Indiana-1611 Post Mortem

November 9, 2016 11:16 AM

Solve the PgUp/PgDn HandoutBrowser bug!

Dror Bar-Natan: Talks: Indiana-1611: (thanks for accepting my invitation!)

ωεβ:=http://drorbn.net/Indiana-1611/

choices will change!

Work in Progress!

A Poly-Time Knot Polynomial Via Solvable Approximation

Abstract. Rozansky [Ro2] and Overbay [Ov] described a spectacular knot polynomial that failed to attract the attention it deserved as the first poly-time-computable knot polynomial since Alexander's [Al, 1928] and (in my opinion) as the second most likely knot polynomial (after Alexander's) to carry topological information. With Roland van der Veen, I will explain how to compute the Rozansky polynomial using some new commutator-calculus techniques and a Lie algebra $\mathfrak{g}_1$ which is at the same time solvable and an approximation of the simple Lie algebra sl_2.

Rozansky

Overbay

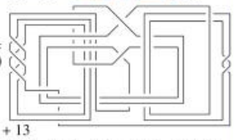
Gompf, Scharlemann, Thompson [GST]

$$U\in\mathcal{T}_n \qquad 1\in\mathcal{A}_n$$

$$\mathcal{T}_{2n} \xrightarrow{z} \mathcal{A}_{2n} \quad \text{with } \mathcal{R} := \kappa(\tau^{-1}(1))$$

ribbon $K\in\mathcal{T}_1 \quad z(K)\in\mathcal{R}\subseteq\mathcal{A}_1$

Faster is better, leaner is meaner!



$A^+ = -t^8+2t^7-t^6-2t^4+5t^3-2t^2-7t+13$

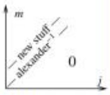
$\rho_1^+ = 5t^{15}-18t^{14}+33t^{13}-32t^{12}+2t^{11}+42t^{10}-62t^9-8t^8+166t^7-242t^6+108t^5+132t^4-226t^3+148t^2-11t-36$

Theorem ([BNG], conjectured [MM], elucidated [Ro1]). Let $J_d(K)$ be the coloured Jones polynomial of K, in the d-dimensional representation of sl_2. Writing

Melvin, Morton, Garoufalidis

$$\left.\frac{(q^{1/2}-q^{-1/2})J_d(K)}{q^{d/2}-q^{-d/2}}\right|_{q=e^\hbar} = \sum_{j,m\geq 0} a_{jm}(K)d^j\hbar^m,$$

"below diagonal" coefficients vanish, $a_{jm}(K)=0$ if $j>m$, and "on diagonal" coefficients give the inverse of the Alexander polynomial: $\left(\sum_{m=0}^\infty a_{mm}(K)\hbar^m\right)\cdot A(K)(e^\hbar)=1$.



"Above diagonal" we have Rozansky's Theorem [Ro3, (1.2)]:

$$J_d(K)(q) = \frac{q^d-q^{-d}}{(q-q^{-1})A(K)(q^d)}\left(1+\sum_{k=1}^\infty \frac{(q-1)^k R_k(K)(q^d)}{A^{2k}(K)(q^d)}\right).$$

The Gold Standard is set by the "Γ-calculus" Alexander formulas [BNS, BN1]. An S-component tangle T has $\Gamma(T)\in R_S\times M_{S\times S}(R_S) = \left\{\begin{array}{c|c}\omega & S\\ \hline S & A\end{array}\right\}$ with $R_S := \mathbb{Z}(\{t_a : a\in S\})$:

$$\left({}_a\!\nearrow_b, {}_b\!\nwarrow_a\right) \to \begin{array}{c|cc} 1 & a & b\\ \hline a & 1 & 1-t_a^{\pm1}\\ b & 0 & t_a^{\pm1}\end{array} \qquad T_1\sqcup T_2 \to \begin{array}{c|cc}\omega_1\omega_2 & S_1 & S_2\\ \hline S_1 & A_1 & 0\\ S_2 & 0 & A_2\end{array}$$

