3 - 3t^2 + 3t - 3$ $3t^7 - 12t^6 + 16t^5 - 20t^4 + 24t^3 - 24t^2 + 27t - 24$	4 / ✘ 3 / ✘
	10_3^a	$13 - 6t$ $11t - 28$	1 / ✔ 2 / ✘
	10_4^a	$-3t^2 + 7t - 7$ $4t^3 - 8t^2 + t + 8$	2 / ✘ 2 / ✘
	10_5^a	$t^4 - 3t^3 + 5t^2 - 5t + 5$ $-2t^7 + 8t^6 - 20t^5 + 28t^4 - 36t^3 + 36t^2 - 39t + 36$	4 / ✘ 2 / ✘

diagram	n_k^t	Alexander's A_+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral	diagram	n_k^t	Alexander's A_+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral
	10^a_6	$-2t^3+6t^2-7t+7$ $9t^5-36t^4+56t^3-72t^2+81t-84$	3 / ✘ 3 / ✘		10^a_7	$-3t^2+11t-15$ $14t^3-72t^2+135t-160$	2 / ✘ 1 / ✘
	10^a_8	$-2t^3+5t^2-5t+5$ $7t^5-20t^4+23t^3-28t^2+26t-24$	3 / ✘ 2 / ✘		10^a_9	$-t^4+3t^3-5t^2+7t-7$ $-t^7+4t^6-10t^5+20t^4-25t^3+28t^2-28t+28$	4 / ✘ 1 / ✘
	10^a_{10}	$3t^2-11t+17$ $-5t^3+24t^2-71t+100$	2 / ✘ 1 / ✘		10^a_{11}	$-4t^2+11t-13$ $16t^3-52t^2+68t-72$	2 / ✘ 2, 3 / ✘
	10^a_{12}	$2t^3-6t^2+10t-11$ $-5t^5+20t^4-50t^3+72t^2-89t+92$	3 / ✘ 2 / ✘		10^a_{13}	$2t^2-13t+23$ $t^3-12t^2+51t-84$	2 / ✘ 2 / ✘
	10^a_{14}	$-2t^3+8t^2-12t+13$ $9t^5-52t^4+119t^3-180t^2+225t-236$	3 / ✘ 2 / ✘		10^a_{15}	$2t^3-6t^2+9t-9$ $-3t^5+12t^4-24t^3+24t^2-17t+12$	3 / ✘ 2 / ✘
	10^a_{16}	$-4t^2+12t-15$ $-16t^3+56t^2-76t+80$	2 / ✘ 2 / ✘		10^a_{17}	$t^4-3t^3+5t^2-7t+9$ 0	4 / ✘ 1 / ✔
	10^a_{18}	$-4t^2+14t-19$ $16t^3-68t^2+121t-140$	2 / ✘ 1 / ✘		10^a_{19}	$2t^3-7t^2+11t-11$ $3t^5-16t^4+35t^3-40t^2+30t-24$	3 / ✘ 2 / ✘
	10^a_{20}	$-3t^2+9t-11$ $14t^3-56t^2+88t-104$	2 / ✘ 2 / ✘		10^a_{21}	$-2t^3+7t^2-9t+9$ $9t^5-44t^4+80t^3-104t^2+121t-124$	3 / ✘ 2 / ✘
	10^a_{22}	$-2t^3+6t^2-10t+13$ $-t^5+4t^4-10t^3+24t^2-37t+44$	3 / ✔ 2 / ✘		10^a_{23}	$2t^3-7t^2+13t-15$ $-5t^5+24t^4-67t^3+108t^2-137t+144$	3 / ✘ 1 / ✘
	10^a_{24}	$-4t^2+14t-19$ $24t^3-116t^2+221t-268$	2 / ✘ 2 / ✘		10^a_{25}	$-2t^3+8t^2-14t+17$ $9t^5-52t^4+131t^3-232t^2+314t-344$	3 / ✘ 2 / ✘
	10^a_{26}	$-2t^3+7t^2-13t+17$ $-t^5+4t^4-10t^3+28t^2-49t+60$	3 / ✘ 1 / ✘		10^a_{27}	$2t^3-8t^2+16t-19$ $5t^5-28t^4+87t^3-164t^2+229t-252$	3 / ✘ 1 / ✘
