# Links in a Pole Dancing Studio: A Reading of Massuyeau, Alekseev, Kawazumi, Kuno, and Naef

Abstract. I will report on joint work with Zsuzsanna Dancso, Tamara Hogan, Jessica Liu, and Nancy Scherich. Little of what we do is original, and much of it is simply a reading of Massuyeau [Ma] and Alekseev, Kawazumi, Kuno, and Naef [AKKN1, AN1, AKKN2].

We study the pole-strand and strand-strand double filtration on the space of links in a pole dancing studio (a punctured disk cross an interval), the corresponding homomorphic expansions, and a strand-only HOMFLY-PT relation. When the strands are transparent or nearly transparent to each other we recover and perhaps simplify substantial parts of the work of the aforementioned authors on expansions for the Goldman-Turaev Lie bi-algebra.

The Kontsevich Integral

$$K = \int \left(\ \right)$$

↓Z

$$\int \left(\ \right) \in A = \langle \ \rangle / _{4T}$$
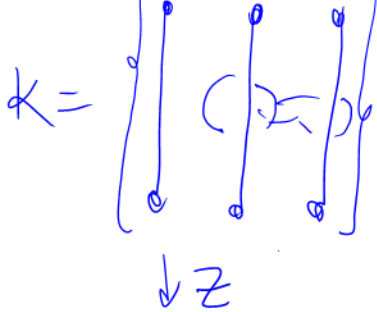
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References.
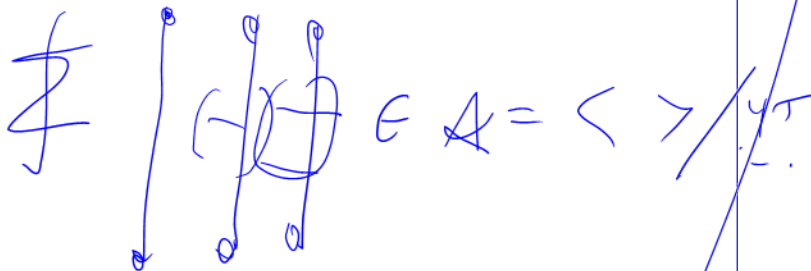
[AKKN1] A. Alekseev, N. Kawazumi, Y. Kuno, and F. Naef, *The Goldman-Turaev Lie Bialgebra in Genus Zero and the Kashiwara-Vergne Problem,* Adv. Math. **326** (2018) 1–53, arXiv:1703.05813.

[AKKN2] A. Alekseev, N. Kawazumi, Y. Kuno, and F. Naef, *Goldman-Turaev formality implies Kashiwara-Vergne,* Quant. Topol. **11-4** (2020) 657-–689, arXiv:1812.01159.

[AN1] A. Alekseev and F. Naef, *Goldman-Turaev Formality from the Knizhnik-Zamolodchikov Connection,* Comp. Rend. Math. **355-11** (2017) 1138–1147, arXiv:1708.03119.

[Ma] G. Massuyeau, *Formal Descriptions of Turaev's Loop Operations,* Quant. Topol. **9-1** (2018) 39-–117, arXiv:1511.03974.

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Homework.

1. What more do we get if we don't mod out by the HOMFLY-PT relation?
2. What more do we get if we allow more than one strand-strand interaction?
3. In this language, recover the double bracket.
4. In this language, recover Kashiwara-Vergne.
5. How is all this related to w-knots?
6. Do the same with associators. Use that to derive formulas for solutions of Kashiwara-Vergne.
7. What's the relationship with the Habiro-Massuyeau invariants of links in handlebodies (same things, different filtration).

Significance 1. In knot/link theory (P=0) this is the "old" Kontsevich integral of knots/links, a "universal F.T. invariant". Many knot polynomials factor through.

2. Z is "an expansion".

Further properties 1. Multiplicative

2. Reflective.

3. Respects the Homflypt relation [Le-Murakami]

4. Respects the SS filtration.

$\implies$ Contains an expansion $Z_6: CGW_p \longrightarrow CAW_p$

$\implies$ Z_6 respects the Goldman bracket & the Turaev Co-bracket.

Further comments...