$$\begin{array}{c|ccc}\omega & a & b & S\\ \hline a & \alpha & \beta & \theta\\ b & \gamma & \delta & \epsilon\\ S & \phi & \psi & \Xi\end{array} \xrightarrow[t_a,t_b\to t_c]{m_c^{ab}} \left(\begin{array}{c|cc}(1-\beta)\omega & c & S\\ \hline c & \gamma+\frac{\alpha\delta}{1-\beta} & \epsilon+\frac{\delta\theta}{1-\beta}\\ S & \phi+\frac{\alpha\psi}{1-\beta} & \Xi+\frac{\psi\theta}{1-\beta}\end{array}\right)$$

(Roland: "add to A the product of column b and row a, divide by $(1-A_{ab})$, delete column b and row a".)

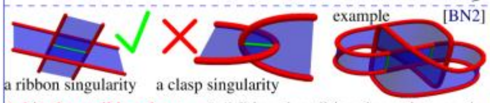
For long knots, ω is Alexander, and that's the fastest Alexander algorithm I know! Dunfield: 1000-crossing fast.

(There are also formulas for strand doubling and strand reversal).

Why "spectacular"? Foremost reason: *OBVIOUSLY.* Cf. proving (incomputable A)=(incomputable B), or categorifying (incomputable C).

Also, will bound genus and may disprove {ribbon} = {slice}.

Genus.



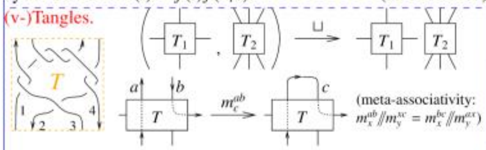
a ribbon singularity a clasp singularity

A bit about ribbon knots. A "ribbon knot" is a knot that can be presented as the boundary of a disk that has "ribbon singularities", but no "clasp singularities". A "slice knot" is a knot in $S^3=\partial B^4$ which is the boundary of a non-singular disk in B^4. Every ribbon knots is clearly slice, yet,

Conjecture. Some slice knots are not ribbon.

Fox-Milnor. The Alexander polynomial of a ribbon knot is always of the form $A(t)=f(t)f(1/t)$. (also for slice)

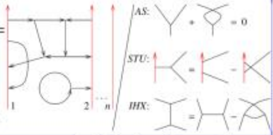
(v-)Tangles.



(meta-associativity: $m_x^{ab}/\!\!/m_y^{xc} = m_x^{bc}/\!\!/m_y^{ax}$)

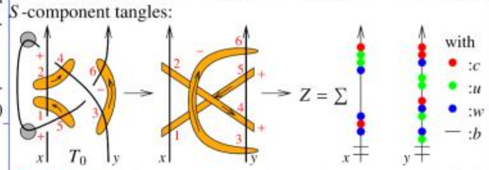
Theorem [EK, Ha, En, Se]. There is a "homomorphic expansion"

$$Z: \left\{\begin{matrix}S\text{-component}\\ (v/b\text{-})\text{tangles}\end{matrix}\right\} \to \mathcal{A}_S^v :=$$



Etingof Kazhdan Haviv Enriquez Ševera

Algebras and Invariants. Given any unital algebra A (even better if A is Hopf; typically, $A\sim\hat{\mathcal{U}}(\mathfrak{g})$), appropriate orange $R\in A\otimes A$, and appropriate cuaps $\in A$, get an $A^{\otimes S}$-valued invariant of pure S-component tangles:



Good News. In theory, enough to know R, the cuaps, and stitching/multiplication $m_k^{ij}: A_i\otimes A_j\to A_k$.

Problem. Extract information out of Z.

Textbook Solution. Use representation theory ...works, slowly.

Today's Solution (with van der Veen). For some specific $\mathfrak{g}$'s, work in a space of "formulas of a specific type" for elements of $\hat{\mathcal{U}}(\mathfrak{g})^{\otimes S}$:

$$\left\{\begin{matrix}\text{ordered perturbed}\\ \text{Gaussian formulas}\end{matrix}\right\} \to \hat{\mathcal{U}}(\mathfrak{g})^{\otimes S}$$

van der Veen

bring to front?