	10^a_{28}	$4t^2-13t+19$ $-8t^3+36t^2-100t+136$	2 / ✘ 2 / ✘		10^a_{29}	$t^3-7t^2+15t-17$ $t^5-12t^4+52t^3-104t^2+124t-128$	3 / ✘ 2 / ✘
	10^a_{30}	$-4t^2+17t-25$ $24t^3-148t^2+345t-440$	2 / ✘ 1 / ✘		10^a_{31}	$4t^2-14t+21$ $-4t^2+9t-12$	2 / ✘ 1 / ✘
	10^a_{32}	$-2t^3+8t^2-15t+19$ $t^5-4t^4+13t^3-40t^2+78t-96$	3 / ✘ 1 / ✘		10^a_{33}	$4t^2-16t+25$ 0	2 / ✘ 1 / ✔
	10^a_{34}	$3t^2-9t+13$ $-5t^3+20t^2-52t+68$	2 / ✘ 2 / ✘		10^a_{35}	$2t^2-12t+21$ $-t^3+12t^2-47t+76$	2 / ✔ 2 / ✘
	10^a_{36}	$-3t^2+13t-19$ $14t^3-88t^2+208t-264$	2 / ✘ 2 / ✘		10^a_{37}	$4t^2-13t+19$ 0	2 / ✘ 2 / ✔
	10^a_{38}	$-4t^2+15t-21$ $24t^3-128t^2+270t-336$	2 / ✘ 2 / ✘		10^a_{39}	$-2t^3+8t^2-13t+15$ $9t^5-52t^4+125t^3-204t^2+263t-280$	3 / ✘ 2 / ✘
	10^a_{40}	$2t^3-8t^2+17t-21$ $-5t^5+28t^4-89t^3+176t^2-258t+288$	3 / ✘ 2 / ✘		10^a_{41}	$t^3-7t^2+17t-21$ $t^5-12t^4+54t^3-120t^2+157t-164$	3 / ✘ 2 / ✘
	10^a_{42}	$-t^3+7t^2-19t+27$ $2t^3-8t^2+11t-12$	3 / ✔ 1 / ✘		10^a_{43}	$-t^3+7t^2-17t+23$ 0	3 / ✘ 2 / ✔
	10^a_{44}	$t^3-7t^2+19t-25$ $t^5-12t^4+56t^3-140t^2+220t-248$	3 / ✘ 1 / ✘		10^a_{45}	$-t^3+7t^2-21t+31$ 0	3 / ✘ 2 / ✔
	10^a_{46}	$-t^4+3t^3-4t^2+5t-5$ $-3t^7+12t^6-21t^5+34t^4-43t^3+52t^2-55t+56$	4 / ✘ 3 / ✘		10^a_{47}	$t^4-3t^3+6t^2-7t+7$ $-2t^7+8t^6-23t^5+38t^4-56t^3+60t^2-68t+64$	4 / ✘ 2, 3 / ✘
	10^a_{48}	$t^4-3t^3+6t^2-9t+11$ $t^5-2t^4+2t^3-3t+4$	4 / ✔ 2 / ✘		10^a_{49}	$3t^3-8t^2+12t-13$ $30t^5-94t^4+196t^3-292t^2+372t-392$	3 / ✘ 3 / ✘
	10^a_{50}	$-2t^3+7t^2-11t+13$ $-9t^5+44t^4-94t^3+150t^2-186t+200$	3 / ✘ 2 / ✘		10^a_{51}	$2t^3-7t^2+15t-19$ $-5t^5+24t^4-73t^3+134t^2-194t+212$	3 / ✘ 2, 3 / ✘
	10^a_{52}	$2t^3-7t^2+13t-15$ $-3t^5+16t^4-37t^3+50t^2-49t+44$	3 / ✘ 2 / ✘		10^a_{53}	$6t^2-18t+25$ $93t^3-346t^2+680t-828$	2 / ✘ 2, 3 / ✘
	10^a_{54}	$2t^3-6t^2+10t-11$ $-3t^5+12t^4-24t^3+26t^2-21t+16$	3 / ✘ 2, 3 / ✘		10^a_{55}	$5t^2-15t+21$ $66t^3-246t^2+488t-596$	2 / ✘ 2 / ✘
	10^a_{56}	$-2t^3+8t^2-14t+17$ $-9t^5+52t^4-133t^3+234t^2-312t+340$	3 / ✘ 2 / ✘		10^a_{57}	$2t^3-8t^2+18t-23$ $-5t^5+28t^4-93t^3+194t^2-300t+340$	3 / ✘ 2 / ✘
