Pensieve header: Testing invariance under changes of the order of operations by permuting tangles.

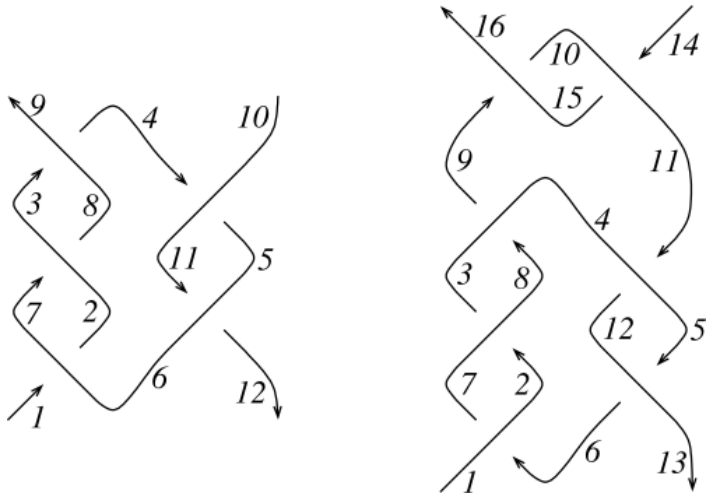
```
In[•]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\ICERM-2305"];
        << Signatures.m
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

Random Permutations

```
In[•]:= RandomPermutation[K_PD] ^:= K[[PermutationList[RandomPermutation@Length@K, Length@K]]]
```

Some Tangles




