

```
In[*]:= SetDirectory[
  "C:\\drorbn\\AcademicPensieve\\Talks\\LearningSeminarOnCategorification-2006";
  << "../..../Projects/Profile/Profile.m";
  << "../..../Projects/SL2Portfolio2/Engine-Speedy.m";
  << "../..../Projects/SL2Portfolio2/Objects.m";
  $k = 0;  $\gamma = \hbar = 1$ ;
```

This is Profile.m of <http://www.drorbn.net/AcademicPensieve/Projects/Profile/>.

This version: April 2020. Original version: July 1994.

» Warning: On Sep 4 2019 I swapped the operations  $\epsilon$  and  $\eta$ . Some incompatibilities may arise in older notebooks.

```
In[*]:= Ri,j
```

```
Out[*]:= E{i}→{i,j} [aj bi, xj yi, 1]
```

```
In[*]:= cmi,j→k
```

```
Out[*]:= E{i,j}→{k} [ak  $\alpha_i$  + ak  $\alpha_j$  + bk  $\beta_i$  + bk  $\beta_j$ , yk  $\eta_i$  +  $\frac{y_k \eta_j}{\mathcal{A}_i} + \frac{x_k \xi_i}{\mathcal{A}_j} + b_k \eta_j \xi_i + x_k \xi_j$ , 1 + O[ $\epsilon$ ]1]
```

```
In[*]:= dmi,j→k
```

```
Out[*]:= E{i,j}→{k} [ak  $\alpha_i$  + ak  $\alpha_j$  + bk  $\beta_i$  + bk  $\beta_j$ , yk  $\eta_i$  +  $\frac{y_k \eta_j}{\mathcal{A}_i} + \frac{x_k \xi_i}{\mathcal{A}_j} + (1 - B_k) \eta_j \xi_i + x_k \xi_j$ , 1 + O[ $\epsilon$ ]1]
```

```
In[*]:= Gsp1 [ $\omega_1$ , L1, Q1] // Esp2 [ $L_2$ , Q2, P2] := E{i}→sp1 [L1, Q1,  $\omega_1 + 0[\epsilon]$ ] // Esp2 [L2, Q2, P2]
```

```
In[*]:= G1 = Expand /@
```

$$G_{\{1,2,3\}} [\omega, \{b_1, b_2, b_3\}] \cdot \begin{pmatrix} l_{11} & l_{12} & l_{13} \\ l_{21} & l_{22} & l_{23} \\ l_{31} & l_{32} & l_{33} \end{pmatrix} \cdot \{a_1, a_2, a_3\}, \{y_1, y_2, y_3\} \cdot \begin{pmatrix} \alpha & \beta & \theta \\ g & \delta & \epsilon \\ \phi & \psi & \varpi \end{pmatrix} \cdot \{x_1, x_2, x_3\}$$

```
Out[*]:= G{1,2,3} [ $\omega$ , a1 b1 l11 + a2 b1 l12 + a3 b1 l13 + a1 b2 l21 + a2 b2 l22 + a3 b2 l23 + a1 b3 l31 + a2 b3 l32 + a3 b3 l33,  $\alpha x_1 y_1 + \beta x_2 y_1 + \theta x_3 y_1 + g x_1 y_2 + \delta x_2 y_2 + \epsilon x_3 y_2 + \phi x_1 y_3 + \psi x_2 y_3 + \varpi x_3 y_3$ ]
```

```
In[*]:= G1 // cm1,2→0 // Simplify
```

```
Out[*]:= E{i}→{0,3} [a0 (b0 (l11 + l12 + l21 + l22) + b3 (l31 + l32)) + a3 (b0 (l13 + l23) + b3 l33),  $\frac{1}{-1 + g b_0} \left( x_3 \left( (-\theta + (-\alpha \epsilon + g \theta) b_0 - \epsilon B_0^{11+121} B_3^{131}) y_0 + (-\varpi + (g \varpi - \epsilon \phi) b_0) y_3 \right) + x_0 \left( (-\beta + (g \beta - \alpha \delta) b_0 - \delta B_0^{11+121} B_3^{131} - \alpha B_0^{12+122} B_3^{132} - g B_0^{11+112+121+122} B_3^{131+132}) y_0 + (-\psi + (-\delta \phi + g \psi) b_0 - \phi B_0^{12+122} B_3^{132}) y_3 \right) \right), \frac{\omega}{1 - g b_0} + 0[\epsilon]^1]$ ]
```

```
In[*]:= E{i}→r [Lr, Qr, Pr}] // G := Gr [Normal@P, L, Q]
```