	10^a_{58}	$3t^2-16t+27$ $3t^3-28t^2+94t-140$	2 / ✘ 2 / ✘		10^a_{59}	$t^3-7t^2+18t-23$ $-t^5+12t^4-55t^3+128t^2-181t+196$	3 / ✘ 1 / ✘

diagram	n_k^t Alexander's A_+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral	diagram	n_k^t Alexander's A_+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral
	10^a_{60} $-t^3+7t^2-20t+29$ $5t^3-40t^2+122t-176$	3 / ✘ 1 / ✘		10^a_{61} $-2t^3+5t^2-6t+7$ $-7t^5+20t^4-27t^3+36t^2-35t+36$	3 / ✘ 2, 3 / ✘
	10^a_{62} $t^4-3t^3+6t^2-8t+9$ $-2t^7+8t^6-23t^5+40t^4-63t^3+76t^2-89t+88$	4 / ✘ 2 / ✘		10^a_{63} $5t^2-14t+19$ $66t^3-220t^2+416t-496$	2 / ✘ 2 / ✘
	10^a_{64} $-t^4+3t^3-6t^2+10t-11$ $-t^7+4t^6-11t^5+24t^4-37t^3+52t^2-60t+64$	4 / ✘ 2 / ✘		10^a_{65} $2t^3-7t^2+14t-17$ $-5t^5+24t^4-71t^3+124t^2-169t+180$	3 / ✘ 2 / ✘
	10^a_{66} $3t^3-9t^2+16t-19$ $30t^5-112t^4+279t^3-480t^2+662t-724$	3 / ✘ 3 / ✘		10^a_{67} $-4t^2+16t-23$ $24t^3-140t^2+312t-392$	2 / ✘ 2 / ✘
	10^a_{68} $4t^2-14t+21$ $8t^3-40t^2+117t-164$	2 / ✘ 2 / ✘		10^a_{69} $t^3-7t^2+21t-29$ $-t^5+12t^4-68t^3+212t^2-397t+476$	3 / ✘ 2 / ✘
	10^a_{70} $t^3-7t^2+16t-19$ $-t^5+12t^4-53t^3+114t^2-146t+152$	3 / ✘ 2 / ✘		10^a_{71} $-t^3+7t^2-18t+25$ t^3-2t^2-t+4	3 / ✘ 1 / ✘
	10^a_{72} $-2t^3+9t^2-16t+19$ $-9t^5+60t^4-167t^3+298t^2-410t+448$	3 / ✘ 2 / ✘		10^a_{73} $t^3-7t^2+20t-27$ $t^5-12t^4+65t^3-194t^2+350t-416$	3 / ✘ 1 / ✘
	10^a_{74} $-4t^2+16t-23$ $24t^3-136t^2+290t-360$	2 / ✘ 2 / ✘		10^a_{75} $-t^3+7t^2-19t+27$ $-4t^3+36t^2-117t+172$	3 / ✔ 2 / ✘
	10^a_{76} $-2t^3+7t^2-12t+15$ $-9t^5+44t^4-104t^3+184t^2-245t+272$	3 / ✘ 2, 3 / ✘		10^a_{77} $2t^3-7t^2+14t-17$ $-5t^5+24t^4-71t^3+132t^2-189t+208$	3 / ✘ 2, 3 / ✘
	10^a_{78} $-t^3+7t^2-16t+21$ $2t^5-24t^4+105t^3-244t^2+390t-448$	3 / ✘ 2 / ✘		10^a_{79} $t^4-3t^3+7t^2-12t+15$ 0	4 / ✘ 2, 3 / ✔
	10^a_{80} $3t^3-9t^2+15t-17$ $30t^5-112t^4+260t^3-426t^2+568t-616$	3 / ✘ 3 / ✘		10^a_{81} $-t^3+8t^2-20t+27$ 0	3 / ✘ 2 / ✔
