Proofs of Taylor

June 26, 2018 9:14 AM

The "trailhead sign" method [remember to scan the terrain at the big rock]

$$f(x) = f(x_0) + \int_{x_0}^{x} f'(x_1)\,dx_1$$

$$= f(x_0) + \int_{x_0}^{x}\left[f'(0) + \int_{x_0}^{x_1} f''(x_2)\,dx_2\right]dx_1 = \dots$$

$$= f(x_0) + \int_{x_0}^{x} dx_1 \left[f'(0) + \int_{x_0}^{x_1} dx_2 \left[f''(0) + \int_{x_0}^{x_2} dx_3 \left[f'''(0) + \int_{x_0}^{x_3} dx_4\, f^{(4)}(x_4)\right]\right]\right] = \dots$$

$$= \sum_{k=0}^{n} \frac{(x-x_0)^k}{k!} f^{(k)}(x_0) + \int_{x_0}^{x} dt\, f^{(n+1)}(t) \cdot \frac{(x_1-t)^n}{n!}.$$

could have started from the end, by integrating $f^{(n+1)}$ on a simplex in two ways.

Remove Fubini?

$$\int_{x_0}^{x} dx_1 \int_{x_0}^{x_1} dt\, f''(t) = \int_{x_0}^{x} dx_1 \left[f'(x_1) - f'(x_0)\right]$$

$$\overset{?}{=} \int_{x_0}^{x} dt\, f''(t) \cdot (x-t)$$

start from this proof. Then.

1. too clever! Discuss origin.
2. Uses Fubini! Remove.
3. Use integrating! Replace w/ MVT.

The "high point" method