In[ ]:= **G1 // cm<sub>1,2→0</sub> // G**

$$\text{Out[ ]} = \mathbb{G}_{\{0,3\}} \left[ -\frac{\omega}{-1 + g b_0}, a_0 b_0 (l_{11} + l_{12} + l_{21} + l_{22}) + a_3 b_0 (l_{13} + l_{23}) + a_0 b_3 (l_{31} + l_{32}) + a_3 b_3 l_{33}, \right. \\ \left. \frac{1}{-1 + g b_0} \left( -\beta x_0 y_0 + g \beta b_0 x_0 y_0 - \alpha \delta b_0 x_0 y_0 - \delta B_0^{111+121} B_3^{131} x_0 y_0 - \alpha B_0^{112+122} B_3^{132} x_0 y_0 - \right. \right. \\ \left. \left. g B_0^{111+112+121+122} B_3^{131+132} x_0 y_0 - \theta x_3 y_0 - \alpha \epsilon b_0 x_3 y_0 + g \theta b_0 x_3 y_0 - \epsilon B_0^{111+121} B_3^{131} x_3 y_0 - \right. \right. \\ \left. \left. \psi x_0 y_3 - \delta \phi b_0 x_0 y_3 + g \psi b_0 x_0 y_3 - \phi B_0^{112+122} B_3^{132} x_0 y_3 - \Xi x_3 y_3 + g \Xi b_0 x_3 y_3 - \epsilon \phi b_0 x_3 y_3 \right) \right]$$

```
In[ ]:= GS [  $\omega$ _, L_, Q_ ] // GForm := Module [ { M, i, j },
M = Table [ Simplify [  $\partial_{x_i, y_j} Q$  ], { i, S }, { j, S } ];
PrependTo [ M,  $y_{\#}$  & /@ S ];
M = Join [
  { Prepend [  $x_{\#}$  & /@ S,  $\omega$  ] },
  Transpose [ M ],
  { Prepend [ Table [  $\partial_{a_i} L$ , { i, S } ], "G" ] }
];
MatrixForm [ M ]
];
```

```
In[ ]:= GS1 [  $\omega 1$ _, L1_, Q1_ ] ≡ GS2 [  $\omega 2$ _, L2_, Q2_ ] :=
(Sort [ S1 ] === Sort [ S2 ] ) ^ Simplify [  $\omega 1 = \omega 2$  ] ^ Simplify [ L1 = L2 ] ^ Simplify [ Q1 = Q2 ]
```

In[ ]:= **G1 // GForm**

Out[ ]//MatrixForm=

$$\begin{pmatrix} \omega & x_1 & x_2 & x_3 \\ y_1 & \alpha & \beta & \theta \\ y_2 & g & \delta & \epsilon \\ y_3 & \phi & \psi & \Xi \\ \mathbb{G} & b_1 l_{11} + b_2 l_{21} + b_3 l_{31} & b_1 l_{12} + b_2 l_{22} + b_3 l_{32} & b_1 l_{13} + b_2 l_{23} + b_3 l_{33} \end{pmatrix}$$

In[ ]:= **G1 // cm<sub>1,2→0</sub> // G // GForm**

Out[ ]//MatrixForm=

$$\begin{pmatrix} -\frac{\omega}{-1+g b_0} & x_0 & x_3 \\ y_0 & \frac{\beta + (-g \beta + \alpha \delta) b_0 + \delta B_0^{111+121} B_3^{131} + \alpha B_0^{112+122} B_3^{132} + g B_0^{111+112+121+122} B_3^{131+132}}{1-g b_0} & \frac{\theta + (\alpha \epsilon - g \theta) b_0 + \epsilon B_0^{111+121} B_3^{131}}{1-g b_0} \\ y_3 & \frac{\psi + (\delta \phi - g \psi) b_0 + \phi B_0^{112+122} B_3^{132}}{1-g b_0} & \frac{\Xi + (-g \Xi + \epsilon \phi) b_0}{1-g b_0} \\ \mathbb{G} & b_0 (l_{11} + l_{12} + l_{21} + l_{22}) + b_3 (l_{31} + l_{32}) & b_0 (l_{13} + l_{23}) + b_3 l_{33} \end{pmatrix}$$