	10^a_{82} $-t^4+4t^3-8t^2+12t-13$ $t^7-6t^6+19t^5-42t^4+64t^3-78t^2+84t-84$	4 / ✘ 1 / ✘		10^a_{83} $2t^3-9t^2+19t-23$ $-5t^5+34t^4-110t^3+214t^2-301t+332$	3 / ✘ 2 / ✘
	10^a_{84} $2t^3-9t^2+20t-25$ $-5t^5+34t^4-116t^3+246t^2-373t+424$	3 / ✘ 1 / ✘		10^a_{85} $t^4-4t^3+8t^2-10t+11$ $2t^7-12t^6+36t^5-68t^4+101t^3-124t^2+138t-140$	4 / ✘ 2 / ✘
	10^a_{86} $-2t^3+9t^2-19t+25$ $-t^5+6t^4-21t^3+58t^2-105t+128$	3 / ✘ 2 / ✘		10^a_{87} $-2t^3+9t^2-18t+23$ $-t^5+6t^4-23t^3+66t^2-125t+152$	3 / ✔ 2 / ✘
	10^a_{88} $-t^3+8t^2-24t+35$ 0	3 / ✘ 1 / ✔		10^a_{89} $t^3-8t^2+24t-33$ $t^5-14t^4+83t^3-264t^2+495t-596$	3 / ✘ 2 / ✘
	10^a_{90} $-2t^3+8t^2-17t+23$ $-t^5+6t^4-21t^3+54t^2-93t+112$	3 / ✘ 2 / ✘		10^a_{91} $t^4-4t^3+9t^2-14t+17$ $t^5-2t^4+2t^3-3t+4$	4 / ✘ 1 / ✘
	10^a_{92} $-2t^3+10t^2-20t+25$ $-9t^5+68t^4-216t^3+428t^2-622t+696$	3 / ✘ 2 / ✘		10^a_{93} $2t^3-8t^2+15t-17$ $3t^5-18t^4+43t^3-58t^2+55t-48$	3 / ✘ 2 / ✘
	10^a_{94} $-t^4+4t^3-9t^2+14t-15$ $-t^7+6t^6-20t^5+46t^4-76t^3+102t^2-115t+120$	4 / ✘ 2 / ✘		10^a_{95} $2t^3-9t^2+21t-27$ $-5t^5+32t^4-114t^3+248t^2-384t+436$	3 / ✘ 1 / ✘
	10^a_{96} $-t^3+7t^2-22t+33$ $-7t^3+50t^2-147t+212$	3 / ✘ 2 / ✘		10^a_{97} $-5t^2+22t-33$ $-37t^3+242t^2-603t+788$	2 / ✘ 2 / ✘
	10^a_{98} $-2t^3+9t^2-18t+23$ $9t^5-60t^4+177t^3-348t^2+501t-564$	3 / ✘ 2 / ✘		10^a_{99} $t^4-4t^3+10t^2-16t+19$ 0	4 / ✔ 2 / ✔
	10^a_{100} $t^4-4t^3+9t^2-12t+13$ $2t^7-12t^6+39t^5-80t^4+128t^3-164t^2+192t-196$	4 / ✘ 2, 3 / ✘		10^a_{101} $7t^2-21t+29$ $-129t^3+480t^2-942t+1148$	2 / ✘ 2, 3 / ✘
	10^a_{102} $-2t^3+8t^2-16t+21$ $-t^5+6t^4-19t^3+50t^2-89t+108$	3 / ✘ 1 / ✘		10^a_{103} $2t^3-8t^2+17t-21$ $5t^5-30t^4+93t^3-178t^2+254t-280$	3 / ✘ 3 / ✘
	10^a_{104} $t^4-4t^3+9t^2-15t+19$ $t^5-2t^4+2t^3-3t+4$	4 / ✘ 1 / ✘		10^a_{105} $t^3-8t^2+22t-29$ $-t^5+14t^4-71t^3+184t^2-292t+332$	3 / ✘ 2 / ✘
	10^a_{106} $-t^4+4t^3-9t^2+15t-17$ $-t^7+6t^6-20t^5+48t^4-82t^3+114t^2-134t+140$	4 / ✘ 2 / ✘		10^a_{107} $-t^3+8t^2-22t+31$ $2t^3-8t^2+13t-16$	3 / ✘ 1 / ✘
