Les Diablerets handout on August 6, 2016

August 6, 2016 8:52 AM

Hour 1: Intro, why I care.
Sketch:

1. Display the Logos as "this is the worst that we will see".
2. Display 1-smidgen sl2 and mention that this should generalize.
3. Display the program and some output.
4. Go over "why expected" and "why paradise" in some detail.

Hour 2: Av etc.

Hour 3: 0-smidgen.

Hour 4: 1-smidgen.

Future directions, in approximate decreasing value:

1. Clean up, make accessible, restate in topological language.
2. Study ribbon knots.
3. Relate to existing invariants.
4. k-smidgen sl(n), etc.
5. 3-manifold invariants.
6. Categorify and appease the gods.

Work in Progress!

The Brute and the Hidden Paradise

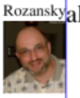
Abstract. There is expected to be a hidden paradise of poly-time computable knot polynomials lying just beyond the Alexander polynomial. I will describe my brute attempts to gain entry.

Why "expected"? Finite-type invariants include all coefficients of all quantum knot polynomials (appropriately parametrized), and each is computable in poly-time. Yet

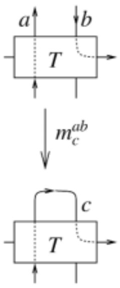
d	2	3	4	5	6	7	8	$\cdots$
known f.t. invts in $O(n^d)$	1	1	∞	3	4	8	11	$\cdots$

This is an unreasonable picture! So there ought to be further poly-time polynomial invariants.

Also. • The line above the Alexander line in the Melvin-Morton [MM, Ro] expansion of the coloured Jones polynomial. • The 2-loop contribution to the Kontsevich integral.

Rozansky


Why "paradise"? Foremost answer: *OBVIOUSLY.* Cf. proving (incomputable A)=(incomputable B), or categorifying (incomputable C).



Why "brute"? Cause it's the only thing I know, for now. There may be better ways in, and it's fair to hope that sooner or later they will be found.

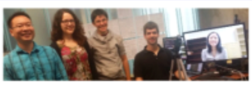
The Gold Standard is set by the formulas [BNS, BN] for Alexander. An S-component tangle T has $\Gamma(T) \in R_S \times M_{S\times S}(R_S) = \left\{\begin{array}{c|c} \omega & S \\ \hline S & A \end{array}\right\}$ with $R_S := \mathbb{Z}(\{t_a : a \in S\})$:

$$\left({}_a\!\nearrow\!\!\!\nwarrow_b,\ {}_b\!\nwarrow\!\!\!\nearrow_a\right) \to \begin{array}{c|cc} 1 & a & b \\ \hline a & 1 & 1-t_a^{\pm 1} \\ b & 0 & t_a^{\pm 1} \end{array} \qquad T_1 \sqcup T_2 \to \begin{array}{c|cc} \omega_1\omega_2 & S_1 & S_2 \\ \hline S_1 & A_1 & 0 \\ S_2 & 0 & A_2 \end{array}$$

$$\begin{array}{c|ccc} \omega & a & b & S \\ \hline a & \alpha & \beta & \theta \\ b & \gamma & \delta & \epsilon \\ S & \phi & \psi & \Xi \end{array} \xrightarrow[t_a, t_b \to t_c]{m_c^{ab}} \left(\begin{array}{c|cc} (1-\beta)\omega & c & S \\ \hline c & \gamma + \frac{\alpha\delta}{1-\beta} & \epsilon + \frac{\delta\theta}{1-\beta} \\ S & \phi + \frac{\alpha\psi}{1-\beta} & \Xi + \frac{\psi\theta}{1-\beta} \end{array}\right)$$

Help Needed! Disorganized videos of talks in a private seminar are at ωεβ/PP.

Vo, Halacheva, Dalvit, Ens, Lee (van der Veen, Schaveling)



For long knots, ω is Alexander, and that's the fastest Alexander algorithm I know!

Dunfield: 1000-crossing fast.

1-Smidgen sl_2 (with van der Veen). Let $\mathfrak{g}_1$ be the 4D Lie algebra $\mathfrak{g}_1 = \langle b, c, u, w\rangle$ over $\mathbb{Q}[\epsilon]/(\epsilon^2 = 0)$, with b central and $[w, c] = w$, $[c, u] = u$, and $[u, w] = b - 2\epsilon c$, with CYBE $a_{ij} = (b_i - \epsilon c_i)c_j + u_i w_j$ in $\mathcal{U}(\mathfrak{g}_1)^{\otimes\{i,j\}}$. In a certain objective sense, $\mathfrak{g}_1$ is more valuable than sl_2.


van der Veen

0-Smidgen sl_2 ☺. Let $\mathfrak{g}_0$ be $\mathfrak{g}_1$ at $\epsilon = 0$, or $\mathbb{Q}\langle b, c, u, w\rangle/([b,\cdot] = 0, [c, u] = u, [c, w] = -w, [u, w] = b$ with $a_{ij} = b_i c_j + u_i w_j$. It is $\mathfrak{a}^* \rtimes \mathfrak{a}$ where $\mathfrak{a}$ is the 2D Lie algebra $\mathbb{Q}\langle b, u\rangle$ and (c, w) is the dual basis of (b, u). It is even more valuable than $\mathfrak{g}_1$, but topology already got by other means almost everything $\mathfrak{g}_0$ has to give.

Work in Progress!

The Brute and the Hidden Paradise

References.

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"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

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