```
In[•]:= T1 = PD[X_{-6,2,7,-1}, X_{-2,8,3,-7}, X_{-8,4,9,-3}, X_{-11,6,12,-5}, X_{-4,11,5,-10}];
        T2 = PD[X_{-6,2,7,-1}, X_{-2,8,3,-7}, X_{-8,4,9,-3}, X_{-12,6,13,-5}, X_{-4,12,5,-11}, X_{-10,15,11,-14}, X_{-15,10,16,-9}];
```

In[•]:=
```
Column@{TL[T1], Kas[T1]}
```

Out[•]=

$$-2\,\Theta\left(u-\frac{\sqrt{3}}{2}\right)+2\,\Theta\left(u+\frac{\sqrt{3}}{2}\right)-1$$

	$(\gamma_{-10}$	γ_9	γ_{-1}	$\gamma_{12})$
$\overline{\gamma}_{-10}$	0	$1-\omega$	0	$\omega-1$
$\overline{\gamma}_9$	$\frac{\omega-1}{\omega}$	$\frac{2\,\omega}{\omega^2-\omega+1}$	$-\frac{\omega-1}{\omega}$	$-\frac{2\,\omega}{\omega^2-\omega+1}$
$\overline{\gamma}_{-1}$	0	$\omega-1$	0	$1-\omega$
$\overline{\gamma}_{12}$	$-\frac{\omega-1}{\omega}$	$-\frac{2\,\omega}{\omega^2-\omega+1}$	$\frac{\omega-1}{\omega}$	$\frac{2\,\omega}{\omega^2-\omega+1}$

$$-2\,\Theta\left(u-\frac{\sqrt{3}}{2}\right)+2\,\Theta\left(u+\frac{\sqrt{3}}{2}\right)-1$$

	$(\gamma_{-10}$	γ_9	γ_{-1}	$\gamma_{12})$
$\overline{\gamma}_{-10}$	$2\,(u-1)\,(u+1)\,\left(4\,u^2-3\right)$	0	$-2\,(u-1)\,(u+1)\,\left(4\,u^2-3\right)$	0
$\overline{\gamma}_9$	0	$\frac{1}{2\,\left(4\,u^2-3\right)}$	0	$-\frac{1}{2\,\left(4\,u^2-3\right)}$
$\overline{\gamma}_{-1}$	$-2\,(u-1)\,(u+1)\,\left(4\,u^2-3\right)$	0	$2\,(u-1)\,(u+1)\,\left(4\,u^2-3\right)$	0
$\overline{\gamma}_{12}$	0	$-\frac{1}{2\,\left(4\,u^2-3\right)}$	0	$\frac{1}{2\,\left(4\,u^2-3\right)}$

In[•]:=
```
Column@{TL[T2], Kas[T2]}
```

Out[•]=

$$0$$

	$(\gamma_{-14}$	γ_{16}	γ_{-1}	$\gamma_{13})$
$\overline{\gamma}_{-14}$	0	$1-\omega$	0	$\omega-1$
$\overline{\gamma}_{16}$	$\frac{\omega-1}{\omega}$	$-\frac{2\,(\omega-1)^2\,\omega}{\omega^4-3\,\omega^3+5\,\omega^2-3\,\omega+1}$	$-\frac{\omega-1}{\omega}$	$\frac{2\,(\omega-1)^2\,\omega}{\omega^4-3\,\omega^3+5\,\omega^2-3\,\omega+1}$
$\overline{\gamma}_{-1}$	0	$\omega-1$	0	$1-\omega$
$\overline{\gamma}_{13}$	$-\frac{\omega-1}{\omega}$	$\frac{2\,(\omega-1)^2\,\omega}{\omega^4-3\,\omega^3+5\,\omega^2-3\,\omega+1}$	$\frac{\omega-1}{\omega}$	$-\frac{2\,(\omega-1)^2\,\omega}{\omega^4-3\,\omega^3+5\,\omega^2-3\,\omega+1}$

$$1$$

	$(\gamma_{-14}$	γ_{16}	γ_{-1}	$\gamma_{13})$
$\overline{\gamma}_{-14}$	$\frac{1}{2}\,\left(-16\,u^4+28\,u^2-13\right)$	0	$\frac{1}{2}\,\left(16\,u^4-28\,u^2+13\right)$	0
$\overline{\gamma}_{16}$	0	$-\frac{2\,(u-1)\,(u+1)}{16\,u^4-28\,u^2+13}$	0	$\frac{2\,(u-1)\,(u+1)}{16\,u^4-28\,u^2+13}$
$\overline{\gamma}_{-1}$	$\frac{1}{2}\,\left(16\,u^4-28\,u^2+13\right)$	0	$\frac{1}{2}\,\left(-16\,u^4+28\,u^2-13\right)$	0
$\overline{\gamma}_{13}$	0	$\frac{2\,(u-1)\,(u+1)}{16\,u^4-28\,u^2+13}$	0	$-\frac{2\,(u-1)\,(u+1)}{16\,u^4-28\,u^2+13}$

In[•]:=
```
Total[TL[T1] == TL[#] & /@ Permutations[T1]]
```

Out[•]=

120 True

In[•]:=
```
Total[TL[T2] == TL[#] & /@ Permutations[T2]]
```

Out[•]=

5040 True

In[•]:=
```
Total[Kas[T1] == Kas[#] & /@ Permutations[T1]]
```

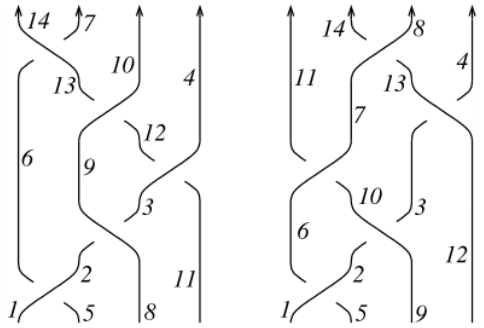
Out[•]=

120 True

```
In[•]:= Total[Kas[T2] == Kas[#] & /@ Permutations[T2]]
Out[•]=
```

5040 True

Some Braids




```
In[•]:= B1 = PD[X_{-5,2,6,-1}, X̄_{-8,3,9,-2}, X_{-11,4,12,-3}, X_{-12,10,13,-9}, X̄_{-13,7,14,-6}];
        B2 = PD[X_{-5,2,6,-1}, X̄_{-9,3,10,-2}, X_{-10,7,11,-6}, X̄_{-12,4,13,-3}, X_{-13,8,14,-7}];