	10^a_{108} $2t^3-8t^2+14t-15$ $-3t^5+18t^4-41t^3+50t^2-40t+32$	3 / ✘ 2 / ✘		10^a_{109} $t^4-4t^3+10t^2-17t+21$ 0	4 / ✘ 2 / ✔
	10^a_{110} $t^3-8t^2+20t-25$ $t^5-14t^4+69t^3-160t^2+219t-236$	3 / ✘ 2 / ✘		10^a_{111} $-2t^3+9t^2-17t+21$ $-9t^5+60t^4-171t^3+316t^2-436t+480$	3 / ✘ 2 / ✘
	10^a_{112} $-t^4+5t^3-11t^2+17t-19$ $t^7-8t^6+29t^5-68t^4+115t^3-152t^2+175t-180$	4 / ✘ 2 / ✘		10^a_{113} $2t^3-11t^2+26t-33$ $-5t^5+42t^4-167t^3+394t^2-623t+720$	3 / ✘ 1 / ✘

diagram	n_k^t Alexander's A_+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral	diagram	n_k^t Alexander's A_+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral
	10^a_{114} $-2t^3+10t^2-21t+27$ $t^5-8t^4+30t^3-78t^2+140t-168$	3 / ✘ 1 / ✘		10^a_{115} $-t^3+9t^2-26t+37$ 0	3 / ✘ 2 / ✔
	10^a_{116} $-t^4+5t^3-12t^2+19t-21$ $t^7-8t^6+30t^5-74t^4+132t^3-184t^2+217t-228$	4 / ✘ 2 / ✘		10^a_{117} $2t^3-10t^2+24t-31$ $-5t^5+38t^4-144t^3+330t^2-522t+600$	3 / ✘ 2 / ✘
	10^a_{118} $t^4-5t^3+12t^2-19t+23$ 0	4 / ✘ 1 / ✔		10^a_{119} $-2t^3+10t^2-23t+31$ $-t^5+6t^4-26t^3+86t^2-175t+220$	3 / ✘ 1 / ✘
	10^a_{120} $8t^2-26t+37$ $166t^3-692t^2+1433t-1788$	2 / ✘ 2, 3 / ✘		10^a_{121} $2t^3-11t^2+27t-35$ $5t^5-42t^4+167t^3-396t^2+634t-732$	3 / ✘ 2 / ✘
	10^a_{122} $-2t^3+11t^2-24t+31$ $-t^5+8t^4-34t^3+104t^2-211t+264$	3 / ✘ 2 / ✘		10^a_{123} $t^4-6t^3+15t^2-24t+29$ 0	4 / ✔ 2 / ✔
	10^n_{124} t^4-t^3+t-1 $-4t^7-6t^4-4t^2-6t$	4 / ✘ 4 / ✘		10^n_{125} t^3-2t^2+2t-1 $-t^5+2t^4-2t^3+3t-4$	3 / ✘ 2 / ✘
	10^n_{126} t^3-2t^2+4t-5 $t^5-2t^4+10t^3-12t^2+22t-20$	3 / ✘ 2 / ✘		10^n_{127} $-t^3+4t^2-6t+7$ $2t^5-14t^4+32t^3-52t^2+67t-72$	3 / ✘ 2 / ✘
	10^n_{128} $2t^3-3t^2+t+1$ $-13t^5+12t^4-3t^3-10t^2-9t+12$	3 / ✘ 3 / ✘		10^n_{129} $2t^2-6t+9$ $-t^3-2t^2+14t-20$	2 / ✔ 1 / ✘
	10^n_{130} $2t^2-4t+5$ $t^3-2t^2+19t-24$	2 / ✘ 2 / ✘		10^n_{131} $-2t^2+8t-11$ $5t^3-38t^2+87t-112$	2 / ✘ 1 / ✘
	10^n_{132} t^2-t+1 $2t^2+5t-4$	2 / ✘ 1 / ✘		10^n_{133} $-t^2+5t-7$ $t^3-14t^2+37t-48$	2 / ✘ 1 / ✘
	10^n_{134} $2t^3-4t^2+4t-3$ $-13t^5+24t^4-33t^3+30t^2-41t+40$	3 / ✘ 3 / ✘		10^n_{135} $3t^2-9t+13$ $t^3-6t^2+18t-24$	2 / ✘ 2 / ✘