"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

www.katlas.org

Improve ?

That way murky

Consider expanding and bringing to Front.

re-embed in PDF

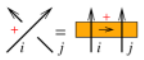
1-Smidgen sl_2 Let $\mathfrak{g}_1$ be the 4-dimensional Lie algebra $\mathfrak{g}_1 = \langle b, c, u, w\rangle$ over the ring $R = \mathbb{Q}[\epsilon]/(\epsilon^2 = 0)$, with b central and with $[w, c] = w$, $[c, u] = u$, and $[u, w] = b - 2\epsilon c$, with CYBE $r_{ij} = (b_i - \epsilon c_i)c_j + u_i w_j$ in $\mathcal{U}(\mathfrak{g}_1)^{\otimes\{i,j\}}$. Over $\mathbb{Q}$, $\mathfrak{g}_1$ is a solvable approximation of sl_2: $\mathfrak{g}_1 \supset \langle b, u, w, \epsilon b, \epsilon c, \epsilon u, \epsilon w\rangle \supset \langle b, \epsilon b, \epsilon c, \epsilon u, \epsilon w\rangle \supset 0$. (note: $\deg(b, c, u, w, \epsilon) = (1, 0, 1, 0, 1)$)

0-Smidgen sl_2 ☺. Let $\mathfrak{g}_0$ be $\mathfrak{g}_1$ at $\epsilon = 0$, or $\mathbb{Q}\langle b, c, u, w\rangle/([b, \cdot] = 0, [c, u] = u, [c, w] = -w, [u, w] = b$ with $r_{ij} = b_i c_j + u_i w_j$. It is $\mathfrak{b}^* \rtimes \mathfrak{b}$ where $\mathfrak{b}$ is the 2D Lie algebra $\mathbb{Q}\langle c, w\rangle$ and (b, u) is the dual basis of (c, w). For topology, it is more valuable than $\mathfrak{g}_1$ / sl_2, but topology already got by other means almost everything $\mathfrak{g}_0$ gives.

How did these arise? $sl_2 = \mathfrak{b}^+ \oplus \mathfrak{b}^-/\mathfrak{h} =: sl_2^+/\mathfrak{h}$, where $\mathfrak{b}^+ = \langle c, w\rangle/[w, c] = w$ is a Lie bialgebra with $\delta\colon \mathfrak{b}^+ \to \mathfrak{b}^+ \otimes \mathfrak{b}^+$ by $\delta\colon (c, w) \mapsto (0, c \wedge w)$. Going back, $sl_2^+ = \mathcal{D}(\mathfrak{b}^+) = (\mathfrak{b}^+)^* \oplus \mathfrak{b}^+ = \langle b, u, c, w\rangle/\cdots$. **Idea.** Replace $\delta \to \epsilon\delta$ over $\mathbb{Q}[\epsilon]/(\epsilon^{k+1} = 0)$. At $k = 0$, get $\mathfrak{g}_0$. At $k = 1$, get $[w, c] = w$, $[w, b'] = -\epsilon w$, $[c, u] = u$, $[b', u] = -\epsilon u$, $[b', c] = 0$, and $[u, w] = b' - \epsilon c$. Now note that $b' + \epsilon c$ is central, so switch to $b := b' + \epsilon c$. This is $\mathfrak{g}_1$.

Ordering Symbols. $\mathbb{O}(\mathit{poly} \mid \mathit{specs})$ plants the variables of *poly* in $S(\oplus_i \mathfrak{g})$ on several tensor copies of $\mathcal{U}(\mathfrak{g})$ according to *specs*. E.g.,
$$\mathbb{O}\left(c_1^3 u_1 c_2 e^{u_3} w_3^9 | x\colon w_3 c_1,\ y\colon u_1 u_3 c_2\right) = w^9 c^3 \otimes u e^u c \in \mathcal{U}(\mathfrak{g})_x \otimes \mathcal{U}(\mathfrak{g})_y$$
This enables the description of elements of $\hat{\mathcal{U}}(\mathfrak{g})^{\otimes S}$ using commutative polynomials / power series.