$$\text{In}[ ] := \left( \mathbf{G2} = \mathbb{G}_{\{0,3\}} \left[ \frac{\omega}{1-g b_0}, \{b_0, b_3\} \cdot \begin{pmatrix} l_{11} + l_{12} + l_{21} + l_{22} & l_{13} + l_{23} \\ l_{31} + l_{32} & l_{33} \end{pmatrix} \cdot \{a_0, a_3\}, \{y_0, y_3\} \cdot \begin{pmatrix} \frac{\beta + (\alpha - g - \beta) b_0 + \delta B_0^{l_{11}+l_{21}} B_3^{l_{31}} + \alpha B_0^{l_{12}+l_{22}} B_3^{l_{32}} + g B_0^{l_{11}+l_{12}+l_{21}+l_{22}} B_3^{l_{31}+l_{32}}}{1-g b_0} & \frac{\theta + (\alpha - g - \theta) b_0 + \epsilon B_0^{l_{11}+l_{21}} B_3^{l_{31}}}{1-g b_0} \\ \frac{\psi + (\delta - \phi - g - \psi) b_0 + \phi B_0^{l_{12}+l_{22}} B_3^{l_{32}}}{1-g b_0} & \frac{\Xi + (\epsilon - \phi - g - \Xi) b_0}{1-g b_0} \end{pmatrix} \cdot \{x_0, x_3\} \right] // \mathbb{G}\text{Form} \right)$$

$$(\mathbf{G1} // \text{cm}_{1,2 \rightarrow 0} // \mathbb{G}) \equiv \mathbf{G2}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{\omega}{1-g b_0} & & X_0 & & & X_3 \\ y_0 & \frac{\beta + (-g - \beta + \alpha \delta) b_0 + \delta B_0^{l_{11}+l_{21}} B_3^{l_{31}} + \alpha B_0^{l_{12}+l_{22}} B_3^{l_{32}} + g B_0^{l_{11}+l_{12}+l_{21}+l_{22}} B_3^{l_{31}+l_{32}}}{1-g b_0} & & & \frac{\theta + (\alpha - g - \theta) b_0 + \epsilon B_0^{l_{11}+l_{21}} B_3^{l_{31}}}{1-g b_0} & \\ y_3 & & \frac{\psi + (\delta - \phi - g - \psi) b_0 + \phi B_0^{l_{12}+l_{22}} B_3^{l_{32}}}{1-g b_0} & & & \frac{\Xi + (-g - \Xi + \epsilon - \phi) b_0}{1-g b_0} \\ \mathbb{G} & b_0 (l_{11} + l_{12} + l_{21} + l_{22}) + b_3 (l_{31} + l_{32}) & & & b_0 (l_{13} + l_{23}) + b_3 l_{33} & \end{pmatrix}$$

Out[ ] = True

$$\left( \frac{\beta + (\alpha - g - \beta) b_0 + \delta B_0^{l_{11}+l_{21}} B_3^{l_{31}} + \alpha B_0^{l_{12}+l_{22}} B_3^{l_{32}} + g B_0^{l_{11}+l_{12}+l_{21}+l_{22}} B_3^{l_{31}+l_{32}}}{1-g b_0} \quad \frac{\theta + (\alpha - g - \theta) b_0 + \epsilon B_0^{l_{11}+l_{21}} B_3^{l_{31}}}{1-g b_0} \right) / \left( \frac{\psi + (\delta - \phi - g - \psi) b_0 + \phi B_0^{l_{12}+l_{22}} B_3^{l_{32}}}{1-g b_0} \quad \frac{\Xi + (\epsilon - \phi - g - \Xi) b_0}{1-g b_0} \right)$$

In[ ] :=  $\mathbf{G1} // \text{dm}_{1,2 \rightarrow 0} // \mathbb{G} // \mathbb{G}\text{Form}$