	10^n_{136} $-t^2+4t-5$ $-t^3+4t^2-2t-4$	2 / ✘ 1 / ✘		10^n_{137} $t^2-6t+11$ $-4t^2+24t-44$	2 / ✔ 1 / ✘
	10^n_{138} t^3-5t^2+8t-7 $-t^5+8t^4-22t^3+24t^2-11t+8$	3 / ✘ 2 / ✘		10^n_{139} t^4-t^3+2t-3 $-4t^7-12t^4+5t^3-4t^2-16t+12$	4 / ✘ 4 / ✘
	10^n_{140} t^2-2t+3 $8t-8$	2 / ✔ 2 / ✘		10^n_{141} $-t^3+3t^2-4t+5$ $t^3-8t^2+16t-20$	3 / ✘ 1 / ✘
	10^n_{142} $2t^3-3t^2+2t-1$ $-13t^5+12t^4-13t^3+4t^2-17t+12$	3 / ✘ 3 / ✘		10^n_{143} t^3-3t^2+6t-7 $t^5-4t^4+15t^3-28t^2+45t-48$	3 / ✘ 1 / ✘
	10^n_{144} $-3t^2+10t-13$ $10t^3-44t^2+80t-96$	2 / ✘ 2 / ✘		10^n_{145} t^2+t-3 $2t^3+8t^2+6t-8$	2 / ✘ 2 / ✘
	10^n_{146} $2t^2-8t+13$ $t^3-8t^2+21t-28$	2 / ✘ 1 / ✘		10^n_{147} $-2t^2+7t-9$ $-3t^3+12t^2-15t+12$	2 / ✘ 1 / ✘
	10^n_{148} t^3-3t^2+7t-9 $t^5-4t^4+18t^3-36t^2+62t-68$	3 / ✘ 2 / ✘		10^n_{149} $-t^3+5t^2-9t+11$ $2t^5-18t^4+55t^3-104t^2+149t-164$	3 / ✘ 2 / ✘
	10^n_{150} $-t^3+4t^2-6t+7$ $-2t^5+12t^4-26t^3+38t^2-45t+44$	3 / ✘ 2 / ✘		10^n_{151} $t^3-4t^2+10t-13$ $-t^5+6t^4-21t^3+42t^2-66t+72$	3 / ✘ 2 / ✘
	10^n_{152} $t^4-t^3-t^2+4t-5$ $4t^7-7t^5+18t^4-7t^3-12t^2+45t-52$	4 / ✘ 4 / ✘		10^n_{153} t^3-t^2-t+3 $t^5-2t^4+t^3+2t^2-t$	3 / ✔ 2 / ✘
	10^n_{154} t^3-4t+7 $-3t^5-6t^4+13t^3-47t+68$	3 / ✘ 3 / ✘		10^n_{155} $-t^3+3t^2-5t+7$ $-2t^3+12t^2-22t+28$	3 / ✔ 2 / ✘
	10^n_{156} t^3-4t^2+8t-9 $t^5-6t^4+19t^3-30t^2+33t-32$	3 / ✘ 1 / ✘		10^n_{157} $-t^3+6t^2-11t+13$ $-2t^5+22t^4-78t^3+148t^2-218t+240$	3 / ✘ 2 / ✘
	10^n_{158} $-t^3+4t^2-10t+15$ $2t^2-7t+12$	3 / ✘ 2 / ✘		10^n_{159} $t^3-4t^2+9t-11$ $t^5-6t^4+26t^3-60t^2+98t-112$	3 / ✘ 1 / ✘
	10^n_{160} $-t^3+4t^2-4t+3$ $-2t^5+12t^4-20t^3+14t^2-16t+12$	3 / ✘ 2 / ✘		10^n_{161} t^3-2t+3 $3t^5+6t^4-3t^3+4t^2+14t-12$	3 / ✘ 3 / ✘
	10^n_{162} $-3t^2+9t-11$ $10t^3-38t^2+58t-68$	2 / ✘ 2 / ✘		10^n_{163} $t^3-5t^2+12t-15$ $-t^5+8t^4-30t^3+62t^2-89t+96$	3 / ✘ 1, 2 / ✘
	10^n_{164} $3t^2-11t+17$ $t^3-10t^2+29t-40$	2 / ✘ 1 / ✘		10^n_{165} $-2t^2+10t-15$ $-5t^3+50t^2-146t+196$	2 / ✘ 2 / ✘