0-Smidgen Invariants. $r = Id \in \mathfrak{b}^- \otimes \mathfrak{b}^+$ solves the CYBE $[r_{12}, r_{13}] + [r_{12}, r_{23}] + [r_{13}, r_{23}] = 0$ in $\mathcal{U}(\mathfrak{g}_0)^{\otimes 3}$ and, by luck,



$$= R_{ij} = e^{r_{ij}} = e^{b_i c_j + u_i w_j} \in \mathcal{U}(\mathfrak{g}_{0,i} \oplus \mathfrak{g}_{0,j})$$
solves YB/R3.

Lemma. $R_{ij} = e^{b_i c_j + u_i w_j} = \mathbb{O}\left(\exp\left(b_i c_j + \frac{e^{b_i}-1}{b_i} u_i w_j\right) | i\colon u_i,\ j\colon c_j w_j\right)$

Example. $Z(T_0) = \qquad = \sum_{m,n} \frac{b_i^{m-n}(e^{b_i}-1)^n}{m!n!} u^n \otimes c^m w^n$.
$$\mathbb{O}\Big(\exp\left(b_5 c_1 + \tfrac{e^{b_5}-1}{b_5} u_5 w_1 + b_2 c_4 + \tfrac{e^{b_2}-1}{b_2} u_2 w_4 - b_3 c_6 + \tfrac{e^{-b_3}-1}{b_3} u_3 w_6\right) |$$
"cuw form"
$$x\colon c_1 w_1 u_2,\ y\colon u_3 c_4 w_4 u_5 c_6 w_6\Big) = \mathbb{O}\left(? | x\colon c_x u_x w_x,\ y\colon c_y u_y w_y\right)$$

Goal. Write ? as a Gaussian: ωe^{L+Q} where L bilinear in b_i and c_i with integer coefficients, Q a balanced quadratic in u_i and w_i with coefficients in $R_S := \mathbb{Q}(b_i, e^{b_i})$, and $\omega \in R_S$.

The Big $\mathfrak{g}_0$ Lemma. Under $[c, u] = u$, $[c, w] = -w$, and $[u, w] = b$:

1a. $N^{uc} := \mathbb{O}(e^{\gamma c + \beta u}|uc) \overset{\rightarrow}{=} \mathbb{O}(e^{\gamma c + e^{-\gamma}\beta u}|cu)$ (means $e^{\beta u}e^{\gamma c} = e^{\gamma c}e^{e^{-\gamma}\beta u}$

1b. $N^{wc} := \mathbb{O}(e^{\gamma c + \alpha w}|wc) \overset{\rightarrow}{=} \mathbb{O}(e^{\gamma c + e^{\gamma}\alpha w}|cw)$ … in the $\{ax + b\}$ group)

2. $\mathbb{O}(e^{\alpha w + \beta u}|wu) = \mathbb{O}(e^{-b\alpha\beta + \alpha w + \beta u}|uw)$ (the Weyl relations)

3. $\mathbb{O}(e^{\delta uw}|wu)e^{\beta u} = e^{\nu\beta u}\mathbb{O}(e^{\delta uw}|wu)$, with $\nu = (1 + b\delta)^{-1}$

(a. expand and crunch. b. use $w = b\hat{x}$, $u = \partial_x$. c. use "scatter and glow".)

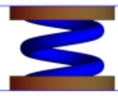
4. $\mathbb{O}(e^{\delta uw}|wu) = \mathbb{O}(\nu e^{\nu\delta uw}|uw)$ (same techniques)

5. $N^{wu} := \mathbb{O}(e^{\beta u + \alpha w + \delta uw}|wu) \overset{\rightarrow}{=} \mathbb{O}(\nu e^{-b\nu\alpha\beta + \nu\alpha w + \nu\beta u + \nu\delta uw}|uw)$

6. $N_k^{c_i c_j} := \mathbb{O}(\zeta|c_i c_j) \overset{\rightarrow}{=} \mathbb{O}(\zeta/(c_i, c_j \to c_k)|c_k)$

Sneaky. α may contain (other) u's, β may contain (other) w's.