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{\omega}{1-g+g B_0} & & X_0 & & & X_3 \\ y_0 & \frac{\beta - g - \beta + \alpha \delta + (g - \beta - \alpha \delta) B_0 + \delta B_0^{l_{11}+l_{21}} B_3^{l_{31}} + \alpha B_0^{l_{12}+l_{22}} B_3^{l_{32}} + g B_0^{l_{11}+l_{12}+l_{21}+l_{22}} B_3^{l_{31}+l_{32}}}{1-g+g B_0} & & & \frac{\alpha \epsilon + \theta - g \theta + (-\alpha \epsilon + g \theta) B_0 + \epsilon B_0^{l_{11}+l_{21}} B_3^{l_{31}}}{1-g+g B_0} & \\ y_3 & & \frac{\delta \phi + \psi - g \psi + (-\delta \phi + g \psi) B_0 + \phi B_0^{l_{12}+l_{22}} B_3^{l_{32}}}{1-g+g B_0} & & & \frac{\Xi - g \Xi + \epsilon \phi + (g \Xi - \epsilon \phi) B_0}{1-g+g B_0} \\ \mathbb{G} & b_0 (l_{11} + l_{12} + l_{21} + l_{22}) + b_3 (l_{31} + l_{32}) & & & b_0 (l_{13} + l_{23}) + b_3 l_{33} & \end{pmatrix}$$

$$\text{In}[ ] := \left( \mathbf{G3} = \mathbb{G}_{\{0,3\}} \left[ \frac{\omega}{1+g(B_0-1)}, \{b_0, b_3\} \cdot \begin{pmatrix} l_{11} + l_{12} + l_{21} + l_{22} & l_{13} + l_{23} \\ l_{31} + l_{32} & l_{33} \end{pmatrix} \cdot \{a_0, a_3\}, \{y_0, y_3\} \cdot \begin{pmatrix} \frac{\beta - g - \beta + \alpha \delta + (g - \beta - \alpha \delta) B_0 + \delta B_0^{l_{11}+l_{21}} B_3^{l_{31}} + \alpha B_0^{l_{12}+l_{22}} B_3^{l_{32}} + g B_0^{l_{11}+l_{12}+l_{21}+l_{22}} B_3^{l_{31}+l_{32}}}{1+g(B_0-1)} & \frac{\alpha \epsilon + \theta - g \theta + (-\alpha \epsilon + g \theta) B_0 + \epsilon B_0^{l_{11}+l_{21}} B_3^{l_{31}}}{1+g(B_0-1)} \\ \frac{\delta \phi + \psi - g \psi + (-\delta \phi + g \psi) B_0 + \phi B_0^{l_{12}+l_{22}} B_3^{l_{32}}}{1+g(B_0-1)} & \frac{\Xi - g \Xi + \epsilon \phi + (g \Xi - \epsilon \phi) B_0}{1+g(B_0-1)} \end{pmatrix} \cdot \{x_0, x_3\} \right] // \mathbb{G}\text{Form} \right)$$

$$(\mathbf{G1} // \text{dm}_{1,2 \rightarrow 0} // \mathbb{G}) \equiv \mathbf{G3}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{\omega}{1+g(-1+B_0)} & & X_0 & & & X_3 \\ y_0 & \frac{\beta - g - \beta + \alpha \delta + (g - \beta - \alpha \delta) B_0 + \delta B_0^{l_{11}+l_{21}} B_3^{l_{31}} + \alpha B_0^{l_{12}+l_{22}} B_3^{l_{32}} + g B_0^{l_{11}+l_{12}+l_{21}+l_{22}} B_3^{l_{31}+l_{32}}}{1+g(-1+B_0)} & & & \frac{\alpha \epsilon + \theta - g \theta + (-\alpha \epsilon + g \theta) B_0 + \epsilon B_0^{l_{11}+l_{21}} B_3^{l_{31}}}{1+g(-1+B_0)} & \\ y_3 & & \frac{\delta \phi + \psi - g \psi + (-\delta \phi + g \psi) B_0 + \phi B_0^{l_{12}+l_{22}} B_3^{l_{32}}}{1+g(-1+B_0)} & & & \frac{\Xi - g \Xi + \epsilon \phi + (g \Xi - \epsilon \phi) B_0}{1+g(-1+B_0)} \\ \mathbb{G} & b_0 (l_{11} + l_{12} + l_{21} + l_{22}) + b_3 (l_{31} + l_{32}) & & & b_0 (l_{13} + l_{23}) + b_3 l_{33} & \end{pmatrix}$$

Out[ ] = True