In[•]:= Total[TL[B1] == TL[#] & /@ Permutations[B1]]
Out[•]=
```

120 True

```
In[•]:= Total[TL[B2] == TL[#] & /@ Permutations[B2]]
Out[•]=
```

120 True

```
In[•]:= Total[Kas[B1] == Kas[#] & /@ Permutations[B1]]
Out[•]=
```

120 True

```
In[•]:= Total[Kas[B2] == Kas[#] & /@ Permutations[B2]]
Out[•]=
```

120 True

```
In[•]:= Column@{TL[B1], Kas[B1]}
```

Out[•]=

$$\begin{array}{c|cccccccc}
 & & & & 0 & & & & \\
 & 1 & 0 & -1 & 0 & \frac{1}{\omega} & 0 & -\frac{1}{\omega} & 0 \\
 & 0 & 0 & 0 & -1 & \frac{1}{\omega} & 0 & -\frac{1}{\omega} & 1 \\
 & (\gamma_{-11} & \gamma_4 & \gamma_{10} & \gamma_7 & \gamma_{14} & \gamma_{-1} & \gamma_{-5} & \gamma_{-8}) \\
\hline
\overline{\gamma}_{-11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\gamma}_4 & 0 & 0 & 0 & 0 & \frac{\omega-1}{\omega^2} & 0 & -\frac{\omega-1}{\omega^2} & 0 \\
\overline{\gamma}_{10} & 0 & 0 & 0 & 0 & -\frac{\omega-1}{\omega} & 0 & \frac{\omega-1}{\omega} & 0 \\
\overline{\gamma}_7 & 0 & 0 & 0 & 0 & \frac{(\omega-1)^2}{\omega^2} & 0 & -\frac{(\omega-1)^2}{\omega^2} & 0 \\
\overline{\gamma}_{14} & 0 & -((\omega-1)\,\omega) & \omega-1 & (\omega-1)^2 & 0 & -\frac{\omega-1}{\omega} & \frac{\omega-1}{\omega} & 0 \\
\overline{\gamma}_{-1} & 0 & 0 & 0 & 0 & \omega-1 & 0 & 1-\omega & 0 \\
\overline{\gamma}_{-5} & 0 & (\omega-1)\,\omega & 1-\omega & -(\omega-1)^2 & 1-\omega & \frac{\omega-1}{\omega} & \frac{(\omega-1)^2}{\omega} & 0 \\
\overline{\gamma}_{-8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}$$

$$\begin{array}{c|cccccccc}
 & & & & 0 & & & & \\
 & 1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 \\
 & (\gamma_{-11} & \gamma_4 & \gamma_{10} & \gamma_7 & \gamma_{14} & \gamma_{-1} & \gamma_{-5} & \gamma_{-8}) \\
\hline
\overline{\gamma}_{-11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\gamma}_4 & 0 & 0 & 0 & -1 & -u & 0 & u & 1 \\
\overline{\gamma}_{10} & 0 & 0 & 0 & -u & 1-2u^2 & 0 & 2u^2-1 & u \\
\overline{\gamma}_7 & 0 & -1 & -u & 2u^2-3 & -u & -1 & 0 & 1 \\
\overline{\gamma}_{14} & 0 & -u & 1-2u^2 & -u & -1 & -u & -2(u-1)(u+1) & u \\
\overline{\gamma}_{-1} & 0 & 0 & 0 & -1 & -u & 0 & u & 1 \\
\overline{\gamma}_{-5} & 0 & u & 2u^2-1 & 0 & -2(u-1)(u+1) & u & 4u^2-3 & 0 \\
\overline{\gamma}_{-8} & 0 & 1 & u & 1 & u & 1 & 0 & 1-2\iota
\end{array}$$