Strand Stitching, m_k^{ij}, is defined as the composition
$$c_i u_i \overline{w_i c_j} u_j w_j \xrightarrow{N_k^{w_i c_j}} c_i \overline{u_i c_k}\, \overline{w_k u_j} w_j \xrightarrow{N_k^{u_i c_k} /\!/ N_k^{w_k w_j}} \overline{c_i c_k}\, \overline{u_k u_k}\, \overline{w_k w_j} \xrightarrow{N_k^{c_i c_k} /\!/ - - /\!/ N_k^{w_k w_j}} c_k u_k w_k$$

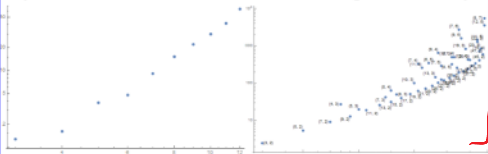


1-Smidgen Invariants. Much is the same:
The Big $\mathfrak{g}_1$ Lemma. Parts 1 and 6 are the same, yet
5. $\mathbb{O}\left(e^{\alpha w + \beta u + \delta uw}|wu\right) = \mathbb{O}\left(\nu(1 + \epsilon\nu\Lambda)e^{\nu(-b\alpha\beta + \alpha w + \beta u + \delta uw)}|cuw\right)$
Here Λ is for Λόγος, "a principle of order and knowledge", a balanced quartic in α, β, c, u, and w:
$$\begin{aligned}\Lambda = &- b\nu\left(\nu^2\alpha^2\beta^2 + 4\delta\nu\alpha\beta + 2\delta^2\right)/2 - \delta\nu^3(3b\delta + 2)\beta^2u^2/2\\ &- b\delta^4\nu^3u^2w^2/2 - \delta^2\nu^3(2b\delta + 1)\beta u^2 w\\ &- \nu^2(2b\delta + 1)(\nu\alpha\beta + 2\delta)\beta u - 2b\delta^2\nu^2(\nu\alpha\beta + \delta)uw\\ &+ \delta\nu^3(b\delta + 2)\alpha^2w^2/2 + 2(\nu\alpha\beta + \delta)c + 2\delta\nu\beta cu + 2\delta^2\nu cuw\\ &+ 2\delta\nu\alpha cw + \delta^2\nu^3\alpha uw^2 + \nu^2(\nu\alpha\beta + 2\delta)\alpha w.\end{aligned}$$
Proof. A brutal hell.
Problem. We now need to normal-order perturbed Gaussians!
Solution. Borrow some tactics from QFT:
$$\mathbb{O}(\epsilon P(c, u)e^{\gamma c + \beta u}|uc) = \mathbb{O}(\epsilon P(\partial_\gamma, \partial_\beta)e^{\gamma c + \beta u}|uc) = \mathbb{O}(\epsilon P(\partial_\gamma, \partial_\beta)e^{\gamma c + e^{-\gamma}\beta u}|cu),$$
and likewise
$$\mathbb{O}\left(\epsilon P(u, w)e^{\alpha w + \beta u + \delta uw}|wu\right) = \mathbb{O}\left(\epsilon P(\partial_\beta, \partial_\alpha)\nu e^{\nu(-b\alpha\beta + \alpha w + \beta u + \delta uw)}|cuw\right)$$
Note. Strand stitching requires a tiny extra step.
Finally, the values of the generators ⤡, ⤢, $\overrightarrow{n}$, $\overleftarrow{n}$, $\underrightarrow{u}$, and $\underleftarrow{u}$, are set by brutally solving many equations, non-uniquely.

Pragmatic Simplifications. Get rid of $\zeta = (e^b - 1)/b$ factors by rescaling $u \to \bar{u} = \zeta u$. Complement this with $\beta \to \bar{\beta} = \zeta^{-1}\beta$, $\delta \to \bar{\delta} = \zeta^{-1}\delta$, $\epsilon \to \bar{\epsilon} = \zeta^{-1}\epsilon$. Simplify further by naming $e^b \to t$; e.g., $\nu \to \bar{\nu} = (1 + (t - 1)\delta)^{-1}$. Get confused by renaming $(\bar{u}, \bar{\beta}, \bar{\delta}, \bar{\nu}) \to (u, \beta, \delta, \nu)$, and more confused by working with $\mu = \nu^{-1}$ and $\mathbb{E}(\omega, L, Q, P) := \omega^{-1}(1 + \epsilon\omega^{-4}P)e^{L + \omega^{-1}Q}$, where $\omega \in R := \mathbb{Q}(t_k)$, $L = \sum l_{ij}b_i c_j$ with $l_{ij} \in \mathbb{Z}$, $Q = \sum q_{ij}u_i w_j$ with $q_{ij} \in R$, and P is a balanced quartic polynomial in c_i, u_i, and w_i with coefficients in R. Magically, all coefficients are now Laurent polynomials in the t_k's.

Rough complexity estimate, after $t_k \to t$. n: xing number; w: width, maybe $\sim \sqrt{n}$.
$$\underbrace{n}_{A}\overbrace{\sum_{d=0}^{4}\underbrace{w^{4-d}}_{E}\underbrace{w^d}_{F}}^{B}\underbrace{n^2}_{G} = n^3w^4 \in [n^5, n^7]$$
A: go over stitchings in order. B: multiplication ops per $N^{u_i w_j}$. d: deg of u_i, w_j in P. E: #terms of deg d in P. F: ops per term. G: cost per polynomial multiplication op.

Experimental Analysis (ωεβ/Exp). Log-log plot of computation time (sec) vs. crossing number, for all knots with up to 12 crossings (mean times) and for all torus knots with up to 48 crossings:



Conjecture (checked on the same collections). Given a knot K with Alexander polynomial A, there is a polynomial ρ_1 such that
$$P = A^2\left((t - 2 + t^{-1})\rho_1 + tAA'\left(\frac{(4 + t - t^2)(uw + (t - 1)c)}{2(t - 1)} - 1\right)\right).$$
Furthermore, A and ρ_1 are symmetric under $t \to t^{-1}$, so let A^+ and ρ_1^+ be their "positive parts", so e.g., $\rho_1(t) = \rho_1^+(t) + \rho_1^+(t^{-1}) - \rho_1^+(0)$.
Power. On the 250 knots with at most 10 crossings, the pair (A, ρ_1) attains 250 distinct values, while (Khovanov, HOMFLY-PT) attains only 249 distinct values. To 11 crossings the numbers are (802, 788, 772) and to 12 they are (2978, 2883, 2786).

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z2 = R⁺_{1,14} R⁻_{5,2} nr_3 ul_4 R⁺_{19,6} R⁻_{7,10} nl_8 ur_9 R⁺_{11,20}
    nr_12 ul_13 R⁻_{15,18} nl_16 ur_17;
(Do[z2 = z2 // m_{1,k→1}, {k, 2, 20}];
 z2 = z2 /. a_1 ↦ a)
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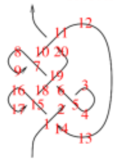
$$\mathbb{E}\Big[-1+\frac{1}{t}+t,\ 0,\ 0,\ -16+\frac{9c}{2}-\frac{2c}{t^4}+\frac{1}{t^3}+\frac{11c}{2t^3}-\frac{4}{t^2}-\frac{8c}{t^2}+\frac{10}{t}+\frac{4c}{t}+18t-10ct-14t^2+8ct^2+7t^3-\frac{3ct^3}{2}-2t^4-2ct^4+2ct^5-\frac{ct^6}{2}-4uw+\frac{2uw}{t^4}-\frac{7uw}{2t^3}+\frac{9uw}{2t^2}+\frac{uw}{2t}+6tuw-2t^2uw-\frac{1}{2}t^3uw+\frac{3}{2}t^4uw-\frac{1}{2}t^5uw\Big]$$