```
In[•]:= Column@{TL[B2], Kas[B2]}
```

Out[•]=

$$0$$

	$(\gamma_{-12}$	γ_4	γ_8	γ_{14}	γ_{11}	γ_{-1}	
$\overline{\gamma}_{-12}$	$\frac{(\omega-1)^2}{\omega}$	$\omega-1$	$-2(\omega-1)$	$\frac{2(\omega-1)^2}{\omega}$	$\frac{2(\omega-1)}{\omega^2}$	0	$-$
$\overline{\gamma}_4$	$-\frac{\omega-1}{\omega}$	0	$\frac{\omega-1}{\omega}$	0	0	0	
$\overline{\gamma}_8$	$\frac{2(\omega-1)}{\omega}$	$1-\omega$	$\frac{(\omega-1)^2}{\omega}$	$-\frac{(\omega-1)(2\omega-3)}{\omega}$	$-\frac{2(\omega-1)}{\omega^2}$	0	$\frac{2}{}$
$\overline{\gamma}_{14}$	$\frac{2(\omega-1)^2}{\omega}$	0	$-\frac{(\omega-1)(3\omega-2)}{\omega}$	$\frac{3(\omega-1)^2}{\omega}$	$-\frac{(\omega-2)(\omega-1)}{\omega^2}$	0	$-$
$\overline{\gamma}_{11}$	$-2(\omega-1)\omega$	0	$2(\omega-1)\omega$	$-((\omega-1)(2\omega-1))$	$\frac{(\omega-1)^2}{\omega}$	$-\frac{\omega-1}{\omega}$	$\frac{2}{}$
$\overline{\gamma}_{-1}$	0	0	0	0	$\omega-1$	0	1
$\overline{\gamma}_{-5}$	$2(\omega-1)\omega$	0	$-2(\omega-1)\omega$	$2(\omega-1)\omega$	$-2(\omega-1)$	$\frac{\omega-1}{\omega}$	$-($
$\overline{\gamma}_{-9}$	$-\frac{(\omega-1)(3\omega-2)}{\omega}$	0	$\frac{2(\omega-1)(2\omega-1)}{\omega}$	$-\frac{2(\omega-1)(2\omega-1)}{\omega}$	$\frac{2(\omega-1)^2}{\omega^2}$	0	$-\frac{(\omega}{}$

$$2\Theta\left(u-\frac{\sqrt{3}}{2}\right)-2\Theta\left(u+\frac{\sqrt{3}}{2}\right)$$

	1	$\frac{1}{2u}$	0	$-\frac{1}{2u}$	-1	$-\frac{1}{2u}$
	$(\gamma_{-12}$	γ_4	γ_8	γ_{14}	γ_{11}	γ_{-1}
$\overline{\gamma}_{-12}$	0	0	0	0	0	0
$\overline{\gamma}_4$	0	$-\frac{(2u-1)(2u+1)(2u^2-1)}{4u^2(4u^2-3)}$	$-\frac{2u^2-1}{2u}$	$\frac{1}{4u^2(4u^2-3)}$	0	$-\frac{(2u-1)(2}{4u^2(4u^2-}$
$\overline{\gamma}_8$	0	$-\frac{2u^2-1}{2u}$	$-2(u-1)(u+1)$	$\frac{2u^2-1}{2u}$	0	$-\frac{1}{2u}$
$\overline{\gamma}_{14}$	0	$\frac{1}{4u^2(4u^2-3)}$	$\frac{2u^2-1}{2u}$	$\frac{(2u^2-1)(16u^4-16u^2+1)}{4u^2(4u^2-3)}$	0	$-\frac{8u^4-10u^2}{4u^2(4u^2-}$
$\overline{\gamma}_{11}$	0	0	0	0	0	0
$\overline{\gamma}_{-1}$	0	$-\frac{(2u-1)(2u+1)}{4u^2(4u^2-3)}$	$-\frac{1}{2u}$	$-\frac{8u^4-10u^2+1}{4u^2(4u^2-3)}$	0	$\frac{8u^4-10u^2-}{4u^2(4u^2-}$
$\overline{\gamma}_{-5}$	0	$-\frac{1}{2u(4u^2-3)}$	0	$\frac{1}{2u(4u^2-3)}$	0	$\frac{8u^4-10u^2+}{2u(4u^2-3}$
$\overline{\gamma}_{-9}$	0	$\frac{8u^4-6u^2-1}{4u^2(4u^2-3)}$	$\frac{1}{2u}$	$\frac{1}{4u^2(4u^2-3)}$	0	$\frac{16u^4-16u^2}{4u^2(4u^2-}$