The 0-Framed Trefoil



Questions and To Do List. • Clean up and write up. • Implement well, compute for everything in sight. • Why are our quantities polynomials rather than just rational functions? • Bounds on their degrees? • Their integrality ($\mathbb{Z}$) properties? • Can everything be re-stated using integrals ($\int$)? • Find the 2-variable version (for knots). How complex is it? • What about links / closed components? • Fully digest the "expansion" theorem; include cuaps. • Explore the (non-)dependence on R. • Is there a canonical R? • What does "group like" mean? • Strand removal? Strand doubling? Strand reversal? • Say something about knot genus. • Find the EK/AT/KV "vertex". • Use as a playground to study associators/braidors. • Restate in topological language. • Study the associated (v-)braid representations. • Study mirror images and the $\mathfrak{b}^+ \leftrightarrow \mathfrak{b}^-$ involution. • Study ribbon knots. • Make precise the relationship with Γ-calculus and Alexander. • Relate to the coloured Jones polynomial. • Relate with "ordinary" q-algebra. • k-smidgen sl_n, etc. • Are there "solvable" CYBE algebras not arising from semi-simple algebras? • Categorify and appease the Gods. Help Needed!

~~Disclaimer. This is all quite new. The overall picture is correct, but many pieces are certainly not in their final form yet.~~

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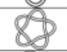
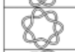
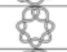
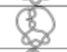
diagram	n^t_k Alexander's A_+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral	diagram	n^t_k Alexander's A_+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral
	0^a_1 1 0	0 / ✔ 0 / ✔		3^a_1 $t-1$ t	1 / ✘ 1 / ✘
	4^a_1 $3-t$ 0	1 / ✘ 1 / ✔		5^a_1 t^2-t+1 $2t^3+3t$	2 / ✘ 2 / ✘
	5^a_2 $2t-3$ $5t-4$	1 / ✘ 1 / ✘		6^a_1 $5-2t$ $t-4$	1 / ✔ 1 / ✘
	6^a_2 $-t^2+3t-3$ t^3-4t^2+4t-4	2 / ✘ 1 / ✘		6^a_3 t^2-3t+5 0	2 / ✘ 1 / ✔
	7^a_1 t^3-t^2+t-1 $3t^5+5t^3+6t$	3 / ✘ 3 / ✘		7^a_2 $3t-5$ $14t-16$	1 / ✘ 1 / ✘
	7^a_3 $2t^2-3t+3$ $-9t^3+8t^2-16t+12$	2 / ✘ 2 / ✘		7^a_4 $4t-7$ $32-24t$	1 / ✘ 2 / ✘
	7^a_5 $2t^2-4t+5$ $9t^3-16t^2+29t-28$	2 / ✘ 2 / ✘		7^a_6 $-t^2+5t-7$ $t^3-8t^2+19t-20$	2 / ✘ 1 / ✘
	7^a_7 t^2-5t+9 $8-3t$	2 / ✘ 1 / ✘		8^a_1 $7-3t$ $5t-16$	1 / ✘ 1 / ✘
	8^a_2 $-t^3+3t^2-3t+3$ $2t^5-8t^4+10t^3-12t^2+13t-12$	3 / ✘ 2 / ✘		8^a_3 $9-4t$ 0	1 / ✘ 2 / ✔
	8^a_4 $-2t^2+5t-5$ $3t^3-8t^2+6t-4$	2 / ✘ 2 / ✘		8^a_5 $-t^3+3t^2-4t+5$ $-2t^5+8t^4-13t^3+20t^2-22t+24$	3 / ✘ 2 / ✘
	8^a_6 $-2t^2+6t-7$ $5t^3-20t^2+28t-32$	2 / ✘ 2 / ✘		8^a_7 t^3-3t^2+5t-5 $-t^5+4t^4-10t^3+12t^2-13t+12$	3 / ✘ 1 / ✘